Tries & Suffix Tries

Tries

A trie ("try") is a tree representing a collection of strings (keys):
the smallest tree such that

Each edge is labeled with a character $c \in \Sigma$

For given node, at most one child edge has label c, for any $c \in \Sigma$

Each key is "spelled out" along some path starting at root

Helpful for implementing a *set* or *map* when the keys are strings

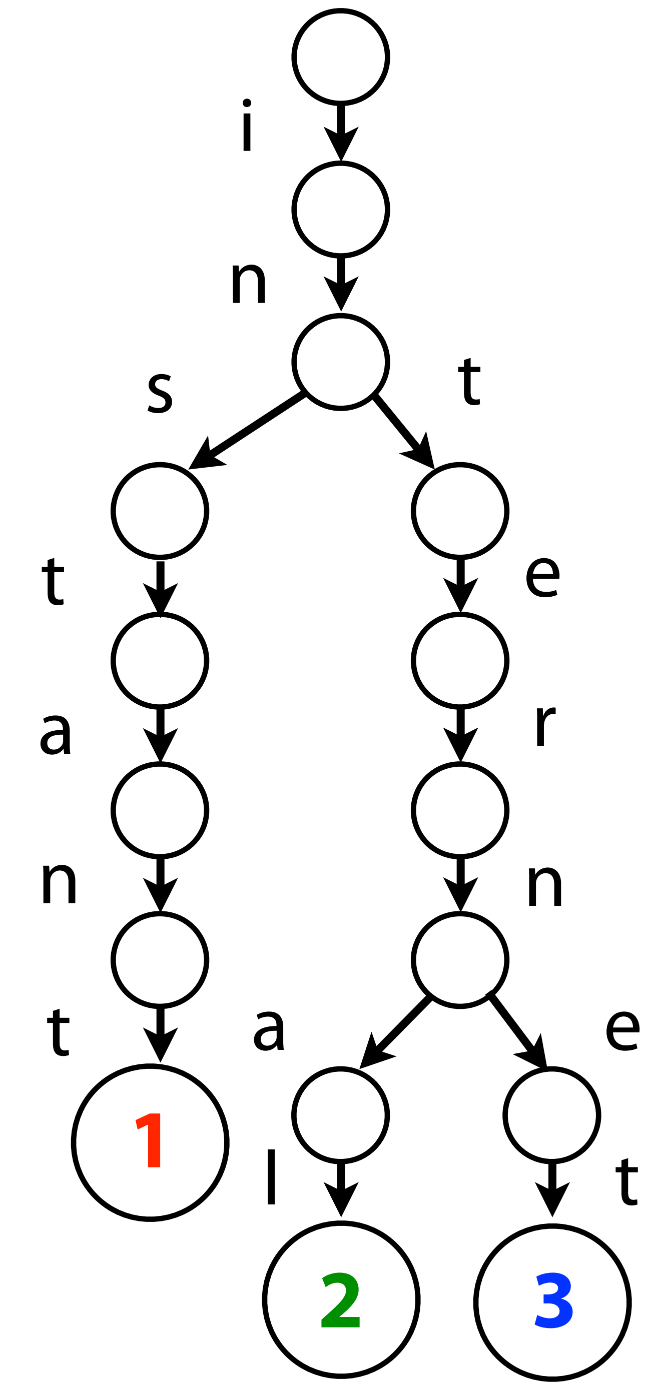
Tries

Keys: instant, internal, internet

Key	Value
instant	1
internal	2
internet	3

Smallest tree such that:

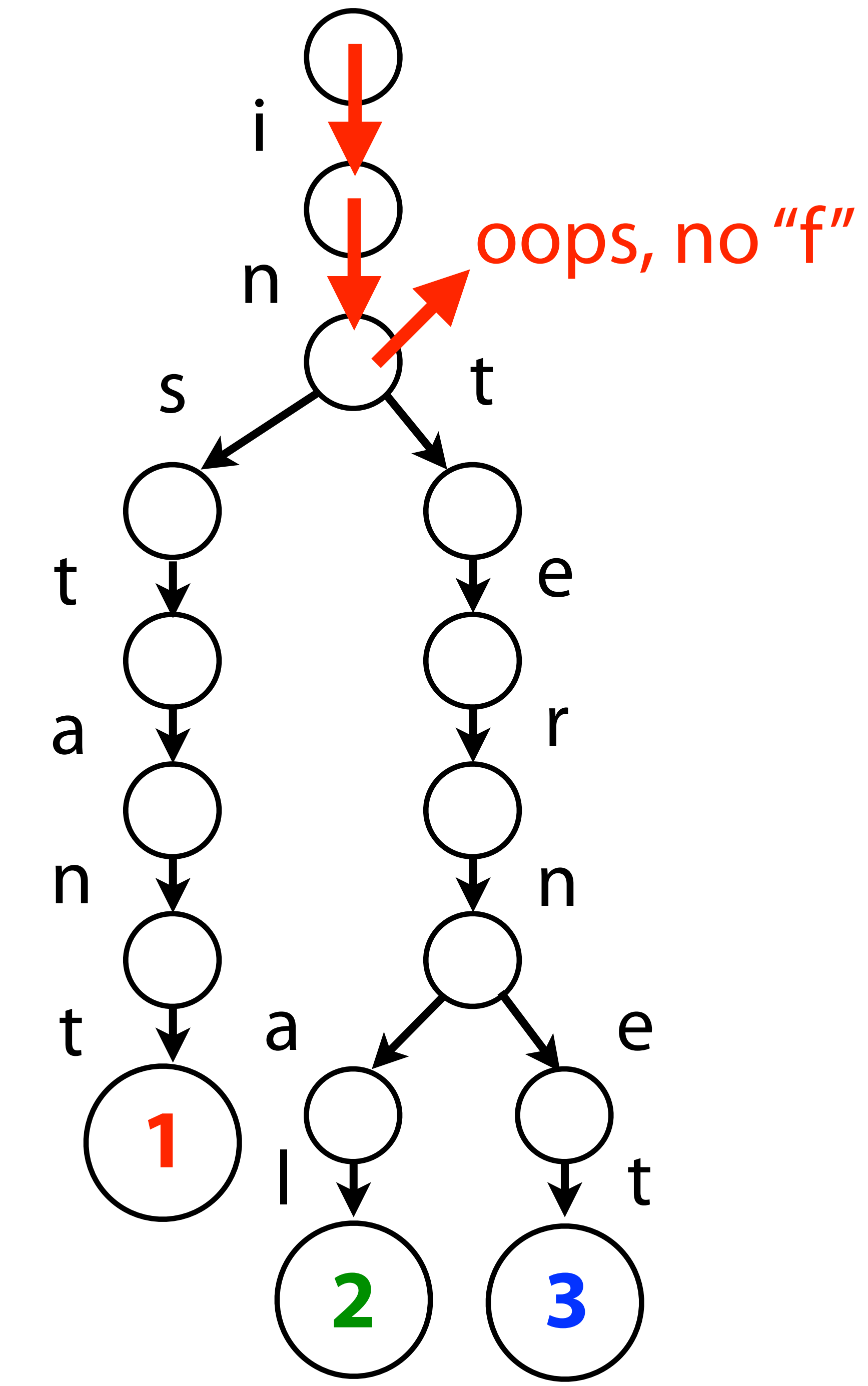
- Each edge is labeled with a character $c \in \Sigma$
- For given node, at most one child edge has label c, for any $c \in \Sigma$
- Each key is "spelled out" along some path starting at root



Tries

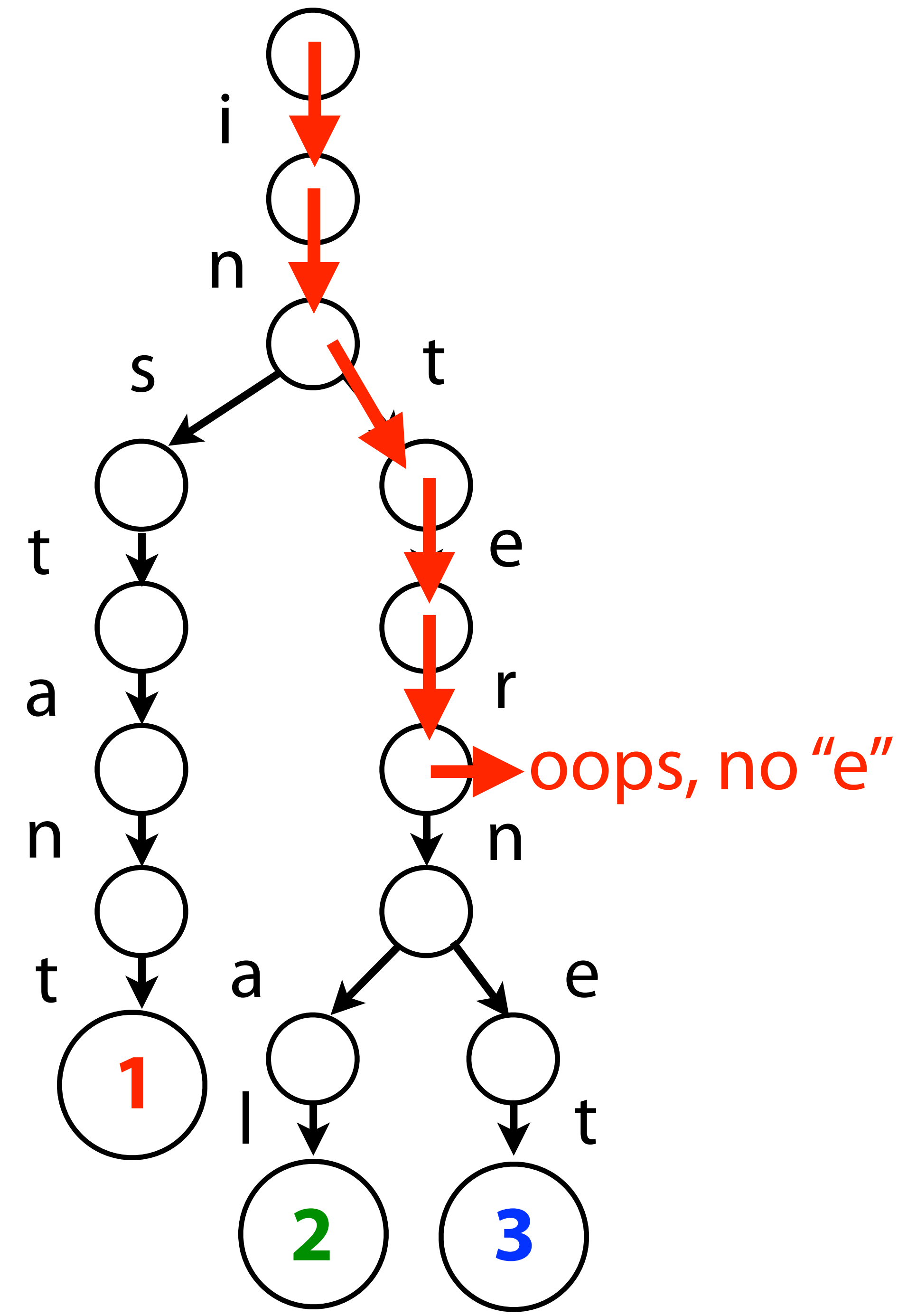
How do we check whether "infer" is in the trie?

