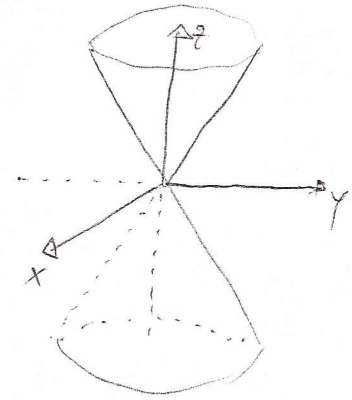
Course 12

**The elliptic cone** $\mathscr{C}: \frac{x^2}{a^2} + \frac{y^2}{b^2} - \frac{z^2}{c^2} = 0$




$(z=\lambda) \cap \mathscr{C}: \begin{cases} \frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{\lambda^2}{c^2} \\ z = \lambda \end{cases}$

$\downarrow$

ellipse for $\lambda \neq 0$ and the point $O(0,0,0)$ for $\lambda = 0$

**The elliptic paraboloid** $\mathscr{P}_e: \frac{x^2}{p} + \frac{y^2}{q} = 2z$, $p, q > 0$

$(z=\lambda) \cap \mathscr{P}_e: \begin{cases} \frac{x^2}{p} + \frac{y^2}{q} = 2\lambda \\ z = \lambda \end{cases}$

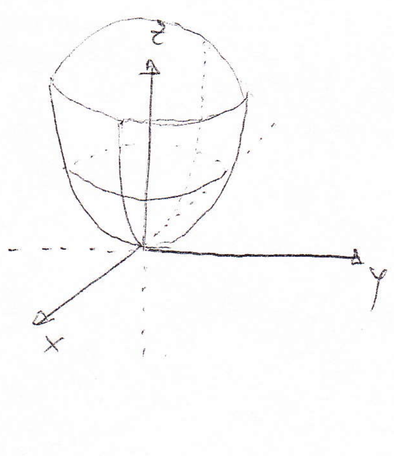
I. $\lambda < 0$, $(z=\lambda) \cap \mathscr{P}_e = \emptyset$

II. $\lambda = 0$, $(xOy) \cap \mathscr{P}_e = \{O(0,0,0)\}$

III. $\lambda > 0$, $(z=\lambda) \cap \mathscr{P}_e \begin{cases} \frac{x^2}{(\sqrt{2p\lambda})^2} + \frac{y^2}{(\sqrt{2p\lambda})^2} = 1 \\ z = \lambda \end{cases}$ ellipse

$(xOz) \cap \mathscr{P}_e: \begin{cases} x^2 = 2pz \\ y = 0 \end{cases}$ parabola

$(yOz) \cap \mathscr{P}_e: \begin{cases} y^2 = 2pz \\ x = 0 \end{cases}$ parabola



The hyperbolic paraboloid $\mathcal{P}_h: \frac{x^2}{p} - \frac{y^2}{q} = 2z \quad , p,q>0$

$$(z=\lambda) \cap \mathcal{P}_h : \begin{cases} \frac{x^2}{p} - \frac{y^2}{q} = 2\lambda \\ z=\lambda \end{cases}$$

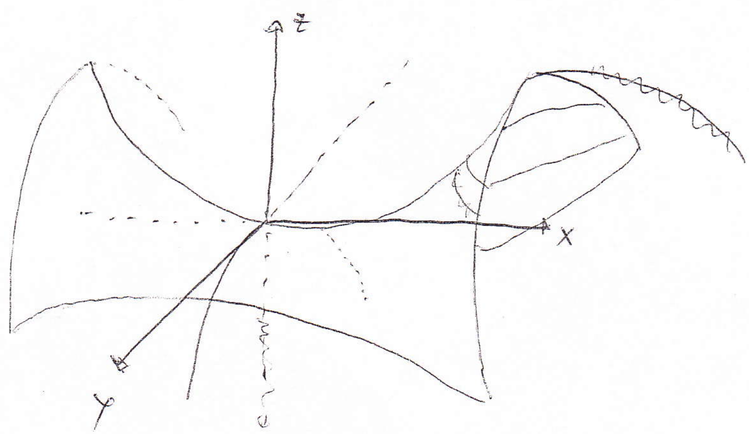
– hyperbola for $\lambda \neq 0$

For $\lambda = 0$, $(xOy) \cap \mathcal{P}_h : \begin{cases} \frac{x^2}{p} - \frac{y^2}{q} = 0 \\ z=0 \end{cases} \Leftrightarrow$

$$\begin{cases} \left(\frac{x}{\sqrt{p}} - \frac{y}{\sqrt{q}}\right)\left(\frac{x}{\sqrt{p}} + \frac{y}{\sqrt{q}}\right) = 0 \\ z=0 \end{cases} \Leftrightarrow \begin{cases} \frac{x}{\sqrt{p}} - \frac{y}{\sqrt{q}} = 0 \\ z=0 \end{cases} \text{ or } \begin{cases} \frac{x}{\sqrt{p}} + \frac{y}{\sqrt{q}} = 0 \\ z=0 \end{cases}$$

$$(y=\mu) \cap \mathcal{P}_h : \begin{cases} \frac{x^2}{p} = 2z + \frac{\mu^2}{q} \\ y=\mu \end{cases} \Leftrightarrow \begin{cases} x^2 = 2p\left(z + \frac{\mu^2}{2pq}\right) \\ y=\mu \end{cases}$$ parabola

