

Forecasting the direction of the US stock market with dynamic binary probit models

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Abstract

Several empirical studies have documented that the signs of excess stock returns are, to some extent, predictable. In this paper, we consider the predictive ability of the binary dependent dynamic probit model in predicting the direction of monthly excess stock returns. The recession forecast obtained from the model for a binary recession indicator appears to be the most useful predictive variable, and once it is employed, the sign of the excess return is predictable in-sample. The new dynamic “error correction” probit model proposed in the paper yields better out-of-sample sign forecasts, with the resulting average trading returns being higher than those of either the buy-and-hold strategy or trading rules based on ARMAX models.

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1. Introduction

There is a considerable amount of evidence in the financial econometric literature that excess stock market returns are, to some extent, predictable. The main objective has been to predict the overall level, that is, the conditional mean, of excess stock returns. It is emphasized that even though the predictability is statistically weak, it can be economically meaningful.

However, many studies have documented that only the direction of excess stock returns or other asset returns are predictable (see, among others, Breen, Glosten, & Jagannathan, 1989; Christoffersen &

Diebold, 2006; Hong & Chung, 2003). A possible explanation for this is that the noise in the observed returns is too high for the accurate forecasting of the overall return. Leitch and Tanner (1991) find that the direction of the change is the best criterion for predictability, because traditional statistical summary statistics may not be closely related to the profits that investors are seeking in the financial market. Directional predictability is also important for market timing, which is crucial for asset allocation decisions between stock and risk-free interest rate investments.

The previous findings of directional predictability are based mainly on time series models for the excess stock return. For instance, Christoffersen and Diebold (2006) and Christoffersen, Diebold, Mariano, Tay, and Tse (2007) considered the theoretical connection

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between asset return volatility and asset return sign predictability, and verified that the volatility and higher-order conditional moments of returns have a statistically significant explanatory power in sign prediction. Even though there is not much previous research, binary dependent time series models provide another way of forecasting the direction of excess stock returns. Various classification-based qualitative models, such as traditional static logit and probit models, were considered by [Leung, Daouk, and Chen \(2000\)](#), whereas [Anatolyev and Gospodinov \(2010\)](#), [Hong and Chung \(2003\)](#), and [Rydberg and Shephard \(2003\)](#) used the so-called autologistic model to predict the return direction. In the last two papers, the return is decomposed into a sign component and an absolute value component, which are modeled separately before the joint forecast is constructed.

We consider various commonly used financial variables as explanatory variables for forecasting the signs of the one-month US excess stock returns from the S&P500 index and of size-sorted small and large firms' stock indices in probit models. The paper introduces a model in which the recession forecast constructed for a binary recession indicator is used as an explanatory variable in the predictive model. To the best of our knowledge, this kind of approach has not previously been applied to forecasting stock return signs. As a motivation for this model, for example, [Chen \(1991\)](#) and [Fama and French \(1989\)](#) propose that business conditions are important determinants of expected stock returns, and therefore, the recession forecast may be a useful predictive variable in our model. Further, [Chauvet and Potter \(2000\)](#) stressed that the stock market "cycle" leads the business cycle. This argument is based on the fact that the expectations about changes in future economic activity could have an important predictive power to predict excess stock returns. If there are expectations of a coming recession, excess stock returns are low, while stock returns should be positive after a recession period.

In this paper, the new dynamic probit models suggested by [Kauppi and Saikkonen \(2008\)](#) are employed and extended. Since there is not much earlier evidence relating to suitable explanatory variables in sign prediction with probit models, various explanatory variables and their in-sample forecasting performances are first evaluated. After that, the out-of-sample

directional predictability for the excess stock return sign is considered. It is not evident, however, how much the in-sample evidence should be emphasized in assessing the overall return predictability, as it does not guarantee out-of-sample predictability, as has been emphasized in many previous studies (see the discussion by [Campbell & Thompson, 2008](#); [Goyal & Welch, 2008](#), for example).

The results show that the probit models have statistically significant in-sample predictive power for the signs of excess stock returns. A proposed new "error correction" model outperforms the other probit and alternative predictive models, such as "continuous" AR-MAX models, out-of-sample. The excess investment returns received over the buy-and-hold trading strategy are also economically significant. Comparisons between different probit models indicate that the forecasting framework based on the constructed recession forecasts yields more accurate sign predictions than the models where only financial explanatory variables are employed. Especially in the case of small and large size firms, the excess stock return signs seem to be predictable out-of-sample as well.

This paper proceeds as follows. The forecasting model employed, together with recession forecasts, suggested dynamic probit models, and, in particular, the new error correction model, is presented in Section 2. In Section 3, the goodness-of-fit evaluation of the sign forecasts and statistical tests for sign predictability are introduced. The empirical evidence on the directional predictability of the US excess stock returns is reported in Section 4. Finally, Section 5 concludes.

2. Forecasting model

2.1. Dynamic probit models in directional forecasting

Let r_t be the excess stock return over the risk-free interest rate. In many studies, the directional predictability of excess stock returns is examined by using models for continuous dependent variables. For example, [Christoffersen et al. \(2007\)](#) proposed a method for forecasting the direction of excess stock returns, where they first model the conditional variance σ_t^2 and the conditional mean μ_t . Assuming that the data generating process of r_t is

$$r_t = \mu_t + \sigma_t \varepsilon_t,$$

where $\varepsilon_t \sim \text{IID}(0, 1)$, the conditional probability of a positive return, given the information set Ω_{t-1} , is

$$\begin{aligned} P_{t-1}(r_t > 0) &= 1 - P_{t-1}(r_t \leq 0) \\ &= 1 - P_{t-1}(\varepsilon_t \leq -\mu_t/\sigma_t) \\ &= 1 - F_\varepsilon(-\mu_t/\sigma_t), \end{aligned} \quad (1)$$

where $F_\varepsilon(\cdot)$ is the cumulative distribution function of the error term ε_t . If the conditional probability of positive excess returns (Eq. (1)) varies with the information set Ω_{t-1} , then the sign of the return should be predictable.

In this paper, our main interest is in studying the directional predictability using probit models, where the dependent variable is the binary sign return indicator

$$I_t = \begin{cases} 1, & \text{if } r_t > 0, \\ 0, & \text{if } r_t \leq 0, \end{cases} \quad (2)$$

which takes the value one when the excess stock return is positive and zero otherwise. Thus, I_t is a binary-valued stochastic process. Conditional on the information set Ω_{t-1} , which includes the predictive variables and lagged values of the stock indicator (Eq. (2)), it has a Bernoulli distribution with probability p_t^I ; that is,

$$I_t | \Omega_{t-1} \sim B(p_t^I).$$

Let E_{t-1} denote the conditional expectation given the information set Ω_{t-1} . In the probit model, the conditional probability of a positive excess stock return ($I_t = 1$) satisfies

$$\begin{aligned} p_t^I &= E_{t-1}(I_t) = P_{t-1}(I_t = 1) \\ &= P_{t-1}(r_t > 0) = \Phi(\pi_t^I), \end{aligned} \quad (3)$$

where $\Phi(\cdot)$ is the standard normal cumulative distribution function. The conditional probability is modeled by specifying a model for π_t^I , which is supposed to be a function of the variables in the information set.¹

The literature indicates that there is not much autocorrelation between any two successive values of excess stock returns. Thus, the benchmark forecasting model is the static model

$$\pi_t^I = \omega + \mathbf{x}_{t-1}'\boldsymbol{\beta}, \quad (4)$$

where the explanatory variables employed are collected in the vector \mathbf{x}_{t-1} . Because of the expected lack of correlation between I_{t-1} and I_t , this static model, without any dynamic structure, might be adequate. In order to investigate this, the value of the lagged return indicator I_{t-1} can be included in the model. This yields the dynamic probit model

$$\pi_t^I = \omega + \delta_1 I_{t-1} + \mathbf{x}_{t-1}'\boldsymbol{\beta}. \quad (5)$$

If the coefficient δ_1 is statistically significant, then the lagged direction of the stock return is a useful predictor of the future direction of excess stock returns.

