Comenius University in Bratislava
Faculty of Mathematics, Physics and Informatics

# Edge colouring of signed cubic graphs

Master's Thesis

2024
Bc. Bohdan Jóža

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Master's Thesis

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|---|---|
| Study Programme: | Computer Science |
| Field of Study: | Computer Science |
| Department: | Department of Computer Science |
| Supervisor: | doc. RNDr. Robert Lukoťka, PhD. |

Bratislava, 2024
Bc. Bohdan Jóža


Univerzita Komenského v Bratislave
Fakulta matematiky, fyziky a informatiky

# ZADANIE ZÁVEREČNEJ PRÁCE

**Meno a priezvisko študenta:** Bc. Bohdan Jóža
**Študijný program:** informatika (Jednoodborové štúdium, magisterský II. st., denná forma)
**Študijný odbor:** informatika
**Typ záverečnej práce:** diplomová
**Jazyk záverečnej práce:** anglický
**Sekundárny jazyk:** slovenský



**Názov:** Edge colourings of signed cubic graphs
*Hranové farbenia signovaných kubických grafov*

**Anotácia:** Signované grafy sú grafy, ktorých hrany sú ohodnotené prvkami z {-1, 1}. Prepínanie signovaného grafu v jeho vrchole v je vynásobenie ohodnotenia incidentných hrán hodnotou -1. Grafy, ktoré možno získať sériou operácií prepínania sú ekvivalentné. Existuje veľa článkov, ktoré skúmajú rozšírenie štandardných grafových invariantov na signované grafy. Jednou zo skúmaných tém je farbenie signovaných grafov. Predmetom práce budú hranové farbenia signovaných kubických grafov. Hranové farbenia signovaných grafov začal skúmať Behr v článku [Edge coloring signed graphs, Discrete Mathematics 343(2020)]. Cieľom práce je začať systematické štúdium hranovej 3-zafarbiteľnosti signovaných grafov.

**Vedúci:** doc. RNDr. Robert Lukoťka, PhD.
**Katedra:** FMFI.KI - Katedra informatiky
**Vedúci katedry:** prof. RNDr. Martin Škoviera, PhD.

**Spôsob sprístupnenia elektronickej verzie práce:**
bez obmedzenia

**Dátum zadania:** 16.11.2022

**Dátum schválenia:** prof. RNDr. Rastislav Kráľovič, PhD.
garant študijného programu

.....
študent

.....
vedúci práce


Comenius University Bratislava
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# THESIS ASSIGNMENT

**Name and Surname:** Bc. Bohdan Jóža
**Study programme:** Computer Science (Single degree study, master II. deg., full time form)
**Field of Study:** Computer Science
**Type of Thesis:** Diploma Thesis
**Language of Thesis:** English
**Secondary language:** Slovak



**Title:** Edge colourings of signed cubic graphs

**Annotation:** Signed graphs are graphs, whose edges have assigned values from {-1, 1}. Switching at a vertex v of a graph is done by multiplying the values of all edges incident with v by -1. Graphs that can be obtained from each other by switching are called equivalent. There are plenty of papers studying generalization of standard graph invariants to signed graphs. One of these invariants is graph colouring. The thesis should focus on edge colourings of signed cubic graphs. The study of edge colourings of signed graphs was started by Behr [Edge coloring signed graphs, Discrete Mathematics 343(2020)]. The aim of the thesis is to initiate the systematic study of 3-edge-colourability of signed cubic graphs.

**Supervisor:** doc. RNDr. Robert Lukoťka, PhD.
**Department:** FMFI.KI - Department of Computer Science
**Head of department:** prof. RNDr. Martin Škoviera, PhD.

**Assigned:** 16.11.2022

**Approved:** prof. RNDr. Rastislav Kráľovič, PhD.
Guarantor of Study Programme

.....
Student

.....
Supervisor

**Acknowledgments:** You can thank anyone who helped you with the thesis here (e.g. your supervisor).

# Abstrakt

Slovenský abstrakt v rozsahu 100–500 slov, jeden odstavec. Abstrakt stručne sumarizuje výsledky práce. Mal by byť pochopiteľný pre bežného informatika. Nemal by teda využívať skratky, termíny alebo označenie zavedené v práci, okrem tých, ktoré sú všeobecne známe.

**Kľúčové slová:** Slovak, keywords, here

# Abstract

Abstract in the English language (translation of the abstract in the Slovak language).

**Keywords:** English, keywords, here

# Contents

# List of Figures

# Introduction

TODO Introduction, for the purposes of Diplomovy seminar (1) I will put some introduction in the Preliminaries chapter

# Chapter 1

# Terminology

## 1.1 Graphs

Here we define the basic terminology used in this thesis.

**Definition 1.** We write $G$ for a graph and $V(G)$ and $E(G)$ for its vertes set and edge set respectively. We assume no graph constraints unless otherwise specified, so loops and duplicit edges are allowed in general.

**Definition 2.** We write $e = vw \in E(G)$ to indicate that the edge $e$ of $G$ has endpoints $v$ and $w$.

**Definition 3.** A *k-regular* graph is a graph where each vertex has degree $k$.

**Definition 4.** A *circle* or a *circuit* is a connected 2-regular subgraph.

**Definition 5.** A *cubic* graph is a 3-regular graph.

## 1.2 Signed graphs

Signed graphs were introduced by Harary[1] in 1953 as a model for social networks. A signed graph has a value of $+1$ or $-1$ assigned to all edges, so each edge is positive or negative. They have proved to be a natural generalization of unsigned graphs in many ways and interesting observations may arise by applying ordinary graph theory to signed graphs.

**Definition 6.** A *signed graph* is a pair $(G, \Sigma)$; $\Sigma \subseteq E(G)$, where $\Sigma$ is a subset of the edge set of $G$ and contains the negative edges.

**Definition 7.** Function $\sigma : E(G) \to \{+1; -1\}$ denotes the sign of an edge $e$.

A signed graph can also be defined as a pair $(G, \sigma)$ using the sign function directly, but I found this definition more natural.

## 1.3 Coloring

Vertex and edge coloring is a deeply explored topic of graph theory, even in the field of signed graphs. The research was initiated by Zaslavsky[2] in the early 1980s and published in multiple seminary papers. He defined a vertex n-coloring of a signed graph.

**Definition 8** (Zaslavsky)**.** A n-coloring of a signed graph $(G, \Sigma)$ is $\phi : V(E) \rightarrow \{-n, -(n-1), \ldots -1, 0, 1, \ldots, (n-1, n)\}$ where for each edge $e = vw \in E(G)$: $\phi(v) \neq \sigma(e)\phi(w)$.

So each vertex in $G$ is assigned a signed color so that the condition of vertex coloring in unsigned graphs (adapted to signed colors) still stands.

# Bibliography

[1] F. Harary. On the notion of balance of a signed graph. *Michigan Math J.*, 2:143–146, 1953.

[2] T. Zaslavsky. Signed graphs. *Discrete Applied Mathematics*, 1:47–74, 1982.