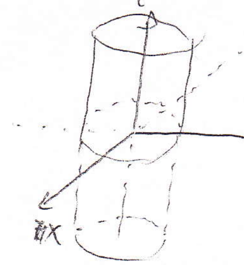
$$(x=\nu) \cap \mathcal{P}_h : \begin{cases} \frac{y^2}{q} = \frac{\nu^2}{p} - 2z \\ x=\nu \end{cases} \Leftrightarrow \begin{cases} y^2 = -2q\left(z - \frac{\nu^2}{2pq}\right) \\ x=\nu \end{cases}$$ – parabola




$$\frac{x^2}{p} - \frac{y^2}{q} = 2z \Leftrightarrow \left(\frac{x}{\sqrt{p}} - \frac{y}{\sqrt{q}}\right)\left(\frac{x}{\sqrt{p}} + \frac{y}{\sqrt{q}}\right) = 2z$$

$$\Delta_\lambda : \begin{cases} \frac{x}{\sqrt{p}} - \frac{y}{\sqrt{q}} = \lambda \\ \lambda\left(\frac{x}{\sqrt{p}} + \frac{y}{\sqrt{q}}\right) = 2z \end{cases}, \quad \Delta'_\mu : \begin{cases} \frac{x}{\sqrt{p}} + \frac{y}{\sqrt{q}} = \mu \\ \mu\left(\frac{x}{\sqrt{p}} - \frac{y}{\sqrt{q}}\right) = 2z \end{cases}$$

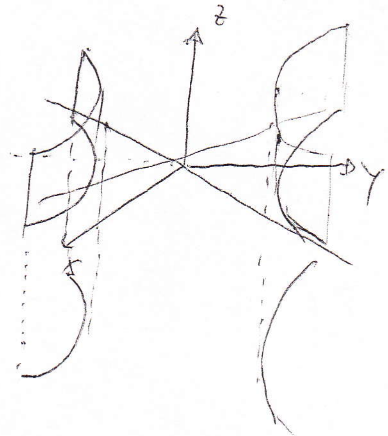
The elliptic cylinder $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$




$\frac{x^2}{a^2}+\frac{z^2}{c^2}=1$ or $\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$

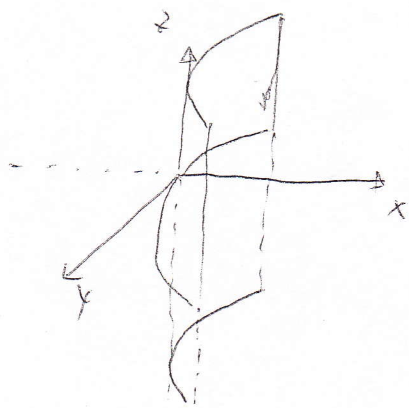
The hyperbolic cylinder $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$

or $\frac{y^2}{b^2}-\frac{z^2}{c^2}=1$ or $\frac{x^2}{a^2}-\frac{z^2}{c^2}=1$

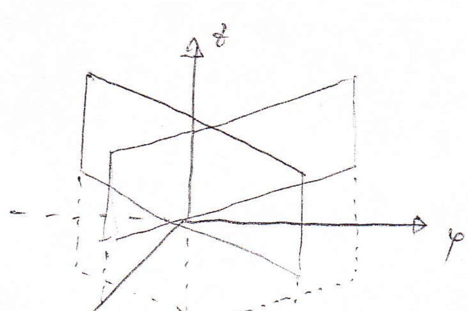



Parabolic cylinder

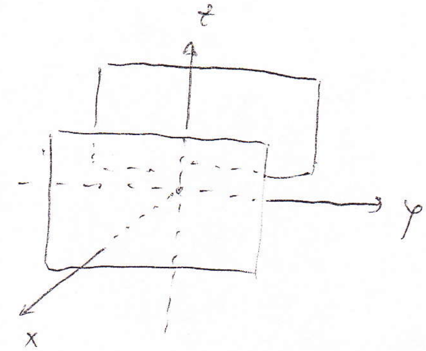
$y^2=2px$




A pair of two concurrent planes $\frac{x^2}{a^2}-\frac{y^2}{b^2}=0$



A pair of parallel planes $x^2=a^2$




Two identical planes $x^2=0$

The line $\frac{x^2}{a^2}+\frac{y^2}{b^2}=0$

The point $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=0$

The empty set $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=-1$