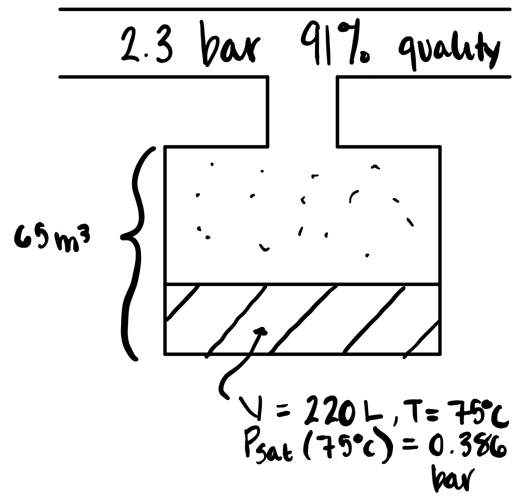
Assignment #4 Soln's

Problem 1



Approach:

Use a system of 3 eqns composed of E and M bals to solve for 3 unks

- well insulated ⇒ essentially adiabatic

→ How much steam enters?
→ How much liquid water at 75°C

$U = (x)U_{vap} + (1-x)U_{Liquid}$
$U_i = (x)(2476\ kJ/kg) + (1-x)(317\ kJ/kg)$

Volume of Vapor: $65000\ L - 220\ L = 64780\ L$
Vapor density $= 0.24\ \frac{kg}{m^3} \times \frac{1m^3}{1000L} \times 64780\ L = 15.547\ kg_{vapor}$

Liquid Density $= 974\ \frac{kg}{m^3} \times \frac{1m^3}{1000L} \times 220L = 214\ kg_{liquid}$

$x = \frac{15.547\ kg_{vap}}{15.547\ kg_v + 214\ kg_L} = 0.0677$

$U_1 = (0.0677)2476\ kJ/kg + (1-0.0677)(317\ kJ/kg)$
$= 463.2\ kJ/kg$
$\Delta U = \dot{m} H_{in} = \dot{m}[0.91 \cdot 2712.7\ kJ/kg + 0.9 \cdot 523.9\ kJ/kg) = \dot{m}(2518.37)\ kJ/kg = U_F - U_I$
$U_F = \dot{m}(2518.37\ kJ/kg) + (463.2\ kJ/kg)(230\ kg)$

$P_F = 2.3$ bar, sat temp $= 124.69°C$ after equilibrium is reached in the tank

3 eqn 3 unk.

$$\begin{cases} U_F = [q_F(2534\ kJ/kg) + (1-q_F)524\ kJ/kg] \\ 65 = q_F(0.775\ kJ/kg)(\dot{m} + 230\ kg) + (1-q_F)(0.0011\ kJ/kg)(\dot{m} + 230\ kg) \\ U_F = \frac{\dot{m}(2518.37\ kJ/kg) + (463.2\ kJ/kg)(230\ kg)}{\dot{m} + 230} \end{cases}$$

⇓

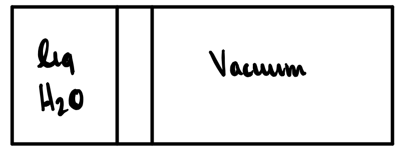
using Mathematica or other solver tool : $U_F = 1048\ kJ/kg$, $q_F = 0.26$, $\dot{m} = 91.6\ kg$

$q_F = 0.26 = \frac{m_{vap}}{m_{tot}} = \frac{m_{vap} \approx 83\ kg}{91.6kg + 230\ kg} \Rightarrow \overbrace{230\ kg - 91.6kg}^{m_{tot}} - \overset{m_v}{83\ kg} = \boxed{240\ kg\ liq}$

$\boxed{\dot{m} = 92\ kg \text{ steam enter the tank}}$

$\boxed{\text{Problem 2}}$

$m_{liq} = 10g = 0.01$ kg
$T = 60°C = 333K$
$V_F = 0.0010\ m^3$



$\Delta U = \cancel{Q}^{\,0} + \cancel{W}^{\,0}$, $U_F = \underset{251.16\ kJ/kg}{U_I}$

$$\hat{V}_F = \frac{0.0010\ m^3}{0.01\ kg} = 0.1\ m^3/kg = x\hat{V}_V + (1-x)\hat{V}_L$$