Over the last few years, various new binary time series models have been introduced to the literature. We concentrate on the model variants suggested by [Kauppi and Saikkonen \(2008\)](#). They add the lagged value π_{t-1}^I , referred to as the autoregressive structure, to the model equation. Thus, the static model (4) and the dynamic model (5) are extended to the autoregressive model²

$$\pi_t^I = \omega + \alpha_1 \pi_{t-1}^I + \mathbf{x}_{t-1}'\boldsymbol{\beta}, \quad (6)$$

and the dynamic autoregressive model

$$\pi_t^I = \omega + \alpha_1 \pi_{t-1}^I + \delta_1 I_{t-1} + \mathbf{x}_{t-1}'\boldsymbol{\beta}, \quad (7)$$

respectively. By recursive substitution, and assuming $|\alpha_1| < 1$, the latter model can be rewritten as

$$\begin{aligned} \pi_t^I &= \sum_{i=1}^{\infty} \alpha_1^{i-1} \omega + \delta_1 \sum_{i=1}^{\infty} \alpha_1^{i-1} I_{t-i} \\ &\quad + \sum_{i=1}^{\infty} \alpha_1^{i-1} \mathbf{x}_{t-i}'\boldsymbol{\beta}. \end{aligned} \quad (8)$$

Therefore, if several lagged values of the stock indicator (Eq. (2)) or the explanatory variables \mathbf{x}_t are useful for forecasting, the autoregressive specifications of Eqs. (6) and (7) could be useful parsimonious forecasting models.

The parameters of the probit models (4)–(7), as well as the case of the new model presented in the next section, can be estimated by the method of maximum likelihood, as described by [de Jong and Woutersen \(in press\)](#) and [Kauppi and Saikkonen \(2008\)](#).

¹ The superscript “I” in π_t^I refers to excess stock return sign forecasting.

² This paper uses the same model attributes as [Kauppi and Saikkonen \(2008\)](#).

2.2. An error correction model

Based on the principles of the efficient market theory, for example, the lagged values of the stock indicator (Eq. (2)) should not have predictive power for predicting future market directions. This indicates that the estimated coefficient of the lagged return indicator δ_1 may be zero or close to zero. Therefore, in the dynamic autoregressive model (Eq. (7)), if $\delta_1 = 0$ and there are no explanatory variables in the model (that is, $\beta = 0$), then the autoregressive parameter α_1 is not identified as in Eq. (8) by imposing the above-mentioned restrictions.³ Even if the coefficient δ_1 is just close to zero, there is a potential identification problem that can affect the parameter estimation and have implications for the forecasting accuracy of excess return sign predictions.

Imposing the restriction $\delta_1 = 1 - \alpha_1$ in the unrestricted dynamic autoregressive model (7) and assuming $|\alpha_1| < 1$, a new “restricted” dynamic autoregressive model can be formulated as

$$\pi_t^I = \omega + \alpha_1 \pi_{t-1}^I + (1 - \alpha_1) I_{t-1} + x'_{t-1} \beta. \quad (9)$$

Because of the assumption $|\alpha_1| < 1$, the coefficient for the lagged return indicator I_{t-1} , $1 - \alpha_1$, is always positive. In model (9), α_1 can also be interpreted as a “weight” between π_{t-1}^I and I_{t-1} . It is expected that α_1 will be positive and quite high in our application (since $\delta_1 \approx 0$). This leads to the fact that the predictive power is distributed over the longer history of I_t and the explanatory variables x_t . On the other hand, if α_1 is “small”, then the first lag I_{t-1} is more useful than in the case of a higher value of α_1 .

For simplicity, we refer to model (9) in this study as an “error correction” model (ecm). This is because, by adding $-\pi_{t-1}^I$ to both sides of Eq. (9), we obtain the error correction form

$$\Delta \pi_t^I = \omega + (1 - \alpha_1)(I_{t-1} - \pi_{t-1}^I) + x'_{t-1} \beta, \quad (10)$$

where $\Delta \pi_t^I = \pi_t^I - \pi_{t-1}^I$. Thus, the difference between I_{t-1} and π_{t-1}^I measures the long-run relationship between the value of the stock return indicator

and the transformed probability $\pi_{t-1}^I = \Phi^{-1}(p_{t-1}^I)$ in the probit model. Rewriting model (10) as

$$\pi_t^I = \omega + \pi_{t-1}^I + (1 - \alpha_1)(I_{t-1} - \pi_{t-1}^I) + x'_{t-1} \beta,$$

it can be seen that, for values of α_1 close to one, the model can be expected to exhibit “near unit root” behavior, implying a rather strong persistence of the variable π_t^I . This means that the conditional probability of a positive excess stock return does not change much between successive time periods.

As can be seen from Eq. (9), the parameter $1 - \alpha_1$ is always positive, and can be interpreted as the proportion of the disequilibrium between I_{t-1} and π_{t-1}^I in period $t - 1$. A positive value of the error correction term $(I_{t-1} - \pi_{t-1}^I)$ increases the probability of a positive excess stock return in the following period, and, of course, vice versa if the error correction term is negative. Later on, we will see that the conditional probability of positive excess stock returns, p_t^I , is typically close to 0.50 in most models, which means that π_{t-1}^I is close to zero.

It is worth noting that the error correction model (9) is closely related to the autoregressive conditional multinomial model (ACM) suggested by Russell and Engle (2005). In their model, the term $(I_{t-1} - \pi_{t-1}^I)$ is replaced by $(I_{t-1} - \Phi(\pi_{t-1}^I))$. Model (9) without the term $x'_{t-1} \beta$ is also similar to the IGARCH model, which was suggested by Engle and Bollerslev (1986) for dealing with conditional heteroskedasticity in models of continuous variables.

2.3. Recession forecast as an explanatory variable

A novel idea in this paper is studying whether recession forecasts have any explanatory power for forecasting the direction of excess stock returns. The empirical finance literature shows that, as forward-looking variables, lagged stock returns should provide information about the future evolution of economic activity and potential recession periods (see, e.g., Estrella & Mishkin, 1998; Nyberg, 2010; Pesaran & Timmermann, 1995). Therefore, if the expectations of future economic activity are correct, movements in the stock market should lead movements in economic activity (see, e.g., Fama 1990). Theoretically, this relationship can be justified by either present value or discounted-cash-flow models, where the price of a stock is equal to expected future dividends, which are

³ Note that there is no such identification problem when the explanatory variables x_{t-1} are included in the model and $\beta \neq 0$, even though $\delta_1 = 0$.

assumed to be related to the future economic activity and profitability of firms.

Our main goal is to forecast periods of recession and to use the potential explanatory power of the recession forecasts thus obtained in order to make better forecasts for the sign of the excess stock returns. This is done by using the binary recession indicator

$$y_t = \begin{cases} 1, & \text{if the economy is in a recession} \\ & \text{at month } t; \\ 0, & \text{if the economy is in an expansion} \\ & \text{at month } t. \end{cases} \quad (11)$$

In this study, the recession dates defined by the NBER are used. As [Chauvet and Potter \(2000\)](#) argue, one feature, but also a potential problem, of the NBER recession dates is that they do not reflect short-lived contraction periods in the economy, which could have a notable explanatory power for predicting excess stock returns. Furthermore, [Chauvet and Potter \(2000\)](#) also construct transition probabilities of the “bear” and “bull” states of the stock market using Markov chain methods. They find that bear markets generally start a couple of months before an economic slowdown or recession period and end some months before the recession period ends. Thus, it seems evident that movements in the stock market should lead the business cycle, and evidence of this can be seen in [Fig. 1](#).⁴ US excess stock returns are often negative before a recession period begins. On the other hand, the returns seem to be positive in the last few recession months, indicating expectations of a recovery in economic activity.

For example, according to this idea, in the general dynamic autoregressive model (7), the vector $\mathbf{x}_{t-1} = (p_{t+5}^y \ \tilde{\mathbf{x}}_{t-1})'$ (where $\tilde{\mathbf{x}}_{t-1}$ contains other financial explanatory variables) may include the estimated recession forecast p_{t+5}^y , constructed using model (12). Therefore, a predictive probit model contains the fitted values of the binary explanatory recession indicator (Eq. (11)) (cf. [Maddala, 1983](#), pp. 122–123). Parameter estimation and forecasting are carried out using a two-step procedure, where the recession and stock return sign prediction models are estimated separately. It is worth noting, however, that the usual asymptotic distribution of the maximum likelihood

estimate may not apply in this kind of model, because the recession probability forecast included in the model is based on the estimated model (cf. [Pagan, 1984](#)).

