Seminar Nr.2, Classical Probability; Geometric Probability; Conditional Probability; Independent Events; Bayes' Formula

Theory Review

Classical Probability: $P(A) = \dfrac{\text{nr. of favorable outcomes}}{\text{total nr. of possible outcomes}}$.

Conditional Probability: $P(A|B) = \dfrac{P(A \cap B)}{P(B)}$, $P(B) \neq 0$.

Independent Events: A, B independent $<=> P(A \cap B) = P(A)P(B) <=> P(A|B) = P(A)$

Total Probability Rule: $\{A_i\}_{i \in I}$ a partition of S, then $P(A) = \sum\limits_{i \in I} P(A_i)P(A_i|A)$

Multiplication Rule: $P\left(\bigcap\limits_{i=1}^{n}\right) A_i = P(A_1)\, P(A_2|A_1)\, P(A_3|A_1 \cap A_2)\ \dots\ P\left(A_n|\bigcap\limits_{i=1}^{n-1}\right)$

Bayes' Formula: $\{A_i\}_{i \in I}$ a partition of S, then $P(A_j|A) = \dfrac{P(A|A_j)\, P(A_j)}{\sum\limits_{i \in I} P(A|A_i)\, P(A_i)}$, $\forall j \in I$

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1. A person buys n lottery tickets. For $i = \overline{1, n}$, let A_i denote the event: the i^{th} ticket is a winning one. Express the following events in terms of $A_1, ..., A_n$.
a) A: all tickets are winning;
b) B: all tickets are losing;
c) C: at least one is winning;
d) D: exactly one is winning;
e) E: exactly two are winning;
f) F: at least two are winning;
g) G: at most two are winning.

2. A firm offers a choice of 10 free software packages to buyers of their new home computer. There are 25 packages available, five of which are computer games, and three of which are antivirus programs.
a) How many selections are possible?
b) How many selections are possible, if exactly three computer games are selected?
c) How many selections are possible, if exactly three computer games and exactly two antivirus programs are selected?

3. The faces of a cube are painted each in a different color (the cube is transparent on the inside). Then the cube is broken into 1000 smaller, equally-sized cubes and one such cube is randomly picked. Find the probability of the following events:
a) A: the cube picked has exactly three colored faces;
b) B: the cube picked has exactly two colored faces;
c) C: the cube picked has exactly one colored face;
d) D: the cube picked has no colored faces.

4. The natural numbers $1, 2, ..., n$ (where n is a fixed natural number) are placed randomly in a sequence. Find the probability of the following events:
a) A: the numbers 1 and 2 are placed consecutively, in increasing order;
b) B: the numbers 1 and 2 are placed consecutively;
c) C: the numbers 1 and 2 are placed in increasing order;
d) D: the numbers i, j, k are placed consecutively, in increasing order.

5. A postman distributes n letters in N mailboxes. What is the probability of the event A: there are m letters in a given (fixed) mailbox ($0 \leq m \leq n$)?

6. A segment of length a is broken into 3 pieces. Determine the probability p that the 3 segments obtained form a triangle .

7. In a study of waters near industrial plants it was found that 30% showed signs of chemical pollution (event C), 25% showed evidence of thermal pollution (event T) and 10% showed signs of chemical and thermal pollution.
a) Are events C and T independent?
b) What is the probability that a stream that shows some thermal pollution will also show signs of chemical pollution (event A)?
c) What is the probability that a stream showing chemical pollution will not show signs of thermal pollution (event B)?

8. A computer center has three printers A, B, and C, which print at different speeds. Documents are routed to the first available printer. The probability that a document is routed to printers A, B, and C are 0.6, 0.3, and 0.1, respectively. Occasionally a printer will jam and destroy a printout. The probability that printers A, B, and C will jam are 0.01, 0.05 and 0.04, respectively. A document has just been destroyed because of a printer jam. What is the probability that printer A is involved? Printer B is involved? Printer C is involved?

9. Three shooters aim at a target. The probability that they hit the target are 0.4, 0.5 and 0.7, respectively. Find the probability that the target is hit exactly once.

Bonus Problems:

10. The probability that n dice are rolled is $\frac{1}{2^n}$, $n \in \mathbb{N}$. Let S_N denote the sum of the numbers shown on N dice. Find the probability of the following events:
a) $S_N = 4$, knowing that N is even;
b) $N = 2$, given that $S_N = 3$;
c) $N = 2$, given that $S_N = 4$ and the first die showed the number 1;
d) the largest number shown on any die is 4, knowing that $N = 3$.

11. There are N people in a room, each wearing a different hat. They all take their hats off, put them together and then each randomly picks one up. What is the probability of no person getting their own hat back (denote this event by A)? What does this probability become as $N \to \infty$?