$\boxed{\text{SEMINAR 12}}$ 16.V.2011

① Find eq of tangent lines to hyp $\mathcal{H}: \frac{x^2}{3} - \frac{y^2}{5} - 1 = 0$ passing through $P(1, -5)$

$tg: (y - y_p) = m(x - x_p)$

$(y+5) = m(x-1)$

$$\begin{cases} \frac{x^2}{3} - \frac{y^2}{5} - 1 = 0 \\ (y+5) = m(x-1) \end{cases} \Leftrightarrow$$

$$\begin{cases} y = m(x-1) - 5 \\ \frac{x^2}{3} - \frac{(mx - (m+5))^2}{5} - 1 = 0 \end{cases}$$

$y = m(x-1) - 5$

$5x^2 - 3(m^2x^2 + m^2 + 25 - 2m^2x - 10mx + 10m)$
$- 15 = 0$

$x^2(5 - 3m^2) + x(6m^2 + 30m) - 90 - 3m^2 -$
$- 30m = 0$

$\Delta = b^2 - 4ac = 16m^2 +$

$\Delta = 0$

$\Delta = m^2 - 5m - 15$

$m^2 - 5m - 15 = 0$

$m_1, m_2 = \frac{5 \pm \sqrt{85}}{2}$

$tg_1: (y+5) = \frac{5+\sqrt{85}}{2}(x-1)$

② Find the area of the Δ det by the asymptots of $H: \frac{x^2}{4} - \frac{y^2}{3} = 1$ and the line $d: 9x + 2y - 24 = 0$

$$\boxed{\begin{aligned} as_1 &: y = \frac{b}{a} x \\ as_2 &\ \ y = -\frac{b}{a} x \end{aligned}}$$

$a = 2$

$b = 3$

$\Rightarrow as_1 : y = \frac{3}{2} x$

$as_2 : y = -\frac{3}{2} x$

$\{A\} = as_1 \cap d$

$\{B\} = as_2 \cap d$

$A: \begin{cases} y = \frac{3}{2} x \\ 9x + 2y - 24 = 0 \end{cases}$

$9x + \frac{3}{2} 2x - 24 = 0$

$9x + 3x - 24 = 0$

$x = 2 \Rightarrow y = 3 \Rightarrow A(2, 3)$

$B: \begin{cases} y = -\frac{3}{2} x \\ 9x + 2y - 24 = 0 \end{cases}$

$x = 4 \Rightarrow y = -6 \Rightarrow B(4, -6)$

$\Delta OAB: \quad A_{\Delta OAB} = \frac{1}{2} \left| \overline{OA} \times \overline{OB} \right|$

$= \frac{1}{2} \begin{vmatrix} i & j & k \\ \end{vmatrix} \quad \frac{1}{2} \left| \bar{k} \cdot (-12 - 12) \right| = \frac{1}{2} \left| \vec{k} \cdot (-24) \right| =$

③ a) Find focus and the director line of parabola $P: y^2 - 24x = 0$
b) Find eq of parabola having as the focus $F(-7, 0)$ and the director line $d: x - 7 = 0$

$P: y^2 - 2pX = 0$

focus $F\left(\frac{p}{2}, 0\right)$

director line $x = -\frac{p}{2}$

a) $p = 12$

$F(6, 0)$

$d: x = -6$

b) $F(-7, 0)$

$-7 = \frac{p}{2} \Rightarrow p = -14$

$P: y + 28x = 0$

④ Find eq of tangent line to the parabola $P: y^2 - 8x = 0$ parallel to $d: 2x + 2y - 3 = 0$

$p = 4$

$tg: y = mx + \frac{p}{2m}$ - eq of tangent to parabola having the slope m

$m_d = -1 = m_{tg}$

$y = -x - 2$

5 Find eq of tangent line to $P: y^2 - 36x = 0$
pass through $P(2, 9)$.

$tg: y = mx + \frac{p}{2m}$

$tg: y = mx + \frac{9}{m}$

$P \in tg$

$9 = 2m + \frac{9}{3}$

$2m^2 + 9 - 9m = 0$

$\Delta = 81 - 72 = 9$

$m_1 = \frac{9+3}{4} = 3$

$m_2 = \frac{9-3}{4} = \frac{3}{2}$

$tg_1: y = \frac{3}{2}x + 6$

$tg_2: y = 3x + 3$

6 Find eq of tg line to $P: y^2 - 4x = 0$
at point $P(1, 2)$

$4 - 4 = 0 \Rightarrow P \in P$

$tg_P: y_P y = p(x + x_P)$

$tg_P: 2y = 2(x+1)$

$tg_P: y = x + 1$

(7) Let $P_1: y^2-18x=0$ and $P_1: y^2=10x$ be 2 parabolas. A moble tg to P_2 intersect P_1 at M_1 and M_2. Find the geometric locus of the midpoint M of $[M_1, M_2]$