In this study, the forecast horizon for recession forecasting is assumed to be six months. A six-month recession forecast for the value of the recession indicator (Eq. (11)) at time $t + 5$, based on the information set Ω_{t-1} at time $t - 1$, is the conditional probability

$$E_{t-1}(y_{t+5}) = P_{t-1}(y_{t+5} = 1) = \Phi(\pi_{t+5}^y) = p_{t+5}^y.$$

In recession forecasting, an autoregressive probit model

$$\pi_{t+5}^y = c + \phi \pi_{t+4}^y + \mathbf{z}_{t-1}' \mathbf{b} \quad (12)$$

is employed where, based on the findings of the recession forecasting literature, the domestic and foreign term spreads and lagged nominal stock returns are used as predictors. The values of these variables are included in the vector \mathbf{z}_{t-1} , that is

$$\mathbf{z}_{t-1} = (SP_{t-1}^{US} \ r_{t-1}^n \ SP_{t-1}^{GE})'. \quad (13)$$

The usefulness of the domestic term spread (SP_t^{US}), defined as the spread between the long-term and short-term interest rates, for predicting recession periods has been demonstrated in many studies (see, among others, [Estrella, 2005](#); [Estrella & Mishkin, 1998](#)). Using dynamic probit models, [Nyberg \(2010\)](#) also suggested that the foreign term spread (SP_t^{GE} , the term spread of Germany) and stock market returns (r_t^n) can be used to forecast coming recession periods (see also, e.g., [Bernard & Gerlach, 1998](#); [Estrella & Mishkin, 1998](#)).⁵ In that study, the recession forecasts obtained were quite accurate for at least six months ahead, and the probit models with the autoregressive structure (see model (12)) yielded the best out-of-sample forecasts of the various probit models used. Therefore, in this study, we take model (12) and the six-month forecast horizon as a given.

One advantage of model (12) is that it does not contain a lagged value of the recession indicator (Eq. (11)). It is important to take into account the fact

⁴ Details of the dataset are given in Section 4.1.

⁵ Further information on the explanatory variables is given in Table 1.

Table 1
Data set of dependent and explanatory variables.

Variable	Description
P_t	Standard&Poor's 500 US stock index
P_t^S	CRSP small size firms index, first decile
P_t^L	CRSP large size firms index, tenth decile
r_t, r_t^S, r_t^L	One-month excess return over the risk-free return (see Eq. (18))
r_t^n	One-month nominal stock return from the S&P500 index
y_t	US recession periods (NBER)
i_t	Three-month US Treasury Bill rate, secondary market
R_t	10-year US Treasury Bond rate, constant maturity
$\Delta i_t, \Delta R_t$	First differences of i_t and R_t
SP_t^{US}	US term spread between R_t and i_t
SP_t^{GE}	German term spread between German long- and short-term interest rates
σ_t	Sum of squared daily stock returns in the S&P500 index within one month
DP_t	Dividends over the past year divided by the current stock index value, $DP_t = D_t/P_t$
EP_t	Earnings over the past year divided by the current stock index value, $EP_t = E_t/P_t$

Notes: The sample period is 1968M1–2006M12. Monthly and daily S&P500 index series are taken from <http://finance.yahoo.com> and <http://www.econstats.com>. Size-sorted CRSP indices are obtained from the Kenneth French Data Library (http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html). Interest rates are from <http://www.federalreserve.gov/releases/h15/data.htm>. The German term spread is constructed as the difference between the 10-year Federal security (series WZ9826, with the missing values between 1971M1–1972M9 replaced by the OECD 10-year interest rate) and the three-month money market rate (series su0107, see <http://www.bundesbank.de/statistik/statistik>). Data for log-dividends D_t and log-earnings E_t were obtained from the homepage of Robert Shiller's book *Irrational exuberance* (<http://www.irrationalexuberance.com>) [January 2009].

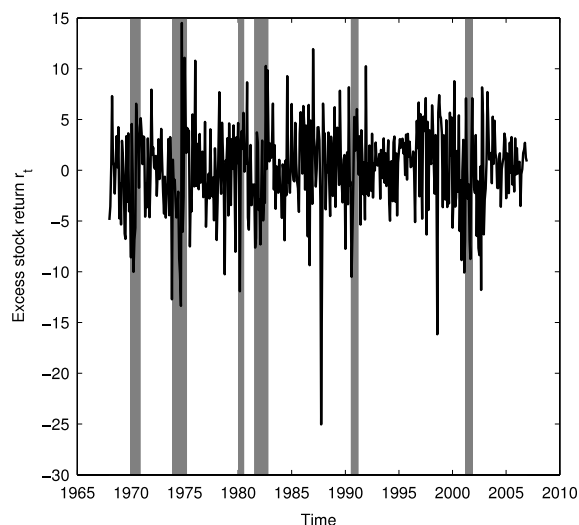


Fig. 1. US excess stock returns r_t and the NBER recession periods y_t (shaded areas) for a sample period from 1968M1 to 2006M12.

that it is several months before, for example, NBER can be sure what the state of the economy really is. Hence, the values of the recession indicator are known with a considerable delay, which indicates that it will be computationally easier to construct recession

forecasts without using the lagged values of the binary recession indicator (Eq. (11)) in an estimated model for stock return signs. This is based on the fact that no multiperiod iterative forecasts (see Kauppi & Saikkonen, 2008) for the recession indicator are made in model (12), because all predictive power comes from the explanatory variables employed, z_t . Thus, it is not necessary to specify the assumed “publication lag” in the known values of y_t exactly.

3. Evaluation of forecasts

3.1. Statistical and economic goodness-of-fit measures

Both the in-sample and out-of-sample performances of the predictive models are evaluated using various frequently used goodness-of-fit measures. One of these is Estrella's (1998) pseudo- R^2 measure

$$psR^2 = 1 - (\hat{l}_u/\hat{l}_c)^{-(2/T)\hat{l}_c}, \quad (14)$$

where \hat{l}_u is the maximum value of the estimated unconstrained log-likelihood function and \hat{l}_c is its constrained counterpart in a model which only

contains a constant term. This measure takes on values between 0 and 1, and can be interpreted in the same way as the coefficient of determination in linear models. The value of the maximized log-likelihood function also enables the comparison of model performances using model selection criteria such as the Schwarz information criterion *BIC* (Schwarz, 1978).

The binary nature of the dependent variable leads to the question of what the percentage of correct “matches” is for the realized values and the forecasts of the stock indicator. This ratio is denoted by *CR*. According to the hypothesis of no predictability in excess stock return signs, the estimated value of *CR* should be close to 0.50, which means that the model employed will be unable to forecast the future market directions correctly. It is desirable to specify a threshold value that translates the probability forecasts into forecasting signals. The most commonly used and natural threshold choice is 0.50, and this is what is used here.

For financial analysts and investors, the most important model evaluation criterion is the return on their investment. There are many different kinds of trading strategies that can be applied. Here, a simple trading simulation similar to that of Leung et al. (2000) is used. At the beginning of each month, the investor makes an asset allocation decision. She can shift her assets into either stocks or risk-free Treasury Bills, and the money that has been invested in either of these alternatives remains there until the next decision date. In this trading strategy, the mentioned 50% threshold value is used. Then, the portfolio consists of the interest rate investment in Treasury Bills ($I_t^f = 0$), if $p_t^I \leq 0.50$, and stocks ($I_t^f = 1$), if $p_t^I > 0.50$. Here, the superscript *f* indicates a forecast.⁶

In this trading simulation, transaction costs are also taken into account. Following Granger and Pesaran (2000), the marginal cost of transactions for asset allocation changes between stocks and interest rates will be denoted by ζ_s and ζ_b , respectively. This means that every time the asset allocation changes, the cost of the transaction is subtracted from the final investment

return. In this paper, the “low cost scenario” suggested by Pesaran and Timmermann (1995), where $\zeta_s = 0.005$ and $\zeta_b = 0.001$, is applied. For example, when the risk-free interest rate investments are switched to stocks, 0.50% of the whole amount of the portfolio value is lost.