Start at root and try to match successive characters of "infer" to edges in trie



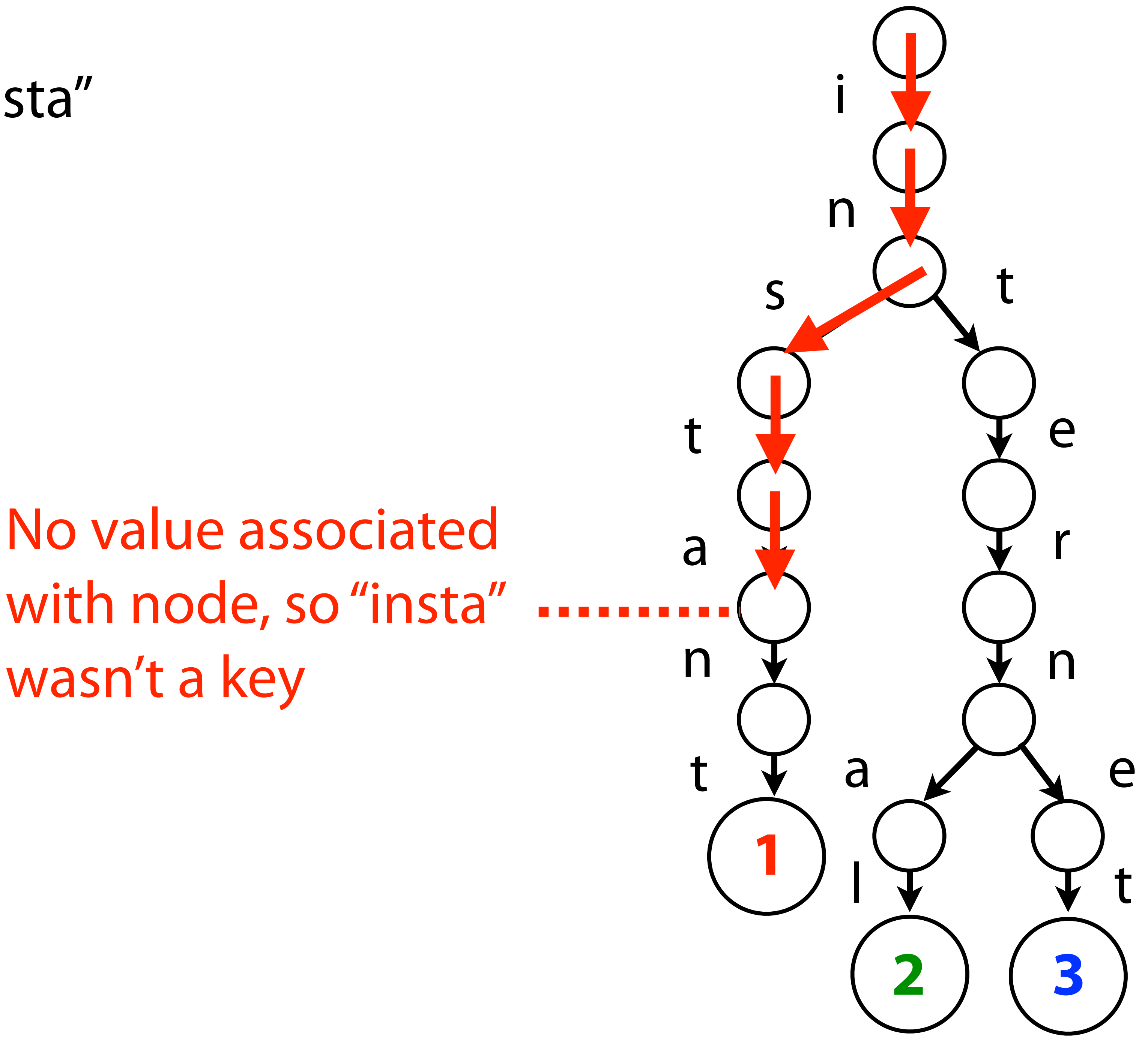
Tries

Matching “interesting”



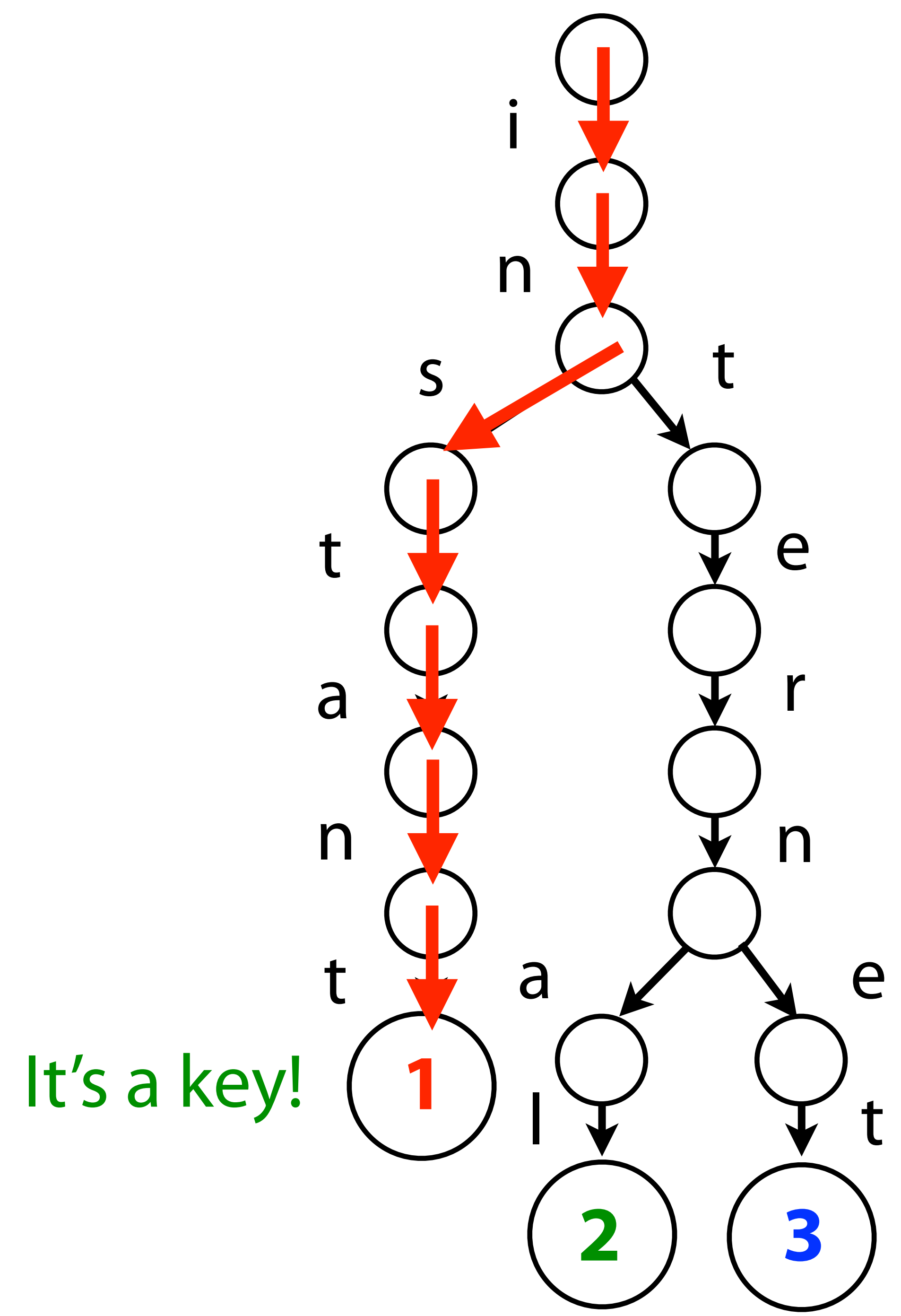
Tries

Matching “insta”

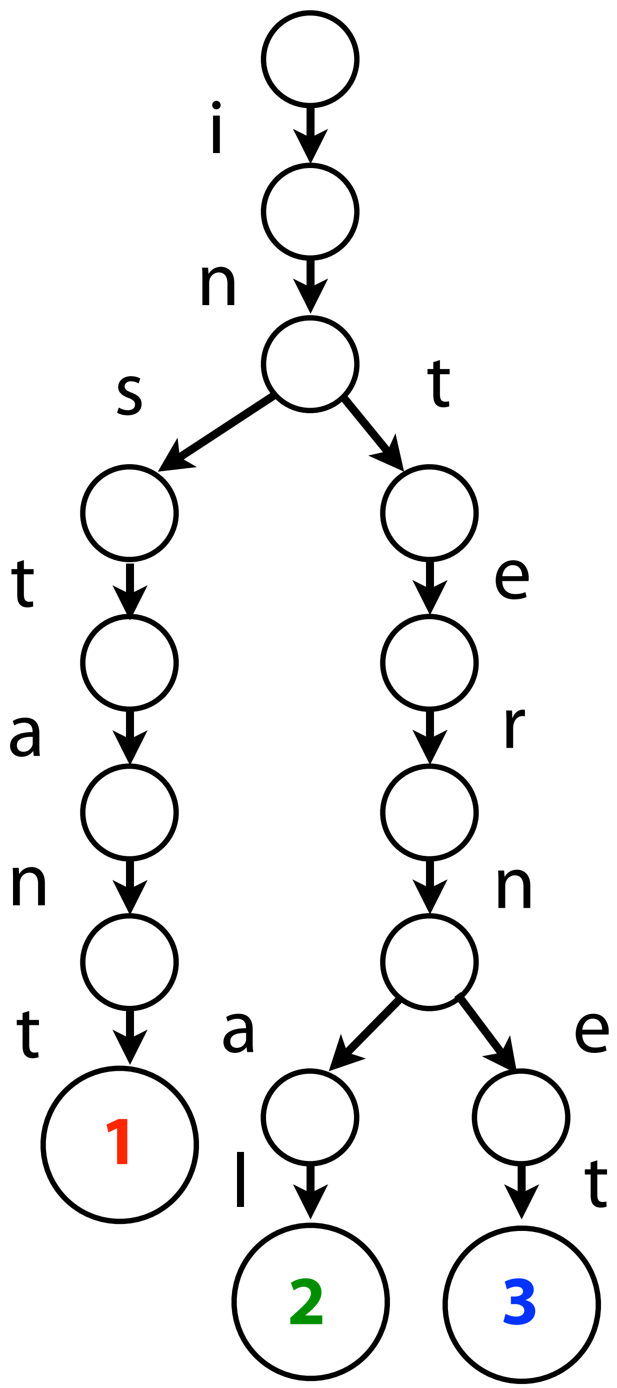


Tries

Matching "instant"



