SEMINAR 13

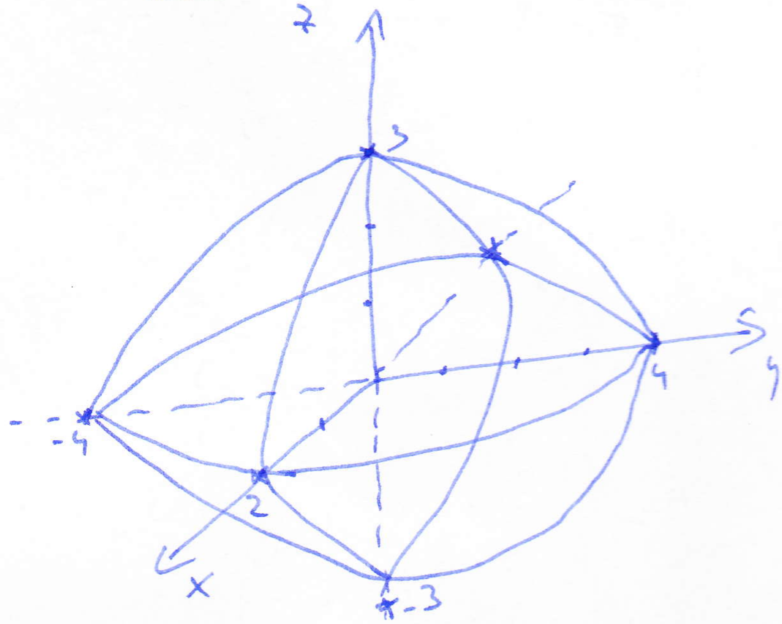
23.V.2011

① Sketch the graph of the ellipsoid

$$\frac{x^2}{4} + \frac{y^2}{16} + \frac{z^2}{9} = 1$$

$$\left[\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1 \right]$$

$a = 2$
$b = 4$
$c = 3$



② S.t. the lines

$$d_1: \begin{cases} x = 3 + t \\ y = 2 + t \\ z = 5 + 2t \end{cases} \quad \text{and}$$

$$d_2: \begin{cases} x = 3 + t \\ y = 2 - t \\ z = 5 + 10t \end{cases}$$

are completely contained into hyperbolic paraboloid $P: z = x^2 - y^2$

* $5 + 2t = (3 + t)^2 - (2 + t)^2$

$5 + 2t = 9 + 6t + t^2 - (4 + 4t + t^2)$

$5 + 2t = 2t + 5$ (T)

$d_1 \in P$

* $5 + 10t = (3+t)^2 - (2-t)^2$

$5 + 10t = t^2 + 6t + 9 - t^2 + 4t - 4$

③ Find the intersection points of $d: \frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ with the ellipsoid

$S: x^2 + \frac{y^2}{4} + \frac{z^2}{9} = 1$

$$d: \begin{cases} x = t \\ y = 2t \\ z = 3t \end{cases}$$

$$t^2 + \frac{4t^2}{4} + \frac{9t^2}{9} = 1$$

$$3t^2 = 1 \Rightarrow t = \pm \frac{\sqrt{3}}{3}$$

$$t = \frac{\sqrt{3}}{3} \Rightarrow \begin{array}{l} x = \frac{\sqrt{3}}{3} \\ y = \frac{2\sqrt{3}}{3} \\ z = \sqrt{3} \end{array} \Rightarrow A\left(\frac{\sqrt{3}}{3}, \frac{2\sqrt{3}}{3}, \sqrt{3}\right)$$

$$t = -\frac{\sqrt{3}}{3} \Rightarrow B\left(-\frac{\sqrt{3}}{3}, -\frac{2\sqrt{3}}{3}, -\sqrt{3}\right)$$

$$d \cap S = \{A, B\}$$

④ Find the eq of surface generated by the rotation of $H: \begin{cases} z = 0 \\ \frac{x^2}{a^2} - \frac{y^2}{b^2} - 1 = 0 \end{cases}$ around the axis Oy

eq of a surface generated by the curve $\mathcal{C}$ in its rotation around a line d:

$$\mathcal{C}: \begin{cases} F_1(x, y, z) = 0 \\ F_2(x, y, z) = 0 \end{cases}$$

$$d: \frac{x - x_0}{p} = \frac{y - y_0}{q} = \frac{z - z_0}{r}$$

$$\Rightarrow (p, q, r) \Rightarrow \overline{v_d}$$

$$\mathcal{C}_{\lambda,\mu} : \begin{cases} (x-x_0)^2+(y-y_0)^2+(z-z_0)^2=\lambda \\ px+qy+rz=\mu \end{cases}$$

$$\left.\begin{cases} \mathcal{C}_{\lambda,\mu} \\ F_1(x,y,z)=0 \\ F_2(x,y,z)=0 \end{cases}\right| \to \varphi(\lambda,\mu)=0$$

