UM–SJTU Joint Institute

Physics Laboratory

(Vp241)

Laboratory Report

Exercise 4

Polarization of Light

Name: Haoming Zhu	**ID: 520021910145**	Group: 1
Partner: Xinkai Wang	ID: 520021910312	Group: 1

Date: 2021.11.26

Contents

1 Introduction **1**
- 1.1 Objective . . . 1
- 1.2 Theoretical Background . . . 1
  - 1.2.1 Polarization of Light . . . 1
- 1.3 Polarizer . . . 1
- 1.4 Malus' Law . . . 1
- 1.5 Generation of Elliptically and Circularly Polarized Light. Half-wave and Quarter-wave Plates . . . 1

2 Experimental Setup **3**

3 Measurement Procedure **3**
- 3.1 Apparatus Adjustment . . . 3
- 3.2 Demonstration of Malus' Law . . . 3
- 3.3 Linearly Polarized Light and the Half-wave Plate . . . 4
- 3.4 Circularly and Elliptically Polarized Light and the 1/4-wave Plate . . . 4

4 Result **4**
- 4.1 Demonstration of Malus' Law . . . 4
- 4.2 Linearly Polarized Light and the Half-wave Plate . . . 6
- 4.3 Circularly and Elliptically Polarized Light and the 1/4-wave Plate . . . 7
  - 4.3.1 Rotation Angle: 0° . . . 7
  - 4.3.2 Rotation Angle: 20° . . . 10
  - 4.3.3 Rotation Angle: 45° . . . 12
  - 4.3.4 Rotation Angle: 70° . . . 14

5 Discussion and Conclusions **14**
- 5.1 Demonstration of Malus' Law . . . 14
- 5.2 Linearly Polarized Light and the Half-wave Plate . . . 15
- 5.3 Circularly and Elliptically Polarized Light and the 1/4-wave Plate . . . 15
  - 5.3.1 Rotation Angle: 0° . . . 16
  - 5.3.2 Rotation Angle: 20° . . . 16
  - 5.3.3 Rotation Angle: 45° . . . 16
  - 5.3.4 Rotation Angle: 70° . . . 16
- 5.4 Causes for errors and uncertainties . . . 16
- 5.5 Suggestions . . . 16
- 5.6 Conclusions . . . 16

A Measurement Uncertainty Analysis **17**
- A.1 Demonstration of Malus' Law . . . 17
- A.2 Linearly Polarized Light and the Half-wave Plate . . . 17
- A.3 Circularly and Elliptically Polarized Light and the 1/4-wave Plate . . . 17

B Data Sheet **17**

1 Introduction

1.1 Objective

The objective of this lab is to study the polarization phenomenon and the Malus's Law of light. Also, the way of how half- and quarter-wave plates work in optical systems will be studied.

1.2 Theoretical Background

1.2.1 Polarization of Light

Light is a kind of electromagnetic wave. The electric field vector $\mathbf{E}$ in the electromagnetic wave is called the light vector. In the plane perpendicular to the propagation direction of a light wave, the light vector may have different directions along which its magnitude oscillates. Specifically, if the light only oscillates in a certain direction, it is called *linear polarized*, and the axis dominating the direction is called the polarization axis.

The light with light vector direction rotating about the propagation direction may generate different kinds of traces at the endpoints. According to the shape of the shape, the light can be categorized into *elliptically polarized* and *circularly polarized*.

1.3 Polarizer

A Ploarizer is a device that can produce polarized light. It polarizes the light by only letting the light polarized in a certain direction pass through, filtrating the others. The light passing through it becomes linearly polarized. Besides, it can detect and analyze linearly polarized, natural, and partially polarized light (it is then called an analyzer).

1.4 Malus' Law

The effect of the polarization can be detected by observing the change of brightness.

If we have two parallel polarizers on the same axis, denote the left one as polarized and the right one as analyzer (Figure 1). Let the angle between their transmission directions (polarization axes) be θ. The light is incident normally on the polarizer and then continues to the analizer. The intensity of the linearly polarized light leaving the analyzer is

$$I_{\text{light}} = I_{\text{light},0} \cos^2 \theta, \tag{1}$$

where $I_{\text{light},0}$ is the intensity of the linearly polarized light incident on the analyzer. This is the so-called Malus' law.

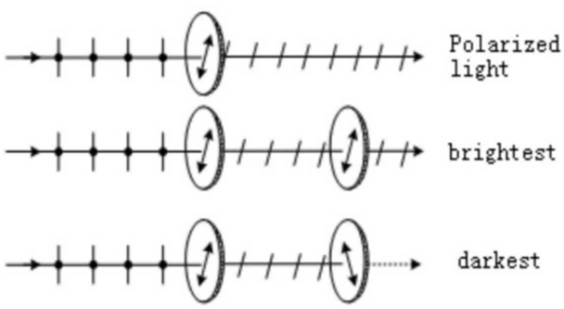


Figure 1: Change in the brightness of the light depends on the mutual orientation of the polarizer and the analyzer.

1.5 Generation of Elliptically and Circularly Polarized Light. Half-wave and Quarter-wave Plates

If a linearly polarized light is incident morally on a crystal plate whose surface is parallel to its optical axis, and the angle between the polarizing axis and the optical axis of the plate is α, then the linearly polarized light is resolved into two waves: an e-wave with the oscillation direction parallel to the optical axis of the plate (extraordinary axis) and an o-wave whose oscillation direction is perpendicular to the optical axis (ordinary axis). They propagate in the same direction, but with different speeds. The resulting optical path difference over the thickness d of the plate is

$$\Delta = (n_\mathrm{e} - n_\mathrm{o})d,$$

and, consequently, the phase difference

$$\delta = \frac{2\pi}{\lambda}(n_\mathrm{e} - n_\mathrm{o})d,$$

where λ is the wavelength, n_e is the refractive index for the extraordinary axis, and n_o is the refractive index for the ordinary axis. In a so-called positive crystal $\delta > 0$, whereas in a negative one $\delta < 0$.

As shown in Figure 4, when the light propagates through the crystal plate, the two components of the light vector are

$$E_x = A_\mathrm{o}\cos\omega t$$
$$E_y = A_\mathrm{e}\cos(\omega t + \delta),$$

where $A_\mathrm{e} = A\cos\alpha$, $A_\mathrm{o} = A\sin\alpha$. Eliminating time from the above equations one obtains

$$\frac{E_x^2}{A_\mathrm{o}^2} + \frac{E_y^2}{A_\mathrm{e}^2} - 2\frac{E_x E_y}{A_\mathrm{o}A_\mathrm{e}}\cos\delta = \sin^2\delta, \tag{2}$$

which is the equation of an ellipse.

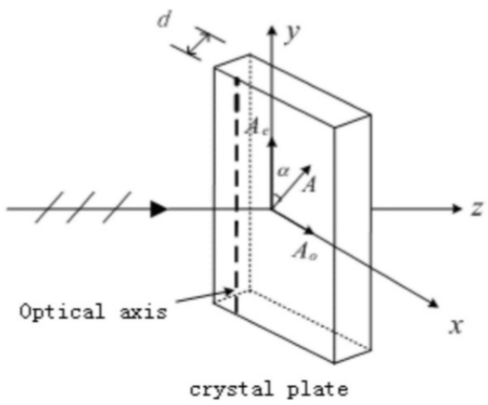


Figure 2: Linearly polarized light passing through a waveplate.