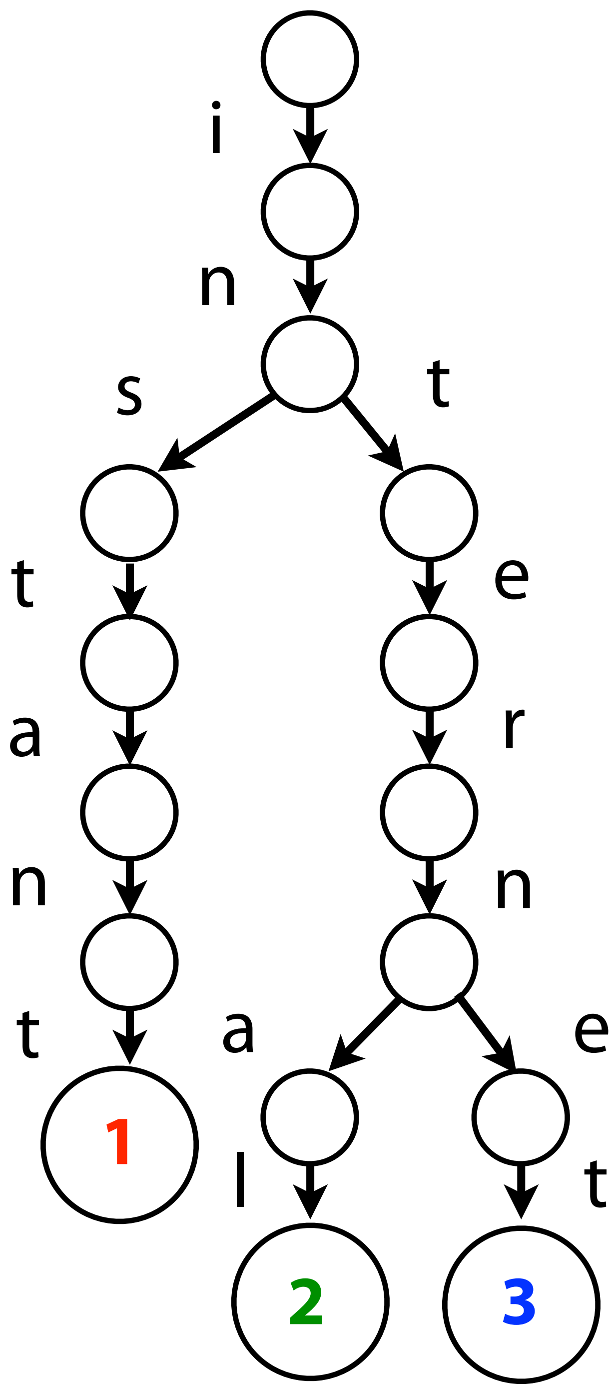
Tries



Checking for presence of key P, where $|P| = n$ traverses $\leq$ ***n*** edges

If total length of all keys is N, trie has $\leq$ ***N*** edges

Tries



How to represent edges between a node and its children?

Map (from characters to child nodes)

Idea 1: Hash table

Idea 2: Sorted lists

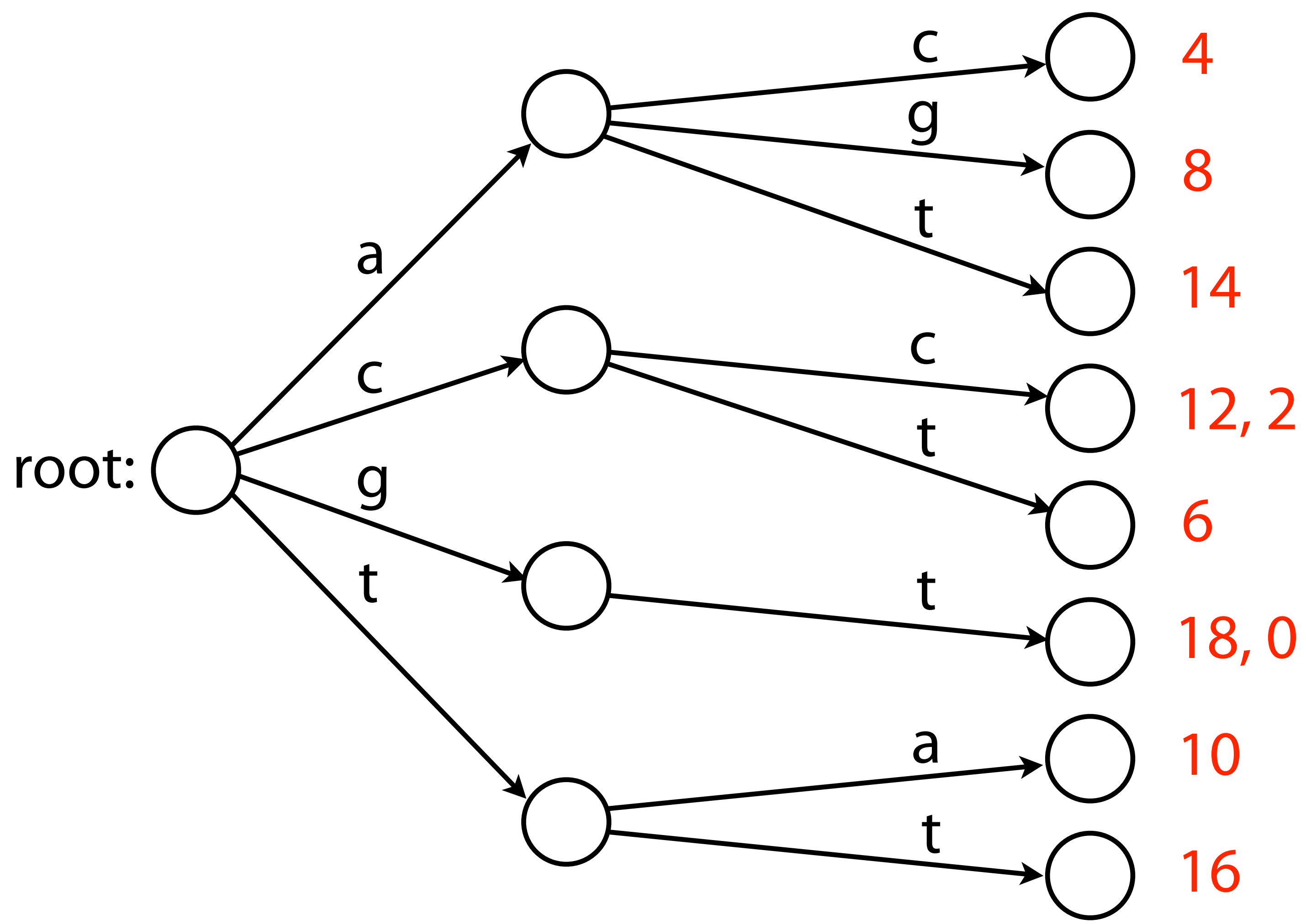
Assuming hash table, it's reasonable to say querying with P, $|P| = n$, is $O(n)$ time

Tries

Could use trie to represent *k*-mer index.
Map *k*-mers to offsets where they occur

ac	4
ag	8
at	14
cc	12
cc	2
ct	6
gt	18
gt	0
ta	10
tt	16

Index



Tries: implementation

Refer to trie-map.ipynb

Tries: alternatives

Tries aren't the only way to encode sets or maps over strings using a tree.

E.g. ternary search tree:

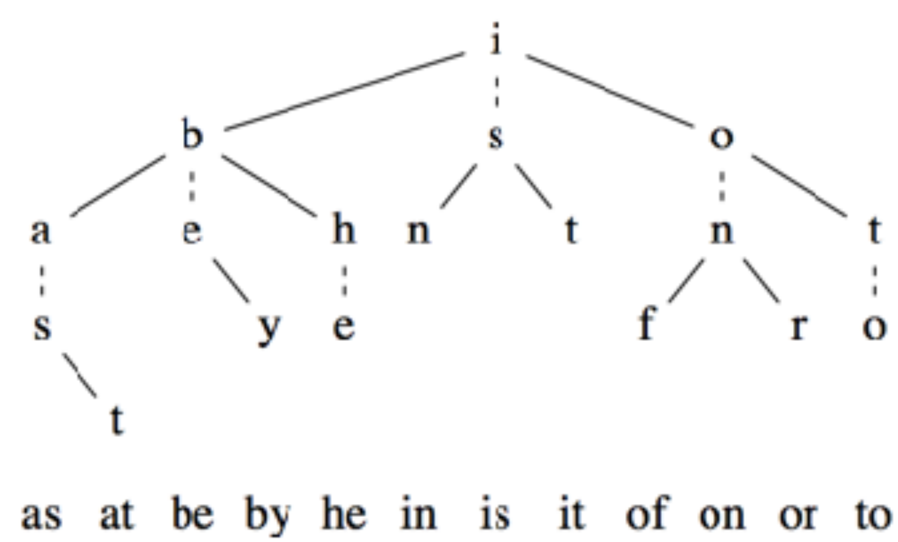


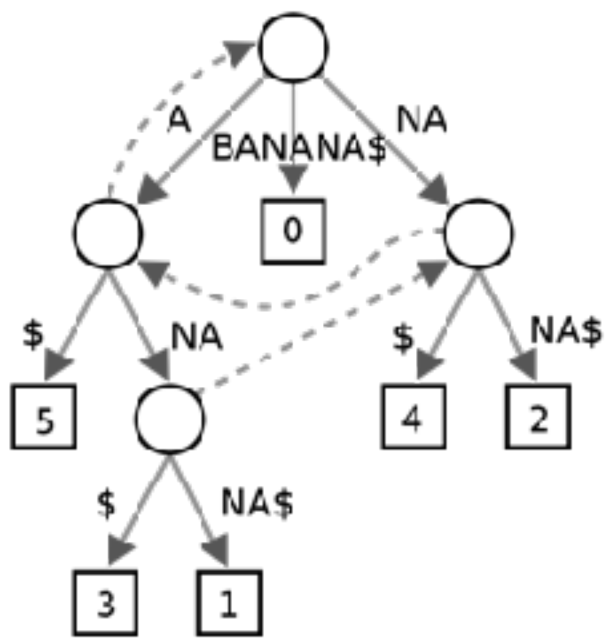
Figure 2. A ternary search tree for 12 two-letter words

