NOTES AND CORRESPONDENCE

The Aerodynamic Roughness of the Sea Surface[1]

E. B. Kraus
Woods Hole Oceanographic Institution, Woods Hole, Mass.

29 October 1965 and 16 February 1966

1. Introduction

The question whether the sea surface is aerodynamically smooth or rough has been the subject of some controversy. Different authors found contradictory flow characteristics. The various results have been discussed in a recent book by Roll (1965), who calls the situation "perplexing".

The difficulty may be a matter of definition and of the fundamental difference between a solid boundary and a fluid interface. It is argued below that in flow of air over water, boundary layer separation can occur only above a critical wind speed of about 6–8 m sec^{-1}. This is due to viscosity and the propagation of waves. It is not related to the Kelvin-Helmholtz instability criterion which once was used to predict initial wave formation and a change in the flow regime at wind speeds in excess of 6.7 m sec^{-1}.

The textbooks by Roll (1965) and Kinsman (1965) list data and theories with a bearing on the following argument. It therefore seemed unnecessary to duplicate detailed references in the present note.

2. The mean translation of a viscous fluid close to a boundary

A viscous sub-layer can separate from an undulating surface only if the relative velocities are large enough for inertial effects to become appreciable. This will not be the case if the Reynolds number is smaller than a critical number

$$R_0 = \frac{du(d)}{\nu}, \tag{1}$$

where d is the sub-layer thickness, ν, the viscosity and $u(z)$, the velocity at distance z from the boundary.

The time averaged velocity at a level $z<d$ is

$$u(z) = \frac{u_*^2 z}{\nu}, \tag{2}$$

where u_* is the friction velocity.

It is possible also to look at the thickness d in a different way. Beyond d the flow becomes turbulent. If density differences are present, they usually become significant only at some appreciable height. At distances slightly larger than d, the flow can be characterized adequately by an eddy viscosity

$$K(z) = \kappa u_* z,$$

and a velocity distribution

$$\frac{u(z)}{u_*} = \frac{1}{\kappa} \ln z + \text{constant},$$

where κ is von Kármán's constant.

With approach to the boundary, K decreases and the flow must become viscous where $K \leqslant \nu$. This occurs on the average at the distance d. Thus, approximately,

$$\nu = K(d) = \kappa u_* d, \tag{3}$$

$$\frac{u(d)}{u_*} = \frac{1}{\kappa} \ln d + \text{constant}. \tag{4}$$

From Eqs. (1) through (4) it follows that

$$R_c = \kappa^{-2}, \tag{5}$$

$$d = \frac{\nu}{\kappa u_*}, \tag{6}$$

$$u(d) = \frac{u_*}{\kappa}, \tag{7}$$

$$\text{constant} = \frac{1}{\kappa}\left(1 - \ln \frac{\nu}{\kappa u_*}\right). \tag{8}$$

[1] Contribution No. 1723 from the Woods Hole Oceanographic Institution. This work has been supported by the U. S. Army Electronics Command and the National Science Foundation under Grant GP 4711.

The average translation of the viscous sub-layer is, therefore,

$$\bar{u}_\nu = \frac{u(d)}{2} = \frac{u_*}{2\kappa}. \tag{9}$$

3. Boundary layer separation at a fluid interface

An expression for the minimum velocity c_* of a surface wave at the boundary of air and still water has been derived by Kelvin. A function of the surface tension Γ (73 dynes cm^{-1}) and of the air and water densities ρ_a, ρ_w, it is given by

$$c_*^4 = \frac{4g\Gamma(\rho_w - \rho_a)}{\rho_w + \rho_a} \approx \frac{4g\Gamma}{\rho_w} = (23 \text{ cm sec}^{-1})^4.$$

Waves moving with that velocity have a length

$$\lambda_* \approx 2\pi\left(\frac{\Gamma}{\rho_w g}\right)^{\frac{1}{2}} = (1.7 \text{ cm}).$$

The average time required by particles in the viscous sub-layer to move along the surface of an obstacle to the point of separation is inversely proportional to $\bar{u}_\nu$. If the obstacle is a wave on the water surface, it will move the same distance in a time inversely proportional to the wave velocity c. From (9) it follows that the point of separation cannot be reached by the bulk of the layer if

$$u_* < 2\kappa c_* = (18 \text{ cm sec}^{-1}). \tag{10}$$

A friction velocity u_* of 18 cm sec^{-1} corresponds frequently to a wind at anemometer level of about 6 m sec^{-1}. It follows that boundary layer separation is unlikely to occur over water at lower wind speeds. At higher wind speeds, that is for $u_* > 18$ cm sec^{-1}, the boundary layer may separate occasionally over gravity waves that are longer than 1.7 cm, or even over some capillary waves. The surface should become increasingly rough as the wind increases and as the range of waves which may cause separation, broadens.

