AI1103 : Assignment 5

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Download all python codes from

https://github.com/BokkaRajaRaviKiranReddy/AI1103/tree/main/Assignment5/codes

and latex codes from

https://github.com/BokkaRajaRaviKiranReddy/AI1103/blob/main/Assignment5/Assignment5.tex

1 GATE 2021 ME-SET1-Q18

Activities A,B,C,D from the critical path for a project with a PERT network. The means an variances of the activity duration for each activity are given below . All activity duration follow the Gaussian (normal) distribution and are independent of each other. The probability that the project will

Activity	A	B	C	D
Mean	6	11	8	15
variance	4	9	4	9

be completed within 40 days is (round off to two decimal places) (Note: Probability is a number between 0 and 1)

2 SOLUTION

Activity	A	B	C	D
μ	6	11	8	15
σ	4	9	4	9

E=A+B+C+D The sum of Gaussian distributions will also give Gaussian distribution.

$$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} \tag{2.0.1}$$

$$f_E(x) = \frac{1}{\sigma_E\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu_E}{\sigma_E}\right)^2} \tag{2.0.2}$$

Definition 1. *For a Gaussian Distribution where X is a random variable with mean μ and variance σ^2 , $Y = \frac{X-\mu}{\sigma}$*

$$\Pr(X > x) = \Pr(Y > y) = Q(y) \tag{2.0.3}$$

$$Q(y) = \frac{1}{\sqrt{2\pi}} \int_y^\infty e^{-\frac{u^2}{2}} du \tag{2.0.4}$$

where,

$$y = \frac{x-\mu}{\sigma}$$

Lemma 2.1. *For a Gaussian Distribution,*

$$\Pr(X > \mu) = \Pr(Y > 0) = Q(0) = \frac{1}{2} \tag{2.0.5}$$

$$\Pr(X \le \mu) = 1 - Q(0) = \frac{1}{2} \tag{2.0.6}$$

Proof.

$$Q(y) = \frac{1}{\sqrt{2\pi}} \int_y^\infty e^{-\frac{u^2}{2}} du \tag{2.0.7}$$

$$\frac{u^2}{2} = t \implies u = \sqrt{2t} \tag{2.0.8}$$

$$\implies du = \frac{dt}{\sqrt{2t}} \tag{2.0.9}$$

$$Q(0) = \frac{1}{\sqrt{2\pi}} \int_0^\infty e^{-t} \frac{dt}{\sqrt{2t}} \tag{2.0.10}$$

$$Q(0) = \frac{1}{2\sqrt{\pi}} \int_0^\infty e^{-t} t^{-\frac{1}{2}} dt \tag{2.0.11}$$

$$\int_0^\infty e^{-t} t^{-\frac{1}{2}} dt = \Gamma\left(\frac{1}{2}\right) \tag{2.0.12}$$

$$\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi} \tag{2.0.13}$$

$$Q(0) = \frac{1}{2\sqrt{\pi}} \times \sqrt{\pi} \tag{2.0.14}$$

$$Q(0) = \frac{1}{2} \tag{2.0.15}$$

□

Mean of E=μ_E ,

$$\mu_E = \mu_A + \mu_B + \mu_C + \mu_D \tag{2.0.16}$$
$$= 40 \tag{2.0.17}$$

Standard deviation of E=σ_E,

$$\sigma_E^2 = \sigma_A^2 + \sigma_B^2 + \sigma_C^2 + \sigma_D^2 \tag{2.0.18}$$
$$\sigma_E = 5.1 \tag{2.0.19}$$

$$f_E(x) = \frac{1}{\sigma_E \sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu_E}{\sigma_E}\right)^2} \tag{2.0.20}$$

here we want $\Pr(X \leq 40)$,

$$\Pr(X \leq 40) = 1 - \Pr(X > 40) \tag{2.0.21}$$

For activity E , $\mu_E = 40$ and from (2.1)

$$\Pr(X > 40) = 0.50 \tag{2.0.22}$$
$$\implies \Pr(x \leq 40) = 0.50 \tag{2.0.23}$$