Bentley, Jon L., and Robert Sedgewick. "Fast algorithms for sorting and searching strings." *Proceedings of the eighth annual ACM-SIAM symposium on Discrete algorithms*. Society for Industrial and Applied Mathematics, 1997

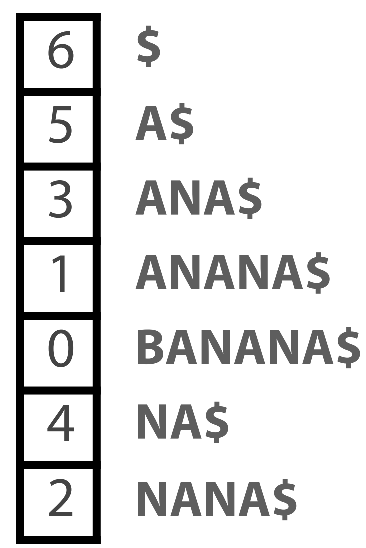
Indexing with suffixes

We studied indexes built over substrings of *T*

Different approach is to index *suffixes* of *T*. This yields surprisingly economical & practical data structures:



Suffix Tree



Suffix Array

FM Index

Suffix trie

Build a **trie** containing all **suffixes** of a text *T*

```
T: GTTATAGCTGATCGCGGCGTAGCGG$
   GTTATAGCTGATCGCGGCGTAGCGG$
    TTATAGCTGATCGCGGCGTAGCGG$
     TATAGCTGATCGCGGCGTAGCGG$
      ATAGCTGATCGCGGCGTAGCGG$
       TAGCTGATCGCGGCGTAGCGG$
        AGCTGATCGCGGCGTAGCGG$
         GCTGATCGCGGCGTAGCGG$
          CTGATCGCGGCGTAGCGG$
           TGATCGCGGCGTAGCGG$
            GATCGCGGCGTAGCGG$
             ATCGCGGCGTAGCGG$
              TCGCGGCGTAGCGG$
               CGCGGCGTAGCGG$
                GCGGCGTAGCGG$
                 CGGCGTAGCGG$
                  GGCGTAGCGG$
                   GCGTAGCGG$
                    CGTAGCGG$
                     GTAGCGG$
                      TAGCGG$
                       AGCGG$
                        GCGG$
                         CGG$
                          GG$
                           G$
                            $
```

m(m+1)/2 chars

Suffix trie

First add special *terminal character* **$** to the end of *T*

$ is a character that does not appear elsewhere in *T*, and we define it to be less than other characters (**$** < **A** < **C** < **G** < **T**)

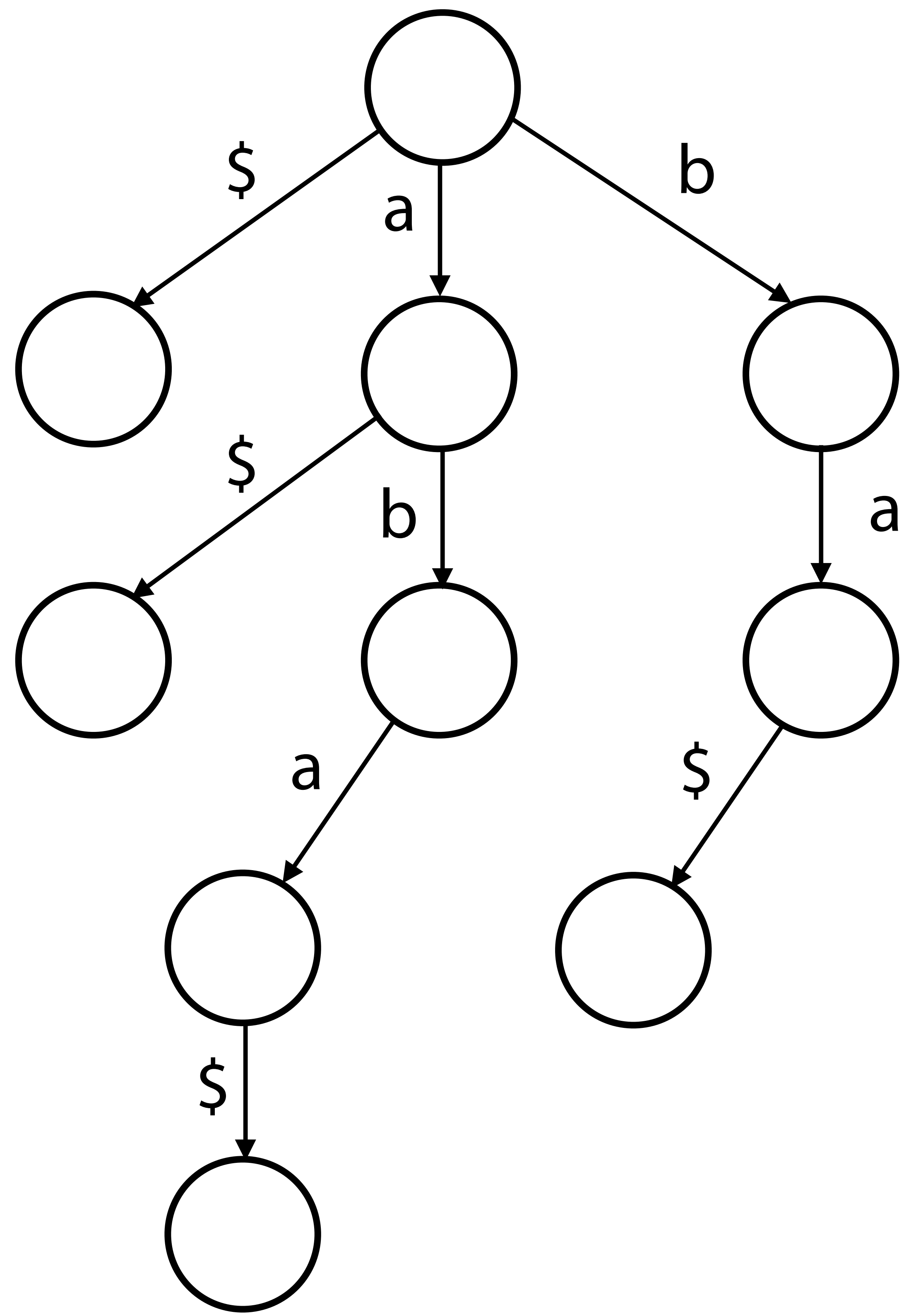
$ enforces a familiar rule: e.g. “as” comes before “ash” in the dictionary.
$ also guarantees no suffix is a prefix of any other suffix.

```
T: GTTATAGCTGATCGCGGCGTAGCGG$
   GTTATAGCTGATCGCGGCGTAGCGG$
    TTATAGCTGATCGCGGCGTAGCGG$
     TATAGCTGATCGCGGCGTAGCGG$
      ATAGCTGATCGCGGCGTAGCGG$
       TAGCTGATCGCGGCGTAGCGG$
        AGCTGATCGCGGCGTAGCGG$
         GCTGATCGCGGCGTAGCGG$
          CTGATCGCGGCGTAGCGG$
           TGATCGCGGCGTAGCGG$
            GATCGCGGCGTAGCGG$
             ATCGCGGCGTAGCGG$
              TCGCGGCGTAGCGG$
               CGCGGCGTAGCGG$
                GCGGCGTAGCGG$
                 CGGCGTAGCGG$
```

Suffix trie

T: aba**$**

What's the suffix trie?



Suffix trie

Excerpt from SuffixTrie class: suffix-trie.ipynb

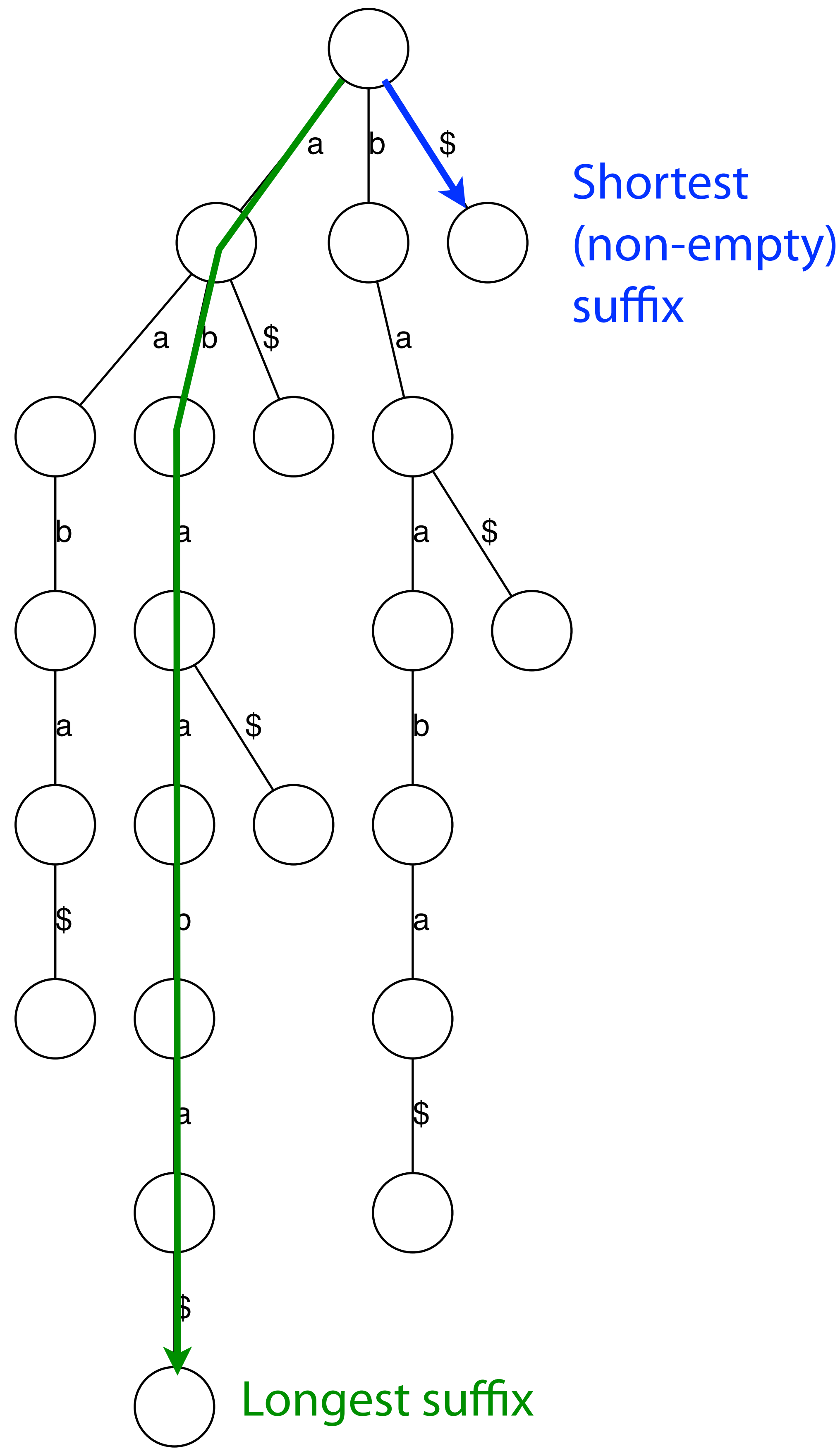
```
def __init__(self, t):
    """ Make suffix trie from t """
    t += '$'                          # add terminator
    self.root = {}
    for i in range(len(t)):  # for each suffix
        cur = self.root
        for c in t[i:]:          # for each character in i'th suffix
            if c not in cur:
                cur[c] = {}  # add outgoing edge if necessary
            cur = cur[c]       # follow the edge and continue
```

