Seminar Nr.3, Probabilistic Models

Theory Review

Binomial Model: The probability of k successes in n Bernoulli trials, with probability of success p, is

$P(n,k) = C_n^k p^k q^{n-k},\ k = \overline{0,n}$

Multinomial Model: The probability that in $n = n_1 + n_2 + ... + n_r$ trials, E_i occurs n_i times, where $p_i = P(E_i)$, $i = \overline{1,r}$, is

$$P(n; n_1, ..., n_r) = \frac{n!}{n_1! n_2! ... n_r!} p_1^{n_1} p_2^{n_2} ... p_r^{n_r}$$

Bernoulli Model Without Replacement (Hypergeometric): The probability that in n trials, we get k white balls out of n_1 and $n-k$ black balls out of $N - n_1$ ($0 \le k \le n_1$, $0 \le n-k \le N-n_1$), is

$$P(n;k) = \frac{C_{n_1}^k C_{N-n_1}^{n-k}}{C_N^n}$$

Bernoulli Model Without Replacement With r States: The probability that in

$M = m_1 + m_2 + ... + m_r$ trials, we get m_i balls of color i out of n_i, $i = \overline{1,r}$, $(n = n_1 + n_2 + ... + n_r)$, is

$$P(n; m_1, ..., m_r) = \frac{C_{n_1}^{m_1} C_{n_2}^{m_2} ... C_{n_r}^{m_r}}{C_n^M}$$

Poisson Model: The probability of k successes $(0 \le k \le n)$ in n trials, with probability of success p_i in the i^{th} trial $(q_i = 1 - p_i)$, $i = \overline{1,n}$, is

$$P(n;k) = \sum_{1 \le i_1 < ... < i_k \le n} p_{i_1} ... p_{i_k} q_{i_{k+1}} ... q_{i_n}, \qquad i_{k+1}, ..., i_n \in \{1, ..., n\} \setminus \{i_1, ..., i_k\}$$
$$= \text{the coefficient of } x^k \text{ in the expansion } (p_1 x + q_1)(p_2 x + q_2) \dots (p_n x + q_n)$$

Pascal Model: The probability of the n^{th} success occurring after k failures in a sequence of Bernoulli trials with probability of success p $(q = 1 - p)$, is

$P(n;k) = C_{n+k-1}^{n-1} p^n q^k = C_{n+k-1}^{k} p^n q^k$

Geometric Model: The probability of the 1^{st} success occurring after k failures in a sequence of Bernoulli trials with probability of success p $(q = 1 - p)$, is

$p_k = pq^k$

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1. It is possible for a computer to pick up an erroneous signal that does not show up as an error on the screen, called a silent error. A particular terminal is defective, and when using the system word processor, it introduces a silent paging error with probability 0.1. The word processor is used 20 times during a given week.
a) Find the probability that no silent paging errors occur.
b) Find the probability that at least one such error occurs.
c) Would it be unusual for more than four such errors to occur? Explain, based on the probability involved.

2. A computer randomly generates passwords consisting only of letters and numbers, distinguishing between capital and lowercase letters. If a password has 8 characters, what is the probability that it will contain 2 lower case letters, 3 capital letters and 3 digits (event A) (their order doesn't count).

3. A person has 40 homing (messenger) pigeons. When released, the probability that a pigeon will come back is 0.7. Find the probability of the events:
a) A: 10 pigeons do not come back;
b) B: all of them come back;
c) C: at least 38 of them come back.

4. There are 200 seats in a theater, 10 of which are reserved for the press. 150 people come to the show one night, and are seated randomly. What is the probability of all the seats reserved for the press to be occupied (A)?

5. At a club, there are $4N$ people from 4 different cities, N from each. Five people are chosen randomly. Find the probability of choosing:
a) A: 4 people from the same city;
b) B: 3 people from one city, and the other 2 from another (but same) city;
c) C: 3 people from one city, and the other 2 from 2 other different cities.

6. A test consists of 3 questions from different areas. A student taking the test can answer correctly each question with probability 0.7, 0.8 and 0.6, respectively. Each question is worth 1 point (no fractions of a point are given). What is the probability that the student gets 2 points on the test (event A)?

7. Students from 3 departments participate in a debate. The boys/girls ratios for the 3 groups are $(6, 4)$, $(4, 5)$ and $(5, 5)$, respectively. If one student is chosen from each department as spokesperson, what is the probability that the spokespersons are 2 boys and 1 girl (event A)?

8. A vaccine for desensitizing patients to bee stings meets specifications with probability 0.9. Would it be unusual if 7 or more vaccines have to be tested to find three that meet specifications (event A)? Explain.

9. (Banach's Problem). A person buys 2 boxes of aspirin, each containing n pills. He takes one aspirin at a time, randomly from one of the two boxes. After a while, he realizes that one box is empty.
a) Find the probability of event A: when he notices that one box is empty, there are k ($k \leq n$) pills left in the other box.
b) Use part a) to find a formula for $S_n = C_{2n}^n + 2\cdot\, C_{2n-1}^n + \ldots + 2^n \cdot C_n^n$.

Bonus Problems:

10. Three contestants participate in a trivia gameshow. Their probability of answering a question correctly are 0.8, 0.9 and 0.75, respectively. If 10 questions are asked and every contestant answers every question, find the probability that one contestant (any one) answers exactly 7 questions correctly, while the other two give any *other* number of correct answers. (event A)?

11. In a department store at the mall, black and brown gloves are on sale. There are N (identical) pairs of black gloves and N (identical) pairs of brown gloves. If N customers come in, one at a time and randomly choose and buy 2 pairs each, find the probability of event A: each customer buys 2 pairs of different colors (one black and one brown).