

AI1103 : Assignment 2

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Download all python codes from

<https://github.com/BokkaRaviKiranReddy/AI1103/tree/main/Assignment2/codes>

and latex codes from

<https://github.com/BokkaRaviKiranReddy/AI1103/blob/main/Assignment2/Assignment2.tex>

GATE 2020 EC-Q54

X is a random variable with uniform probability density function in the interval $[2, 10]$. For $Y = 2X - 6$, The conditional probability $P(Y \leq 7 | X \geq 5)$ (rounded off to three decimal places) is

SOLUTION

$$\Pr(Y \leq 7 | X \geq 5) = \frac{\Pr(Y \leq 7, X \geq 5)}{\Pr(X \geq 5)} \quad (54.1)$$

$$\begin{aligned} Y &= 2X - 6 \leq 7 \\ \Rightarrow 2X &\leq 13 \\ \Rightarrow X &\leq 6.5 \end{aligned} \quad (54.2)$$

So, From Equation 54.1 and 54.2

$$\Pr(Y \leq 7 | X \geq 5) = \frac{\Pr(X \leq 6.5, X \geq 5)}{\Pr(X \geq 5)} \quad (54.3)$$

As $X \in [-2, 10]$ with uniform probability density function,
PDF of X is

$$f_X(x) = \begin{cases} \frac{1}{12} & \text{if } -2 \leq x \leq 10 \\ 0 & \text{otherwise} \end{cases} \quad (54.4)$$

CDF of X,

$$F_X(x) = \int_{-\infty}^x f_X(x) dx$$

$$F_X(x) = \begin{cases} 0 & \text{if } x \leq -2 \\ \frac{1}{12}(x+2) & \text{if } -2 \leq x \leq 10 \\ 1 & \text{if } x \geq 10 \end{cases} \quad (54.5)$$

So, From Equation 54.3

$$\Pr(Y \leq 7 | X \geq 5) = \frac{\Pr(X \leq 6.5, X \geq 5)}{\Pr(X \geq 5, X \leq 10)} \quad (54.6)$$

$$= \frac{\Pr(5 \leq X \leq 6.5)}{\Pr(5 \leq X \leq 10)} \quad (54.7)$$

$$= \frac{F_X(6.5) - F_X(5)}{F_X(10) - F_X(5)} \quad (54.8)$$

$$= 0.300 \quad (54.9)$$

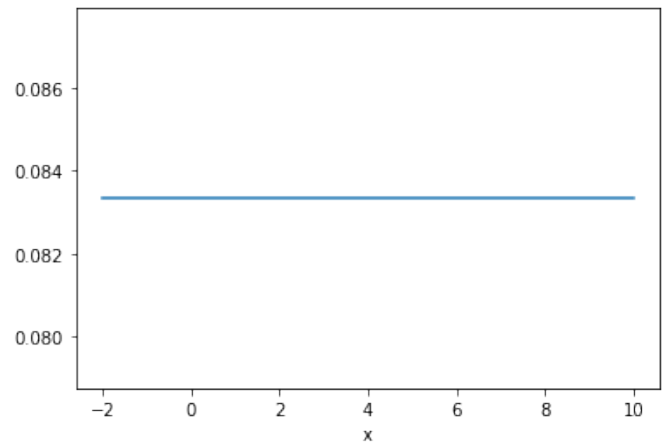


Fig. 1: PDF of X

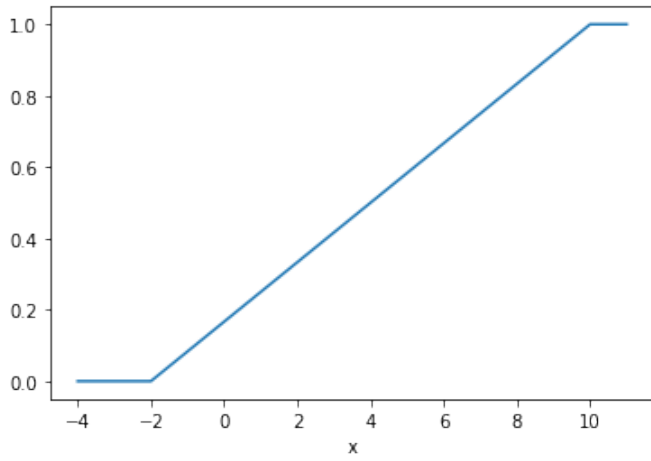


Fig. 2: CDF of X

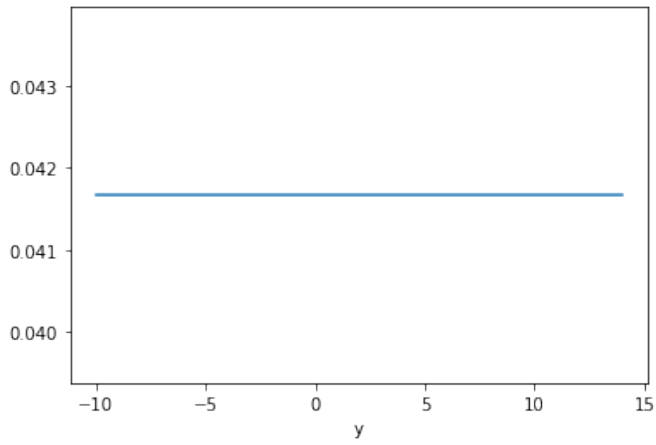


Fig. 3: PDF of Y

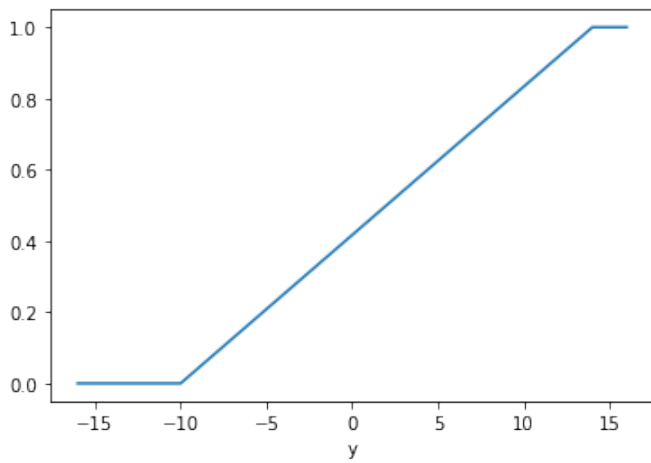


Fig. 4: CDF of Y