Seminar 4

1. Cercetati natura urmatoarelor serii cu termeni pozitivi utilizand criteriile indicate

   i) criteriul comparatiei

   a) $\sum\limits_{n=1}^{\infty} \frac{1}{\sqrt{4n^2-1}}$

   b) $\sum\limits_{n=1}^{\infty} \ln\left(1+\frac{1}{n^2}\right)$

   c) $\sum\limits_{n=1}^{\infty} \sin^3 \frac{1}{n}$

   ii) consecinte ale criteriului lui Kummer

   a) $\sum\limits_{n=0}^{\infty} \frac{2^n}{n!}$

   b) $\sum\limits_{n=0}^{\infty} \left(\frac{1}{2}\right)^{\sqrt{n}}$

   c) $\sum\limits_{n=1}^{\infty} \left[\frac{(2n)!!}{(2n+1)!!}\right]^2$

   d) $\sum\limits_{n=1}^{\infty} \frac{(an)^n}{n!}, \quad a>0$

   iii) criteriul radicalului

   $\sum\limits_{n=1}^{\infty} \frac{n^2}{\left(2+\frac{1}{n}\right)^n}$

   iv) criteriul condensarii

   $\sum\limits_{n=2}^{\infty} \frac{1}{n(\ln n)^p}, \quad p>0$

2. Studiati convergenta si absolut convergenta urmatoarelor serii

   a) $\sum\limits_{n=0}^{\infty} (-1)^n \frac{2n+1}{3^n}$

   b) $\sum\limits_{n=1}^{\infty} (-1)^{n+1} \frac{\sqrt{n}}{n+\sqrt{2}}$

   c) $\sum\limits_{n=1}^{\infty} \frac{\sin n}{2^n}$

   d) $\sum\limits_{n=1}^{\infty} (-1)^n n$

   e) $\sum\limits_{n=1}^{\infty} \sin(\pi\sqrt{n^2+1})$

3. (**criteriul raportului pentru siruri**) Fie $(x_n)_{n\in\mathbb{N}}$ un sir cu termeni strict pozitivi pentru care exista limita $\lim\limits_{n\to\infty} \frac{x_{n+1}}{x_n} = l \geq 0$. Au loc afirmatiile

   i) Daca $l<1$ atunci $\lim\limits_{n\to\infty} x_n = 0$

   ii) Daca $l>1$ atunci $\lim\limits_{n\to\infty} x_n = \infty$

4. Calculati limita sirului $x_n = \frac{3^n\, n!}{n^n}, n \geq 1$.