

AI1103 : Assignment 5

Raja Ravi Kiran Reddy - CS20BTECH11009

Download all python codes from

<https://github.com/BokkaRajaRaviKiranReddy/AI1103/tree/main/Assignment5/codes>

and latex codes from

<https://github.com/BokkaRajaRaviKiranReddy/AI1103/blob/main/Assignment5/Assignment5.tex>

1 GATE 2021 ME-SET1-Q18

Activities A,B,C,D from the critical path for a project with a PERT network. The means and variances of the activity duration for each activity are given below. All activity durations follow the Gaussian (normal) distribution and are independent of each other. The probability that the project will

Activity	A	B	C	D
Mean	6	11	8	15
variance	4	9	4	9

be completed within 40 days is (round off to two decimal places) (Note: Probability is a number between 0 and 1)

2 SOLUTION

Activity	A	B	C	D
μ	6	11	8	15
σ	4	9	4	9

$E = A + B + C + D$ The sum of Gaussian distributions will also give Gaussian distribution.

$$f(x) = \frac{1}{\sigma \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x-\mu}{\sigma} \right)^2} \quad (2.0.1)$$

$$f_E(x) = \frac{1}{\sigma_E \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x-\mu_E}{\sigma_E} \right)^2} \quad (2.0.2)$$

Definition 1. For a Gaussian Distribution where X is a random variable with mean μ and variance σ^2 , $Y = \frac{X-\mu}{\sigma}$

$$\Pr(X > x) = \Pr(Y > y) = Q(y) \quad (2.0.3)$$

$$Q(y) = \frac{1}{\sqrt{2\pi}} \int_y^{\infty} e^{-\frac{u^2}{2}} du \quad (2.0.4)$$

where,

$$y = \frac{x - \mu}{\sigma}$$

Lemma 2.1. For a Gaussian Distribution,

$$\Pr(X > \mu) = \Pr(Y > 0) = Q(0) = \frac{1}{2} \quad (2.0.5)$$

$$\Pr(X \leq \mu) = 1 - Q(0) = \frac{1}{2} \quad (2.0.6)$$

Proof.

$$Q(y) = \frac{1}{\sqrt{2\pi}} \int_y^{\infty} e^{-\frac{u^2}{2}} du \quad (2.0.7)$$

$$\frac{u^2}{2} = t \implies u = \sqrt{2t} \quad (2.0.8)$$

$$\implies du = \frac{dt}{\sqrt{2t}} \quad (2.0.9)$$

$$Q(0) = \frac{1}{\sqrt{2\pi}} \int_0^{\infty} e^{-t} \frac{dt}{\sqrt{2t}} \quad (2.0.10)$$

$$Q(0) = \frac{1}{2\sqrt{\pi}} \int_0^{\infty} e^{-t} t^{-\frac{1}{2}} dt \quad (2.0.11)$$

$$\int_0^{\infty} e^{-t} t^{-\frac{1}{2}} dt = \Gamma\left(\frac{1}{2}\right) \quad (2.0.12)$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} \quad (2.0.13)$$

$$Q(0) = \frac{1}{2\sqrt{\pi}} \times \sqrt{\pi} \quad (2.0.14)$$

$$Q(0) = \frac{1}{2} \quad (2.0.15)$$

□

Mean of E= μ_E ,

$$\mu_E = \mu_A + \mu_B + \mu_C + \mu_D \quad (2.0.16)$$

$$= 40 \quad (2.0.17)$$

Standard deviation of E= σ_E ,

$$\sigma_E^2 = \sigma_A^2 + \sigma_B^2 + \sigma_C^2 + \sigma_D^2 \quad (2.0.18)$$

$$\sigma_E = 5.1 \quad (2.0.19)$$

$$f_E(x) = \frac{1}{5.1 \sqrt{2\pi}} e^{-\frac{1}{2} \left(\frac{x-40}{5.1} \right)^2} \quad (2.0.20)$$

here we want $\Pr(X \leq 40)$,

$$\Pr(X \leq 40) = 1 - \Pr(X > 40) \quad (2.0.21)$$

For activity E , $\mu_E = 40$ and from (2.1)

$$\Pr(X > 40) = 0.50 \quad (2.0.22)$$

$$\implies \Pr(x \leq 40) = 0.50 \quad (2.0.23)$$

