SEMINAR 11

1. Find the foci of the hyperbola $\mathcal{H}: \frac{x^2}{9} - \frac{y^2}{4} - 1 = 0$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$; $c^2 - a^2 = b^2$

$c = \pm\sqrt{a^2+b^2}$

$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

$a^2 - c^2 = b^2$

$F(\sqrt{a^2-b^2}, 0), F'(-\sqrt{a^2-b^2}, 0)$

$a = 3$
$b = 2$

$F(\sqrt{13}, 0)$ $F'(-\sqrt{13}, 0)$

2. Find the of the triangle determined by the asymptotes of the hyperbola $\mathcal{H}: \frac{x^2}{4} - \frac{y^2}{1} = 1$ and the line

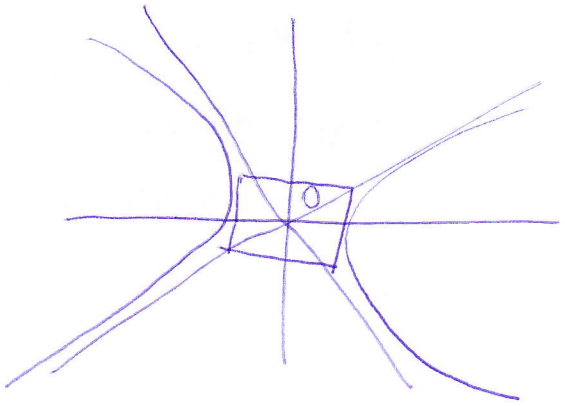
$d: 9x + 2y - 24 = 0$

$\begin{cases} 9x + 2y = 24 \\ x = 2y \end{cases} \Leftrightarrow 18y + 2y = 24$

$20y = 24$

$y = \frac{6}{5}$

$x = \frac{12}{5} \Rightarrow A\left(\frac{6}{5}, \frac{12}{5}\right)$



$\begin{cases} 9x + 2y = 2y \\ x = 2y \end{cases} \quad -18y + 2y = 20$

$-16y = 24$

$y = -\frac{3}{4} \Rightarrow x = \frac{3}{2} \Rightarrow B\left(-\frac{3}{4}, \frac{3}{2}\right)$

$\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$, $c^2 - a^2 = b^2$

$c = \pm\sqrt{a^2+b^2} \Rightarrow F'(-\sqrt{a^2+b^2}, 0)$

$y = \pm\frac{b}{a}x$

$\mathcal{A}_{OAB} = \frac{1}{2}\left|\begin{matrix} 0 & 0 & 1 \\ \frac{6}{5} & \frac{12}{5} & 1 \\ -\frac{3}{4} & \frac{3}{2} & 1 \end{matrix}\right| = \frac{1}{2}\left|\frac{9}{5} + \frac{9}{5}\right| = \frac{9}{5}$

3. Det the coords of the foci of the ellipse $\mathcal{E}: 3x^2+25y^2=225$

$\frac{x^2}{25}+\frac{y^2}{9}=1 \Rightarrow a=5 \quad F(4,0)$

$b=3 \quad F'(-4,0)$

4. Find the intersection points between the line:

$d: x+2y-7=0$

ellipse $\mathcal{E}: x^2+5y^2-25=0$

$\begin{cases} x+2y=7 \Rightarrow x=7-2y \\ x^2+5y^2=25 \Rightarrow x=7-2y \end{cases}$

$49-28y+4y^2+5y^2=25$

$\Rightarrow 9y^2-28y+24=0$

$\Delta = 784-864 \Rightarrow \sqrt{\Delta} = 40i$

$\Rightarrow y_2 = \frac{28+40i}{2} = 14+20i$

$y_2 = 14-20i$

$x_1 = -21+40i$

$x_2 = -21-40i$

$\Rightarrow A(-21+40i, 14+20i)$

$B(-21-40i, 14-20i)$

5. Find the eq of the tangent lines to the hiperbola $H: \frac{x^2}{3}-\frac{y^2}{5}=1$

$y-y_0 = m(x-x_0)$

$y+\sqrt{5} = m(x-1) \Rightarrow \begin{cases} 5x^2-3y^2=15 \\ y=-\sqrt{5}+m(x-1) \end{cases}$

$5x^2-3(-\sqrt{5}+m(x-1))=0$

$5x^2-3(25-10m(x-1)+(x-1)^2m^2)=0$

$5x^2-75+30m(x-1)+3m^2(x-1)^2=0$

$5x^2-75+30mx-30m+3m^2(x^2-2x+1)=0$

$\Rightarrow x^2(5-3m^2)+x(30m+6m^2)-75-30m-3m^2=0$

$\Delta = (30m+6m^2)-4(5-3m^2)(-75-30m-3m^2)=0$

$\Delta = 900m^2+36m^4+360m^2-4(-375-150m-15m^2+$

$+225m^2+90m^3+45m)$

$\Delta = 36m^4+360m^3+1500$

$y = -x + m$

$x^2 + 4(-x+m)^2 - 20 = 0$

$x^2 + 4x^2 - 8xm + 4m^2 - 20 = 0$

$5x^2 - 8xm + 4m^2 - 20 = 0$

$\Delta' = 16m^2 - 5(4m^2 - 20)$

$= 16m^2 - 20m^2 + 100 = -4m^2 + 100$

$\Delta' = 0$

$-4m^2 + 100 = 0$

$m = \pm 5$

$y = -x \pm 5$

7. Find the ~~eq of the parabola~~ focus and the directrix line of the parabola

$P: y^2 - 24x = 0$

$y^2 - 24x = 0$

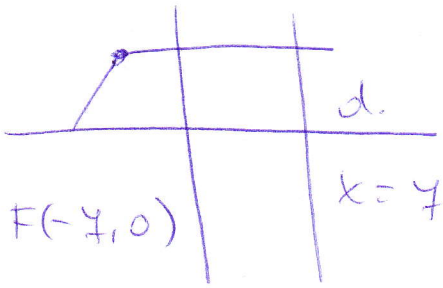
$d: x = -6$

$F(6, 0)$

$y^2 = 2px$; $d: x = -\frac{p}{2}$; $F(\frac{p}{2}, 0)$

$p = y^2$

$y = 24x \Rightarrow p = 12$



8. Find the eq of the parabola having the focus $F(-7, 0)$ and the directrix line $x - 7 = 0$

$F(-7, 0)$

$x - 7 = 0$

$\delta(M, F) = \delta(M, d) \Leftrightarrow \sqrt{(x+7)^2 + y^2} = (x-7)(.)^2$

$x^2 + 14x + 49 + y^2 = x^2 - 14x + 49$

$28x + y^2 = 0$

9. Find the eq of the tangent line to the parabola

$P: y^2 - 8x = 0$ parallel to $d: 2x + 2y - 3 = 0$

$D\lambda: x + y + \lambda = 0$

$\begin{cases} x + y + \lambda = 0 \\ y^2 - 8x = 0 \end{cases} \Rightarrow x = -\frac{y^2}{8}$

$y^2 + 8y + 8\lambda = 0$

$\Delta = 64 - 32\lambda$

$64 - 32\lambda = 0 \Rightarrow \lambda = 1$