When the thickness of the plate changes, the optical path difference changes as well. Some cases of particular interest, are discussed below:

- ► If $\Delta = k\lambda$, where $k = 0, 1, 2, ...$, the phase difference $\delta = 0$, and Eq. (2) reduces to

  $$E_y = \frac{A_\mathrm{e}}{A_\mathrm{o}}E_x,$$

  which is a linear equation. Hence the transmitted light is linearly polarized with the oscillation direction remaining unchanged. A waveplate that satisfies this condition is called a *full-wave* plate. The light goes through a full-wave plate without changing its polarization state.

- ► If $\Delta = (2k+1)\lambda/2$, where $k = 0, 1, 2, ...$, the phase difference $\delta = \pi$, and Eq. (2) simplifies to

  $$E_y = -\frac{A_\mathrm{e}}{A_\mathrm{o}}E_x.$$

  The transmitted light is also linearly polarized with the polarization axis rotated by the angle of 2α. A waveplate that satisfies the condition is called *1/2-wave plate* or *half-wave plate*. When a polarized light passes through a half-wave plate, its polarization axis gets rotated by an angle 2α. If $\alpha = \pi/4$, then the polarization axis of the transmitted light is perpendicular to that of the incident light.

- Finally, if $\Delta = (2k+1)\lambda/4$, where $k = 0, 1, 2, ...$, the phase difference $\delta = \pm\pi/2$, and Eq. (2) transforms into

$$\frac{E_x^2}{A_\mathrm{o}^2} \pm \frac{E_y^2}{A_\mathrm{e}^2} = 1.$$

  The transmitted light is elliptically polarized with a waveplate that satisfies the above condition is called a *1/4-wave plate* or a *quarter-waveplate* and is an important optical element in many polarization experiments.

If $A_\mathrm{e} = A_\mathrm{o} = A$, then $E_x^2 + E_y^2 = A^2$, and the transmitted light is circularly polarized. Since the amplitudes of the *o*-wave and the *e*-wave are both functions of α, the polarization state after passing through a 1/4-wave plate will vary, depending on the angle:

- if $\alpha = 0$, the transmitted light is linearly polarized with the polarization axis parallel to the optical axis of the 1/4-wave plate;
- if $\alpha = \pi/2$, the transmitted light is linearly polarized with the polarization axis perpendicular to the optical axis of the 1/4-wave plate;
- if $\alpha = \pi/4$, the transmitted light is circularly polarized;
- otherwise, the transmitted light is elliptically polarized.

2 Experimental Setup

The experimental setup in this lab consists of a laser, a silicon photo-cell, a UT51 digital universal meter, two polarizers, a 1/2-wave plate, a 1/4-wave plate.

The precision of the devices is shown in Table 1.

Instrument	Measurement Quantity	Uncertainty
Scale on the element	Angle θ	$\pm 2°$
Universal meter	Current I	$\pm 0.001 \mu A$ or $\pm 0.01 \mu A$

Table 1: Precision of the measurement instruments.

3 Measurement Procedure

3.1 Apparatus Adjustment

1. Adjust the laser and the photo-cell so that the light can pass through the ϕ 6.0 aperture
2. Fix the laser at one of the ends of the bench and place the glass sheet and lens in front of it. Adjust the position so that the light passes through the center of the lens. Make sure that the light spot on the lens won't change significantly if the glass sheet moves along the bench.
3. Remove the glass sheet. Set the digital universal meter in the appropriate mode and range.

3.2 Demonstration of Malus' Law

1. Place an analyzer between the polarizer and the photo-cell as is shown in Figure 3.
2. Adjust the angle of the analyzer until the electric current reaches its minimum. This position is considered to be $\theta = 90°$.
3. Rotate the analyzer from 90° to 0° and measure the corresponding current I every 5°.
4. Plot the graph I/I_0 vs. $\cos^2\theta$ and perform linear fitting.

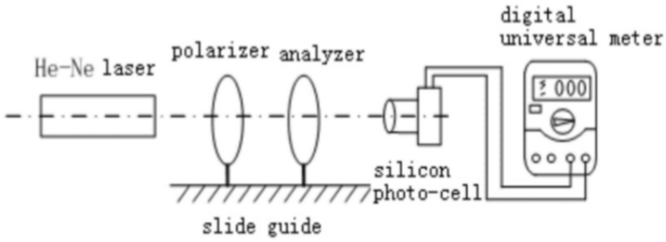


Figure 3: Experimental setup for a demonstration of Malus' law.

3.3 Linearly Polarized Light and the Half-wave Plate

1. Place the 1/2-wave plate between the polarizer and the analyzer.
2. Rotate the 1/2-wave plate to make the light extinction appear again and set this position as the initial position.
3. Rotate the 1/2-wave plate for $\alpha = 10°$ from the initial position and rotate the analyzer to make the light extinction appear again, record the angle of rotation $\Delta\theta$. Repeat this operation for 8 times, where $\Delta\theta$ becomes 90° at the end.
4. Plot the graph $\Delta\theta$ vs. θ.

3.4 Circularly and Elliptically Polarized Light and the 1/4-wave Plate

1. Maintain the previous setup. Replace the 1/2-wave plate with a 1/4-wave plate and rotate it to make the light extinction appear and set this position as the initial position. At this point $\alpha = 0°$. Then rotate the 1/4-wave plate and observe the change in the light intensity.
2. Rotate the analyzer for 360° for every 10°, and record the light intensity indicated by the current I in the table.
3. Rotate the 1/4-wave plate for 20°, repeat Step 2.
4. Rotate the 1/4-wave plate for 45°, repeat Step 2.
5. Rotate the 1/4-wave plate for 70°. Rotate the analyzer and record its position and the magnitude of the current when the light intensity reaches a maximum.
6. Plot the curves between the rotation angle of the analyzer and the light amplitude in polar coordinates. Normalize the amplitude by its maximum value. Mark the position recorded in Step 5 and compare it with the data recorded.
7. Compare the data with the circular polarization, plot a linear fit to the data when the angle is 45°.

4 Result

4.1 Demonstration of Malus' Law

In this part, the current corresponding to different angles of analyzer are measured. And the measurement data are presented in Table 2.

Then, corresponding $\cos^2\theta$ and I/I_0 can be calculated. Take the first set of data as an calculation example.

$$\cos^2\theta = \cos^2(0 \times \frac{\pi}{180}) = 1 \pm 0,$$

$$\frac{I}{I_0} = \frac{1.037}{1.037} = 1.0000 \pm 0.0013.$$

The rest of the results are shown in Table 3.

Maximum Electric Current I_0: $2.00 \pm\ 0.01[\mu A]$	
θ [°]	$I[\mu A]\ \pm\ 0.01[\mu A]$
0	2.00
5	1.97
10	1.97
15	1.92
20	1.84
25	1.73
30	1.60
35	1.46
40	1.33
45	1.18
50	1.01
55	0.82
60	0.69
65	0.48
70	0.34
75	0.23
80	0.11
85	0.04
90	0.01

Table 2: Measurement data for Malus' law demonstration.