Suffix trie

T: abaaba**$**

Each path from root to leaf represents a suffix; each suffix is represented by some path from root to leaf

Would this still be the case if we hadn't added **$**?



Suffix trie

T: abaaba**$**

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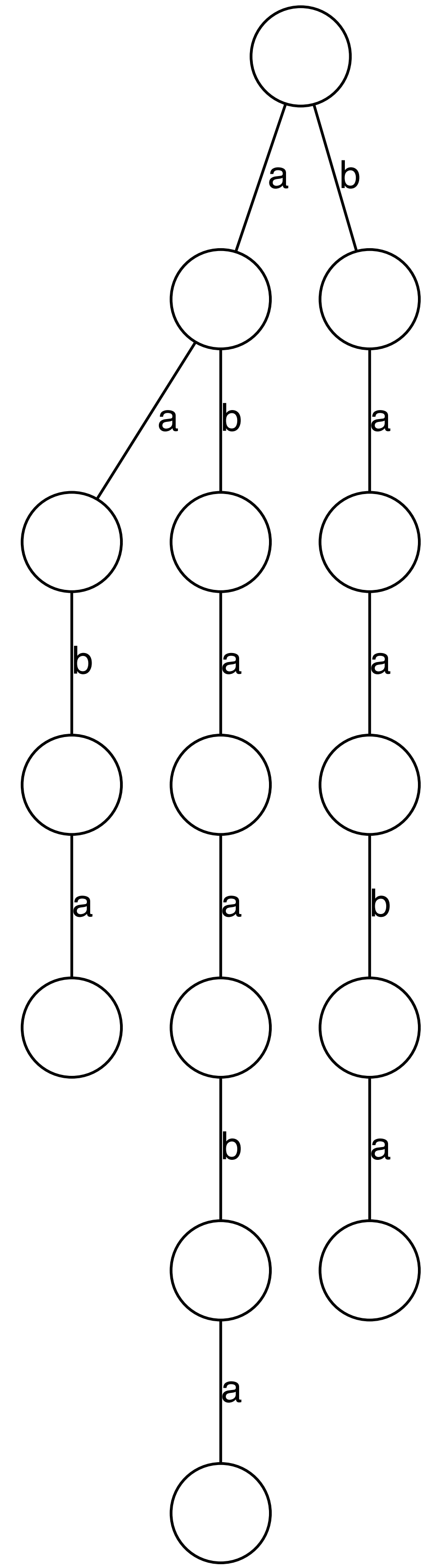
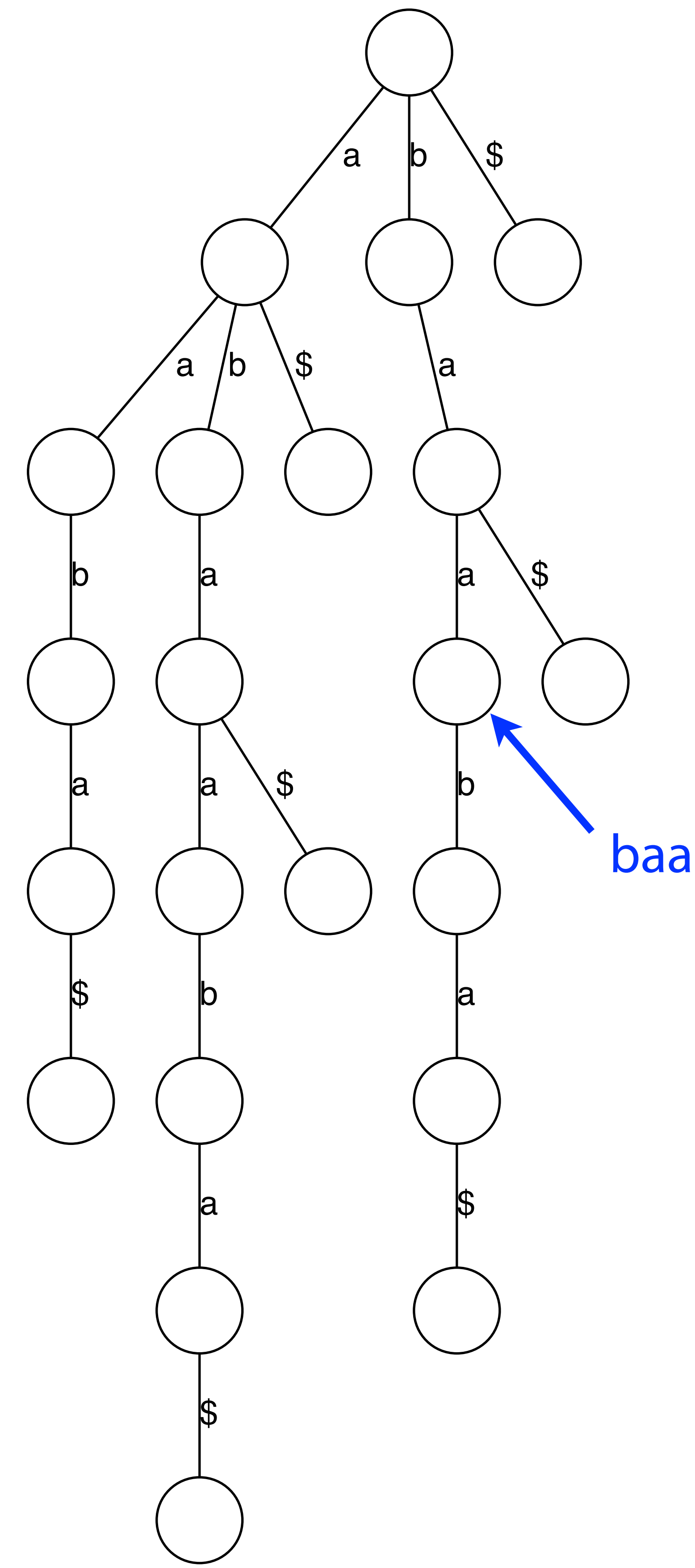
Would this still be the case if we hadn't added **$**? **No**



Suffix trie

Think of each node as having a **label**, spelling out characters on path from root to node



Suffix trie

How do we check whether a string *S* is a substring of *T*?

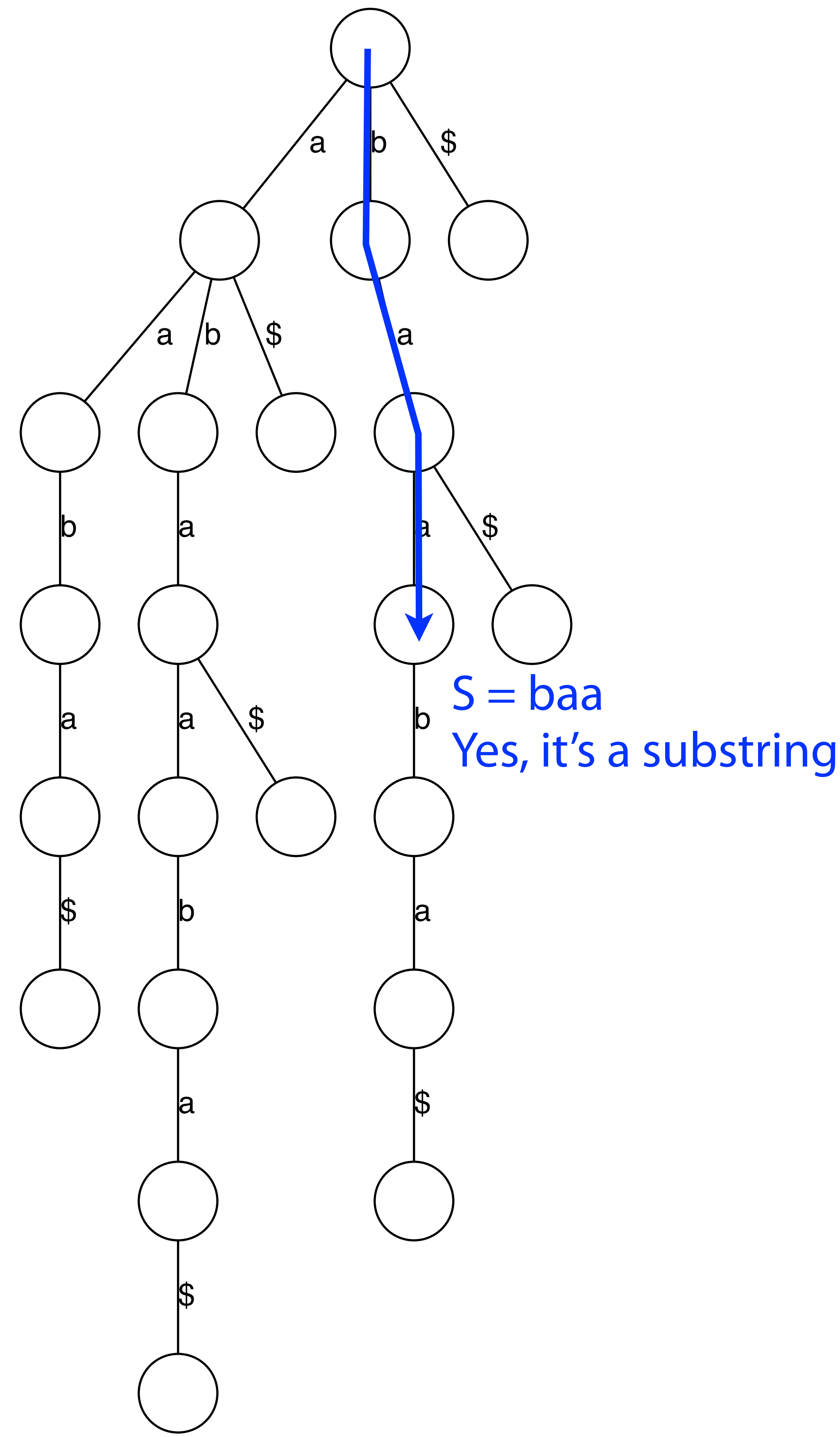
Note: Each of *T*'s substrings is spelled out along a path from the root.