$\to$ replace λ, μ $\Rightarrow$ eq of surface

$\overline{v_{Oy}}=(0,1,0)$

$O(0,0,0)\in d$

$$\mathcal{C}_{\lambda,\mu} \begin{cases} x^2+y^2+z^2=\lambda \\ y=\mu \end{cases}$$

$$\left.\begin{cases} x^2+y^2+z^2=\lambda \\ y=\mu \\ z=0 \\ \frac{x^2}{a^2}-\frac{y^2}{b^2}-1=0 \end{cases}\right| \begin{aligned} & x^2+\mu^2=\lambda \Rightarrow x^2=\lambda-\mu^2 \\ \Rightarrow\ & \frac{\lambda-\mu^2}{a^2}-\frac{\mu^2}{b^2}-1=0 \end{aligned}$$

$$\frac{x^2+y^2+z^2-y^2}{a^2}-\frac{y^2}{b^2}-1=0$$

$$\frac{x^2+z^2}{a^2}-\frac{y^2}{a^2}-1=0$$

$$\boxed{\frac{x^2}{a^2}-\frac{y^2}{b^2}+\frac{z^2}{a^2}-1=0}$$

(5) Find eq of revolution surface generated by the curve $\mathcal{C}: \begin{cases} x^2-2y^2+z^2-5=0 \\ x+z+3=0 \end{cases}$

in its rotation around $d: x=y=z$

$\vec{v}_d\,(1,1,1)$

$O(0,0,0) \in d$

$$\mathcal{C}: \begin{cases} x^2+y^2+z^2=\lambda \\ x+y+z=\mu \end{cases} \Rightarrow \begin{cases} x^2+y^2+z^2=\lambda \\ x^2-2y^2+z^2-5=0 \\ x+z+3=0 \\ x+y+z=\mu \end{cases}$$

$$\begin{cases} 3y^2=\lambda-5 \\ 3-y=-\mu \end{cases} \Rightarrow y=3+\mu$$

$$3(3+\mu)^2=\lambda-5$$

$$3(3+\mu)^2-\lambda+5=0$$

$$\boxed{3(3+x+y+z)^2-x^2-y^2-z^2+5=0}$$

(6) Find the eq of the cylindrical surface having the circle

$\mathcal{C}: \begin{cases} x^2+y^2-a^2=0 \\ z=0 \end{cases}$ as director curve

and the generatrix parallel to $d: x=y=z$

the director curve $\mathcal{C}: \begin{cases} F_1(x,y,z)=0 \\ F_2(x,y,z)=0 \end{cases}$

the generatrix $\parallel$ to $d: \begin{cases} \pi_1=0 \\ \pi_2=0 \end{cases}$

$$d_{\lambda,\mu} \begin{cases} \pi_1=\lambda \\ \pi_2=\mu \end{cases} \qquad \begin{cases} d_{\lambda,\mu} \\ F_1=0 \\ F_2=0 \end{cases} \Rightarrow \varphi(\lambda,\mu)$$

$$d: \begin{cases} x = y = 0 \\ x - z = 0 \end{cases}$$

$$d_{\lambda,\mu}: \begin{cases} x - y = \lambda \\ x - z = \mu \end{cases}$$

$$d: \begin{cases} x - y = \lambda \\ x - z = \mu \\ x^2 + y^2 - a^2 = 0 \\ z = 0 \end{cases}$$

$x = \mu$

$\mu - y = \lambda \Rightarrow y = \mu - \lambda$

$$\boxed{\mu^2 + (\mu - \lambda)^2 - a^2 = 0}$$

$(x-z)^2 + (x - z - x + y)^2 - a^2 = 0$

$$\boxed{(x-z)^2 + (y-z)^2 - a^2 = 0}$$

(7) Find eq of conical surface with vertex $V(0, -a, 0)$ and director curve

$$\mathcal{C}: \begin{cases} x^2 + y^2 + z^2 = 4 \\ y + z = 2 \end{cases}$$

director curve $\mathcal{C}: \begin{cases} F_1(x, y, z) = 0 \\ F_2(x, y, z) = 0 \end{cases}$

$V(x_0, y_0, z_0)$ vertex

$$d_{\lambda,\mu}: \begin{cases} x - x_0 = \lambda(z - z_0) \\ y - y_0 = \mu(z - z_0) \end{cases}$$

$\begin{cases} d_{\lambda,\mu} \\ F_1 = 0 \\ F_2 = 0 \end{cases} \Big| \rightarrow \varphi(\lambda, \mu) = 0$