$cos^2(\theta)$	$\mu_{cos^2(\theta)}$	I/I_0	μ_{I/I_0}
1	0	1.000	0.007
0.992	0.006	0.985	0.007
0.970	0.012	0.985	0.007
0.933	0.017	0.960	0.007
0.88	0.02	0.920	0.007
0.82	0.03	0.865	0.007
0.75	0.03	0.800	0.006
0.67	0.03	0.730	0.006
0.59	0.03	0.665	0.006
0.50	0.03	0.590	0.006
0.41	0.03	0.505	0.006
0.33	0.03	0.410	0.005
0.25	0.03	0.345	0.005
0.18	0.03	0.240	0.005
0.12	0.02	0.170	0.005
0.067	0.017	0.115	0.005
0.030	0.012	0.055	0.005
0.008	0.006	0.020	0.005
0	0	0.005	0.005

Table 3: Result for $\cos^2\theta$ and I/I_0.

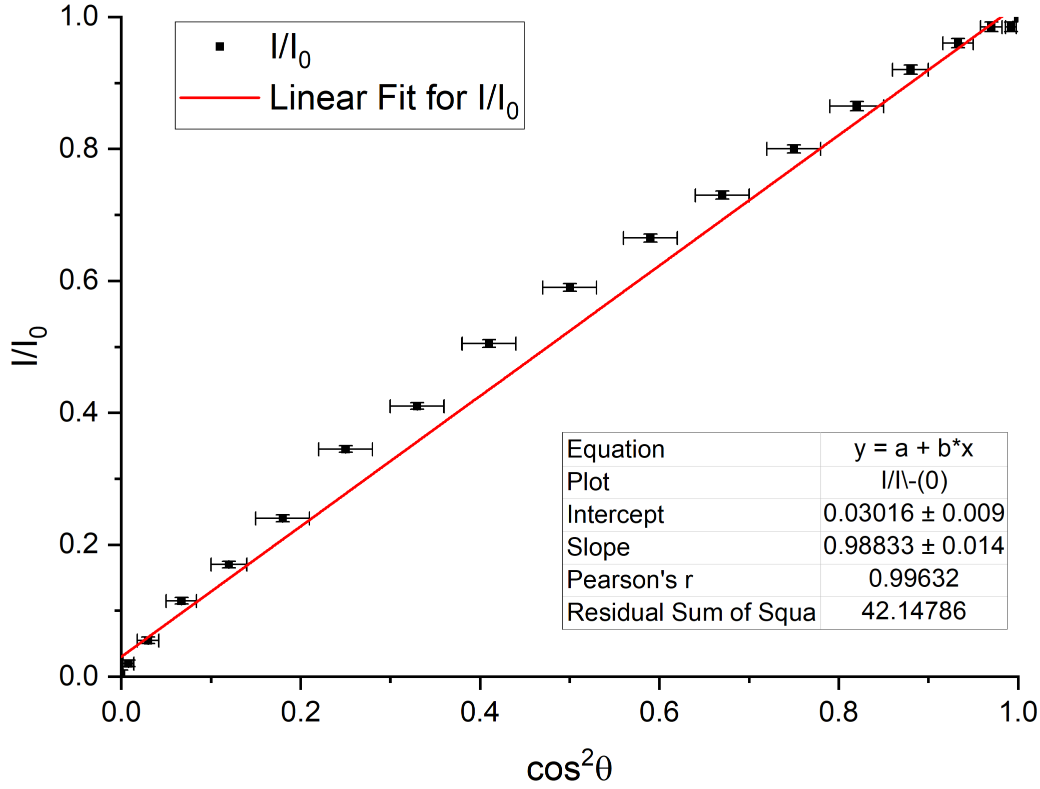


Figure 4: Linear fit for I/I_0 v.s $cos^2\theta$

Then, a linear fit of the plot I/I_0 vs. $\cos^2\theta$ are performed (Figure 4). The slope of the linear fitting is 0.988 ± 0.014. And the Pearson's r is 0.996, which is very close to 1.

4.2 Linearly Polarized Light and the Half-wave Plate

In this part, the rotation angles of the 1/2 plate and the analyzer are measured. The measurement data are presented in Table 4.

Rotation angle of the 1/2-wave plate $[°] \pm 2[°]$	Rotation angle of the analyzer $[°] \pm 2[°]$
initial	0
10	18
20	36
30	56
40	74
50	92
60	112
70	132
80	152
90	172

Table 4: Measurement data for the 1/2-wave plate.

To find the relation between $\Delta\theta$ and θ, the data are plotted in Figure 5 and linear fit is performed. The slope of the linear fit is 1.91 ± 0.05 with Pearson's r=0.999.

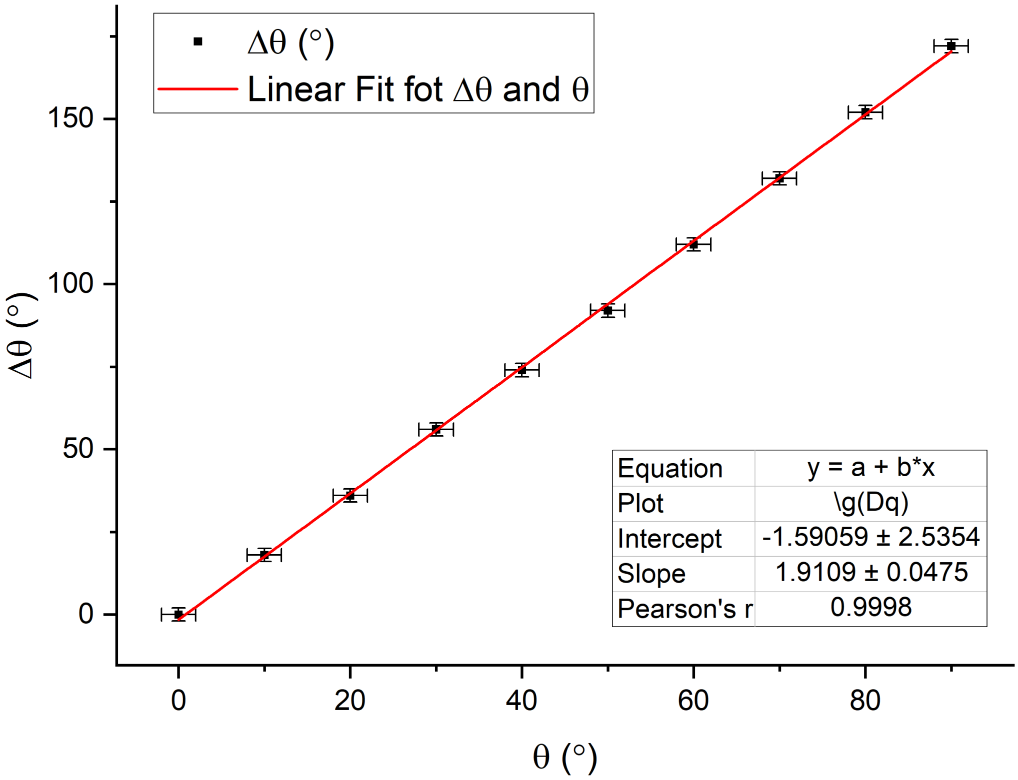


Figure 5: Linear fit of $\Delta\theta$ vs. θ.

