Seminar 2

1. Calculati limita sirului $(x_n)_{n\in\mathbb{N}}$ pentru
   a) $x_n = 5n - n^3$
   b) $x_n = \frac{2^n+3^n}{3^n+4^n}$
   c) $x_n = \frac{\sin(n!)}{n^2}, \quad n \geq 1$

2. Justificati cu definitia valoarea limitelor
   a) $\lim\limits_{n\to\infty} \frac{1}{n^2} = 0$
   b) $\lim\limits_{n\to\infty} \sqrt[n]{n} = 1$
   c) $\lim\limits_{n\to\infty} \frac{n^2}{n+1} = \infty$

3. Studiati convergenta sirului $(x_n)_{n\in\mathbb{N}}$ si calculati limita sa acolo unde ea exista
   a) $x_n = a^n, \quad a \in \mathbb{R}$
   b) $x_n = 1.\underbrace{99\ldots9}_{n \text{ ori}}, \quad n \geq 1$
   c) $x_n = \frac{2^n}{n!}$
   d) $x_{n+1} = \frac{x_n}{2} + \frac{1}{x_n}, \quad n \geq 1, x_1 > 0$
   e) $x_{n+1} = 1 + \frac{1}{x_n}, \quad n \geq 1, x_1 = 1$
   f) $x_n = \frac{\sin(1!)}{1\cdot2} + \frac{\sin(2!)}{2\cdot3} + \ldots + \frac{\sin(n!)}{n\cdot(n+1)}, \quad n \geq 1$
   g) $x_n = 1 + \frac{1}{2^2} + \frac{1}{3^2} + \ldots + \frac{1}{n^2}, \quad n \geq 1$
   h) $x_n = \left(1 + \frac{1}{n}\right)^n, \quad n \geq 1$
   i) $x_n = 1 + \frac{1}{1!} + \frac{1}{2!} + \ldots + \frac{1}{n!}$

4. Determinati multimea punctelor limita ale sirului $(x_n)_{n\in\mathbb{N}}$ pentru
   a) $x_n = (-1)^n\, n \sin\frac{n\pi}{2}$
   b) $x_n = \left(1 + \frac{\cos(n\pi)}{n}\right)^n, \quad n \geq 1$
   c) $x_n = \frac{1}{2+\sqrt{n}\cos(n\pi)}$

5. $e \notin \mathbb{Q}$.