$\Rightarrow$ replace $\lambda, \mu \Rightarrow$ eq

$$d_{\lambda,\mu} : \begin{cases} x - 0 = \lambda(z-0) \\ y + a = \mu(z-0) \end{cases}$$

$$\begin{cases} x = \lambda z \Rightarrow \lambda = \frac{x}{z} \\ y + a = \mu z \Rightarrow y = \mu z - a \\ x^2 + y^2 + z^2 = 4 \\ y + z = 2 \quad y = 2 - z \end{cases}$$

$\Rightarrow \mu z - a = 2 - z$

$\mu z + z = 2 + a$

$z = \frac{2+a}{\mu+1}$

$\lambda = \frac{y+a}{z}$

$y = 2 - z = \frac{2\mu + 2 - 2 - a}{\mu+1} = \frac{2\mu - a}{\mu+1}$

$x = \lambda \frac{2+a}{\mu+1}$

$x^2 + y^2 + z^2 = 4 \qquad \lambda = \frac{x}{z} \; ; \; \lambda = \frac{y+a}{z}$

$\cancel{\frac{y+a}{z} \cdot \frac{2+a}{}}$

$$\left(\frac{2 \cdot \frac{x}{z} + \frac{x}{z} \cdot a}{\frac{y+a}{z}}\right)^2 + \left(\frac{2 \cdot \frac{y+a}{z} - a}{\frac{y+a}{z}}\right)^2 + \left(\frac{2+a}{\frac{y+a}{z} + 1}\right)^2 = 4$$

(8) Find eq of the conical system surface with vertex $V(2,2,2)$ and director curve

$$\mathcal{C}: \begin{cases} y^2 - 4x + 1 = 0 \\ z + 1 = 0 \end{cases}$$

$$d_{\lambda,\mu}: \begin{cases} x-2 = \lambda(z-2) \\ y-2 = \mu(z-2) \end{cases}$$

$$\begin{cases} x-2 = \lambda(z-2) \\ y-2 = \mu(z-2) \\ y^2 - 4x + 1 = 0 \\ z+1 = 0 \end{cases} \rightarrow \begin{cases} x-2 = \lambda(z-2) \\ y-2 = -3\mu \\ y^2 - 4x + 1 = 0 \\ z = -1 \end{cases} \Rightarrow$$

$$\Rightarrow \begin{cases} x = 3\lambda + 2 \\ y = -3\mu + 2 \\ 9\mu^2 - 12\mu + 4 + 12\lambda - 8 + 1 = 0 \end{cases}$$

$$\lambda = \frac{x-2}{z-2} \; ; \; \mu = \frac{y-2}{z-2}$$

$$\boxed{9 \cdot \left(\frac{y-2}{z-2}\right)^2 - 12 \cdot \frac{y-2}{z-2} + 12 \cdot \frac{x-2}{z-2} - 3 = 0}$$

(9) Find the $\underbrace{\text{surface}}_{\text{conoidal}}$ generated by a line which intersects Oz and the line

$$d: \begin{cases} x - z = 0 \\ x + 2y - 3 = 0 \end{cases}$$ and stays $\parallel$ to xOy

Conoidal surface whose generatix intersects

$$d: \begin{cases} \pi_1 = 0 \\ \pi_2 = 0 \end{cases}$$ and $$\mathcal{C}: \begin{cases} F_1(x,y,z) = 0 \\ F_2(x,y,z) = 0 \end{cases}$$

and stays $\parallel$ to $\pi = 0$

$$d_{\lambda\mu}: \begin{cases} \pi_1 = \lambda \\ \pi_{\cancel{x}} = \mu\,\pi_2 \end{cases} \Rightarrow \begin{cases} d_{\lambda,\mu} \\ F_1 = 0 \rightarrow \varphi(\lambda,\mu) = 0 \\ F_2 = 0 \end{cases}$$

$d: \begin{cases} x = 0 \\ y = 0 \end{cases}$ $\quad xOy: z = 0$

$d_{\lambda,\mu}: \begin{cases} z = \lambda \\ x = \mu \cdot y \end{cases}$

$\begin{cases} z = \lambda \\ x = \mu \cdot y \\ x - z = 0 \\ x + 2y - 3 = 0 \end{cases}$ $\Rightarrow x = \lambda$, $y = \frac{\lambda}{\mu}$

$\Rightarrow \boxed{\lambda + \frac{2\lambda}{\mu} - 3 = 0}$ $\quad z = \lambda$, $\mu = \frac{x}{y}$

$z + \frac{2zy}{x} - 3 = 0 \quad | \cdot x$

$\boxed{xz + 2zy - 3x = 0}$