1. When the half-wave plate rotates for 360°, 4 times of light extinction are observed.
2. When the analyzer rotates for 360°, 2 times of light extinction during the analyzer rotating 360°.
3. Explanation: after the light passes through the 1/2-wave plate, the polarization axis is rotated by twice of the origin angle.

4.3 Circularly and Elliptically Polarized Light and the 1/4-wave Plate

4.3.1 Rotation Angle: 0°

The measurement data for 0° rotation angle of 1/4-wave plate are presented in Table 5.

Rotation angle of 1/4-wave plate: 0°			
Maximum Electric Current I_0		1.50 ± 0.01 [μA]	
θ [°] ± 2[°]	I [μA] ± 0.01 [μA]	θ [°] ± 2[°]	I [μA] ± 0.01 [μA]
0	0.02	180	0.02
10	0.07	190	0.08
20	0.22	200	0.20
30	0.43	210	0.44
40	0.69	220	0.69
50	0.93	230	0.94
60	1.15	240	1.10
70	1.33	250	1.31
80	1.38	260	1.43
90	1.42	270	1.50
100	1.35	280	1.39
110	1.19	290	1.25
120	0.99	300	1.06
130	0.82	310	0.85
140	0.57	320	0.60
150	0.34	330	0.35
160	0.17	340	0.15
170	0.05	350	0.02

Table 5: Measurement data for the 1/4-wave plate (rotation angle 0°).

As described in the procedure part, $\sqrt{I/I_0}$ is calculated. Take the first set of data as an example,

$$\sqrt{\frac{I}{I_0}} = \sqrt{\frac{0.02}{1.50}} = 0.12 \pm 0.08.$$

Perform similar calculations to each of the other sets of data and the results are presented in Table 6.

θ [°] ± 2[°]	$\sqrt{I/I_0}$	$\mu_{\sqrt{I/I_0}}$	θ [°] ± 2[°]	$\sqrt{I/I_0}$	$\mu_{\sqrt{I/I_0}}$
0	0.12	0.03	180	0.12	0.03
10	0.22	0.015	190	0.231	0.014
20	0.383	0.009	200	0.365	0.009
30	0.535	0.006	210	0.542	0.006
40	0.678	0.005	220	0.678	0.005
50	0.787	0.005	230	0.792	0.005
60	0.876	0.005	240	0.856	0.005
70	0.942	0.005	250	0.935	0.005
80	0.959	0.005	260	0.976	0.005
90	0.973	0.005	270	1.000	0.005
100	0.949	0.005	280	0.963	0.005
110	0.891	0.005	290	0.913	0.005
120	0.812	0.005	300	0.841	0.005
130	0.739	0.005	310	0.753	0.005
140	0.616	0.006	320	0.632	0.006
150	0.476	0.007	330	0.483	0.007
160	0.337	0.010	340	0.316	0.011
170	0.18	0.02	350	0.12	0.03

Table 6: Results for $\sqrt{I/I_0}$ when rotation angle is 0°.

Then the relationship of $\sqrt{I/I_0}$ and θ are plotted in polar coordinate (Figure 6).

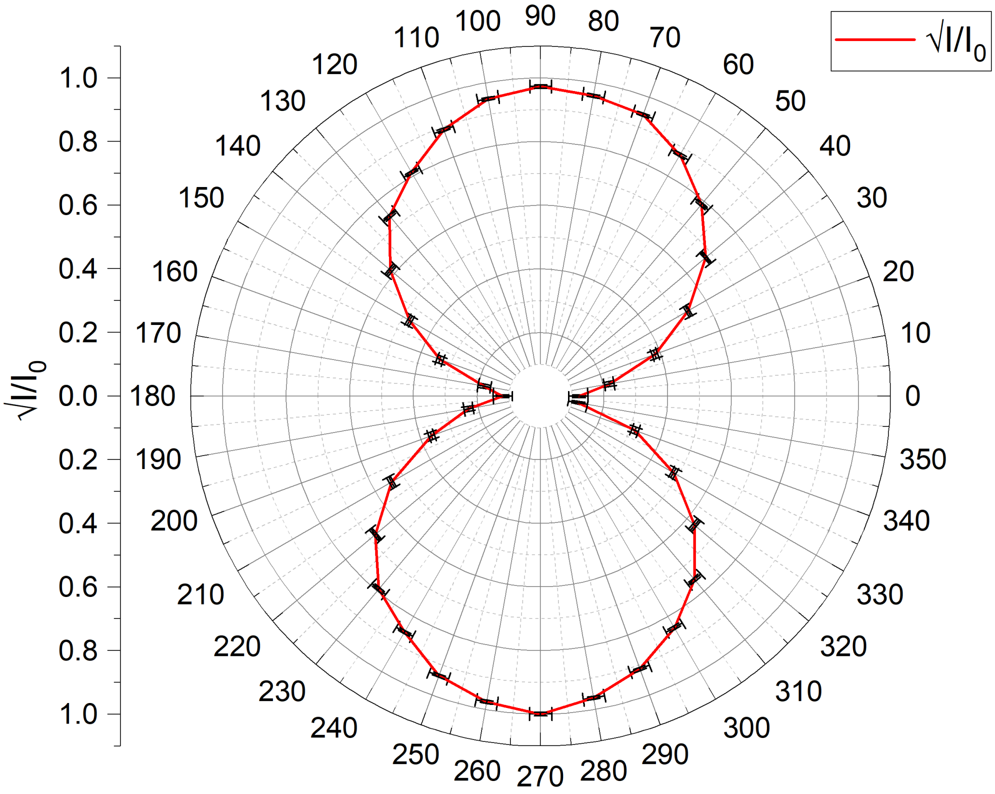


Figure 6: $\sqrt{I/I_0}$ vs. θ relation in polar coordinate when rotation angle is $0°$.

4.3.2 Rotation Angle: 20°

The measurement data for 20° rotation angle of 1/4-wave plate are shown in Table 7.

Rotation angle of 1/4-wave plate: 20°			
Maximum Electric Current I_0		0.707 ± 0.001 [μA]	
θ [°] ± 2[°]	I [μA] ± 0.001 [μA]	θ [°] ± 2[°]	I [μA] ± 0.001 [μA]
0	0.19	180	0.18
10	0.22	190	0.22
20	0.32	200	0.31
30	0.44	210	0.45
40	0.60	220	0.61
50	0.77	230	0.76
60	0.89	240	0.93
70	1.03	250	1.02
80	1.09	260	1.16
90	1.11	270	1.19
100	1.07	280	1.17
110	1.03	290	1.08
120	0.92	300	0.97
130	0.78	310	0.80
140	0.61	320	0.60
150	0.45	330	0.43
160	0.31	340	0.31
170	0.21	350	0.23

Table 7: Measurement data for the 1/4-wave plate (rotation angle 20°).

Similarly, $\sqrt{I/I_0}$ is calculated and the results are presented in Table 8.

