Title: Notions of identity for and in higher dimensional categories

Abstract: FOLDS, first-order logic with dependent sorts, provides a general concept of identity (examples of which are: isomorphism for classical structures, equivalence for categories, biequivalence for bicategories, etc.) for categorical structures, by (1) re-coding the structures in question as *L*-structures: SET-valued functors from a well-chosen FOLDS signature (a 1-way, finitary category) *L*, and (2) deploying the general concept of *L*-equivalence, defined uniformly for all FOLDS signatures *L*. This is the notion of identity *for* a specific kind of a higher structure, say bicategory, mentioned in the title.

On the other hand, internally in a bicategory, we talk about equivalence of objects (0-cells), and also of 1-cells, and even of diagrams indexed by a fixed bicategory, as the "right" notion of identity. It turns out that FOLDS equivalence helps to give a simple uniform way of defining internal identity valid in a wide variety of cases. FOLDS equivalence as the notion of identity for higher structures is the main topic of my 1995 – unpublished but available on my web-site – monograph on FOLDS. The material for FOLDS equivalence used for identity internally in higher structures, examples and certain general facts, is unpublished, and some of it needs polishing, although I am ready to soon provide notes and give lectures, rather technical lectures and notes at that, on the subject.

The right initial context for the general theory of FOLDS equivalence "in and for" higher structures is one that eliminates usual syntax completely, It is based on the consideration of classes (I use *PHI* as a variable denoting a class in question) of *X* -augmented *L*-structures, for varying FOLDS signatures *L* and augmentation types *X*; *X* here is a finite SET-valued functor on *L*; and an augmented structure is a functor *M:L*-->SET (*L*-structure), together with an augmentation (natural transformation) *a:X-->M* . The only condition imposed on *PHI* is that it be invariant under *L*-equivalence (which is defined naturally for augmented structures as well). The context described is one familiar in model-theoretic-logic, where a formula – any formula in any logic such as first-order, second order, etc – is replaced by the class of augmented structures satisfying it; the above *X* is the (context-organized) set of (typed by dependent sorts) free variables in the formula. I would like to call this the Lindstroem context for FOLDS. Originally introduced by Per Lindstroem for ordinary classical one-sorted logic, Lindstroem's context for model-theoretic logic gives rise to his (once?) celebrated (but nowadays quite forgotten …?) theorem characterizing first-order logic "from above". It turns out that Lindstroem's theorem has a smooth, even elegant, generalization to FOLDS, with a proof that is a careful, but quite straightforward, lifting Lindstroem's proof, by replacing the uniform classical notion of isomorphism – accepted exclusively as the right notion of identity for structures in model-theoretic logic – by FOLDS *L*-equivalence, for varying FOLDS signatures *L*.

Let me note that the Lindstroem theorem for FOLDS and its proof are quite independent from the main meta-theorem for FOLDS (let me call it the invariance theorem) which asserts, roughly and slightly more weakly than necessary, that, up to logical equivalence, the first-order (multi-sorted) statements over *L* (as a plain multisorted signature) that are invariant under FOLDS equivalence are exactly the ones formulated in the FOLDS syntax over *L*: see the 1995 monograph. On the other hand, similarly to the invariance theorem, the FOLDS-version of the Lindstroem theorem is a demonstration of the tight relation of the FOLDS concept of identity and the FOLDS syntax. One is reminded of the Leibnizian definition of identity: two things are identical if and only if they satisfy the same meaningfully applicable properties.