As Granger and Pesaran (2000) have shown, it is possible to form non-constant “payoff” probability ratios of switches between stocks and interest rates as an alternative to this 50% threshold. However, in this study, these payoff ratios are not very useful, because, according to these threshold ratios, the asset allocation decision is to stick to stocks almost all of the time. Therefore, probability forecasts, even if accurate, have little economic value.

When considering the predictability of excess stock return signs with different trading rules, one important evaluation criterion is the overall portfolio return, denoted by *RET*. Nevertheless, as Hong and Chung (2003) emphasize, it is also worth considering risk-adjusted returns, as different trading rules involve different levels of risk. One common measure which we use in this evaluation is the Sharpe ratio (Sharpe, 1966, 1994):

$$SR = \frac{\overline{RET}^k - \overline{RET}^{rf}}{\hat{\sigma}^k}, \quad (15)$$

where \overline{RET}^k is the average portfolio return based on the model and the trading rule *k*, \overline{RET}^{rf} is the average risk-free portfolio return (bond investment strategy), and $\hat{\sigma}^k$ is the sample standard deviation of portfolio returns RET^k . The higher the Sharpe ratio is, the higher the return and the lower the volatility. Portfolios with a high Sharpe ratio are preferable to those with a low Sharpe ratio.

3.2. Testing the statistical predictability

For the evaluation of the directional forecasting performance and market timing, a test proposed by Pesaran and Timmermann (1992) is used. The null hypothesis is that the value of the correct prediction ratio, *CR*, does not differ statistically significantly from the ratio that would be obtained in the case of no predictability, where the forecasts and the realized values of the return indicator I_t are independent. Granger and Pesaran (2000) show that the market

⁶ One alternative method of parameter estimation and determining the cutoff threshold, which is assumed to be 50% above, is the maximum utility estimation method proposed by Elliott and Lieli (2007).

timing test can be based on the test statistic

$$PT = \frac{\sqrt{m}KS}{\left(\frac{\bar{P}_I(1-\bar{P}_I)}{\bar{I}(1-\bar{I})}\right)^{1/2}}. \quad (16)$$

Here, KS is the Kuipers score $KS = HR - FR$ between the “hit rate”

$$HR = \frac{\hat{I}^{uu}}{\hat{I}^{uu} + \hat{I}^{du}}$$

and the “false rate”

$$FR = \frac{\hat{I}^{ud}}{\hat{I}^{ud} + \hat{I}^{dd}},$$

where the forecast classification is denoted by

$$\hat{I}^{uu} = \sum_{t=1}^m \mathbf{1}(I_t^f = 1, I_t = 1),$$

$$\hat{I}^{ud} = \sum_{t=1}^m \mathbf{1}(I_t^f = 1, I_t = 0),$$

$$\hat{I}^{du} = \sum_{t=1}^m \mathbf{1}(I_t^f = 0, I_t = 1),$$

$$\hat{I}^{dd} = \sum_{t=1}^m \mathbf{1}(I_t^f = 0, I_t = 0),$$

where f refers to a forecast, u is an “up” signal ($I_t = 1$) and d a “down” signal ($I_t = 0$), and $\mathbf{1}(\cdot)$ is an indicator function. Furthermore, in the test statistic in Eq. (16), \bar{I} is the sample average of the sign indicator I_t values in the m -month sample period and $\bar{P}_I = \bar{I}HR + (1 - \bar{I})FR$. Under the null hypothesis of no predictability, the PT test statistic has an asymptotic standard normal distribution.

However, the directional predictability of an underlying data generating process is not the same thing as a successful trading strategy. To evaluate the forecasts of the best forecasting models, we test the significance of the differences between the investment returns on the best models and trading strategies. This is tested by means of the Diebold-Mariano test (Diebold & Mariano, 1995). Because the forecast horizon is one month ($h = 1$), the test statistic is

$$DM = \frac{\sqrt{m}\bar{d}}{\sqrt{\text{var}(\bar{d})}}, \quad (17)$$

where \bar{d} is the average difference between the predicted excess returns of the models considered. As with the PT test, under the null hypothesis of equal forecast accuracy, the DM statistic also has an asymptotic $N(0, 1)$ distribution.

4. Empirical results

4.1. Data and previous findings

The monthly data set contains various financial variables which have previously been used to predict the overall level and the direction of excess stock returns in the literature. The data set covers the period from January 1968 to December 2006, and is obtained from the sources mentioned in Table 1. The first 12 observations are used as initial values, and the total number of observations, T , is 468. For out-of-sample forecasting, the data set is divided into two subsamples: the estimation and forecasting samples. The first out-of-sample forecasts will be made for January 1989 and the last for December 2006.

For out-of-sample forecasting, the parameters are estimated recursively using an expanding window of observations, where the models are estimated using data from the start of the data set through to the present time to obtain a new one-period-ahead forecast. This procedure is repeated until the end of the forecasting sample is reached. The use of an alternative rolling estimation window is problematic, because there are not many recession periods in the post-1970 time period, and therefore there would be estimation samples with no recession period in them at all.

The one-month excess stock return is defined as the continuously compounded return of the price index P_t minus the risk-free interest rate rf_t :

$$r_t = 100 \log \left(\frac{P_t}{P_{t-1}} \right) - rf_t. \quad (18)$$

Here, P_t is the value of the S&P500 stock index, and the one-month risk-free return rf_t is approximated by the three-month US Treasury Bill rate i_t . Using excess stock returns r_t , the values of the binary stock return indicator described in Eq. (2) can be constructed.

Several explanatory variables for forecasting the direction of excess stock returns will be considered.

As was confirmed by Leung et al. (2000), the majority of useful information for forecasting stock returns is contained in the interest rates and lagged stock returns. Hence, the financial explanatory variables that are considered in the predictive models are the short-term and long-term interest rates and their first differences, the US term spread, earnings/price and dividends/price variables, and the realized volatility (see Table 1).

In previous studies, both the lagged excess stock returns and the lagged values of the return indicator (Eq. (2)) have been used as predictors. Leung et al. (2000) used the first differences of interest rates and lagged excess stock returns in their comparison between the sign and the overall return forecasting models, and concluded that several past returns should be included in probit and logit models. If the explanatory power is distributed among many lags of past returns, then the autoregressive models in Eqs. (6) and (7) could be useful for forecasting. Anatolyev and Gospodinov (2010) used the lagged sign return indicator I_{t-1} in their dynamic logit model for the direction of the future excess stock return. The corresponding estimated regression coefficient was positive, but not statistically significant.

Interest rate spreads between data of different maturities may offer information about future expectations in financial markets (see, for instance, Fama & French, 1989). In recession forecasting, the term spread (SP_t^{US}) is expected to transmit the expectations for future monetary policy. The smaller the difference between the long-term and short-term interest rates, the more restrictive the current monetary policy. The term spread could also have its own impact on the stock market, not on the real economic activity only.

Dividends (D_t) and earnings (E_t), divided by the value of the price index (P_t), have been among the most commonly used explanatory variables (see, e.g., Campbell & Shiller, 1988; Cochrane, 1997). The dividend-price (earnings-price) ratio is computed using the dividends (earnings) of S&P500 stock index companies over the past year. Since the monthly dividends and earnings data are not available, DP_t and EP_t are constructed as the sums of dividends and earnings over the past year, divided by the current, monthly price level P_t .

Numerous studies have also documented a notable dependence of stock return and stock return volatility, with important implications for asset pricing. The

realized monthly volatility (σ_t), based on the sum of squared daily observations within one month (see Christoffersen et al., 2007), is also examined as a predictor in probit models.

4.2. In-sample results

Even though our main interest lies in the out-of-sample prediction of the direction of future excess stock returns, the in-sample performances of different probit models and combinations of explanatory variables were first examined using the sample period from January 1968 until December 1988. The main objective was to find the best predictive variables to be used in out-of-sample forecasting in Section 4.3. In the following analysis, explanatory variables are included in the model one by one. In particular, we are interested in models containing the recession forecast as a predictor of the stock return sign.

