SEMINAR 12

1. Find the intersection points of the ellipsoid $: \mathcal{E} : \frac{x^2}{16} + \frac{y^2}{12} + \frac{z^2}{4} = 1$ the line $\frac{x-4}{2} = \frac{y+6}{-3} = \frac{z+2}{-2}$ Find the equations of the tangent planes to $\mathcal{E}$ at those points

$$\begin{cases} x-4 = 2t \Rightarrow x = 2t+4 \\ y+6 = -3t \Rightarrow y = -3t-6, t \in \mathbb{R} \\ z+2 = -2t \Rightarrow z = -2t-2 \end{cases}$$

$$\frac{4t^2+16t+16}{16} + \frac{9t^2+36t+36}{12} + \frac{4t^2+8t+4}{4} = 1$$

$$\Rightarrow \frac{t^2+4t+4}{4} + \frac{3t^2+12t+12}{4} + t^2+2t+1 = 1$$

$$t^2+3t+2 = 0 \Rightarrow t_{1,2} = \frac{-3+1}{2} = -1$$

$$\Delta = 9-8 = 1 \Rightarrow t_2 = \frac{-4}{2} = -2$$

$$A(2,-3,0),\ B(0,0,2)$$

$S, F(x,y,z) = 0 \qquad M(x_0,y_0,z_0) \in S$

$$T_M(S): \frac{\partial F}{\partial x}(M)(x-x_0) + \frac{\partial F}{\partial y}(M)(y-y_0) + \frac{\partial F}{\partial z}(M)(z-z_0) = 0$$

$$F(x,y,z): \frac{2x_0}{a^2}(x-x_0) + \frac{2y_0}{b^2}(y-y_0) + \frac{2z_0}{c^2}(z-z) = 0$$

$$T_M(\mathcal{E})\ \frac{x_0x}{a^2} + \frac{y_0y}{b^2} + \frac{z_0z}{c^2} = \frac{x_0^2}{a^2} + \frac{y_0^2}{b^2} + \frac{z_0^2}{c^2}$$

$$T_M(\mathcal{E})\ \frac{x_0x}{a^2} + \frac{y_0y}{b^2} + \frac{z_0z}{c^2} = 1$$

$$T_M(\mathcal{E}): \frac{2x}{16} + \frac{-3y}{12} = 1 \Rightarrow T_{M_B}(\mathcal{E}): \frac{2z}{4} = 1$$

2. Write the eq of the tangent to the ellipse paraboloid

$P_e: \frac{x^2}{2} + \frac{y^2}{4} = 9z$ and the hyperbolic paraboloid $P_h: \frac{x^2}{2} - \frac{y^2}{4} = 9z$

and the intersection points with the line $d: x = y = z$

$$\frac{t^2}{2} + \frac{t^2}{4} = \frac{9t}{4} \Rightarrow 2t^2 + t^2 = 36t \Rightarrow t(t-12) = 0$$

$\Rightarrow t = 12$
$t = 0$

$d \cap P_e: A(0,0,0), B(12,12,12)$

Π and $\frac{t^2}{2} - \frac{t^2}{4} = \frac{9t}{4}$

$2t^2 - t^2 - 36t = 0$

$t^2 - 36t = 0$

$t(t-36) = 0 \Rightarrow t_1 = 0, t_2 = 36 \Rightarrow d \cap P_h: A(0,0,0), C(36,36,36)$

$P_e: \frac{x^2}{p} + \frac{y^2}{q} = 2z, P_e = F^{-1}(0), M(x_0, y_0, z_0) \in P_e$

