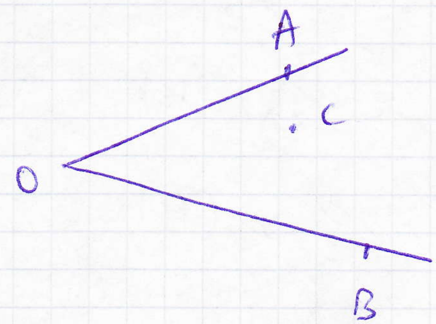
SEMINAR 9

18.IV.2011

① Let C be a point inside the angle $\widehat{AOB}$. Det 2 points $P \in (OA$ and $Q \in (OB$ s.t. the perimeter of ΔCPQ is minimum.



$C' = s_{OA}(C)$

$C'' = s_{OB}(C)$

$\{P\} = C'C'' \cap OA$

$\{Q\} = C'C'' \cap OB$

$P_{CPQ} = PC + CQ + QP = PC' + QC'' + PQ = C'C''$

$P_{\Delta PCQ} = C'C'' < C'P' + P'Q' + Q'C'' =$

$= P'C + P'Q' + Q'C = P_{\Delta Q'CP'}$

$\Rightarrow P_{\Delta PCQ}$ – minim.

(2) Let ΔABC, H - orthocenter. Prove that the symmetrical of H with respect to AB, AC, BC are situated on the circumscribed circle of ΔABC

let $M = s_{AB}(H)$

$N = s_{AC}(H)$

$P = s_{BC}(H)$

? $M, N, P \in \mathcal{C}(A,B,C)$

We'll show that $M \in \mathcal{C}(A,B,C)$

$\Leftrightarrow$ $MACB$ is a cyclic quadrilater

$\Leftrightarrow m(\widehat{AMB}) + m(\widehat{BCA}) = 180^\circ$

$m(\widehat{BMA}) = m(\widehat{BHA}) = m(\widehat{B'HA'})$ (1)

in $A'CB'H$: $m(\widehat{AB'C}) + m(\widehat{CA'H}) = 180^\circ$

$\Rightarrow$ cyclic. $\Rightarrow m(\widehat{B'HA'}) + m(\widehat{A'CB'}) = 180$ (*)

$\Rightarrow m(\widehat{BMA}) + m(\widehat{ACB}) = 180^\circ$

$\Rightarrow MACB$ cyclic $\Rightarrow M \in \mathcal{C}(A,B,C)$

③ Find eq of circle of center $I(-1,2)$ and which passes through $A(2,6)$

eq. of cycle of center $I(x_I, y_I)$ and radius R

$$\mathcal{C}: (x-x_I)^2 + (y-y_I)^2 = R^2$$

$$IA = \sqrt{(x_I - x_A)^2 + (y_I - y_A)^2} = \sqrt{25} = 5$$

$$\mathcal{C}: (x+1)^2 + (y-2)^2 = 25$$

$$x^2 + 2x + 1 + y^2 - 4y + 4 - 25 = 0$$

$$\mathcal{C}: x^2 + y^2 + 2x - 4y - 20 = 0$$

④ Find eq of circle of diameter $[AB]$

$A(1,2)$ $B(-3,-1)$

$$AB = \sqrt{(-3-1)^2 + (-1-2)^2} = \sqrt{16+9} = \sqrt{25} = 5$$

$$x_O = \frac{x_A + x_B}{2} = -1$$

$$y_O = \frac{y_A + y_B}{2} = \frac{1}{2}$$

$O(-1; \frac{1}{2})$ $OB = \frac{5}{2}$

$$(x+1)^2 + (y-\frac{1}{2})^2 = \frac{25}{4}$$

$$x^2 + 2x + 1 + y^2 - y + \frac{1}{4} = \frac{25}{4}$$

⑤ Find eq of the circle centered at the origin and tangent to $d: 3x - 4y + 20 = 0$

$$d(O, d) = \frac{|20|}{5} = 4$$

$$C: x^2 + y^2 = 16$$

⑥ Find eq of circle det $A(1,1)$, $B(1,-1)$
$C(2,0)$

$$\begin{vmatrix} x^2 + y^2 & x & y & 1 \\ x_A^2 + y_A^2 & x_A & y_A & 1 \\ x_A^2 + y_B^2 & x_B & y_B & 1 \\ x_C^2 + y_C^2 & x_C & y_C & 1 \end{vmatrix} = 0$$

$$\begin{vmatrix} x^2 + y^2 & x & y & 1 \\ 2 & 1 & 1 & 1 \\ 2 & 1 & -1 & 1 \\ 4 & 2 & 0 & 1 \end{vmatrix} = 0$$

