Seminar 3

1. Verificati aplicabilitatea teoremei Stolz-Cesaro in cazul sirurilor

$$a_n = \sum_{k=1}^{n} \frac{1+(-1)^k}{2},\ b_n = n, \quad \forall n \in \mathbb{N}^\star$$

2. Daca $(x_n)_{n\in\mathbb{N}}$ este un sir cu termeni strict pozitivi si daca exista limita $\lim\limits_{n\to\infty} \frac{x_{n+1}}{x_n} = l$ atunci $\lim\limits_{n\to\infty} \sqrt[n]{x_n} = l$

3. Calculati $\lim\limits_{n\to\infty} x_n$ pentru

a) $x_n = \sqrt[n]{n!}$
b) $x_n = \frac{\sqrt[n]{n!}}{n}$
c) $x_n = \frac{1+\frac{1}{2}+\ldots+\frac{1}{n}}{\ln n}$
d) $x_n = \frac{1+\sqrt{2}+\ldots+\sqrt{n}}{n\sqrt{n}}$
e) $x_n = \frac{\sqrt[n]{(n+1)(n+2)\cdot\ldots\cdot(n+n)}}{n}$

4. Scrieti urmatoarele serii infinite cu ajutorul simbolului suma

a) $1 + \frac{1}{3} + \frac{1}{5} + \frac{1}{7} + \ldots$
b) $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} + \ldots$
c) $1 - \frac{1}{4} + \frac{1}{9} - \frac{1}{16} + \frac{1}{25} - \ldots$

5. Calculati suma urmatoarelor serii

a) $\sum\limits_{n=0}^{\infty} \frac{1}{n!}$
b) $\sum\limits_{n=1}^{\infty} \frac{1}{5^n}$
c) $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n-1}}$
d) $\sum\limits_{n=1}^{\infty} \frac{1}{4n^2-1}$
e) $\sum\limits_{n=2}^{\infty} \ln\left(1-\frac{1}{n^2}\right)$
f) $\sum\limits_{n=2}^{\infty} \frac{1}{\mathrm{C}_n^2}$
g) $\sum\limits_{n=1}^{\infty} \mathrm{arctg}\frac{1}{n^2+n+1}$
h) $\sum\limits_{n=1}^{\infty} \frac{n2^n}{(n+2)!}$