θ [°] ± 2[°]	$\sqrt{I/I_0}$	$\mu_{\sqrt{I/I_0}}$	θ [°] ± 2[°]	$\sqrt{I/I_0}$	$\mu_{\sqrt{I/I_0}}$
0	0.400	0.009	180	0.389	0.009
10	0.430	0.008	190	0.430	0.008
20	0.519	0.005	200	0.510	0.005
30	0.608	0.004	210	0.615	0.004
40	0.710	0.003	220	0.716	0.003
50	0.804	0.002	230	0.799	0.002
60	0.865	0.002	240	0.884	0.002
70	0.930	0.002	250	0.926	0.002
80	0.957	0.002	260	0.987	0.002
90	0.966	0.002	270	1.000	0.002
100	0.948	0.002	280	0.992	0.002
110	0.930	0.002	290	0.953	0.002
120	0.879	0.002	300	0.903	0.002
130	0.810	0.002	310	0.820	0.002
140	0.716	0.003	320	0.710	0.003
150	0.615	0.004	330	0.601	0.004
160	0.510	0.005	340	0.510	0.005
170	0.420	0.008	350	0.440	0.007

Table 8: Results for $\sqrt{I/I_0}$ when rotation angle is 20°.

Then the relationship of $\sqrt{I/I_0}$ and θ are plotted in polar coordinate (Figure 7).

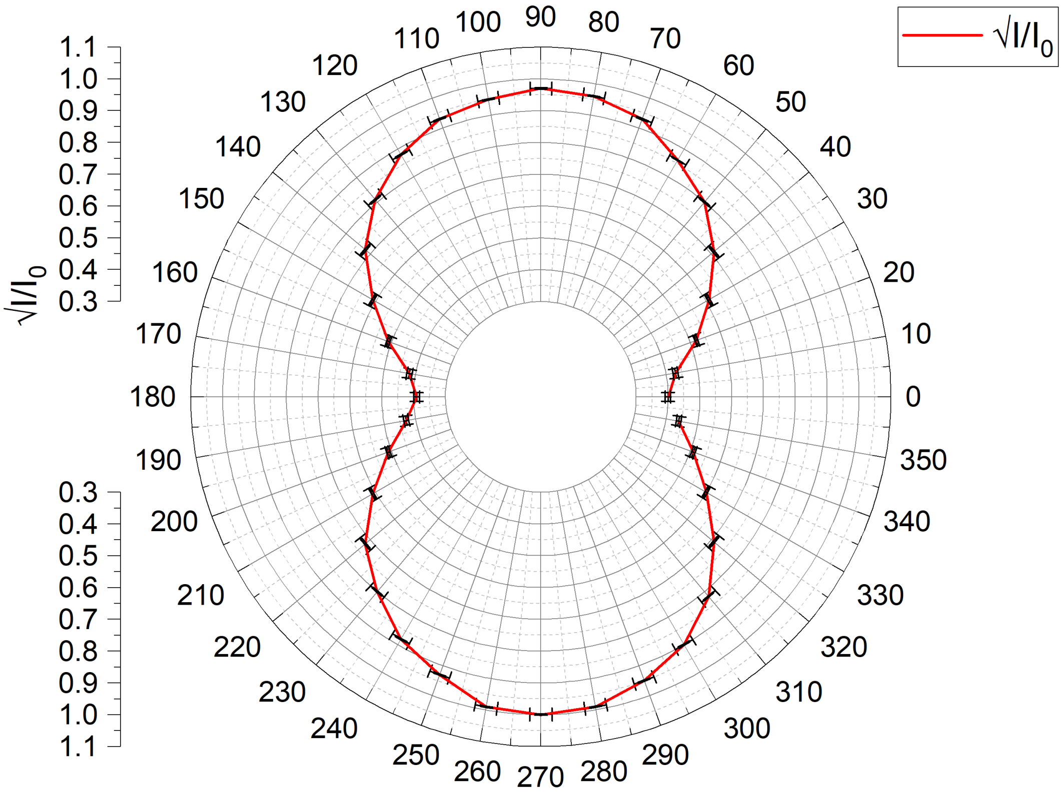


Figure 7: $\sqrt{I/I_0}$ vs. θ relation in polar coordinate when rotation angle is 20°.

4.3.3 Rotation Angle: 45°

The measurement data for 45° rotation angle of 1/4-wave plate are shown in Table 9.

Rotation angle of 1/4-wave plate: 45°			
Maximum Electric Current I_0		0.395 ± 0.001 [μA]	
θ [°] ± 2[°]	I [μA] ± 0.001 [μA]	θ [°] ± 2[°]	I [μA] ± 0.001 [μA]
0	0.566	180	0.553
10	0.572	190	0.606
20	0.568	200	0.600
30	0.573	210	0.609
40	0.596	220	0.617
50	0.624	230	0.646
60	0.614	240	0.648
70	0.627	250	0.648
80	0.664	260	0.650
90	0.664	270	0.666
100	0.646	280	0.680
110	0.624	290	0.672
120	0.633	300	0.673
130	0.635	310	0.645
140	0.654	320	0.620
150	0.646	330	0.605
160	0.627	340	0.587
170	0.602	350	0.576

Table 9: Measurement data for the 1/4-wave plate (rotation angle 45°).

Similarly, $\sqrt{I/I_0}$ is calculated and the results are presented in Table 10.

θ [°] ± 2[°]	$\sqrt{I/I_0}$	$\mu_{\sqrt{I/I_0}}$	θ [°] ± 2[°]	$\sqrt{I/I_0}$	$\mu_{\sqrt{I/I_0}}$
0	0.6897	0.0003	180	0.6817	0.0003
10	0.6933	0.0003	190	0.7136	0.0003
20	0.6909	0.0003	200	0.7101	0.0003
30	0.6939	0.0003	210	0.7154	0.0003
40	0.7077	0.0003	220	0.7201	0.0003
50	0.7241	0.0003	230	0.7368	0.0003
60	0.7183	0.0003	240	0.7379	0.0003
70	0.7259	0.0003	250	0.7379	0.0003
80	0.7470	0.0003	260	0.7391	0.0003
90	0.7470	0.0003	270	0.7481	0.0003
100	0.7368	0.0003	280	0.7559	0.0003
110	0.7241	0.0003	290	0.7515	0.0003
120	0.7293	0.0003	300	0.7520	0.0003
130	0.7305	0.0003	310	0.7362	0.0003
140	0.7413	0.0003	320	0.7218	0.0003
150	0.7368	0.0003	330	0.7130	0.0003
160	0.7259	0.0003	340	0.7023	0.0003
170	0.7113	0.0003	350	0.6957	0.0003

Table 10: Results for $\sqrt{I/I_0}$ when rotation angle is 45°.

Then the relationship of $\sqrt{I/I_0}$ and θ are plotted in polar coordinate (Figure 8).

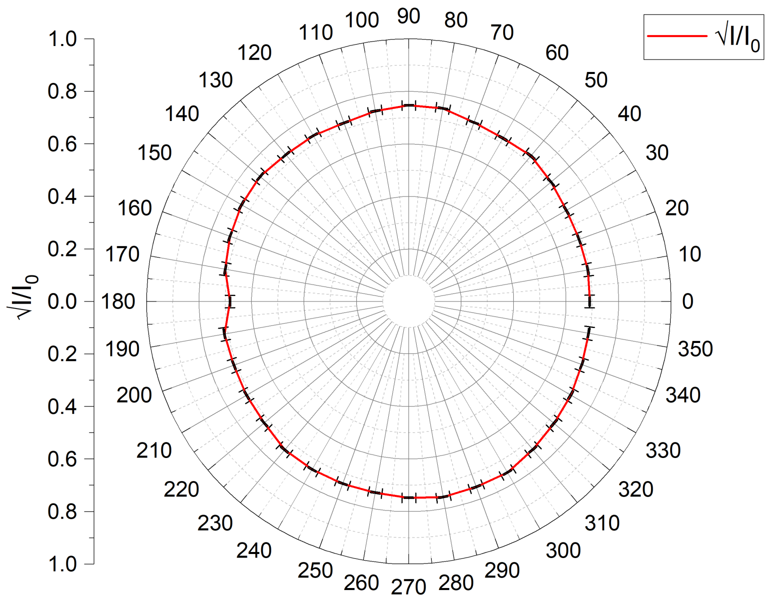


Figure 8: $\sqrt{I/I_0}$ vs. θ relation in polar coordinate when rotation angle is 45°.