$C_1 - 4C_4 \to C_1$

$C_2 - 2C_4 \to C_2$

$$\begin{vmatrix} x^2+y^2-4 & x-2 & y & 1 \\ -2 & -1 & 1 & 1 \\ -2 & -1 & -1 & 1 \\ 0 & 0 & 0 & 1 \end{vmatrix} = 0$$

$$\Leftrightarrow \begin{vmatrix} x^2+y^2-4 & x-2 & y \\ -2 & -1 & 1 \\ -2 & -1 & -1 \end{vmatrix} = 0$$

$$= x^2+y^2-4 \cancel{+2y} -2(x-2) \cancel{-2y} + x^2+y^2-4$$
$$-2(x-2)$$

$$= 2x^2+2y^2-4x-8+4+4$$
$$= 2x^2+2y^2-4x = 0$$
$$x^2+y^2-2x=0$$

(7) Det the position of $A(1,-2)$ relative to the circle $\mathcal{C}: x^2+y^2-8x-4y-5=0$

$$x^2-8x+16+y^2-4y+4-20-5=0$$
$$(x-4)^2+(y-2)^2=25$$
$$\Rightarrow R=5$$
$$O(4,2)$$

$OA=\sqrt{9+16}=5=R \Rightarrow A$ points on $\mathcal{C}$

⑧ Det the position of $d: 2x-y-3=0$ relative to the circle $\mathcal{C}: x^2+y^2-3x+2y-3=0$

$$\begin{cases} y = 2x-3 \\ x^2+y^2-3x-2y-3=0 \end{cases}$$

$x^2+(2x-3)^2-3x+2(2x-3)-3=0$

$x^2+4x^2-12x+9-3x+4x-6-3=0$

$5x^2-11x=0$

$\Rightarrow$ 2 sol. $\Rightarrow d \cap \mathcal{C}\ \{A, B\}\quad A \neq B$

⑨ Find the intersection between $d: 7x-y+12=0$ and $\mathcal{C}: (x-2)^2+(y-1)^2-25=0$

$$\begin{cases} y = 7x+12 \\ (x-2)^2+(y-1)^2-25=0 \end{cases}$$

$(x-2)^2+(7x+11)^2-25=0$

$x^2-4x+4+49x^2+154x+121-25=0$

$50x^2+150x+100=0 \quad | :50$

$x^2+3x+2=0$

$x^2+2x+x+2$

$x(x+2)+x+2=0$

$(x+2)(x+1)=0$

$x=-2 \Rightarrow y=-2 \qquad A(2,-2)$

$x=-1 \Rightarrow y=5 \qquad B(-1,5)$

22.
$\frac{7}{4}$

(d)

⑩ Find eq of the tangent to $\mathcal{C}: x^2+y^2-5=0$ at the point $A(-1,2)$

$O(0,0)$

$A(-1,2)$

$$OA: \frac{x-x_A}{x_0-x_A} = \frac{y-y_A}{y_0-y_A}$$

$$OA: \frac{x+1}{1} = \frac{y-2}{-2}$$

$$-2x-2 = y-2$$

$$OA: 2x+y=0$$

$$m_{OA} = -2$$

$$OA \perp d \Leftrightarrow m_d = \frac{1}{2}$$

$$d: y-y_A = m_d(x-x_A)$$

$$y-2 = \frac{1}{2}(x+1) \quad | \cdot 2$$

$$2y-4 = x+1$$

$$d: x-2y+5=0$$

$$\mathcal{C}: x^2+y^2-R^2=0$$

$$P_0(x_0, y_0)$$

$$tg_P: x_0x + y_0y - R^2 = 0$$