AI1103 : Assignment 2

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Download all python codes from

https://github.com/BokkaRajaRaviKiranReddy/AI1103/tree/main/Assignment2/codes

and latex codes from

https://github.com/BokkaRajaRaviKiranReddy/AI1103/blob/main/Assignment2/Assignment2.tex

GATE 2020 EC-Q54

X is a random variable with uniform probability density function in the interval [2,10] .For Y=2X-6,The conditional probability P($Y \leq 7|X \geq 5$) (rounded off to three decimal places) is

Solution

$$\Pr(Y \leq 7|X \geq 5) = \frac{\Pr(Y \leq 7, X \geq 5)}{\Pr(X \geq 5)} \tag{54.1}$$

$$X = \frac{Y+6}{2} \geq 5$$
$$\implies Y + 6 \geq 10$$
$$\implies Y \geq 4$$

So, From Equation 54.1

$$\Pr(Y \leq 7|X \geq 5) = \frac{\Pr(Y \leq 7, Y \geq 4)}{\Pr(X \geq 5)} \tag{54.2}$$

As $X \in [-2, 10]$ with uniform probability density function,
PDF oF X is

$$f_X(x) = \begin{cases} \frac{1}{12} \text{ if } -2 \leq x \leq 10 \\ 0 \text{ otherwise} \end{cases} \tag{54.3}$$

Given Y=2X-6 $\implies Y \in [-10, 14]$
So, PDF of Y is

$$f_Y(y) = \begin{cases} \frac{1}{24} \text{ if } -10 \leq y \leq 14 \\ 0 \text{ otherwise} \end{cases} \tag{54.4}$$

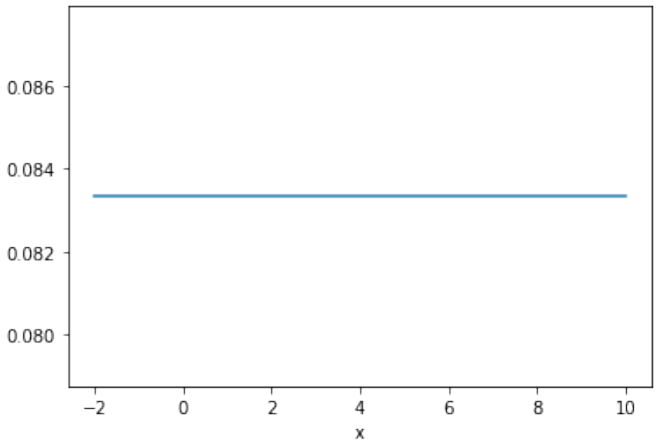

Fig. 1: PDF of X

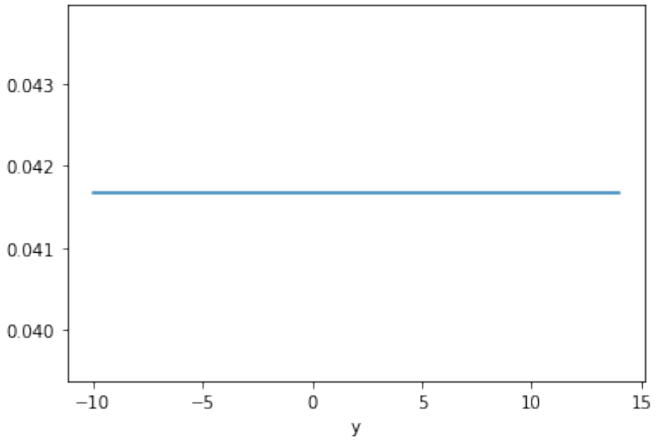

Fig. 2: PDF of Y

CDF of X,

$$F_X(x) = \int_{-\infty}^{x} f_X(x)\,dx$$

$$F_X(x) = \begin{cases} 0 \text{ if } x \leq -2 \\ \frac{1}{12}(x+2) \text{ if } -2 \leq x \leq 10 \\ 1 \text{ if } x \geq 10 \end{cases}$$

CDF of Y,

$$F_Y(y) = \int_{-\infty}^{y} f_Y(y)\,dx$$

$$F_y(y) = \begin{cases} 0 \text{ if } y \leq -10 \\ \frac{1}{24}(y+10) \text{ if } -10 \leq y \leq 14 \\ 1 \text{ if } y \geq 14 \end{cases}$$

So, From Equation 54.2

$$\Pr(Y \leq 7 | X \geq 5) = \frac{\Pr(Y \leq 7, Y \geq 4)}{\Pr(X \geq 5, X \leq 10)}$$
$$= \frac{F_Y(7) - F_Y(4)}{F_X(10) - F_X(5)}$$
$$= \frac{\frac{17}{24} - \frac{14}{24}}{\frac{12}{12} - \frac{7}{12}}$$
$$= 0.300$$

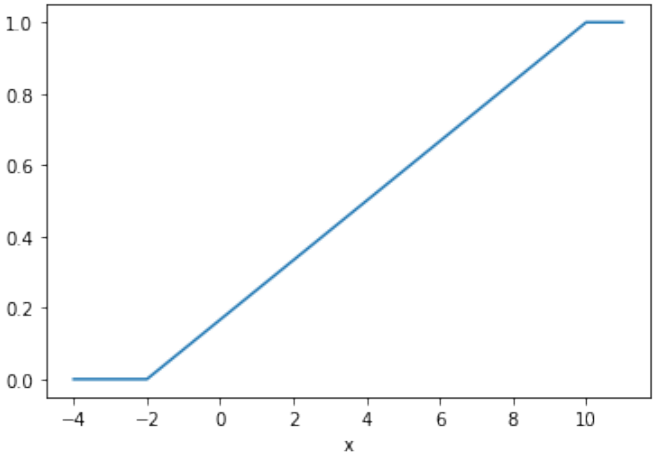

Fig. 3: CDF of X

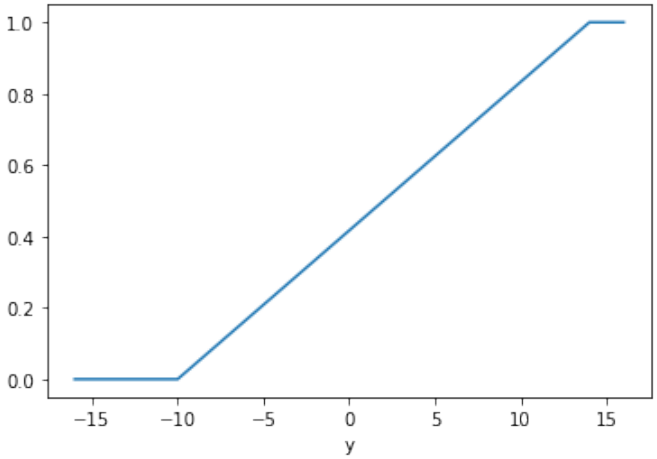

Fig. 4: CDF of Y