The data is also plotted in Cartesian coordinate and linear fit is performed (Figure 9). The slope of the linear fitting is $6 \times 10^{-5} \pm 3 \times 10^{-5}$ with Pearson's r = 0.309.

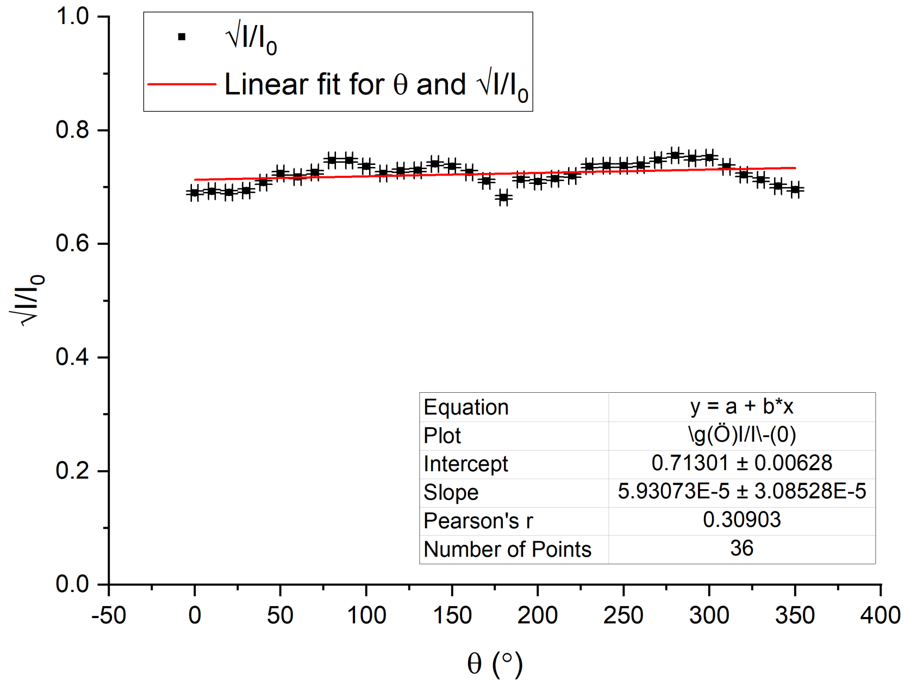


Figure 9: $\sqrt{I/I_0}$ vs. θ relation in the Cartesian coordinate.

4.3.4 Rotation Angle: 70°

When rotation angle is 70°, the data of maximum light intensity are presented in Table 11. The position of this point is marked in blue star in Figure 10. From the figure, it can be seen that the value of θ of maximum light intensity of 70° rotation angle is actually symmetric about the origin with that ($\theta = 270°$) of 20°.

Rotation angle of the 1/4-wave plate: 70°	
θ [°] ± 2[°]	90
I [μA] ± 0.001 [μA]	1.111

Table 11: Measurement data for the 1/4-wave plate (rotation angle 70°).

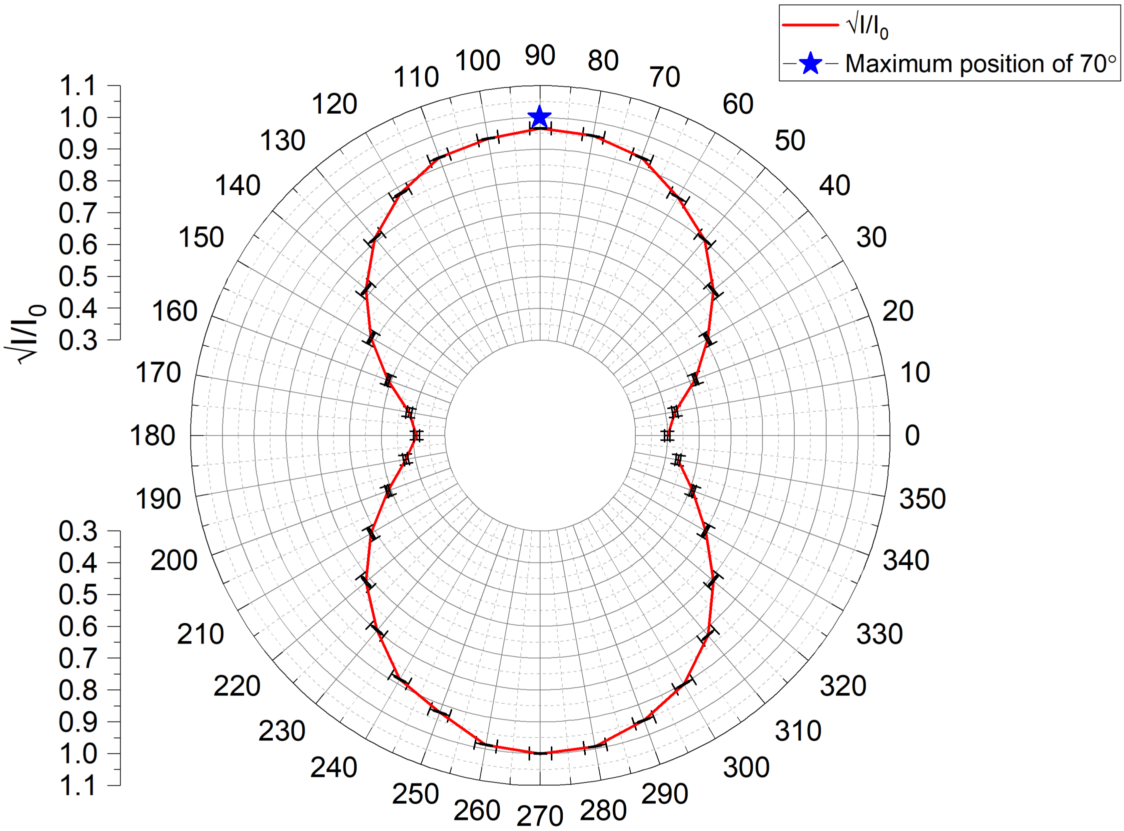


Figure 10: $\sqrt{I/I_0}$ vs. θ relation in polar coordinate when rotation angle is 20°, together with the maximum point of rotation angle 70°(marked in blue star).

5 Discussion and Conclusions

5.1 Demonstration of Malus' Law

The slope of the fitting for I/I_0 v.s. $cos^2\theta$ is 0.988 ± 0.014 and the Pearson's r is 0.996, which is very close to 1. This indicates a strong linear proportional relationship between the two values. Theoretically, by Eq. 1, the slope should be

$$\frac{I/I_0}{\cos^2\theta} = 1,$$

Therefore, the deviation from theoretical value is

$$u_r = \frac{1 - 0.988}{1} \times 100\% = 2\%.$$

Which is relatively small and can be used to verified Malus's Law.

