

AI1103 : Assignment 2

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Download all python codes from

<https://github.com/BokkaRajaRaviKiranReddy/AI1103/tree/main/Assignment2/codes>

and latex codes from

<https://github.com/BokkaRajaRaviKiranReddy/AI1103/blob/main/Assignment2/Assignment2.tex>

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X is a random variable with uniform probability density function in the interval $[2, 10]$. For $Y = 2X - 6$, The conditional probability $P(Y \leq 7 | X \geq 5)$ (rounded off to three decimal places) is

SOLUTION

$$\Pr(Y \leq 7 | X \geq 5) = \frac{\Pr(Y \leq 7, X \geq 5)}{\Pr(X \geq 5)} \quad (54.1)$$

$$Y = 2X - 6 \leq 7$$

$$\Rightarrow 2X \leq 13$$

$$\Rightarrow X \leq 6.5$$

So, From Equation 54.1

$$\Pr(Y \leq 7 | X \geq 5) = \frac{\Pr(X \leq 6.5, X \geq 5)}{\Pr(X \geq 5)} \quad (54.2)$$

As $X \in [-2, 10]$ with uniform probability density function,
PDF of X is

$$f_X(x) = \begin{cases} \frac{1}{12} & \text{if } -2 \leq x \leq 10 \\ 0 & \text{otherwise} \end{cases} \quad (54.3)$$

CDF of X,

$$F_X(x) = \int_{-\infty}^x f_X(x) dx$$

$$F_X(x) = \begin{cases} 0 & \text{if } x \leq -2 \\ \frac{1}{12}(x+2) & \text{if } -2 \leq x \leq 10 \\ 1 & \text{if } x \geq 10 \end{cases}$$

So, From Equation 54.2

$$\begin{aligned} \Pr(Y \leq 7 | X \geq 5) &= \frac{\Pr(X \leq 6.5, X \geq 5)}{\Pr(X \geq 5, X \leq 10)} \\ &= \frac{F_X(6.5) - F_X(5)}{F_X(10) - F_X(5)} \\ &= \frac{\frac{8.5}{12} - \frac{7}{12}}{\frac{12}{12} - \frac{7}{12}} \\ &= 0.300 \end{aligned}$$

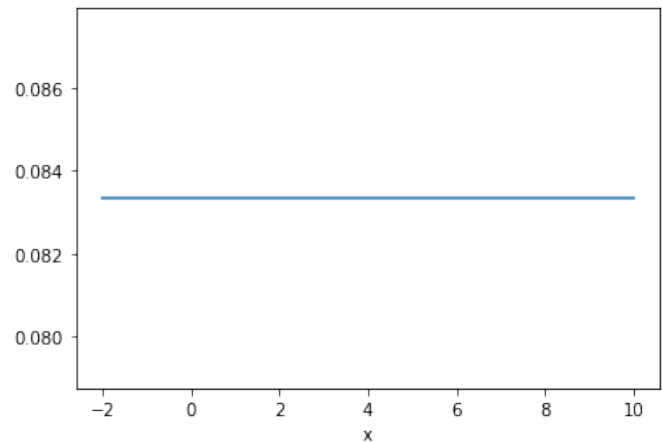


Fig. 1: PDF of X

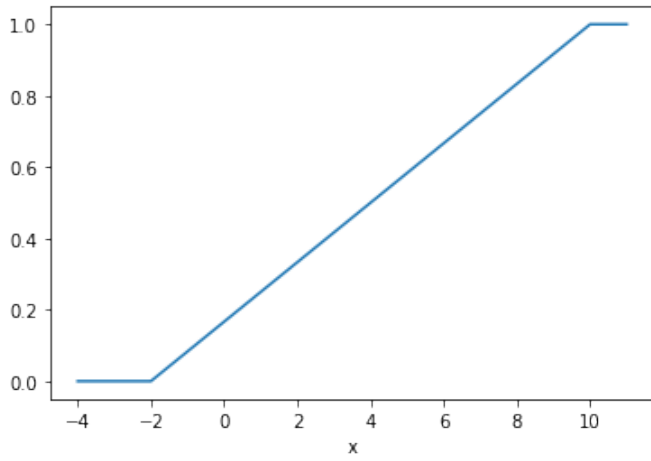


Fig. 2: CDF of X

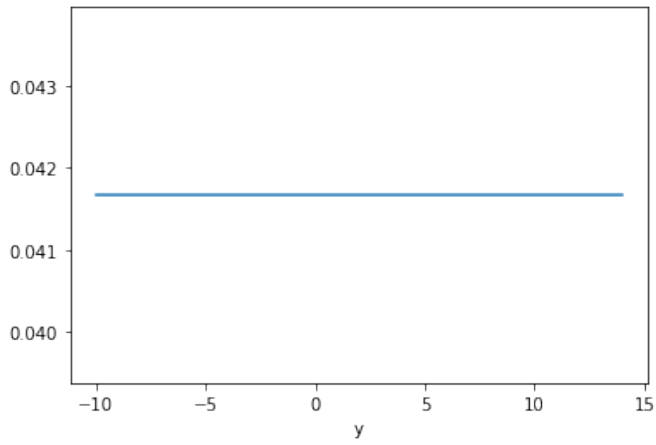


Fig. 3: PDF of Y

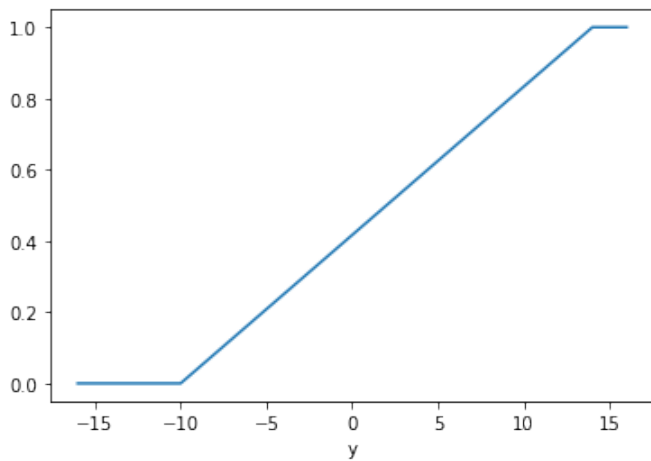


Fig. 4: CDF of Y