$U_F = 251.16\ kJ/kg = xU_V + (1-x)U_L$

$\boxed{\text{APPROACH}}$

Must iteratively find solution that satisfies Volume balance

Try $T_1 \rightarrow$ find $q_1(T)$

use $V = m(qV_V + (1-q)V_L) = 0.001\ m^3$

Check q by solving for U_F to see if $= 251.16\ kJ/kg$

Guess 45°C $\Rightarrow$ $U_V = 2436\ kJ/kg$
$U_L = 188.43$ " "
$V_V = 15.2521\ m^3/kg$
$V_L = 0.00101$ " "

$\Rightarrow 0.1\ m^3/kg = q(15.25\ m^3/kg) + (1-q)(0.00101\ \frac{m^3}{kg})$
$q = 0.0065$

Check: $U_F = (0.0065)(2436\ kJ/kg) + 0.9935(188.43\ kJ/kg) = 203\ kJ/kg \neq 251\ \frac{kJ}{kg}$

Guess 55°C $\Rightarrow$ $U_V = 2499.34\ kJ/kg$
$U_L = 230.24$ " "
$V_V = 9.5643\ m^3/kg$
$V_L = 0.001015$ " "

$\Rightarrow 0.1\ m^3/kg = q(9.56\ m^3/kg) + (1-q)(0.001\ \frac{m^3}{kg})$
$q = 0.0104$

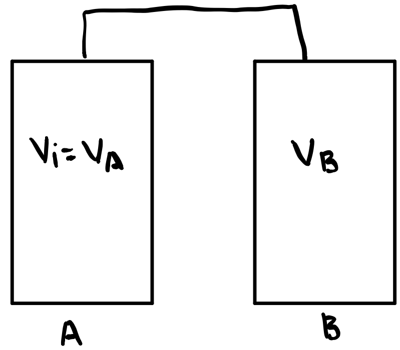
Check: $U_F = (0.0104)(2436.08\ \frac{kJ}{kg}) + 0.9896(230.24\ kJ/kg) = 253\ kJ/kg \approx 251\ \frac{kJ}{kg}$

$T = 55°C$
$P \approx 0.16$ bar
$m_{vap} = 0.10$ g
$m_{liq} = 9.9$ g
$m_{ice} = 0$ g

Problem 3

$P_i = 12$ bar

$T_i = 35°C = 308\,K$



adiabatic, reversible expansion

Approach: CANNOT assume $V_B = 2V_A$

use 4 governing equations:

→ Mbal: $n_i = n_A + n_B$

→ Ebal: $n_i u_i = n_A u_A + n_B u_B$

→ Ideal gas: $PV = nRT$

→ reversible adiabatic expansion: $\frac{T_A}{T_i} = \left(\frac{P_A}{P_i}\right)^{R/C_p}$

$$n_i = n_A + n_B$$

$$\frac{P_i V_i}{\cancel{R} T_i} = \frac{P_A V_A}{\cancel{R} T_A} + \frac{P_B V_B}{\cancel{R} T_B} \Rightarrow \frac{P_i}{T_i} = \frac{P_A}{T_A} + \frac{P_B}{T_B}$$

$$n_i u_i = n_A u_A + n_B u_B$$

$$\frac{P_i V_i}{R T_i}(C_v T_i - C_p T_R) = \frac{P_A V_A}{R T_A}(C_v T_A - C_p T_R) + \frac{P_B V_B}{R T_B}(C_v T_B - C_p T_R)$$

$$P_i C_v - \frac{P_i C_p T_R}{T_i} = P_A C_v - \frac{P_A C_p T_R}{T_A} + P_B C_v - \frac{P_B C_p T_R}{T_B}$$

$$P_i C_v - P_A C_v - P_B C_v - \cancelto{0}{\left(\frac{P_I}{T_i} - \frac{P_A}{T_A} - \frac{P_B}{T_B}\right)} C_p T_R = 0$$

$$C_v (P_i - P_A - P_B) = 0$$

$$P_i = P_A + P_B$$

$$P_A = P_B$$

$$P_i = 2P_A$$

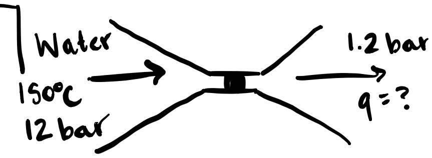
$$P_A = \frac{12}{6} = 2\,\text{bar}$$

$$\frac{T_A}{T_i} = \left(\frac{P_A}{P_i}\right)^{R/C_p}$$

$$T_A = T_i \left(\frac{P_A}{P_i}\right)^{R/C_p} = 308\left(\frac{6\,\text{bar}}{12\,\text{bar}}\right)^{8.314/28}$$