5.2 Linearly Polarized Light and the Half-wave Plate

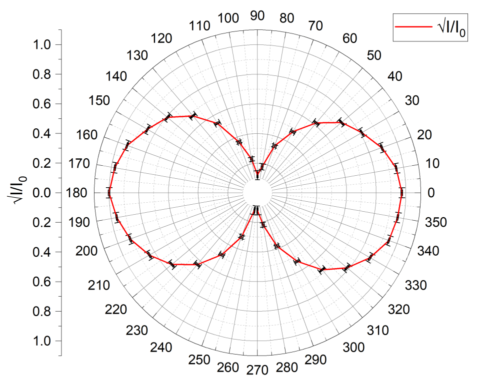
The slope of the linear fit for $\Delta\theta$ v.s. θ is 1.91 ± 0.05 and the Pearson's r is 0.999. Theoretically, the rotation angle of the polarization axis should be twice of the origin angle for a half-wave plate. Therefore the theoretical value of the slope of the linear fitting should be 2. The standard deviation is then

$$u_r = \frac{2 - 1.91}{2} \times 100\% = 5\%.$$

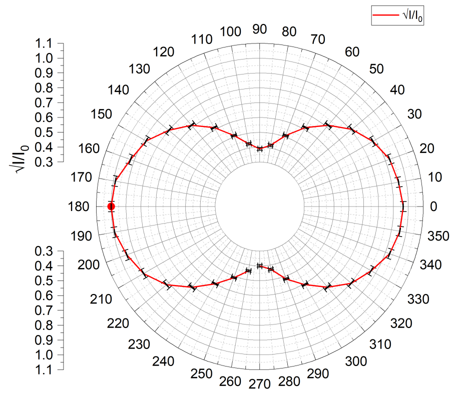
Which is relatively small and it can be said that the experimental result fits with the theoretical one.

5.3 Circularly and Elliptically Polarized Light and the 1/4-wave Plate

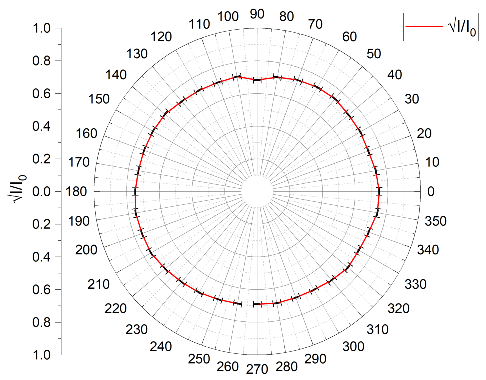
In this section, we mistakenly treated the extinction angle 90° as 0°. Therefore, all the curves should be rotated for 90° to correct the mistake. The corrected figures are shown below.



(a) Correction: $\sqrt{I/I_0}$ vs. θ relation in polar coordinate when rotation angle is 0°.



(b) Correction: $\sqrt{I/I_0}$ vs. θ relation in polar coordinate when rotation angle is 20°.



(c) Correction: $\sqrt{I/I_0}$ vs. θ relation in the polar coordinate when rotation angle is 45°.

Figure 11: Corrected versions of the figure of $\sqrt{I/I_0}$ vs. θ

5.3.1 Rotation Angle: 0°

From Table 5 and Figure 11(a), it can be found that the maximum light intensity is at about $\theta = 0°$. This suggests that polarizing axis is in parallel with the optical axis of the plate.

5.3.2 Rotation Angle: 20°

From Table 7 and Figure 11(b), it can be found that the maximum light intensity is still at about $\theta = 0°$. However, according to [1], the angle of maximum light intensity should be 20°, which has relatively big deviation from the experimental one. Here are the possible reasons for the error:

1. Since the rotation angle is only determined by naked eye, and the tick marks on the analyzer are thin and small, it is difficult to precisely rotate exactly 10° every time. According to the provided uncertainty of analyzer ($\pm 2°$), it is reasonable that after many times of rotations, small errors accumulate to cause this big error.
2. The reading on the current meter is always unstable, it is hard to determine when the current reaches a certain value, causing mistaken judgement of the initial angle.

5.3.3 Rotation Angle: 45°

From Table 9 and Figure 11(c), it can be found that the value of $\sqrt{I/I_0}$ hardly changes when θ varies.

In addition, the slope of the linear fit for $\sqrt{I/I_0}$ vs. θ in Figure 9 is only $6 \times 10^{-5} \pm 3 \times 10^{-5}$ and the Pearson's coefficient is only 0.309, which is considerably small. This indicates that there may not be an linear relation between $\sqrt{I/I_0}$ and θ, suggesting that the amplitude is possibly a constant. According to the definition of *circularly polarized*, it can be preliminary concluded that when the rotation angle is 45°, the light is circularly polarized.

5.3.4 Rotation Angle: 70°

From Table 11 and Figure 10, it can be found that the light intensity reaches maximum at $\theta = 90°$. It is symmetric about the origin with that of 20° rotation angle. Theoretically, the angle of maximum light intensity should be 70°. The reasons for this error are similar to that of rotation angle of 20° (see 5.3.2).

5.4 Causes for errors and uncertainties

Other possible causes for errors and uncertainties are listed below:

1. This experiment is performed in dark environment, so in order to adjust the apparatus, torch is needed. However, the light from the torch may possible interfere the light intensity detection.
2. On the other hand, the dark environment adds more errors to the readings by naked eyes.
3. The scale on the optical elements are not so precise, with uncertainty 2°, which results in a relatively large errors accumulatively.

5.5 Suggestions

Use digital scale to measure the angle of rotation. This not only minimizes the occasional errors but allows the experiment to be performed in a completely dark environment.

5.6 Conclusions

In this lab, the polarization phenomenon of light is studied and Malus' law is verified. The working mechanism of half- and quarter-wave plates is also studied. Although the results of circularly polarized light are not accurate due to mistaken operations, the general trend can be verified.

References

[1] VP241 Exercise 4: Polarization of Light, Department of Physics, Shanghai Jiaotong University.

A Measurement Uncertainty Analysis

A.1 Demonstration of Malus' Law

The uncertainty of $\cos^2\theta$ can be calculated as

$$\begin{aligned} u_{\cos^2\theta} &= \sqrt{(\frac{\partial\cos^2\theta}{\partial\theta}u_\theta)^2} \\ &= |-2u_\theta\cos\theta\sin\theta| \\ &= |u_\theta\sin 2\theta|, \end{aligned}$$

Here $u_\theta = 2[^\circ] = \pi/90$. Take $\theta = 0$ as an example,

$$u_{\cos^2\theta} = |\sin 2\theta u_\theta| = |\sin(2\times 5^\circ)\times\frac{\pi}{90}| = 0.006,$$

The uncertainty of I/I_0 is

$$\begin{aligned} u_{I/I_0} &= \sqrt{(\frac{\partial I/I_0}{\partial I}u_I)^2 + (\frac{\partial I/I_0}{\partial I_0}u_{I_0})^2} \\ &= \sqrt{(\frac{u_I}{I_0})^2 + (-\frac{I}{I_0^2}u_{I_0})^2}, \end{aligned}$$

where $u_I = u_{I_0} = 0.01$ [μA], $I_0 = 2.00\pm 0.01$ [μA]. Take the second set of data as an example,

$$u_{I/I_0} = \sqrt{(\frac{u_I}{I_0})^2 + (-\frac{I}{I_0^2}u_{I_0})^2} = \sqrt{(\frac{0.01}{2.00})^2 + (-\frac{1.97}{2.00^2}\times 0.01)^2} = 0.007,$$

The uncertainties results of $\cos^2\theta$ and I/I_0 are shown in Table 3 in the **Result** section.

