# AI1103 : Assignment 3

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Download the latex codes from

https://github.com/BokkaRajaRaviKiranReddy/AI1103/blob/main/Assignment3/Assignment3.tex

## GATE 2014 MA -Q37

Let $X, Y$ be continuous random variables with joint density function

$$f_{X,Y}(x,y) = \begin{cases} e^{-y}(1-e^{-x}) & \text{if } 0 < x < y < \infty \\ e^{-x}(1-e^{-y}) & \text{if } 0 < y \leq x < \infty \end{cases}$$

Then The value of $E[X+Y]$ is

## SOLUTION

Let $g(X,Y) = X+Y$
We know that,

$$E[g(X,Y)] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} g(x,y)f_{X,Y}(x,y)dxdy$$

Then,

$$E[X+Y] = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} (x+y)f_{X,Y}(x,y)\,dxdy$$
$$= \int_{0}^{+\infty}\int_{0}^{+\infty} (x+y)f_{X,Y}(x,y)\,dxdy$$
$$= \int_{0}^{+\infty}\left(\int_{0}^{+\infty} xf_{X,Y}(x,y)\,dx + \int_{0}^{+\infty} yf_{X,Y}(x,y)\,dx\right)dy$$

First we will calculate the $\int_0^{+\infty} yf_{X,Y}(x,y)\,dx$, $\int_0^{+\infty} xf_{X,Y}(x,y)\,dx$ seperately.
consider,

$$\int_0^{+\infty} yf_{X,Y}(x,y)\,dx$$
$$= \int_0^{y} ye^{-y}(1-e^{-x})\,dx + \int_y^{+\infty} ye^{-x}(1-e^{-y})\,dx$$
$$= (ye^{-y})(y+e^{-y}-1) + y(1-e^{-y})e^{-y}$$
$$= y^2e^{-y}$$

So,

$$\int_0^{+\infty} yf_{X,Y}(x,y)\,dx = y^2e^{-y} \tag{37.1}$$

Now consider,

$$\int_0^{+\infty} xf_{X,Y}(x,y)\,dx$$
$$= \int_0^{y} xe^{-y}(1-e^{-x})\,dx + \int_y^{+\infty} xe^{-x}(1-e^{-y})$$
$$= e^{-y}\left(\frac{y^2}{2} + e^{-y}(y+1) - 1\right) + (1-e^{-y})(e^{-y}(y+1))$$
$$= \frac{y^2e^{-y}}{2} + ye^{-y}$$

So,

$$\int_0^{+\infty} xf_{X,Y}(x,y)\,dx = \frac{y^2e^{-y}}{2} + ye^{-y} \tag{37.2}$$

From Eq 37.1 and 37.2

$$E[X+Y] = \int_0^{+\infty}\left(\frac{y^2e^{-y}}{2} + ye^{-y} + y^2e^{-y}\right)dy$$
$$= \int_0^{+\infty}\left(\frac{3}{2}y^2e^{-y} + ye^{-y}\right)dy$$
$$= \left(-\frac{3}{2}(y^2+2y+2)e^{-y} + (-e^{-y}(y+1))\right)\Big|_0^{+\infty}$$
$$= \frac{3}{2}\times 2 + 1$$
$$= 4$$

So,

$$E[X+Y] = 4$$