$$P(x,y,z) = \frac{x^2}{p} + \frac{y^2}{q} - 2z$$

$$T_M(P_e): \frac{2x_0}{p}(x - x_0) + \frac{2y_0}{q}(y - y_0) - 2(z - z_0) = 0$$

$$T_M(P_e): \frac{x_0 x}{p} + \frac{y_0 y}{q} = z + z_0$$

$P_h: \frac{x^2}{p} - \frac{y^2}{q} = 2z, M_0(x_0, y_0, z_0) \in P_h$

$x^2 \to x_0 x, y^2 \to y_0 y, z \to z_0 z$

$x \to \frac{x + x_0}{2}; y = \frac{y + y_0}{2}; z \to \frac{z + z_0}{2}$

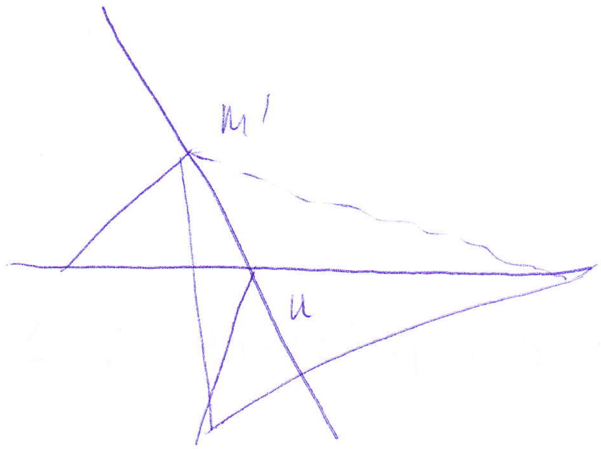
$$T_M(h): \frac{x_0 x}{p} - \frac{y_0 y}{q} = z + z_0$$

$T_A(P_e): z = 0$

$T_B(P_e): \frac{12x}{2} + \frac{12y}{4} = z + 12$

$T_0(P_h): z = 0$

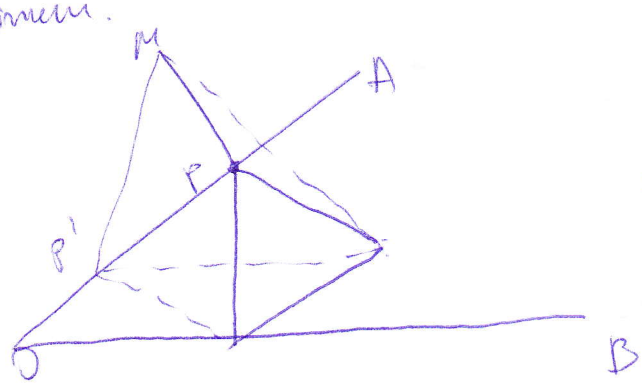
3. Let d be a line and A, B two points on the plane. Det the position of $M \in d$ such that the sum $|AM| + |MB|$ is minimum



$$|AM'| + |BM'| = |AM'| + |M'B'| \geq |AB'|$$

$$= |AM| + |MB'| = |AM| + |BM|$$

4. Let C be a point inside the angle $A\hat{O}B$. Determine the points $P \in]OA$ and $Q \in]OB$ such that perimeter of ΔCPQ is minimum.



$$P_{CQ'P'} = CP' + P'Q' + Q'C =$$

$$= MP' + Q'N + P'Q' \geq MA + PQ + QN$$

$$= P_{CPQ}$$

5. Write the eq of the families of lines which are contained in hyperbolic paraboloid.

$$Ph: \frac{x^2}{16} - \frac{y^2}{4} = z$$

and chose those lines in each family which are parallel to the plane π

$$3x + 2y - 4z = 0$$

$$Ph\left(\frac{x}{4} - \frac{y}{2}\right)\left(\frac{x}{4} + \frac{y}{2}\right) = z$$

$D_\lambda \subseteq Ph, \forall \lambda \in \mathbb{R}$

$D_\mu' \subseteq Ph \; \forall \mu \in \mathbb{R}$

$$D_\lambda': \begin{cases} \frac{x}{4} + \frac{y}{2} = \lambda \\ \lambda\left(\frac{x}{4} - \frac{y}{2}\right) = z \end{cases}$$

$$D_\mu' = \begin{cases} \frac{x}{4} - \frac{y}{2} = \mu \\ \mu\left(\frac{x}{4} + \frac{y}{2}\right) = z \end{cases}$$

$$P_\lambda = \left(\frac{x}{4} \quad 0 \right)$$

$$q\lambda = \begin{vmatrix} 0 & \frac{1}{4} \\ -1 & \frac{\lambda}{4} \end{vmatrix} = \frac{1}{4}$$

$$r\lambda = \begin{vmatrix} -\frac{1}{4} & \frac{1}{2} \\ \frac{\lambda}{4} & -\frac{\lambda}{2} \end{vmatrix} = -\frac{x}{4}$$

$D\lambda \parallel \pi \Leftrightarrow \vec{n}_\pi \perp \vec{D}\lambda \Leftrightarrow \vec{n}_\pi \cdot \vec{D}\lambda = 0 \Leftrightarrow 3\left(-\frac{1}{2}\right) + 2\left(\frac{1}{2}\right) - 4\left(-\frac{\lambda}{4}\right) = 0$

$-\frac{3}{2} + \frac{1}{2} + \lambda = 0$

$-3 + 1 + 2\lambda = 0 \qquad \lambda = -1$

$T_m(P_1)\ \frac{x_0 x}{p} + \frac{y_0 y}{q} = z + z_0$