A.2 Linearly Polarized Light and the Half-wave Plate

The uncertainty is the precision of the device, which is

$$u_\theta = 2^\circ.$$

A.3 Circularly and Elliptically Polarized Light and the 1/4-wave Plate

The uncertainty of $\sqrt{I/I_0}$ is

$$\begin{aligned} u_{\sqrt{I/I_0}} &= \sqrt{(\frac{\partial\sqrt{I/I_0}}{\partial I}u_I)^2 + (\frac{\partial\sqrt{I/I_0}}{\partial I_0}u_{I_0})^2} \\ &= \sqrt{\frac{1}{4II_0}u_I^2 + \frac{I}{4I_0^3}u_{I_0}^2}, \end{aligned}$$

where
$u_I = u_{I_0} = 0.01$ [μA], $I_0 = 1.50\pm 0.01$ [μA] for rotation angle of 0°,
$u_I = u_{I_0} = 0.01$ [μA], $I_0 = 1.19\pm 0.01$ [μA] for rotation angle of 20°,
$u_I = u_{I_0} = 0.001$ [μA], $I_0 = 0.680\pm 0.001$ [μA] for rotation angle of 45°.
Take the first set of data in rotation angle of 0° as an example,

$$u_{\sqrt{I/I_0}} = \sqrt{\frac{1}{4\times 0.02\times 1.50}\times 0.01^2 + \frac{0.02}{4\times 1.50^3}\times 0.01^2} = 0.03.$$

The uncertainties results of $\sqrt{I/I_0}$ are shown in Table 5, Table 7 and Table 9 in the **Result** part following the values of $\sqrt{I/I_0}$.

B Data Sheet

Please find the original data sheet attached at the end of the report.

UM-SJTU Physics Laboratory
Data Sheet (Exercise 4)

Name: 朱浩铭 Haoming Zhu Student ID: 520021910145

Name: 王新凯 Xinkai Wang Student ID: 520021910312

Group: 1 Date: 2021.11.19

NOTICE. Please remember to show the data sheet to your instructor before leaving the laboratory. The data sheet will not be accepted if the data are recorded with pencil or modified by correction fluid/tape. If a mistake is made in recording a datum item, cancel the wrong value by drawing a fine line through it, record the correct value legibly, and ask your instructor to confirm the correction. Please remember to take a record of the precision of the instruments used.

You are required to hand in the original data with your lab report, so please keep the data sheet properly.

Uncertainty of θ is [2]°.

Maximum Electric Current I_0		~~2.55~~ 2.00 ± 0.01 [μA]	
θ	I [μA] ± 0.01 [μA]	θ	I [μA] ± 0.01 [μA]
0°	2.00	50°	1.01
5°	1.97	55°	0.82
10°	1.97	60°	0.69
15°	1.92	65°	0.48
20°	1.84	70°	0.34
25°	1.73	75°	0.23
30°	1.60	80°	0.11
35°	~~1.49~~ 1.46	85°	0.04
40°	1.33	90°	0.01
45°	1.18		

Table 1. Measurement data Malus' law demonstration.

Instructor's signature: [illegible]

Rotation angle of the 1/2–wave plate	Rotation angle of the analyzer [°] ±[2]°
initial	0
10°	18
20°	~~18~~ 36
30°	~~20~~ 56
40°	~~18~~ 74
50°	~~18~~ 92
60°	~~20~~ 112
70°	~~20~~ 132
80°	~~20~~ 152
90°	~~20~~ 172

Table 2. Measurement data for the 1/2–wave plate.

Instructor's signature: ________

Rotation angle of 1/4–wave plate: 0°			
Maximum Electric Current I_0		1.50 ± 0.01 [μA]	
θ	I [μA] $\pm$ 0.01 [μA]	θ	I [μA] $\pm$ 0.01 [μA]
0°	0.02	180°	0.02
10°	0.07	190°	0.08
20°	0.22	200°	0.20
30°	0.43	210°	0.44
40°	0.69	220°	0.69
50°	0.93	230°	0.94
60°	1.15	240°	1.10
70°	1.33	250°	1.31
80°	1.38	260°	1.43
90°	1.42	270°	1.50
100°	1.35	280°	1.39
110°	1.19	290°	1.25
120°	0.99	300°	1.06
130°	0.82	310°	0.85
140°	0.57	320°	0.60
150°	0.34	330°	0.35
160°	0.17	340°	0.15
170°	0.05	350°	0.02

Table 3. Measurement data for the 1/4–wave plate (rotation angle 0°).

Instructor's signature: ______ Yuan

Rotation angle of the 1/4–wave plate: 20°			
Maximum Electric Current I_0		1.19 ± 0.01 [uA]	
θ	I [uA] ± 0.01 [uA]	θ	I [uA] ± 0.01 [uA]
0°	0.19	180°	0.18
10°	0.22	190°	0.22
20°	0.32	200°	0.31
30°	0.44	210°	0.45
40°	0.60	220°	0.61
50°	0.77	230°	0.76
60°	0.89	240°	0.93
70°	1.03	250°	1.02
80°	1.09	260°	1.16
90°	1.11	270°	1.19
100°	1.07	280°	1.17
110°	1.03	290°	1.08
120°	0.92	300°	0.97
130°	0.78	310°	0.80
140°	0.61	320°	0.60
150°	0.45	330°	0.43
160°	0.31	340°	0.31
170°	0.21	350°	0.23

Table 4. Measurement data for the 1/4–wave plate (rotation angle 20°).

Instructor's signature: Yuan

Rotation angle of the 1/4–wave plate: 45°			
Maximum Electric Current I_0		0.680 ± 0.001 [uA]	
θ	I [uA] ± 0.001 [uA]	θ	I [uA] ± 0.001 [uA]
0°	0.566	180°	0.553
10°	0.572	190°	0.606
20°	0.568	200°	0.600
30°	0.573	210°	0.609
40°	0.596	220°	0.617
50°	0.624	230°	0.646
60°	0.614	240°	0.648
70°	0.627	250°	~~0.650~~ 0.648
80°	0.664	260°	~~0.666~~ 0.650
90°	0.664	270°	0.666
100°	0.646	280°	0.680
110°	0.624	290°	0.672
120°	0.633	300°	0.613
130°	0.635	310°	0.645
140°	0.654	320°	0.620
150°	0.646	330°	0.605
160°	0.627	340°	0.587
170°	0.602	350°	0.576

Table 5. Measurement data for the 1/4–wave plate (rotation angle 45°).

Rotation angle of the 1/4–wave plate: 70°	
θ [°] ± [0.7]°	90
I [uA] ± 0.001 [uA]	1.111

Table 6. Measurement data for the 1/4–wave plate (rotation angle 70°).

Instructor's signature:

rev. 2