

# Research Paper Presentation

Bokka Raja Ravi Kiran Reddy

CS20BTECH11009

## Title

A Bayesian Inference Approach for Location-Based Micro Motions using Radio Frequency Sensing

## Authors

David A. Maluf, Amr Elnakeeb, Matt Silverman

## Abstract

- ① A Bayesian framework is proposed for the surface tracking objective
- ② The resulting optimization problem, in which a non-linear forward model is linearized, is solved via gradient descent methods
- ③ The location of the moving surface, the fractional area and the diffuse reflection coefficient as well as the velocity are inferred, versus time.

# Introduction

- ➊ In the context of wireless localization ,Received Signal Strength Indication (RSSI) is used but it requires Line of sight (LOS) which is not guaranteed, especially in indoor environments. So, we use Channel State Information (CSI) which suits for multipath environments and Non-Line of Sight (NLoS) communication—unlike RSSI.
- ➋ In the context of target tracking; herein, the proposed model is a parameterized signal strength model, from which synthetic observations are generated.
- ➌ For the case of wireless signals, we demonstrate how the moving micro surface areas, diffuse coefficients, locations and velocities can be inferred, based on the proposed model.

## PROPOSED MODEL

We assume that the transmitter is an isotropic source, and the signal is reflected via the object surface then arrives at the receiving radio end. We follow the free space attenuation model that is, the received power at the receiving antenna is given by,

$$P_{RX} = \frac{A_{RX}}{4\pi R^2} P_{TX} \quad (1)$$

where,  $R$  is the distance between that object (surface) and the receiving end. In terms of electric field strength ,

$$P_{RX} = \frac{|E_r|^2}{Z_0} A_{RX} \quad (2)$$

$$P_{TX} = \frac{|E_s|^2}{Z_0} \rho A_e \quad (3)$$

where,,  $Z_0$  is the impedance of free space  $|E_r|$  and  $|E_s|$  are the magnitude of the electric field strength at the receiving and transmitting ends,  $\rho A_e$  effective surface area of the object.

# PROPOSED MODEL

By combining ,

$$|E_r| = \frac{\sqrt{\rho A_e}}{2\sqrt{\pi}R} |E_s| \quad (4)$$

Define the normalized magnitude of electric field strengths as M,

$$M = \frac{|E_r|}{|E_s|} = \frac{\sqrt{\rho A_e}}{2\sqrt{\pi}R} \quad (5)$$

The phase difference between the received and transmitted electric field is defined as,

$$\psi = \psi_{E_r} - \psi_{E_s} \quad (6)$$

The measured phase is folded/wrapped due to the recurrence characteristic of phase. Therefore, the measured phase has to be transformed into the true “unwrapped” value.

# PROPOSED MODEL

## Phase Unwrapping Algorithm

- ① **Input**: Measured (wrapped) phase  $\psi_l$  of  $L$  subcarriers;  $l = 1, 2, \dots, L$ .
- ② **output** : Unwrapped phase  $\phi_l$  of  $L$  subcarriers
- ③ **Initialization** : Initialize  $\psi_l = \phi_l$ ,  $d = 0$
- ④ for  $l = 2 : L$  do  
    If  $\psi_l - \psi_{l-1} > \pi$  then  $d = d + 1$   
     $\phi_l = \psi_l - 2\pi d$
- ⑤ **endfor**

Then, we write the difference between the unwrapped phases as follows,

$$\phi = \phi_{E_r} - \phi_{E_s} = -\frac{2\pi}{\lambda}(2R) = -\frac{4\pi}{\lambda}R \quad (7)$$

# PROPOSED MODEL

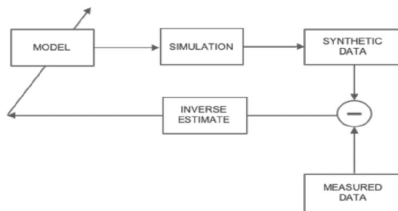


Figure: Signal Model

The error between the real measurements and the synthetic observations is fed to the simulator, through which the system parameters are accordingly adjusted, to minimize the error in the subsequent instances of time. Put differently, the estimation of the errors means the system parameters are updated whenever more measurements are available.



# BAYESIAN FRAMEWORK AND PROPOSED ALGORITHM

The object of interest has an effective surface area  $\rho A_e$ , and is located at distance  $R_i$  from the  $i$ -th access point,  $i = 1, 2, \dots, N$ , where  $N$  denotes the total number of access points. We assume that not all matter moves at once, and for an infinitesimal period of time, motion is characterized by fractional effective surface  $\rho A_e$  that has moved a distance  $d\lambda$ .