$P_1: y^2 = 18x \Rightarrow p_1 = 9$

$P_2: y^2 = 10x \Rightarrow p_2 = 5$

$tg: \; y = mx + \frac{p_2}{2m}$

$y = mx + \frac{5}{2m}$

$$\begin{cases} y^2 = 18x \\ y = mx + \frac{5}{2m} \end{cases}$$

$$x = \frac{y - \frac{5}{2m}}{m} = \frac{y}{m} - \frac{5}{2m^2}$$

$$\Rightarrow y^2 = \frac{18y}{m} - \frac{45}{m^2} \quad \Big| \cdot m^2$$

$$y^2m^2 - 18my + 45 = 0$$

$$\Delta = 324m^2 - 4\cdot 45m^2 = 324m^2 - 180m^2 = 144m^2$$

$$y_{1,2} = \frac{18m \pm 12m}{2m^2} = \frac{9m \pm 6m}{m^2} \begin{cases} \frac{15}{m} \\ \frac{3}{m} \end{cases}$$

$x_1 = \frac{1}{2m^2} \qquad y_1 = \frac{3}{m} \qquad M_1\left(\frac{1}{2m^2} ; \frac{3}{m}\right)$

$x_2 = \frac{25}{2m^2} \qquad y_2 = \frac{15}{m} \qquad M_2\left(\frac{25}{2m^2} ; \frac{15}{m}\right)$

$$\boxed{y_M^2 = 2\cdot p\cdot x_M}$$

$$\Rightarrow M\left(\frac{13}{2m^2} , \frac{9}{m}\right)$$

(8) Let $A(1,2)$ $B(4,-4)$ $C(0,0)$ 3 points on a parabola P. The tangent lines at A, B, C determine a triangle $\Delta A'B'C'$. Prove that the line passing through the centers of gravity of ΔABC and $\Delta A'B'C'$ is parallel to Oy

$P: y^2 = 2px$

$A \in P \Rightarrow 4 = 2p$

$p = 2$

$P: y^2 = 4x$

$tg_C = Oy : x = 0$

$tg_A : y_A \cdot y = p(x + x_A)$

$2 \cdot y = 2x + 2$

$tg_A : y = x + 1$

$tg_B : -4y = 2x + 8$

$tg_B : -2y = x + 4$

$GG' : y = -\frac{2}{3}$

$\{C'\} = tg_B \cap tg_A$

$\begin{cases} y = x + 1 \\ -2y = x + 4 \end{cases}$

$\Rightarrow x = -2 \quad y = -1 \qquad C'(-2,-1)$

$\{A'\} = tg_C \cap tg_B$

$\begin{cases} x = 0 \\ -2y = x + 4 \end{cases} \qquad A'(0,-2)$

$\{B'\} = tg_A \cap tg_B$

$\begin{cases} x = 0 \end{cases}$

$\Rightarrow B'(0,1)$

(9) Find the parametric locus of the orthogonal projection of la parabola on the tangent lines to the parabola.

$P: y^2 = 2px$

$F\left(\frac{p}{2}, 0\right)$

$t_{g_m}: y = mx + \frac{p}{2m}$

$FF_1 \perp t_{g_m} \Rightarrow m_{FF_1} \cdot m_{t_{g_m}} = -1$

$\cancel{FF_1}: \quad m_{FF_1} = -\frac{1}{m}$

$FF_1: \quad y - y_F = m_{FF_1}(x - x_F)$

$FF_1 \quad y = -\frac{1}{m}\left(x - \frac{p}{2}\right)$

$\{F_1\} = t_g \cap FF_1$

$$\begin{cases} y = mx + \frac{p}{2m} \\ y = -\frac{1}{m}\left(x - \frac{p}{2}\right) \end{cases}$$

$$\begin{cases} mx + \frac{p}{2m} = -\frac{1}{m}\left(x - \frac{p}{2}\right) \\ mx + \frac{p}{2m} = -\frac{x}{m} + \frac{p}{2m} \end{cases}$$

$mx + \frac{x}{m} = 0$

$x\left(m + \frac{1}{m}\right) = 0 \Rightarrow \boxed{x = 0}$