$$\boxed{T_A = 251\,K}$$

Problem 4



throttle $\therefore H_1 = H_2$

Approach:
Use $\Delta H = 0 \Rightarrow q$
Steam tables $\Rightarrow T$

$$H_1 = 632.698 \text{ kJ/kg} = H_2$$

@ 1.2 bar, $H^L_{sat} = 439.4$ kJ/kg, $H^V_{sat} = 2683$ kJ/kg; $H^L_{sat} < H_2 < H^V_{sat}$ $\therefore$ 2 phases

Outlet saturated so $\boxed{T_f = 104°C \approx 100°C}$

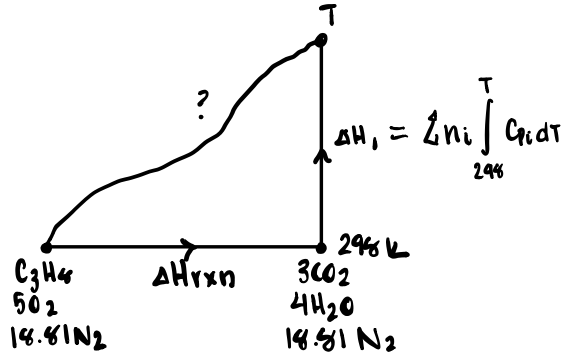
$$H_f = H^L_{sat}(1-x) + xH^V_{sat} \Rightarrow 632.698 = 439.4 \text{ kJ/kg}(1-x) + 2683 \text{ kJ/kg}(x)$$

$$x = 0.086$$

$\therefore$ $\boxed{9\% \text{ vapor, } 91\% \text{ liquid}}$

$\boxed{\text{Problem 5}}$ $C_3H_8 + 5O_2 + 18.81\,N_2 \rightarrow 3CO_2 + 4H_2O_{(g)} + 18.81\,N_2$

Adiabatic, $\Delta H = 0$



Approach:

Use $\Delta H_{rxn} + \Delta H_1 = 0$ to find T

Calculate ΔH_{rxn}: $\Delta H_{rxn} = \sum_i \Delta H^\circ_{f,prod} - \sum_i \Delta H^\circ_{f\,rxts} = 3\Delta H_{f,CO_2} + 4\Delta H_{f,H_2O} - \Delta H_{f,C_3H_8}$

$\hookrightarrow \Delta H_{f,O_2} = \Delta H_{f,N_2} = 0$

$$\Delta H_{rxn} = (3\,\text{mol})(-393.51\,\tfrac{kJ}{mol}) + 4\,\text{mol}\,(-241.835\,kJ/mol) - (1\,\text{mol})(-104.68\,\tfrac{kJ}{mol})$$

$$\Delta H_{rxn} = -2043.19\,kJ$$

$$\left.\begin{aligned} C_{p,CO_2} &= 19.8 + 0.07344\,T - 5.602\times10^{-5}\,T^2 + 1.715\times10^{-8}\,T^3 \\ C_{p,H_2O} &= 32.24 + 0.001924\,T + 1.055\times10^{-5}\,T^2 - 3.596\times10^{-9}\,T^3 \\ C_{p,N_2} &= 31.15 - 0.001357\,T + 2.680\times10^{-5}\,T^2 - 1.168\times10^{-8}\,T^3 \end{aligned}\right\}\ \text{From Textbook}$$

$$\sum_i n_i C_{pi} = 3C_{p,O_2} + 4C_{p,H_2O} + 18.81\,C_{p,18.8} = 773.98 - 2.7E\text{-}2\,T + 3.78E\text{-}4\,T^2 - 1.825\,E\text{-}7\,T^3$$

$$\int_{298}^{T} \sum_i n_i C_{pi}\,dT = 773.98\Big|_{298}^{T} - \frac{2.7E\text{-}2}{2}T^2\Big|_{298}^{T} + \frac{3.78E\text{-}4}{3}T^3\Big|_{298}^{T} - \frac{1.84E\text{-}7}{4}T^4\Big|_{298}^{T}$$

$$\int_{298}^{T} \sum_i n_i C_{pi}\,dT = -4.65E\text{-}8\,T^4 + 0.000126\,T^3 - 0.0135\,T^2 + 773.98\,T - 232841\,J$$

$$0 = \Delta H_{rxn} + \int_{298}^{T} \sum n_i C_p\,dT$$

$$0 = -2041.190\,J/mol - 4.56E\text{-}8\,T^4 + 0.000126\,T^3 - 0.0135\,T^2 + 773.98\,T - 232841\,J$$

Solve for T $\Rightarrow$ imaginary solutions b/c the heat capacity of nitrogen $\rightarrow \infty$ as T$\uparrow$.

