

# AI1103 : Assignment 2

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Download all python codes from

<https://github.com/BokkaRajaRaviKiranReddy/AI1103/tree/main/Assignment2/codes>

and latex codes from

<https://github.com/BokkaRajaRaviKiranReddy/AI1103/blob/main/Assignment2/Assignment2.tex>

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$X$  is a random variable with uniform probability density function in the interval  $[2, 10]$ . For  $Y = 2X - 6$ , The conditional probability  $P(Y \leq 7 | X \geq 5)$  (rounded off to three decimal places) is

SOLUTION

As  $X \in [-2, 10]$  with uniform probability density function,  
PDF of  $X$  is

$$f_X(x) = \begin{cases} \frac{1}{8} & \text{if } -2 \leq x \leq 10 \\ 0 & \text{otherwise} \end{cases} \quad (54.1) \quad \text{So,}$$

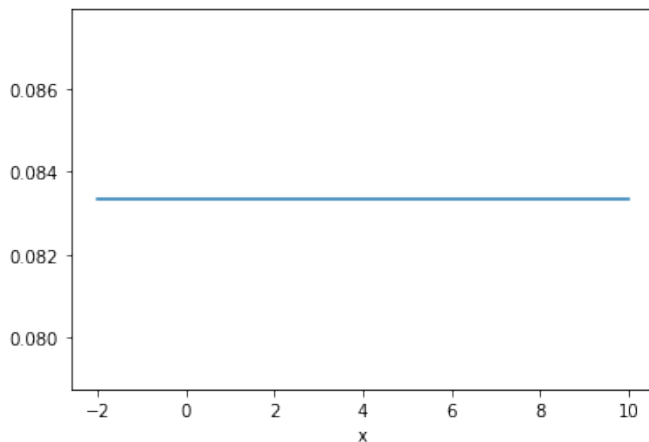


Fig. 1: PDF of  $X$

Given  $Y = 2X - 6 \implies Y \in [-10, 14]$

So, PDF of  $Y$  is

$$f_Y(y) = \begin{cases} \frac{1}{24} & \text{if } -10 \leq y \leq 14 \\ 0 & \text{otherwise} \end{cases} \quad (54.2)$$

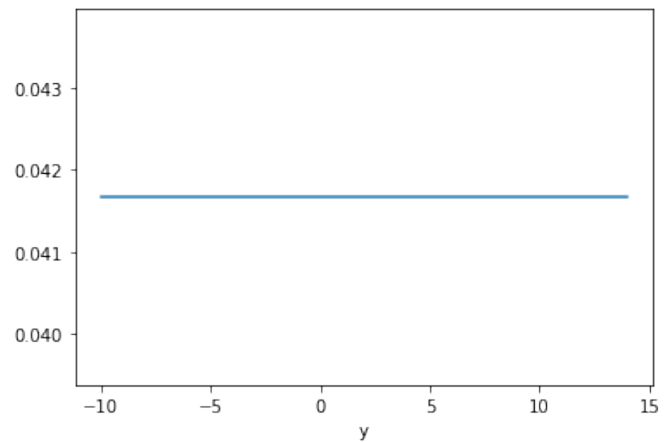


Fig. 2: PDF of  $Y$

$$\Pr(Y \leq 7 | X \geq 5) = \frac{\Pr(Y \leq 7, X \geq 5)}{\Pr(X \geq 5)} \quad (54.3)$$

$$\begin{aligned} X &= \frac{Y + 6}{2} \geq 5 \\ \implies Y + 6 &\geq 10 \\ \implies Y &\geq 4 \end{aligned}$$

So, From Equation 54.3

$$\begin{aligned}
 \Pr(Y \leq 7 | X \geq 5) &= \frac{\Pr(Y \leq 7, Y \geq 4)}{\Pr(X \geq 5)} \\
 &= \frac{\int_4^7 f_Y(y) dy}{\int_5^{10} f_X(x) dx} \\
 &= \frac{\int_4^7 \frac{1}{24} dy}{\int_5^{10} \frac{1}{12} dx} \\
 &= \frac{\frac{3}{24}}{\frac{5}{12}} \\
 &= 0.300
 \end{aligned}$$

$$F_X(x) = \int_{-\infty}^x f_X(x) dx$$

CDF of X,

$$F_X(x) = \begin{cases} 0 & \text{if } x \leq -2 \\ \frac{1}{12}(x+2) & \text{if } -2 \leq x \leq 10 \\ 1 & \text{if } x \geq 10 \end{cases}$$

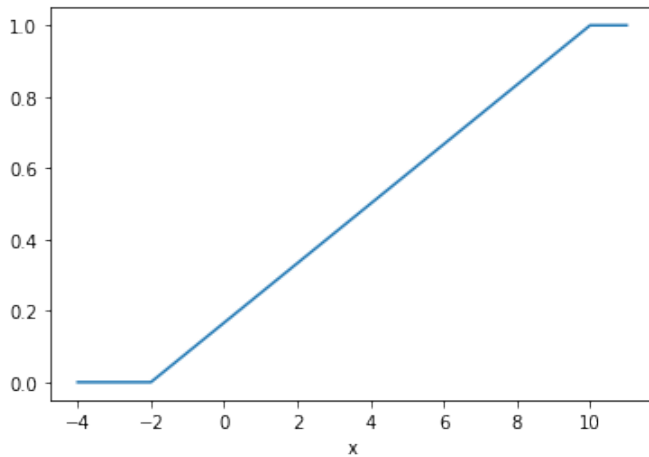


Fig. 3: CDF of X

CDF of Y,

$$F_Y(y) = \begin{cases} 0 & \text{if } y \leq -10 \\ \frac{1}{24}(y+10) & \text{if } -10 \leq y \leq 14 \\ 1 & \text{if } y \geq 14 \end{cases}$$

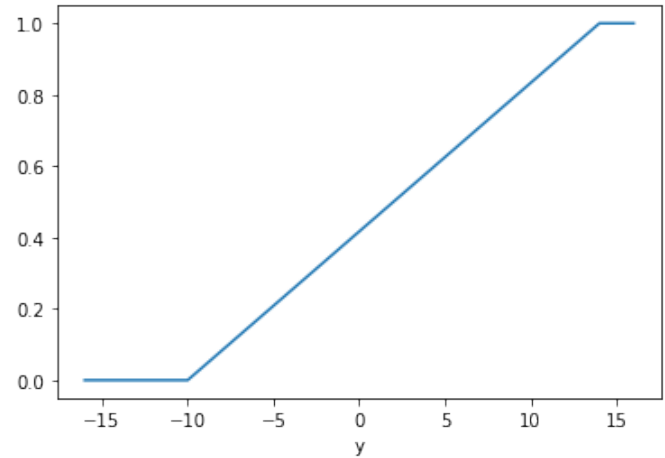


Fig. 4: CDF of Y