Every *substring* is a *prefix* of some *suffix* of T.

Start at the root and follow the edges labeled with the characters of *S*

If we "fall off" the trie -- i.e. there is no outgoing edge for next character of *S*, then *S* is not a substring of *T*

If we exhaust *S* without falling off, *S* is a substring of *T*



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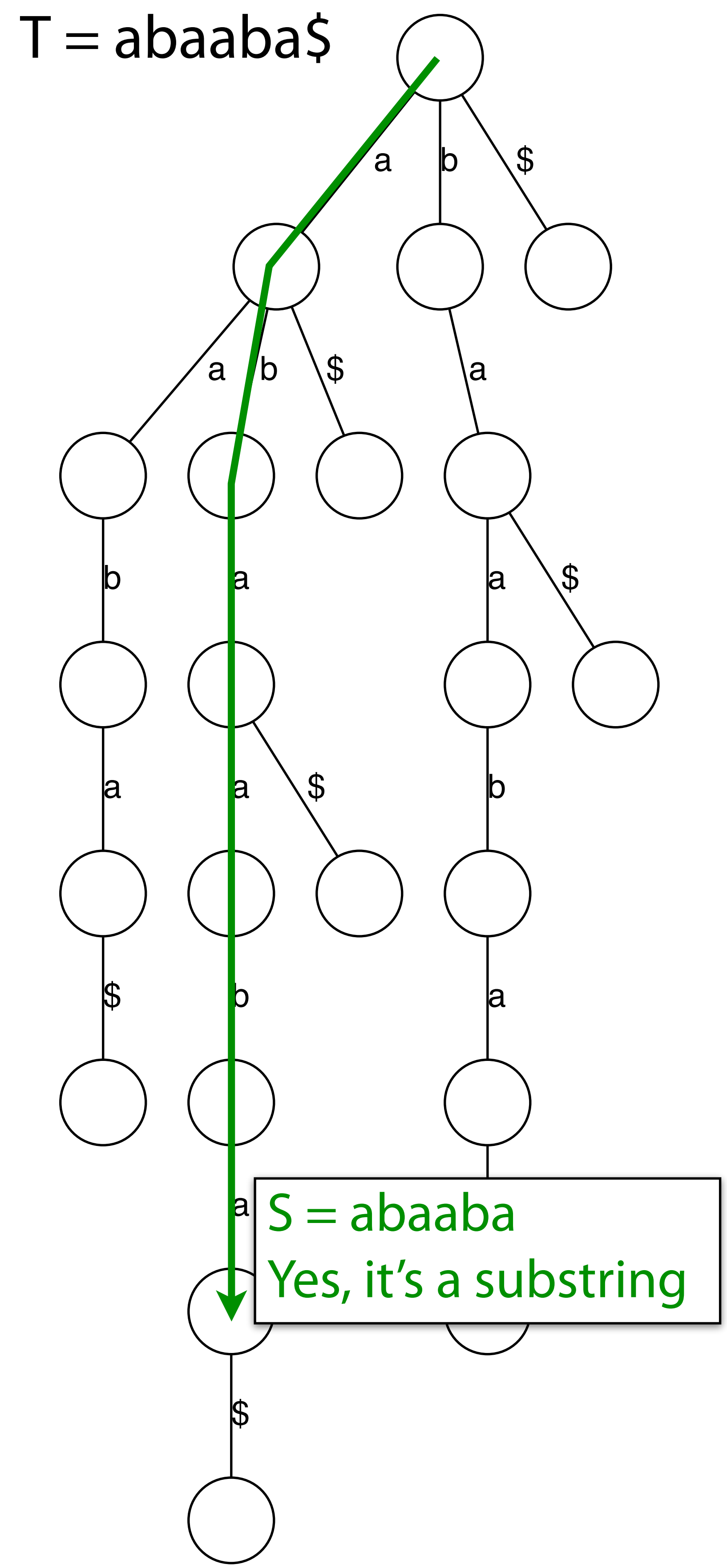

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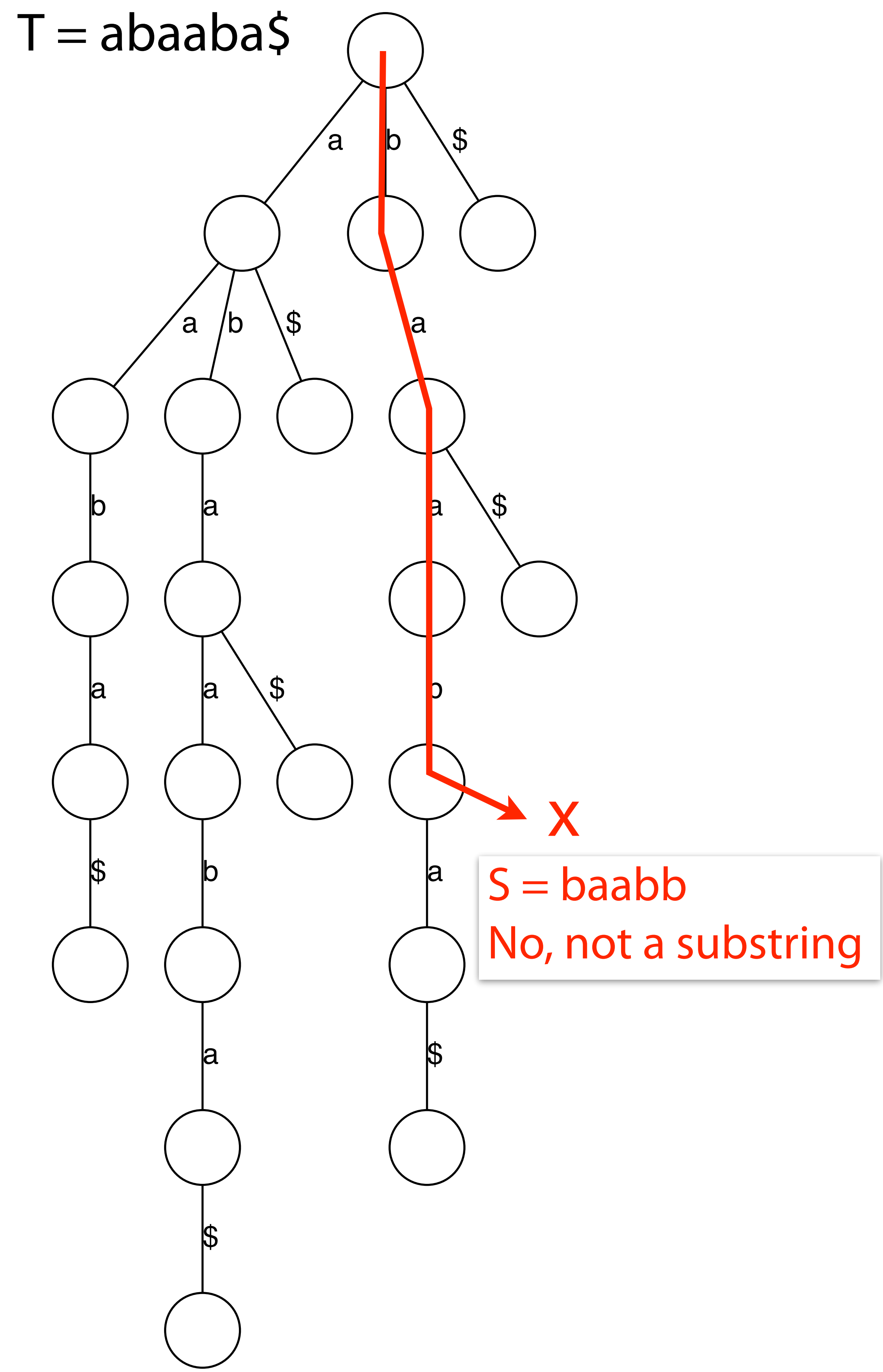