The main results and findings are as follows. According to the psR^2 and CR values, and the returns on the trading strategies in the chosen in-sample period, the recession forecast and the first difference of the short-term interest rate are the best predictive variables. When these variables are employed, there seems to be evidence that the excess stock return signs are predictable in-sample. The first difference of the long-term interest rate and the realized volatility also have some predictive power. Interestingly, the corporate earning and dividend variables, which have been used in many previous studies, are not particularly useful predictors.⁷ When the recession forecast is employed with different financial explanatory variables in the model equation, the evidence is very much the same as that above. The first difference of the short-term interest rate appears to be the best predictor for the recession forecast in-sample.

Table 2 presents details of the parameter estimates in different probit models when the recession forecast (p_{t+5}^y) and the first difference of the short-term interest rate (Δi_{t-1}) are used as explanatory variables. The robust standard errors suggested by Kauppi and Saikkonen (2008) are also presented. However, it should be noted that these standard errors may be

⁷ Further information on the in-sample performances of the different models is available upon request.

Table 2
Estimation results of in-sample predictive models.

	Static model (4)	Dynamic model (5)	Auto. model (6)	Dyn. auto. model (7)	ecm model (9)
Constant	0.07 (0.10)	0.02 (0.12)	0.07 (0.09)	0.03 (0.13)	−0.07 (0.03)
π_{t-1}^I			0.04 (0.24)	−0.03 (0.28)	0.85 (0.05)
I_{t-1}		0.08 (0.11)		0.08 (0.19)	
Δi_{t-1}	−0.24 (0.04)	−0.30 (0.12)	−0.30 (0.12)	−0.29 (0.12)	−0.16 (0.07)
p_{t+5}^y	−0.50 (0.25)	−0.47 (0.25)	−0.49 (0.24)	−0.49 (0.25)	−0.02 (0.05)
Log-L	−161.33	−161.22	−160.72	−160.64	−162.10
psR^2	0.041	0.041	0.046	0.046	0.034
<i>BIC</i>	169.55	172.18	171.68	174.34	173.06
<i>CR</i>	0.580	0.591	0.579	0.579	0.579
<i>RET</i>	10.50	9.98	9.88	9.57	8.12
<i>SR</i>	0.95	0.86	0.79	0.72	0.25
<i>PT</i>	0.003	0.002	0.006	0.006	0.008
<i>DM</i>	0.009	0.017	0.017	0.023	0.123
DM_{ra}	0.000	0.000	0.000	0.000	0.003
$DM_{I_t=0}^{I_t=0}$	0.000	0.000	0.000	0.000	0.000
$DM_{ra}^{I_t=0}$	0.000	0.000	0.000	0.000	0.000

Notes: The models are estimated using the in-sample data from 1969M1 to 1988M12. Robust standard errors, given in parentheses, are computed using the procedures suggested by Kauppi and Saikkonen (2008). *RET* is the average annualized in-sample portfolio return in the considered model and *SR* is the corresponding Sharpe ratio (Eq. (15)). The in-sample return in the B&H strategy *RET* is 3.60%. The corresponding Sharpe ratio is negative because the average return in the pure risk-free interest rate investment strategy is higher ($\overline{RET}^{rf} = 7.16\%$). The *p*-values of the market timing test (Eq. (16)) and the Diebold and Mariano (1995) test (Eq. (17)) are reported. In the *DM* tests, the buy-and-hold trading strategy is the benchmark. Furthermore, *ra* means the risk-adjusted returns, where the average return is standardized by the standard deviation of returns. The values of the test statistics $DM_{I_t=0}^{I_t=0}$ and $DM_{ra}^{I_t=0}$ are obtained when only months with negative excess stock returns ($I_t = 0$) are considered.

inaccurate because the estimated recession forecast is employed in the model.⁸

It seems that the lagged stock indicator I_{t-1} has no statistically significant predictive ability for the sign of the stock return. The autoregressive coefficient π_{t-1}^I is clearly statistically significant in the error correction model, but in other dynamic models it is not. As was expected, the estimated coefficients of the first difference of the short-term interest rate and the recession forecast are negative. In this case, the recession forecast is not statistically significant in the error correction model, but the first difference of the short-term interest rate is. In the other models, both of

these predictors are statistically significant according to the robust standard errors presented.

Fig. 2 depicts the estimated probability of a positive excess stock return in the static model (4) and the error correction model (9), whose estimation results are shown in Table 2 (the first and fifth models). The two models seem to give much the same in-sample predictions. In recession periods, both models suggest investing in a risk-free interest rate. More or less the only significant difference between the models is in the time period from approximately 1976 to 1979. At that time, the probability forecast in the static model is typically above the 0.50 threshold value, while in the error correction model it is below the threshold.

As the psR^2 values in Table 2 indicate, the statistical predictive power for the sign of the excess stock return is, as expected, quite low. Although the statistical

⁸ In addition, no formal proofs of the asymptotic distributions of the maximum likelihood estimator in models (6), (7) and (9) are available at present.

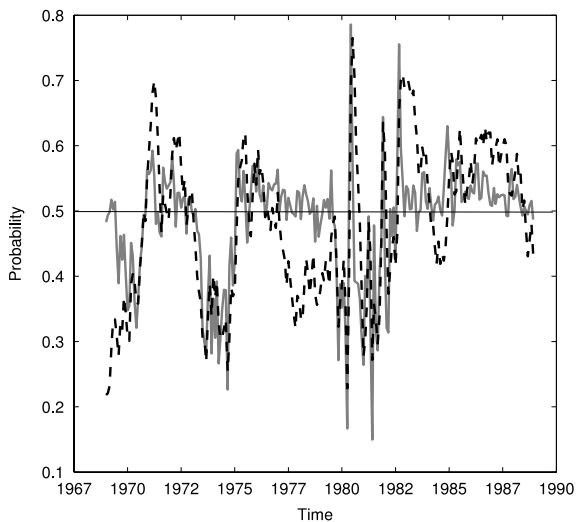


Fig. 2. In-sample probabilities p_t^I (see Eq. (3)) of the static model (4) (solid line) and the error correction model (9) (dashed line). The 50% threshold is also shown.

predictability is weak, the portfolio investment performance yields evidence of a useful degree of sign predictability in excess stock returns. The average rates of return for the different models and trading strategies are higher than in the “passive” buy-and-hold trading strategy (hereafter the B&H strategy), where one is investing only in stocks. This annualized benchmark return is 3.60%. In the best models, the returns, including the transaction costs, are between 9.50% and 10.50%. The presented error correction probit model seems to yield smaller in-sample returns than its counterparts.

The statistical significance of return differences between an examined model and B&H returns was tested. Table 2 presents the values of the statistical test statistics introduced in Section 3.2. Since we are only interested in cases where the proportion of correctly predicted signs and the portfolio returns in the estimated models are higher than under the null hypothesis of no predictability, only the positive and statistically significant values of the PT and DM test statistics (see Eqs. (16) and (17)) provide evidence of predictability. The values of the market timing test statistic PT are statistically significant at the 1% level under all models examined in Table 2. Thus, the null hypothesis of no predictability is rejected, providing in-sample evidence that excess return signs are predictable. In the DM tests, the null hypothesis

of equal performance between the returns in the model considered and those from the B&H strategy is rejected for all models at the 5% level, except for the error correction model, where the p -value of the test statistic is 0.123. When risk-adjusted returns are considered, the DM_{ra} test statistics are statistically significant for all models, providing evidence of profitable trading strategies based on the forecasts from the probit models.