The full vector of distances and differential surface area is defined as,

$$\mathbf{u} = [R_1, R_2, \dots, R_N, \bar{A}_e] \in \mathbb{R}^{N+1} \quad (8)$$

To estimate  $\mathbf{u}$  from the collected measurements, we apply Bayes theorem,

$$p(\mathbf{u} | dM_1, \dots, dM_N) \propto p(dM_1, \dots, dM_N | \mathbf{u}) p(\mathbf{u}) \quad (9)$$

where,  $dM_i$ ;  $i = 1, 2, \dots, N$ , is the differential component of the magnitude of the signal strength on the  $i$ -th access point.  $p(\mathbf{u})$  is prior distribution of the distances and surface area.

Furthermore, the prior distribution is assumed to be zero-mean Gaussian, that is,

# BAYESIAN FRAMEWORK AND PROPOSED ALGORITHM

$$p(\mathbf{u}) \propto \exp\left(-\frac{1}{2}\mathbf{u}\Sigma^{-1}\mathbf{u}^\top\right) \quad (10)$$

where,  $\Sigma^{-1} \in \mathbb{R}^{N+1 \times N+1}$  is the inverse covariance matrix

$$\Sigma^{-1} = \frac{4\pi}{\lambda} \begin{pmatrix} \chi_1 & 0 & \cdots & 0 & 0 \\ 0 & \chi_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & \chi_N & 0 \\ 0 & 0 & \cdots & 0 & 1 \end{pmatrix} \quad (11)$$

We penalize over the average difference, on each access point, between the differential distances  $dR_{ij}$  and the phase shifts  $\frac{\lambda}{4\pi}d\phi_{ij}$  from angles of arrival.

# BAYESIAN FRAMEWORK AND PROPOSED ALGORITHM

$$\chi_i = \frac{1}{\binom{V_i}{2}} \sum_{k,j,k \neq j}^{\binom{V_i}{2}} \left( dR_{kj_i} - \frac{\lambda}{4\pi} d\phi_{kj_i} \right)^2 \quad (12)$$

where  $k, j$  denote two antennas' indices on any particular access point  $i$ ;  $i = 1, 2, \dots, N$ ,  $V_i$  denotes the number of antennas on the  $i$ -th access point.

We will take 2 Asssssumptions.

- 1 The difference between the measured real data and the synthesized observed data follows a zero mean Gaussian distribution
- 2 The measurements are conditionally independent

from this we get,

# BAYESIAN FRAMEWORK AND PROPOSED ALGORITHM

$$p(dM_1, \dots, dM_N | \mathbf{u}) \propto \exp \left( -\frac{\sum_{i=1}^N (dM_i - \hat{m}_i(\mathbf{u}))^2}{2\sigma^2} \right) \quad (13)$$

where,  $\hat{m}_i(\mathbf{u})$  denotes estimated differential magnitude on the  $i$ -th AP, and  $\sigma^2$  is the noise variance. Therefore, the negative-log posterior is written as,

$$\mathcal{L}(\mathbf{u}) \propto -\frac{\sum_{i=1}^N (dM_i - \hat{m}_i(\mathbf{u}))^2}{2\sigma^2} + \mathbf{u} \Sigma^{-1} \mathbf{u}^T \quad (14)$$

(14) is a nonlinear function of  $\mathbf{u}$ ; and the MAP estimate is that value of  $\mathbf{u}$  which minimizes  $\mathcal{L}(\mathbf{u})$ . The vector of synthesized magnitudes of the motion is denoted by  $\hat{\mathbf{m}}(\mathbf{u})$ , which is linearized about the current estimate, that is,

$$\hat{\mathbf{m}}(\mathbf{u}) = \hat{\mathbf{m}}(\mathbf{u}_0) + \mathbf{D}(\mathbf{u} - \mathbf{u}_0) \quad (15)$$

# BAYESIAN FRAMEWORK AND PROPOSED ALGORITHM

D is written as,

$$\mathbf{D} = \begin{pmatrix} -\frac{\sqrt{\bar{A}_e}}{R_1^2} & \frac{\sqrt{\bar{A}_e}}{R_1 R_2} & \cdots & \frac{\sqrt{\bar{A}_e}}{R_1 R_N} & \frac{1}{2R_1 \sqrt{\bar{A}_e}} \\ \frac{\sqrt{\bar{A}_e}}{R_2 R_1} & -\frac{\sqrt{\bar{A}_e}}{R_2^2} & \cdots & \frac{\sqrt{\bar{A}_e}}{R_2 R_N} & \frac{1}{2R_2 \sqrt{\bar{A}_e}} \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ \frac{\sqrt{\bar{A}_e}}{R_N R_1} & \frac{\sqrt{\bar{A}_e}}{R_N R_2} & \cdots & -\frac{\sqrt{\bar{A}_e}}{R_N^2} & \frac{1}{2R_N \sqrt{\bar{A}_e}} \end{pmatrix} \quad (16)$$