Suffix trie

Excerpt from SuffixTrie class: suffix-trie.ipynb

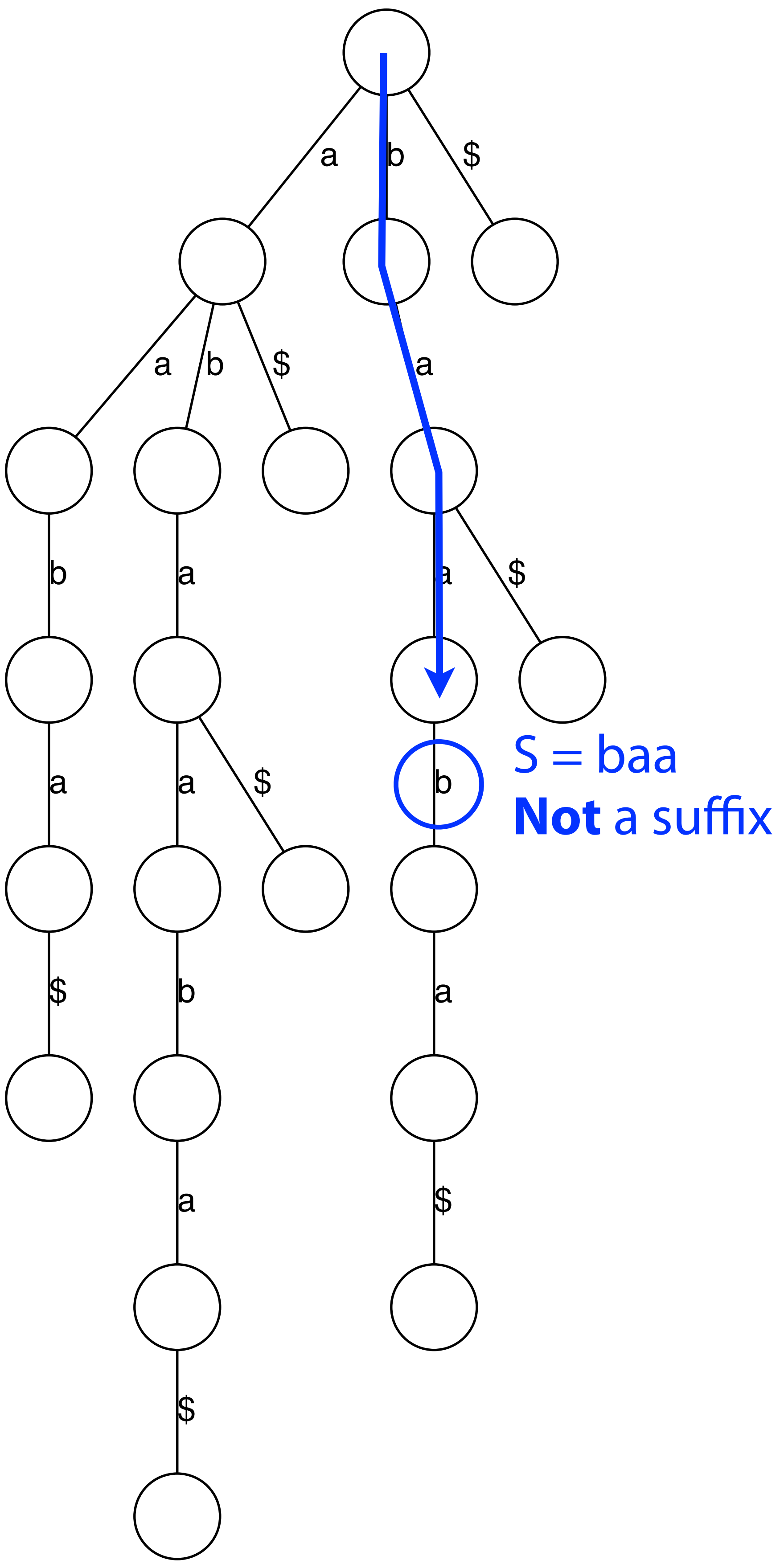
```
def follow_path(self, s):
    """ Follow path given by characters of s.  Return node at
        end of path, or None if we fall off. """
    cur = self.root
    for c in s:
        if c not in cur:
            return None  # no outgoing edge on next character
        cur = cur[c]     # descend one level
    return cur

def has_substring(self, s):
    """ Return true if s appears as a substring of t """
    return self.follow_path(s) is not None
```

Suffix trie

How do we check whether a string *S* is a **suffix** of *T*?

Same procedure as for substring, but additionally check terminal node for **$** child



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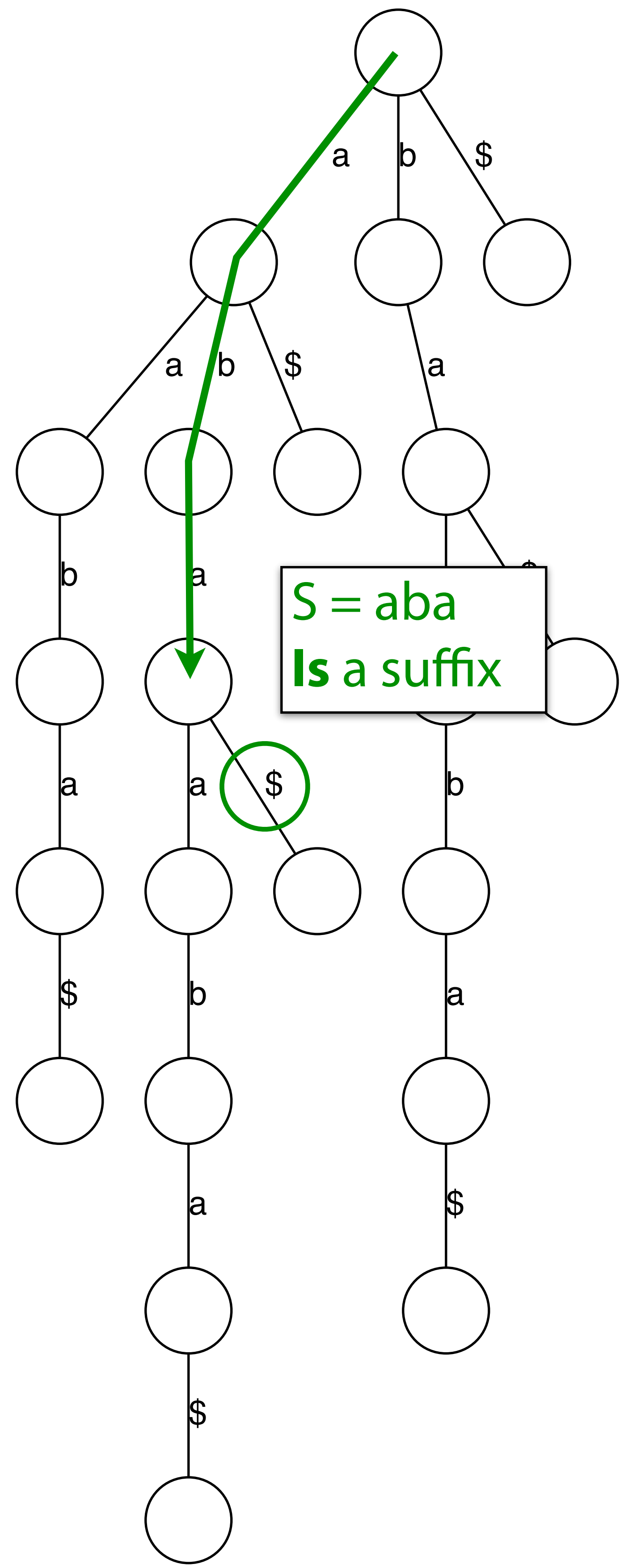


Suffix trie

Excerpt from SuffixTrie class: suffix-trie.ipynb

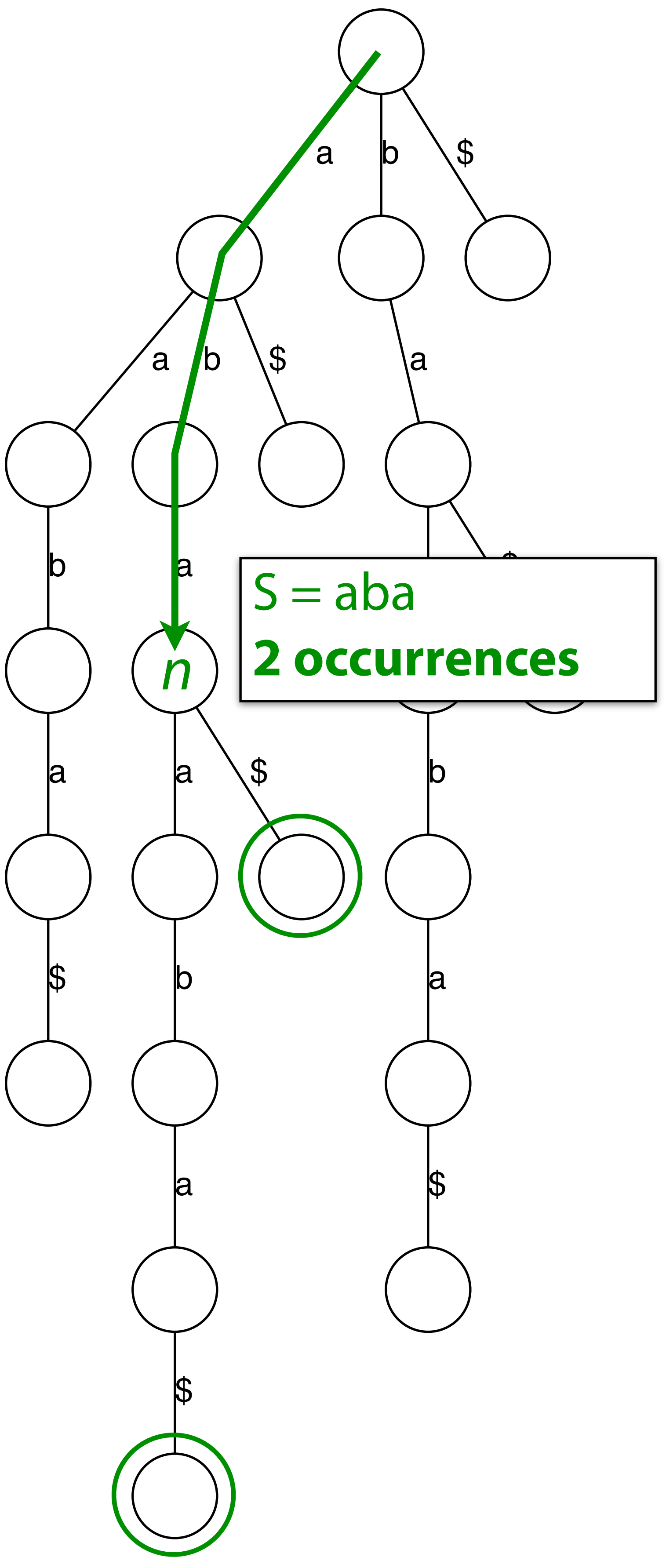
```
def has_suffix(self, s):
    """ Return true if s is a suffix of t """
    node = self.follow_path(s)
    return node is not None and '$' in node
```

Suffix trie

How do we count the **number of times** a string *S* occurs as a substring of *T*?

Follow path labeled with *S*. If we fall off, answer is 0. If we end up at node *n*, answer equals # of leaves in subtree rooted at *n*.