Although the unrestricted dynamic autoregressive probit model (7) gives a better in-sample fit than the error correction model in the models presented in Table 2, in some other models with different explanatory variables the error correction model gives higher psR^2 and CR values. In contrast to the error correction model (9), in many other unrestricted dynamic autoregressive models, the autoregressive coefficient α_1 is typically negative. Thus, if the probability of a positive excess stock return has been high in some period, it tends to be lower in the next period. As an example, consider a model in which the first difference of the short-term interest rate (Δi_{t-1}) is the only explanatory variable in x_{t-1} . Fig. 3 shows the estimated probabilities of positive excess stock returns in the unrestricted dynamic autoregressive model (7) and in the error correction model (9) in this example case. As the estimate of the autoregressive coefficient α_1 is negative in model (7), the probability of excess stock returns fluctuates heavily around the threshold value 0.50. On the other hand, with a high, positive estimate of α_1 , the probability forecasts follow a relatively persistent swing. Therefore, it seems that the error correction model yields fewer transactions between stocks and bonds, and therefore also lower transaction costs, than the unrestricted dynamic autoregressive model. This is particularly striking in the models presented in Fig. 3, and could be an important property in out-of-sample forecasting.

4.3. Out-of-sample results

When forecasting the signs of excess stock returns, it is important to compare the different models out-of-sample. Previous results on predictive models for overall excess stock returns suggest that in-sample predictability does not necessarily imply out-of-sample predictability. For example, Han (2007) found that a VAR-GARCH model which had a statistically

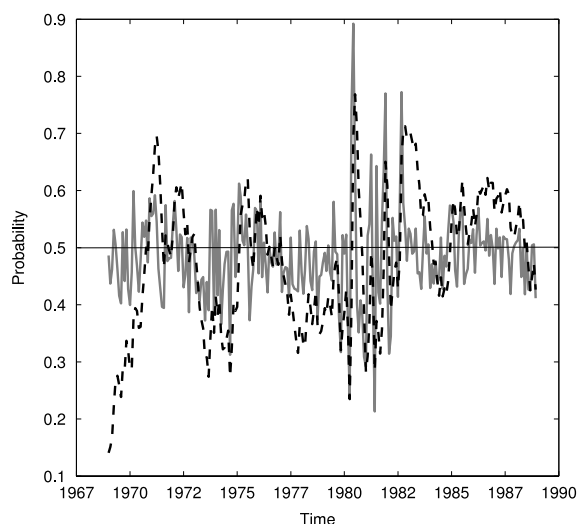


Fig. 3. In-sample predictions of the dynamic autoregressive model (7) (solid line) and the error correction model (9) (dashed line) when Δi_{t-1} is the only predictive variable.

superior predictive performance in-sample did not consistently outperform its competitors in terms of portfolio investment returns out-of-sample. Goyal and Welch (2008) argue that traditional predictive models for the excess return cannot beat the historical average return out-of-sample and that there is no single variable that has theoretically meaningful and robust explanatory power. On the other hand, Campbell and Thompson (2008) show that some predictors perform better than the historical average when restrictions are imposed on the regression coefficients. They and Anatolyev and Gospodinov (2010), among others, have stressed that while the out-of-sample predictive power is small, it can still be utilized in market timing decisions to earn economically higher excess returns than the B&H strategy, even out-of-sample.

In this study, the out-of-sample period consists of 216 months from January 1989 to December 2006. The out-of-sample recession forecast p_{t+5}^y is constructed before making any forecasts of excess return signs. As was described in Section 4.1, the parameters in the recession prediction and sign models are estimated recursively. Table 3 shows the out-of-sample performances of the best in-sample predictive models, and also of some other probit models. The idea is to compare the predictive performances of different models when the same combinations of explanatory

Table 3

Out-of-sample performances of the different probit models.

Model	x_{t-1}	psR^2	CR	RET	SR
B&H				8.07	0.79
Static (4)	p_{t+5}^y	0.015	0.593	7.63	0.75
Static (4)	$\Delta R_{t-1}, p_{t+5}^y$	0.013	0.588	8.02	0.86
Static (4)	$\Delta i_{t-1}, p_{t+5}^y$	0.014	0.588	7.02	0.61
Dynamic (5)	–	neg.	0.528	3.92	neg.
Dynamic (5)	p_{t+5}^y	0.005	0.569	6.16	0.46
Dynamic (5)	$\Delta R_{t-1}, p_{t+5}^y$	0.002	0.574	7.32	0.76
Dynamic (5)	$\Delta i_{t-1}, p_{t+5}^y$	0.004	0.579	7.46	0.79
Auto (6)	p_{t+5}^y	neg.	0.514	2.30	neg.
Auto (6)	$\Delta R_{t-1}, p_{t+5}^y$	0.015	0.583	7.20	0.68
Auto (6)	$\Delta i_{t-1}, p_{t+5}^y$	0.014	0.579	6.53	0.50
Dyn. auto (7)	–	neg.	0.514	4.11	neg.
Dyn. auto (7)	p_{t+5}^y	0.008	0.574	5.42	0.28
Dyn. auto (7)	$\Delta R_{t-1}, p_{t+5}^y$	0.006	0.565	7.03	0.63
Dyn. auto (7)	$\Delta i_{t-1}, p_{t+5}^y$	0.011	0.565	5.46	0.27
ecm (9)	–	0.014	0.588	8.62	0.97
ecm (9)	p_{t+5}^y	0.018	0.606	10.33	1.46
ecm (9)	$\Delta R_{t-1}, p_{t+5}^y$	0.016	0.588	9.09	1.12
ecm (9)	$\Delta i_{t-1}, p_{t+5}^y$	0.017	0.588	9.78	1.28

Notes: See also the notes to Table 2. The average return of the buy-and-hold trading strategy (B&H) is 8.07% (annual), with a corresponding Sharpe ratio of $SR = 0.79$. The risk-free return on interest rate investments is 4.21%. “neg.” means a negative psR^2 value and “–” in x_{t-1} indicates that there are no explanatory variables in the model. Note that the transaction costs are also taken into account in RET.

variables, which turned out to be the best out-of-sample predictive variables, are examined.⁹

According to commonly used statistical model evaluation measures, there is not much out-of-sample predictability in excess stock return signs. The values of the out-of-sample psR^2 measures (see Eq. (14)) are small or even negative, even in the best models.¹⁰ The percentage of correct forecasts, CR, varies between 0.51 and 0.61. Contrary to the statistical measures employed, the results of the portfolio returns RET and the Sharpe ratios SR exhibit evidence of useful predictability for asset allocation decisions, even

⁹ The models with other financial variables, presented in Section 4.1, were also considered, and the results are available upon request.

¹⁰ A negative psR^2 value means a very poor out-of-sample forecasting performance (Estrella, 1998).

though the average portfolio returns vary strongly between different models. As in the in-sample evidence, the models with recession forecasts generate better sign forecasts than the models without these forecasts. It is worth noting that the sign prediction models containing the constructed recession forecast outperform the models including the variables used in recession forecasting (see Eq. (13)), especially out-of-sample.

It is interesting that the error correction model (9) clearly outperforms the corresponding unrestricted dynamic autoregressive model (7) out-of-sample. As can be seen from the results in Table 2, the autoregressive model (6) and the dynamic autoregressive model (7) outperform the error correction model (9) in-sample, but the out-of-sample evidence seems to be very different. In Table 3, the psR^2 values of the error correction models are clearly positive, and the ratios of correct predictions, CR , are higher than in the other probit models considered. Above all, the error correction models generate more profitable trading strategies than the other probit models. Perhaps the most striking finding is the performance of the model with no explanatory variables (“–” in Table 3). The psR^2 values, CR ratios, average excess returns, and Sharpe ratios are clearly higher in the error correction model. A possible explanation for this superior out-of-sample performance of the error correction model could be the potential identification problem in the dynamic autoregressive model (7) discussed in Section 2.2.

Overall, compared with the dynamic models (5)–(7), the static probit model (4), without the autoregressive model structure π_{t-1}^I or the lagged I_{t-1} , seems to be an adequate model for the excess stock return sign. The error correction model (9) appears to be the only dynamic model which yields better forecasts than the static model in this data set.

The recession forecast is the main predictive variable in different models. The first differences of the short-term and long-term interest rates are also fairly good predictors in almost all probit models, and perform consistently better than the other financial explanatory variables examined. For instance, the realized volatility σ_t was quite a good predictive variable in-sample, but its out-of-sample performance in probit models is very poor.