The minimization of  $\mathcal{L}(\mathbf{u})$  can be replaced with the minimization of the following quadratic form

$$\mathcal{L}' = \frac{1}{2} \mathbf{x} \mathbf{A} \mathbf{x}^\top - \mathbf{b} \mathbf{x} \quad (17)$$

$$\mathbf{x} = \mathbf{u} - \mathbf{u}_0 \quad (18)$$

$$(19)$$

# BAYESIAN FRAMEWORK AND PROPOSED ALGORITHM

$$\mathbf{A} = \Sigma^{-1} - \frac{\mathbf{D}\mathbf{D}^{\top}}{\sigma^2} \quad (20)$$

$$\mathbf{b} = \mathbf{D} \frac{(m(\mathbf{u}) - \hat{m}(\mathbf{u}))}{\sigma^2} + \Sigma^{-1} \mathbf{u}_0 \quad (21)$$

We search for the minimum in  $\mathbf{x}$  in using a conjugate gradient method. At the minimum, we update the current estimate  $\mathbf{u} = \mathbf{u}_0 + \mathbf{x}$ , recompute  $\hat{m}(\mathbf{u})$  and  $\mathbf{D}$ , and repeat the minimization procedure iteratively until the current estimate converges.

# BAYESIAN FRAMEWORK AND PROPOSED ALGORITHM

Once again, using Bayes theorem

$$p(x, y, z, s|\mathbf{u}) \propto p(\mathbf{u}|x, y, z, s)p(x, y, z, s) \quad (22)$$

where  $(x, y, z)$  is the position of the moving surface and  $s$  is a scale. The position of the moving surface and the scale are estimated by solving the following set of  $N$  quadratic equations.

$$(x - x_i)^2 + (y - y_i)^2 + (z - z_i)^2 = sR_i^2, \quad i = 1, \dots, N \quad (23)$$

The method of choice is Levenberg–Marquardt minimization. The corresponding velocities are estimated as follows,

$$v_{x_i}^2 + v_{y_i}^2 + v_{z_i}^2 = \left( \frac{\frac{\lambda}{4\pi} d\phi_{t,t-1}}{dt} \right)_i^2, \quad i = 1, \dots, N \quad (24)$$

where  $t$  is the time index

# Results

In our experiments, the CSI is collected from Cisco 4800 WiFi APs (Cisco Systems Inc., San Jose, CA, USA) on the 5 GHz radio frequency for WiFi. We use three APs for 2D inference. The three APs are placed to form a triangle, and they are put on desks

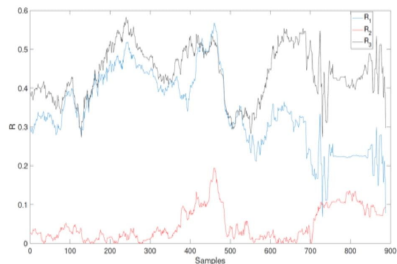


Figure: Distances from the three APs in meters.



# Results

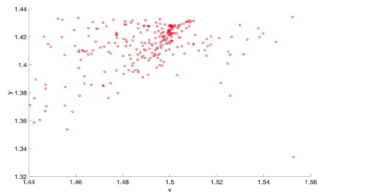


Figure:  $(x, y)$  positions in meters

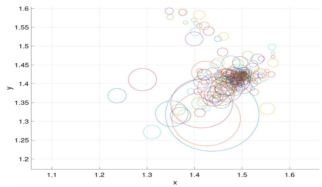


Figure: Point spread function in meters

# Results

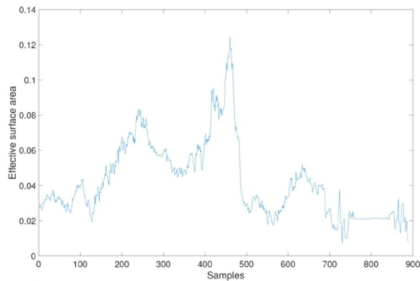


Figure: . Effective surface area of the moving surface in  $m^2$ .

# Conclusion

- ① A general inference framework has been proposed for the tracking of general moving micro surfaces, where the model choice is determined by the natural and physical properties of the surface we infer.
- ② Bayesian inference is formulated, and a combination of gradient descent and Levenberg–Marquardt algorithms is conducted.