

Performing a transposed convolution

$$\text{stride } (s) = 2, \text{ padding } (p) = 1, \text{ kernel} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{pmatrix}, \text{ input} = \begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix}$$

Step 1. Compute parameters

$$z = s - 1 = 2 - 1 = 1, \quad p' = k - p - 1 = 3 - 1 - 1 = 1$$

Step 2. Insert zeros

$$\begin{pmatrix} 1 & 1 \\ 2 & 3 \end{pmatrix} \longrightarrow \begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 2 & 0 & 3 \end{pmatrix}$$

Step 3. Pad with zeros

$$\begin{pmatrix} 1 & 0 & 1 \\ 0 & 0 & 0 \\ 2 & 0 & 3 \end{pmatrix} \longrightarrow \begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Step 4. Perform standard convolution with stride 1

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} * \begin{pmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{pmatrix} = \begin{pmatrix} -4 & 2 & -4 \\ 3 & 0 & 4 \\ -8 & 5 & -12 \end{pmatrix}$$

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} * \begin{pmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{pmatrix} = -4; \quad \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} * \begin{pmatrix} 0 & 1 & 0 \\ 1 & -4 & 1 \\ 0 & 1 & 0 \end{pmatrix} = 1 + 1 = 2 \text{ etc.}$$

The output size is as expected: $o = (2 - 1) \cdot 2 + 3 - 2 \cdot 1 = 2 + 3 - 2 = 3$.