Fig. 4 depicts the out-of-sample probability forecasts of the positive excess stock returns from two

models, which are presented in Table 4. The most notable difference is that in 2001–2003 the error correction model gives a signal to invest in the risk-free interest rate when the monthly stock returns are negative most of the time. Furthermore, Table 4 shows the values of the PT and DM test statistics in these best error correction and static models, in terms of investment return (RET). In the error correction model, the p -value of the PT test statistic is 0.053 and that of the DM test statistic is 0.121. The risk-adjusted returns are statistically significantly higher than the returns from the B&H strategy (p -value = 0.000). On the other hand, the p -values of the test statistics in the best static probit model show that the excess stock return signs are not predictable with this model.

It should be pointed out that all models suggest investing in stocks most of the time. For instance, in Fig. 4, the conditional probabilities of a positive excess stock return are typically above the 0.50 threshold. Thus, the return differences between probit models and the B&H trading strategy are zero in most months. In addition, because the probability of recession is principally almost zero when the economy is in an expansionary state, the recession forecast should be a particularly useful predictor of negative excess stock return months when the economic activity is declining. Hence, the value of the DM test statistic is also calculated based on only those months when the excess stock returns have been non-positive (that is, $I_t = 0$). There are 86 months with a negative excess return in the out-of-sample period. In Table 4, the values of the test statistic $DM^{I_t=0}$, and $DM_{ra}^{I_t=0}$ in the case of risk-adjusted returns, are strongly statistically significant in the best models. The in-sample results in Table 2 are similar.

Furthermore, when the investment returns of the best error correction model and the best static model are compared, the p -values of the DM test statistic are 0.111 and 0.048, respectively, when the risk-adjusted returns are considered. Therefore, the error correction model yields higher returns, but the statistical significance between return differences is relatively weak in the out-of-sample period considered. However, according to the “asymmetric” DM test statistics discussed above ($DM^{I_t} = 0$ and $DM_{ra}^{I_t=0}$), the best error correction model outperforms the best static model at all traditional statistical significance levels.

Table 4

Statistical tests for the best error correction probit model and the best static probit model.

Model	x_{t-1}	CR	RET	SR	PT	DM	DM_{ra}	$DM^{I_t=0}$	$DM_{ra}^{I_t=0}$
ecm (9)	p_{t+5}^y	0.61	10.33	1.46	0.053	0.121	0.011	0.000	0.000
Static (4)	$p_{t+5}^y, \Delta R_{t-1}$	0.59	8.02	0.86	0.326	0.616	0.184	0.000	0.000

Notes: The best error correction model (9) (ecm) and the best static model (4), reported in Table 3 and depicted in Fig. 4, are presented. The p -values of the PT and DM tests are reported. In the DM tests the B&H trading strategy is the alternative asset allocation strategy. In the table, ra means the risk-adjusted returns and the test statistics $DM^{I_t=0}$ and $DM_{ra}^{I_t=0}$ are obtained when only months with negative excess stock returns ($I_t = 0$) are taken into account.

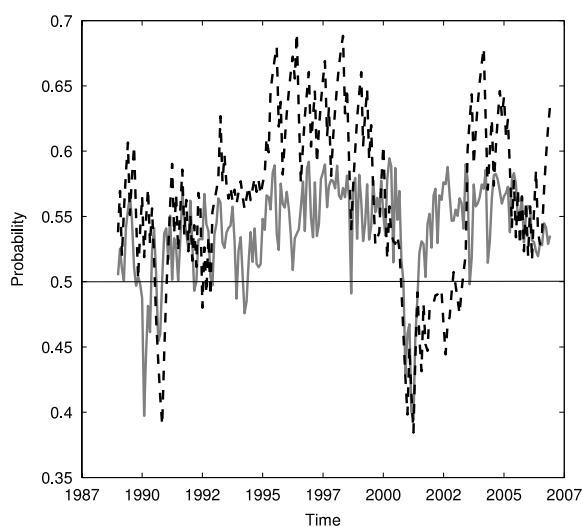


Fig. 4. Out-of-sample predictions of the static (4) model with ΔR_{t-1} and p_{t+5}^y ($RET = 8.02\%$, solid line). The dashed line is the error correction model (9) with p_{t+5}^y ($RET = 10.33\%$).

4.4. Comparison between probit and alternative predictive models

It is interesting to compare probit models with alternative models, such as ARMAX models and models where forecasts of the asset return volatility are employed to produce sign forecasts for excess stock returns. In fact, there are not many previous studies which have compared the predictive performances of these models. Leung et al. (2000) found some evidence that qualitative response models, including logit and probit models, outperform models for the continuous dependent variables in their out-of-sample forecasting. They considered a sample of US, UK and Japanese stock indices from January 1991 to December 1995. In their study, both the ratios of correct sign predictions and the investment returns are higher for

qualitative dependent models than for models for continuous variables.

ARMAX models consider the same explanatory variables as probit models. The dependent variable is the excess stock return r_t , and it is assumed that a positive forecast gives the signal to buy stocks (i.e. $I_t^f = 1$). This is consistent with the definition of the stock return indicator (Eq. (2)). As in probit models, the in-sample predictive performances of different ARMAX models are first analyzed.¹¹ The estimated values of the BIC model selection criterion (Schwarz, 1978) suggested an ARMAX(2, 0) model,¹² with the first difference of the short-term interest rate (Δi_{t-1}) and the recession forecast (p_{t+5}^y) as explanatory variables. An ARMAX(1, 0) model with the recession forecast and the US term spread (SP_t^{US}) generates the highest in-sample investment return. As with the probit models, the models which include the recession forecast outperform the models containing the variables used in recession forecasting (Table 5).

The out-of-sample forecasting performances of the best in-sample models and some other ARMAX models are shown in Table 5. On the whole, the percentage of correct forecasts appears to be somewhat higher among the best probit models than in the best ARMAX models, but the investment return performance in particular is clearly better among the best probit models. In Table 5, only the two best ARMAX models yield considerably higher returns than the other ARMAX models. When the return differences between the best error correction probit model and the best ARMAX models are tested, the return differences are also statistically significant at the 5% level, based on the DM

¹¹ The UCSD.GARCH toolbox package for Matlab is used in the estimation of ARMAX models.

¹² The ARMAX(2, 0) model is the same as the AR(2) model with explanatory variables.

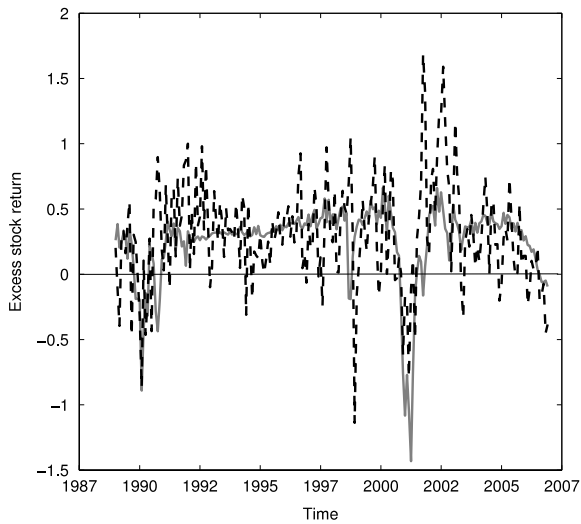


Fig. 5. Out-of-sample predictions of the ARMAX(1, 0) model with p_{t+5}^y (solid line) and the ARMAX(2, 0) model (dashed line) with p_{t+5}^y and Δi_{t-1} .

Table 5
Out-of-sample performances of the ARMAX and volatility models.

Model	x_{t-1}	CR	RET	SR
B&H			8.07	0.79
ARMAX(1, 0)	p_{t+5}^y	0.565	7.12	0.67
ARMAX(1, 0)	p_{t+5}^y, SP_{t-1}^{US}	0.542	6.07	0.45
ARMAX(1, 0)	$p_{t+5}^y, \Delta R_{t-1}$	0.556	4.98	0.19
ARMAX(1, 0)	$p_{t+5}^y, \Delta i_{t-1}$	0.569	6.16	0.45
ARMAX(2, 0)	p_{t+5}^y	0.576	5.83	0.37
ARMAX(2, 0)	p_{t+5}^y, SP_{t-1}^{US}	0.514	4.36	0.04
ARMAX(2, 0)	$p_{t+5}^y, \Delta R_{t-1}$	0.532	4.76	0.13
ARMAX(2, 0)	$p_{t+5}^y, \Delta i_{t-1}$	0.588	6.28	0.47
Volatility models				
Non-parametric		0.481	5.57	0.65
Extended		0.514	4.93	0.16

Notes: The ARMAX(p , 0) model for r_t is $r_t = a + \sum_{i=1}^p b_i r_{t-i} + x'_{t-1} d$. “Non-parametric” and “Extended” refer to the predictive models proposed by Christoffersen et al. (2007), which are based on the volatility forecasts $\hat{\sigma}_{t|t-1}$. See also the notes to Table 3.

test statistics shown in Table 6. Therefore, the error correction probit model also seems to be a superior predictive model relative to the alternative ARMAX models.