Recommend Cp expressions from Perry's handbook.
If you use other source, T will range from 1000K to 3000K.

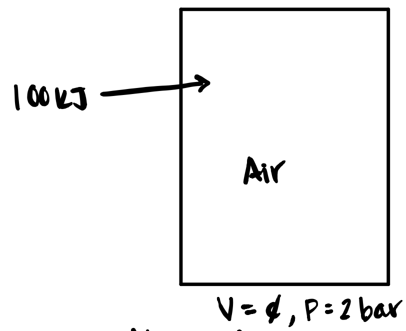
Problem 6

$H_f >, <, \neq 100 \text{ kJ}?$

$$dH = dU + P\cancel{dV}^{\,0} + VdP$$
$$dU = dQ + d\cancel{W}_{0} = 100$$

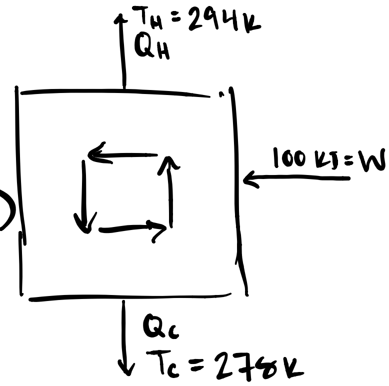
$$dH = \underbrace{(100 \text{ kJ})}_{dU} + \underbrace{(+\#)}_{VdP}$$

$$\therefore \quad dH > dU \quad \text{so} \quad \boxed{H_f > 100 \text{ kJ}}$$



The change in internal energy = 100 kJ because there is no work. By adding heat, T↑ so P↑. So, H is going to be dependent on the internal energy <u>and</u> the change in pressure. So H must be > Q

Problem 7



Second Law

$$\Delta S_{eng} = 0$$

$$\frac{Q_c}{T_c} + \frac{Q_H}{T_H} = 0$$

$$\frac{Q_c}{278K} + \frac{Q_H}{294K} = 0$$

First Law

$$Q_H + Q_c + W = 0$$

$$Q_H = -(Q_c + W)$$

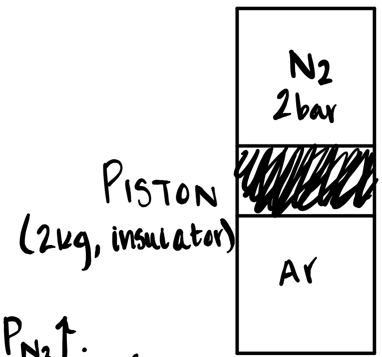
Combine: $$\frac{Q_c}{278K} + \frac{(-Q_c + W)}{294K} = 0 \Rightarrow 294 Q_c = 278(Q_c + W)$$

$$1.06\, Q_c = Q_c + 100 \text{ kJ}$$
$$0.06\, Q_c = 1738 \text{ kJ}$$
$$Q_c = 1738 \text{ kJ}$$
$$Q_H = -1838 \text{ kJ}$$

$$\boxed{Q_H = 1800 \text{ kJ} \overset{\text{max}}{\text{heat}} \text{ added}}$$

The carnot cycle achieves maximum efficiency, which implies maximum heat

$\boxed{\text{Problem 8}}$ if $P_{N_2}\downarrow$, P_{Ar}, $T_{AR} = ?$



insulated ∴ adiabatic

$dU = \cancel{Q} + W$

$dU = W = \int P dV$

$\Delta U \uparrow \therefore T \uparrow$

Work is being done on the Ar as $P_{N_2}\uparrow$.
Therefore $dU\uparrow$. U is proportional to T so $T_{AR}\uparrow$.

equilibrium

$P_{AR}\uparrow$ because $T_{AR}\uparrow$ as $V_{AR}\downarrow$.