The breaking of waves and the formation of white-caps is due to interaction between waves. It is not necessarily the direct result of wind stress and can occur at quite low wind speeds. At high wind velocities, however, when boundary layer separation occurs, the resulting pressure differences may cause the mechanical disruption of wave crests and spindrift can be formed in this way.

It is not possible to assign a precise value to the critical wind speed at which the sea surface may become rough. The preceding argument has a statistical character. It reflects the effect of wave interactions on the velocity of the shorter waves. It does suggest, however, the existence of a mechanism for momentum transfer from air to water which increases with wind speeds above some critical value and which is zero for lower wind velocities.

4. The roughness of the sea surface

A rather rapid, not necessarily discontinuous, change in the flow characteristics of air over the sea surface at a critical wind velocity of about 6–8 m sec^{-1}, has been reported by Munk, Mandelbaum, Deacon and a number of other authors (Roll, 1965). It was explained initially by the Kelvin-Helmholtz instability criterion which deals with irrotational, potential flow and states that waves can only begin to form on a water surface at wind speeds in excess of

$$u_1 \approx \left(\frac{\rho_w}{\rho_a}\right)^{\frac{1}{2}} c_* = (6.7 \text{ m sec}^{-1}).$$

The agreement of this velocity value with the critical wind speed derived above from boundary layer theory is fortuitous. Potential theory predicts that waves could not form at all at lower wind speeds. In fact, they are observed to do so at wind speeds as low as 1 m sec^{-1} and an explanation of this has been given by Phillips (Kinsman, 1965).

The change in the air flow regime would be compatible with the "contradictory" results obtained by different investigators. In particular, the sea surface was considered to be aerodynamically rough by Charnock (Roll, 1965) with a roughness parameter

$$z_0 \propto \frac{u_*^2}{g}.$$

This implies an increase of roughness with wind velocity which is supported by the profile observations of Hay, Francis, Deacon and others. These observations were taken at anemometer wind velocities in excess of 5–8 m sec^{-1}.

On the other hand, Roll, Motzfeld, Portman, Bruce and others used profile data to obtain roughness parameters z_0 of the order of 0.01 cm or less. From Roll's summary (page 139) it can be seen that this group of observations was associated with friction velocities $u_* < 20$ cm sec^{-1}. The thickness of the viscous sub-layer, $d \approx \nu / u_*\kappa$ is then at least 0.02 cm which is larger than the quoted values of z_0 and, therefore, implies aerodynamically smooth flow. It follows that the sea surface tends to behave not unlike a smooth solid surface at wind velocities smaller than 6–8 m sec^{-1}.

The analogy is not complete because there are fundamental differences at all wind speeds between a rigid and a flexible or fluid interface. A traveling wave on a water surface will give rise to pressure perturbations in the air above which travel with the wave. In certain circumstances the pressure perturbation may further excite the boundary displacement by which it was produced in the first instance. The resulting exponential growth of wave energy was first discussed by Miles and has since been the object of many studies (Kinsman, 1965). Traveling waves have momentum which has been derived from the air flow above by this interaction. The associated drain of momentum from the wind has been discussed by Stewart (1961) and by Miles (1965).

4. Conclusions

The drag of the sea surface on the wind is associated with three principal processes:

1. A viscous drag which may be relatively important at low wind speeds.
2. A pressure or roughness drag associated with boundary layer separation at wind speeds above 6–8 m sec^{-1}. The part of the wave spectrum associated with boundary layer separation becomes wider with increasing wind speeds. The roughness length and the drag coefficient increase, therefore, with wind velocity above a critical value. The roughness will be associated at all times with relatively short waves. These can develop quickly. It can be expected, therefore, that the roughness drag will not be strongly dependent on wind fetch or duration.
3. A wave drag due to the dynamic instability of flow over a flexible surface. This drag depends on the whole of the locally existing wave spectrum. The resulting drag coefficient is a function not only of the wind speed but also of the fetch and wind duration. It only can become independent of these over a fully developed sea.

REFERENCES

Kinsman, B., 1965: *Wind Waves*. New Jersey, Prentice-Hall, Inc., 676 pp.

Miles, J. W., 1965: A note on the interaction between surface waves and wind profiles. *J. Fluid Mech.*, **22**, 823–827.

Roll, H. U., 1965: *Physics of the Marine Atmosphere*. New York, Academic Press Inc., 426 pp.

Stewart, R. W., 1961: The wave drag of wind over water. *J. Fluid Mech.*, **10**, 189–194.