As Eq. (1) shows, if the volatility σ_t , conditional on the information at time $t - 1$, is predictable, then

the signs of stock returns should also be predictable, provided that $\mu_t \neq 0$, although the conditional mean μ_t could be unpredictable. Using the same terminology as Christoffersen et al. (2007) for predictive models based on volatility forecasts, a “non-parametric” model indicates a model where the one-step-ahead volatility forecast is also used to compute the conditional mean forecast, which then, together with the volatility forecast, determines the probability forecast for a positive excess return. In an “extended” model, the skewness and kurtosis of excess returns are also taken into account in the model. The percentages of correct forecasts CR given in Table 5 show that the volatility models do not produce out-of-sample sign predictability, and that, relative to the best error correction probit model, the latter produces higher values of CR and higher investment returns. The p -values of the DM test statistics between the models are 0.012 (non-parametric model) and 0.008 (extended model), indicating that the return differences are statistically significant at the 5% level.

4.5. Sign predictability of small and large size firms' returns

Finally, we extend our analysis by considering the sign predictability of small and large sized firms. Perez-Quiros and Timmermann (2000), among others, find that there is a close link between the stock returns of different sized firms and the state of the economy. They propose that recessions, which indicate, for instance, worsening credit market conditions, would be expected to affect the expected returns of small firms more strongly than those of large firms. In their model, Perez-Quiros and Timmermann (2000) employed a Markov switching model where the continuous excess stock return is modeled as a function of the lagged Treasury Bill rate, the default spread, changes in the money stock growth and the dividend yield.

In our limited study, we employ the presented error correction probit model (9) with the six-month recession forecast employed for both return indicator series. The values of the return indicators (Eq. (2)) are constructed by using the excess stock returns r_t^S and r_t^L (details in Table 1) from the size-sorted CRSP decile portfolios. It is worth noting that there is a significant correspondence with the binary values of stock indicator series between S&P500 and these size-sorted stock indices, as expected. In the case of large

Table 6

Diebold-Mariano tests between the best error-correction probit model and the best ARMAX models.

Model	x_{t-1}	DM	DM_{ra}	$DM^{I_t=0}$	$DM_{ra}^{I_t=0}$
ARMAX(1, 0)	p_{t+5}^y	0.046	0.019	0.000	0.000
ARMAX(2, 0)	$p_{t+5}^y, \Delta i_{t-1}$	0.015	0.006	0.000	0.000

Notes: The p -values of the Diebold and Mariano (1995) tests between the investment returns from the error correction probit model presented in Table 4 and the ARMAX models mentioned in the first column are presented. Forecasts from the ARMAX models are also depicted in Fig. 5.

Table 7

Out-of-sample sign predictions for small and large sized firms' returns.

Firm size	Model	x_{t-1}	psR^2	CR	RET	SR	PT	DM	DM_{ra}	$DM^{I_t=0}$	$DM_{ra}^{I_t=0}$
Small	B&H				13.76	1.37					
	ecm (9)	p_{t+5}^y	0.029	0.625	15.72	2.24	0.002	0.267	0.014	0.000	0.000
Large	B&H				10.89	1.36					
	ecm (9)	p_{t+5}^y	0.007	0.634	12.91	2.07	0.014	0.116	0.009	0.000	0.000

Notes: As in Tables 3 and 5, the average annual risk-free interest rate return is 4.21% in both series. See also the notes to Table 3.

firms in particular, the correspondence is about 96%. There is more variation between the values of return indicators in the returns from small firms, but overall the mean and volatility of small firms' returns are higher than in the case of large firms.

Table 7 presents the out-of-sample forecasting performances of the error correction probit models considered. The p -values of the PT market timing test statistic (Eq. (16)) are statistically significant, which shows that the signs are predictable out-of-sample in both cases, and the percentage of correct forecasts is even higher than in the case of S&P500 returns. Furthermore, this sign predictability also converts to higher investment returns in our simple trading simulation relative to the B&H strategy. However, the differences are statistically significant only in terms of risk-adjusted returns where the standard deviations of the portfolio returns are taken into account.

There does not seem to be much difference between the results for large and small firms in terms of predictive accuracy. For small firms, the error correction model produces a higher value of the psR^2 measure. On the other hand, the percentage of correct forecasts is higher in the case of large firms. However, one significant difference between the models can be seen in Fig. 6. There, the conditional probability forecast for small firms fluctuates much more than that for large firms. In that model, the estimated value for the coefficient α_1 is between 0.30–0.50 for the out-of-sample period, whereas for large firms it is about the

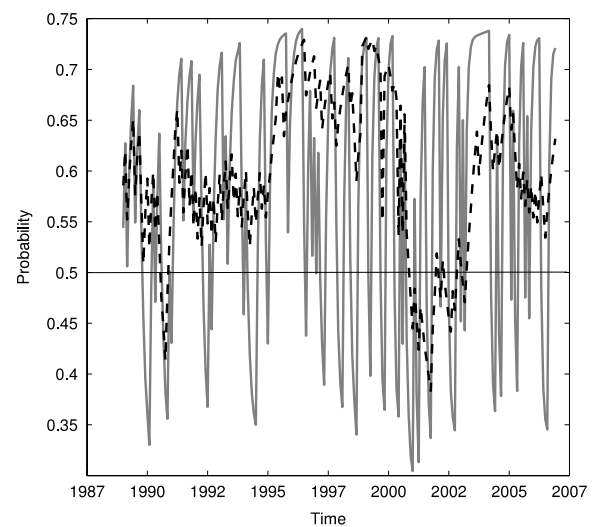


Fig. 6. Out-of-sample predictions of the error correction probit models (9) with the recession forecast p_{t+5}^y for small (solid line) and large (dashed line) firms.

same magnitude as in the analysis of the S&P500 index ($\alpha_1 \approx 0.90$). Despite the fact that this fluctuation around the 50% threshold indicates higher transaction costs (about a 2% deficit in investment returns relative to the returns without transaction costs), as discussed in Section 4.2, the investment returns from our trading simulation are still higher than for the B&H strategy.

5. Conclusions

We examine the predictability of the US excess stock return signs using dynamic binary probit models. The proposed forecasting method, where the six-month recession forecast for the recession indicator is used as an explanatory variable, seems to outperform other predictive models. Using the S&P500 stock index, the direction of the excess stock return is predictable, and it is possible to earn statistically significantly higher investment returns than the buy-and-hold trading strategy in-sample. However, the out-of-sample predictability turns out to be weaker. This is in line with previous findings related to stock return forecasting. In fact, in out-of-sample forecasting, the best dynamic probit model appears to be the error correction model proposed in this paper. Using this model, both the number of correct sign predictions and investment returns are higher than in other probit models, ARMAX models, or predictive models based on volatility forecasts. In the best error correction model, the average investment returns are also higher than in the buy-and-hold trading strategy. Compared to the evidence from S&P500 returns for small and large size-sorted firms' returns, it appears that the out-of-sample sign predictability is higher using the error correction probit model with recession forecast.

The analysis performed in this paper can be extended in various ways. A system analysis in which the recession and sign forecasts are determined endogenously using the same model is of particular interest. In this study, the six-month-ahead recession forecast is taken as a given, but this selection may not be optimal in terms of predictive power in sign predictions. It could also be interesting to form a somewhat more complicated trading strategy rule that could take the sign predictability, and perhaps also the risks related to different models, into account even better in investment allocation decisions.

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