Seminar 11

1. Calculati matricea Jacobi $J(f)(1,0)$ pentru functia $f:\mathbb{R}^2\to\mathbb{R}^3$,

$$f(x,y)=(x^2-y,3x-2y,2xy+y^2)$$

2. Calculati diferentiala $df(-1,1)$ pentru functia $f:\mathbb{R}^\star\times\mathbb{R}^\star\to\mathbb{R}$,

$$f(x,y)=\mathrm{arctg}\frac{x^2}{y}+\mathrm{arctg}\frac{y^2}{x}$$

3. Fie $f,g:\mathbb{R}^m\to\mathbb{R}$ functii de clasa C^1 intr-un punct $x^0\in\mathbb{R}^m$. Justificati egalitatea

$$\nabla(fg)(x^0)=\nabla(f)(x^0)\cdot g(x^0)+f(x^0)\cdot\nabla(g)(x^0)$$

4. Stiind ca $g=g(u,v,w):\mathbb{R}^3\to\mathbb{R}$ este o functie diferentiabila pe $\mathbb{R}^3$, calculati derivatele partiale ale functiei compuse

$$\varphi(x,y,z)=g(x,xy,xyz)$$

5. Stiind ca $g=g(u,v):\mathbb{R}^2\to\mathbb{R}$ este o functie diferentiabila pe $\mathbb{R}^2$ si ca

$$u\,\frac{\partial g}{\partial v}(u,v)=v\,\frac{\partial g}{\partial u}(u,v),\quad\forall(u,v)\in\mathbb{R}^2$$

aratati ca functia compusa $\varphi(x,y)=g(x,\sqrt{y-x^2})$ satisface egalitatea

$$\frac{\partial\varphi}{\partial x}(x,y)=0,\quad\forall(x,y)\in\mathbb{R}^2,\ y>x^2$$

6. Studiati continuitatea in origine, existenta derivatelor dupa directie in origine si diferentiabilitatea in origine a functiilor

a) $f(x,y)=\sqrt{|xy|},\quad\forall(x,y)\in\mathbb{R}^2$

b) $f(x,y)=\begin{cases}x^2\,\mathrm{arctg}\frac{y}{x} & ,x\neq 0\\ 0 & ,x=0\end{cases}$

c) $f(x,y)=\begin{cases}\frac{xy(x^2-y^2)}{x^2+y^2} & ,(x,y)\neq(0,0)\\ 0 & ,(x,y)=(0,0)\end{cases}$