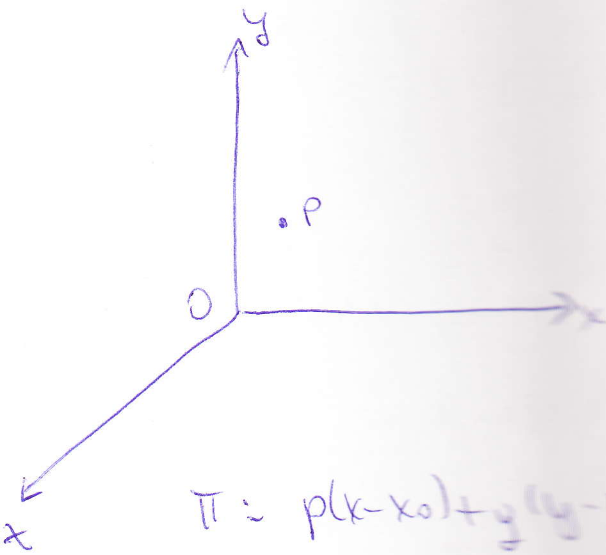
# SEMINAR 8

① Find the distance from the point $P(1,2,-1)$ to the line $d: x=y=z$




$\pi: (x+1)+(y-2)+(z+1)=0$

$\pi: x+y+z-2=0$

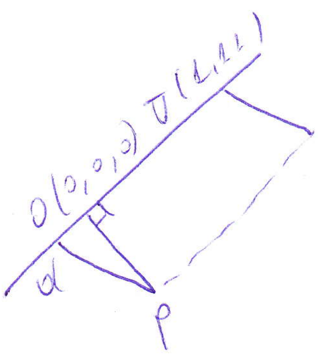
$d: \frac{x-x_0}{p} = \frac{y-y_0}{q} = \frac{z-z_0}{p}$

$p(x_p, y_p, z_p)$

$\pi: p(x-x_0)+q(y-y_0)+p(z-z_0)$

$\begin{cases} x+y+z-2=0 \\ x=y=z \end{cases}$

$\begin{cases} x=\frac{2}{3} \\ y=\frac{2}{3} \\ z=\frac{2}{3} \end{cases}$ $\quad P\left(\frac{2}{3},\frac{2}{3},\frac{2}{3}\right)$




$\delta(P,d) = \frac{\text{area of the parallelogram}}{|\bar{v}|}$

$= \frac{|\bar{OP} \times \bar{v}|}{|\bar{v}|} = \frac{\sqrt{3^2+(-2)^2+(-1)^2}}{\sqrt{3}}$

$\bar{OP} \times \bar{v} = \begin{vmatrix} \bar{i} & \bar{j} & \bar{k} \\ 1 & 2 & -1 \\ 1 & 1 & 1 \end{vmatrix} = 2\bar{i} + \bar{k} - \bar{j} - 2\bar{k} + \bar{i} - \bar{j} = 3i - 2j - k$

$\delta(P,d) = \frac{\sqrt{14}}{\sqrt{3}} = \frac{\sqrt{42}}{3} \Rightarrow PP' = \sqrt{\frac{42}{3}}$

2. Find the distance between the lines

$d_1: \frac{x-1}{2} = \frac{y+1}{3} = \frac{z}{1}$ and

$d_2: \frac{x+1}{3} = \frac{y}{4} = \frac{z-1}{3}$

$A_1(1,-1,0) \in d_1$, $A_2(-2,0,1) \in d_2$, $\bar{v}_1(2,3,1)$, $\bar{v}_2(3,4,3)$

$d_1, d_2$ - coplanar iff $(\overline{A_1A_2}, \bar{v}_1, \bar{v}_2) = \begin{vmatrix} -2 & 1 & 1 \\ 2 & 3 & 1 \\ 3 & 4 & 3 \end{vmatrix} = -18+8+3-9+8 = -14$
$\neq 0$

$\Rightarrow$ they are not coplanar $\Rightarrow$ they are skew lines

$A = \begin{vmatrix} q_1 & r_1 \\ q_2 & r_2 \end{vmatrix} = \begin{vmatrix} 3 & 1 \\ 4 & 3 \end{vmatrix} = 5$

$B = \begin{vmatrix} r_1 & p_1 \\ r_2 & p_2 \end{vmatrix} = \begin{vmatrix} 1 & 2 \\ 3 & 3 \end{vmatrix} = -3$

$C = \begin{vmatrix} p_1 & q_1 \\ p_2 & q_2 \end{vmatrix} = -1$

$\delta(d_1, d_2) = \delta(\pi_1, \pi_2) = \delta(A_1, \pi_2) = \delta(A_2, \pi_1) =$

$= \frac{|A(x_2-x_1) + B(y_2-y_1) + C(z_2-z_1)|}{\sqrt{A^2+B^2+C^2}}$

$d(d_1, d_2) = \frac{|A(x_1-x_2) + B(y_1-y_2) + C(z_1-z_2)|}{\sqrt{A^2+B^2+C^2}}$