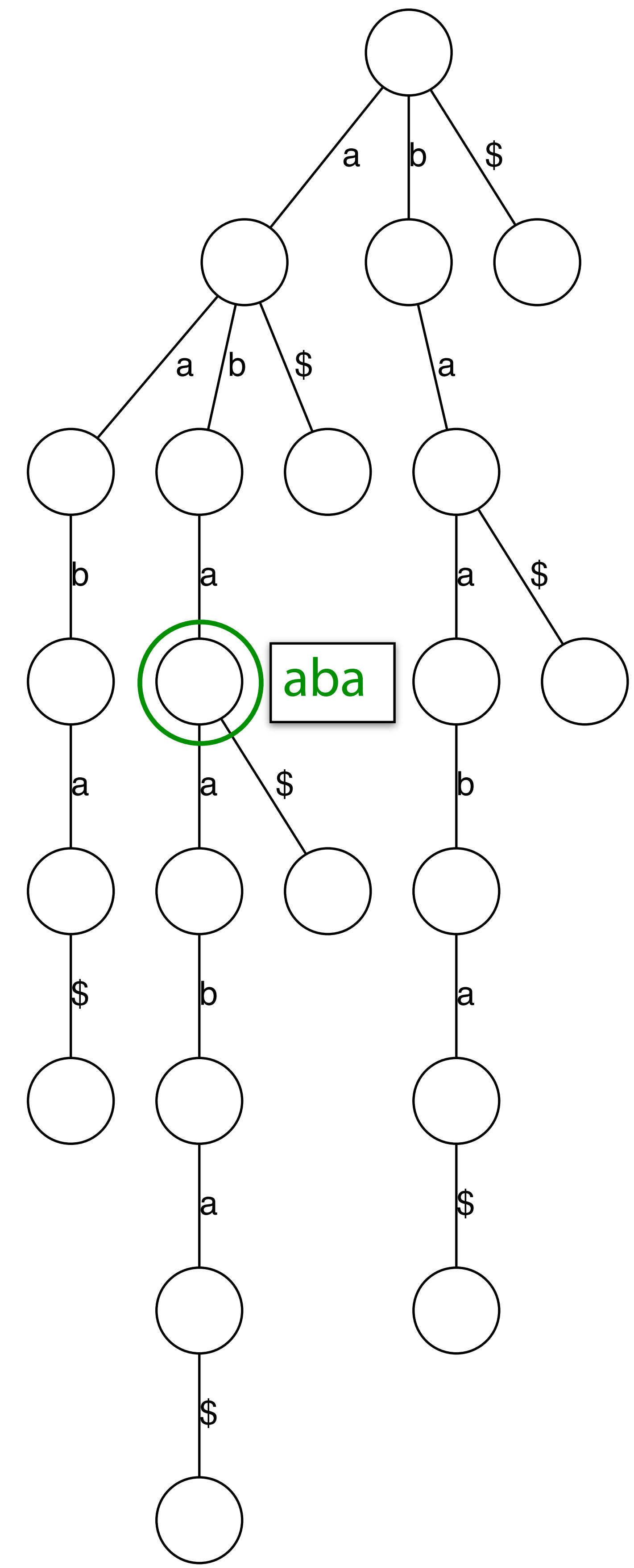
Leaves can be counted with depth-first traversal.



Suffix trie

How do we find the **longest repeated substring** of *T*?

Find the deepest node with more than one child



Suffix trie

How does the suffix trie grow with $|T| = m$?

Suffix trie

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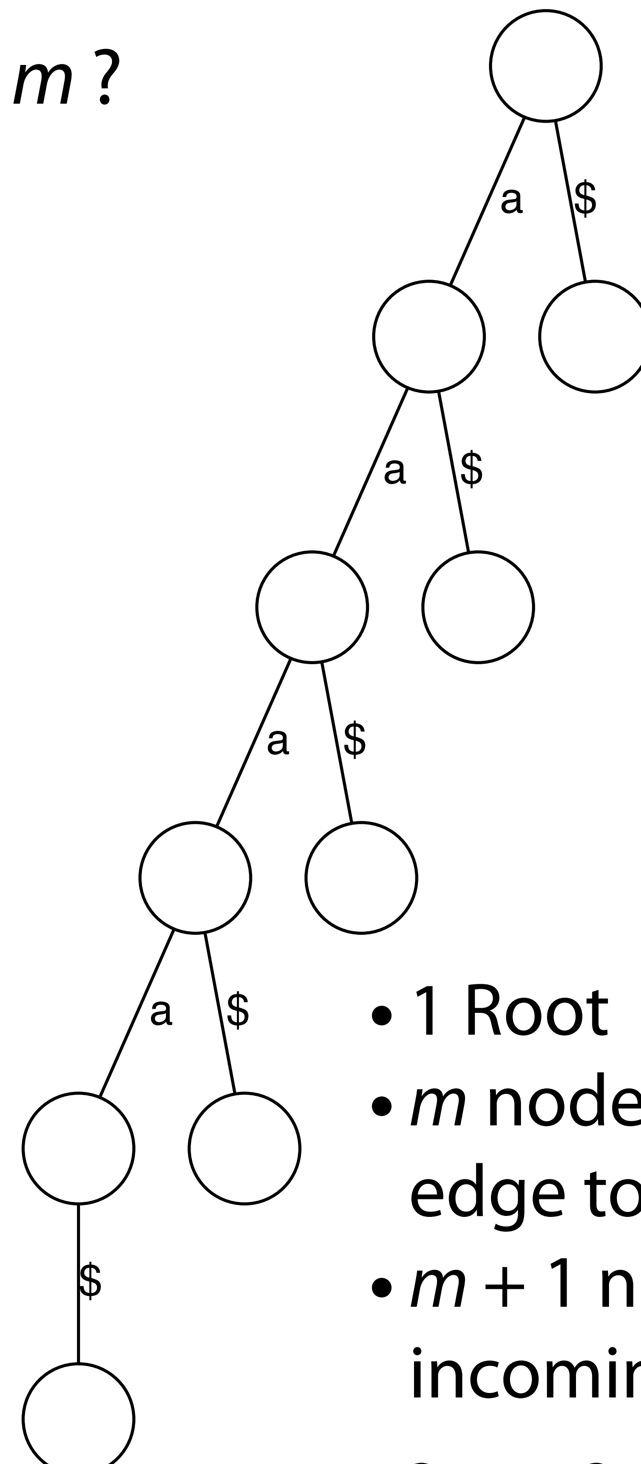
Is there a class of string where the number of suffix trie nodes grows linearly with m?

Yes: a string of m a's in a row (a^m)

T = aaaa$



- 1 Root
- m nodes with "a" edge to parent
- $m + 1$ nodes with incoming **$** edge

$2m + 2$ nodes

Suffix trie

T = aaabbb$

How does the suffix trie grow with $|T| = m$?

Is there a class of string where the number of suffix trie nodes grows with m^2 ?

Yes: $a^n b^n$ where $2n = m$

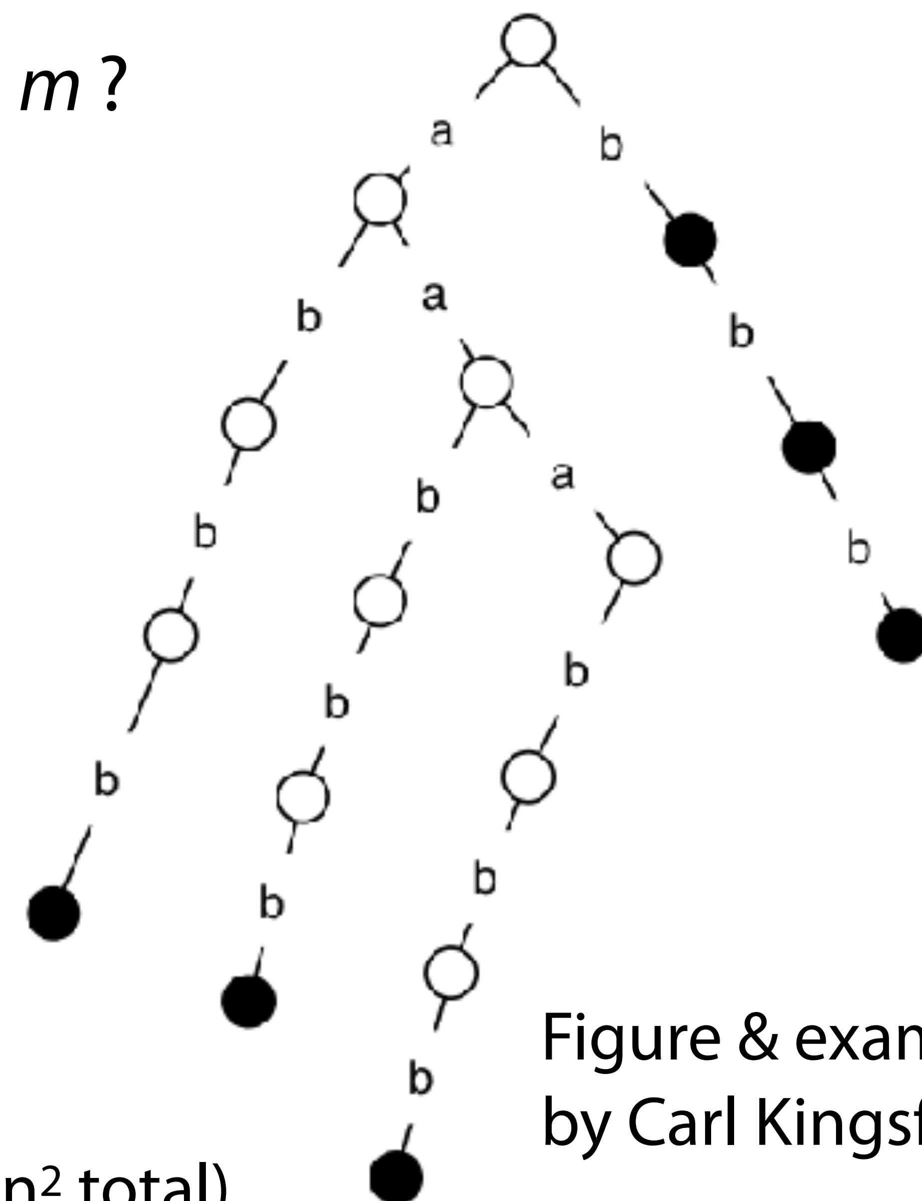


Figure & example by Carl Kingsford

- 1 root
- n nodes along "b chain," right
- n nodes along "a chain," middle
- n chains of n "b" nodes hanging off "a chain" (n^2 total)
- $2n + 1$ **$** leaves (not shown)

$n^2 + 4n + 2$ nodes, where $m = 2n$