Problem set 3

Linear algebra 6

September 25, 2025

1. Suppose the we have square matrices B and C are invertible, find inverse of the matrix in block form.
$$\begin{bmatrix} A & B \\ C & 0 \end{bmatrix}.$$

2. Use block multiplication rules for matrices to find the inverses of following matrices:
   (a) $\begin{bmatrix} 1 & -1 & 0 & 0 \\ 2 & 1 & 0 & 0 \\ 0 & 0 & 2 & 5 \\ 0 & 0 & 1 & 3 \end{bmatrix}$
   (b) $\begin{bmatrix} 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 5 \\ 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix}$
   (c) $\begin{bmatrix} 1 & 0 & \cdots & 0 & a_n \\ a_1 & 0 & \cdots & 0 & 0 \\ 0 & a_2 & \cdots & 0 & 0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & a_{n-1} & 0 \end{bmatrix}$, if all a_i's are non-zero.

3. Show that any non-singular $n \times n$ matrix A that admits a LDU factorization can be written as a product of a lower triangular matrix with a symmetric matrix. (Hints: start with the LU factorization $A = LDU$)

4. Let u, v be vectors in $\mathbb{R}^n$, assume that $I - uv^T$ is invertible with inverse $I + \lambda uv^T$:
   (a) find λ in terms of u and v;
   (b) Suppose A are $A - uv^T$ are invertible, express the inverse of $A - uv^T$ in terms of A^{-1}, u and v;

(c) use the above to find the inverse of $\begin{bmatrix} 3 & 1 & 1 & 1 \\ 1 & 2 & 1 & 1 \\ 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 2 \end{bmatrix}$.

5. Let $P_{\leq 2}$ to be the vector space consisting of polynomials $f(x) = a_2x^2 + a_1x + a_0$ with degree less than or equal to 2.

   (a) Show that $W = \{f(x) \in P_{\leq 2} \mid f(1) = 0,\ f'(1) = 0\}$ is a subspace of $P_{\leq 2}$ (Here $f'(x)$ is the derivative of $f(x)$).

   (b) Find all possible solutions $f(x)$ of the system of equations:

$$\begin{aligned} f(1) &= 1 \\ f'(1) &= 1. \end{aligned}$$