$= \frac{|5(2)+3+1|}{\sqrt{25+9+1}} = \frac{14}{\sqrt{35}}$

3. Find the distance between the planes

$\pi_1 : 2x-3y+4z-7=0$

$\pi_2 : 4x-6y+8z-3=0$

$\pi_1 \parallel \pi_2 \qquad \delta(\pi_1,\pi_2)=\delta(P_1,\pi_2),\ P_1 \in \pi$

$2x-3+4-7=0$

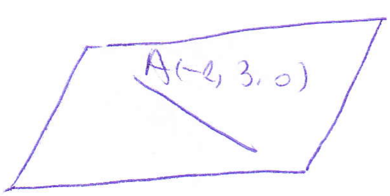
$2x=6$

$x=3 \qquad P_1(3,1,1) \in \pi_1$

$$\delta(P_1,\pi_2)=\frac{|Ax_0+By_0+Cz_0+D|}{\sqrt{A^2+B^2+C^2}}=\frac{|43-6+8-3|}{\sqrt{4+9+16}}=$$

$$=\frac{11}{\sqrt{116}}=\frac{11}{2\sqrt{29}}$$

4. Show that the line $d: \frac{x+1}{1}=\frac{y-3}{2}=\frac{z}{1}$ and the plane $\pi: 2x-2y-2z+5=0$ are parallel and find the distance between them.




$d \parallel \pi \Leftrightarrow \bar{v} \perp \bar{n} \Leftrightarrow \bar{v}\cdot\bar{n}=0$

$\bar{v}\cdot\bar{n}=2-4+2=0 \Rightarrow$ they are parallel

$\bar{v}(1,2,-1)$

$\bar{n}(2,-2,-2) \qquad d(d,\pi)=d(A,\pi)=\frac{|2\cdot(-1)-2\cdot3-2\cdot0+3|}{\sqrt{4+4+4}}=\frac{-5}{\sqrt{12}}=\frac{-5}{2\sqrt{3}}$

5. Find the eq. of the circle

a. of diameter [AB] where A(1,2) and B(-3,-1)

b. of center I(2,-3) and radius R=9

c. of center I(-1,2) which passes through A(2,6)

d. centered at the origin and tangent to $d: 3x-4y+20=0$

e. passing through A(3,1) and B(-1,3) and having the center of the line $d: 3x-y-2=0$

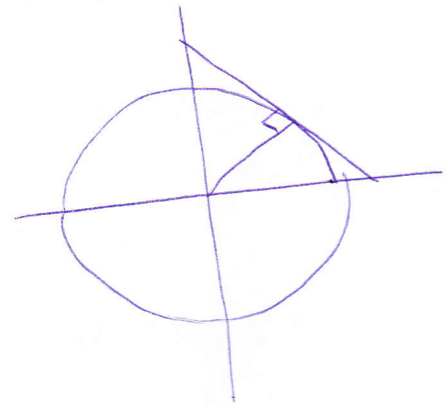
a) $Ox = \frac{x_A + x_B}{2} = -\frac{2}{2} = -1$ $\quad O(-1, \frac{1}{2})$

$Oy = \frac{y_A + y_B}{2} = \frac{1}{2}$ $\quad (x+1)^2 + (y - \frac{1}{2})^2 = \sqrt{\frac{5}{2}}$

b) $(x-2)^2 + (y+3)^2 = 9$

$x^2 + 4x + 4 + y^2 + 6x + 9 - 9 = 0$

$x^2 + 4x - y^2 + 6y + 4 = 0$



c) $(x+1)^2 + (y-2)^2 = r$

$(x+1)^2 + (y-2)^2 = (r+2)^2 + (6-2)^2$

$(x+1)^2 + (y-2)^2 = 25$

d) $\delta(O, d) = \frac{20}{\sqrt{3^2 + (-4)^2}} = \frac{20}{5} = 4$

$x^2 + y^2 = r^2 \Rightarrow x^2 + y^2 = 16$

e) $\frac{x_A + x_B}{2} = 1$

$\frac{y_A + y_B}{2} = 2$ $\quad M(1,2)$ $\quad \frac{x - x_A}{x_B - x_A} = \frac{y - y_A}{y_B - y_A}$

$AB: \frac{x-3}{-1-3} = \frac{y-1}{3-1}$

$\Rightarrow 2x - 6 = -4y + 4$

$\Rightarrow 2x + 4y - 10 = 0$

$D: \frac{x-1}{2} = \frac{y-2}{4}$ $\quad M(1,2)$ $\quad \Rightarrow 4x - 4 - 2y + 4 = 0 \Rightarrow 2x - y = 0$

$$\begin{cases} 3x - y - 2 = 0 \\ 2x - y = 0 \end{cases} \Rightarrow y = 2x \Rightarrow 3x - 2x - 2 = 0 \Rightarrow x = 2 \Rightarrow y = 4$$

$d(N, A) = \sqrt{1 + 9} = \sqrt{10}$

$(x-2)^2 + (y-4)^2 = 10$