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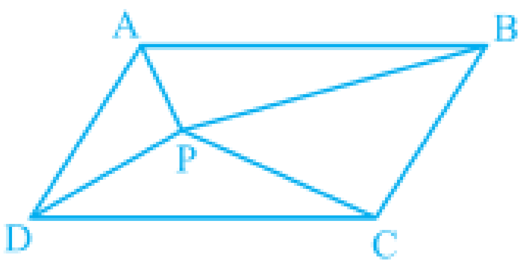
MATRIX ASSIGNMENT

Problem Statement:

For a given parallelogram ABCD, show that for any point 'P' inside the parallelogram,

i. $Ar(\Delta APB) + Ar(\Delta PCD) = \frac{1}{2}Ar(ABCD)$

ii. $Ar(\Delta APB) + Ar(\Delta PCD) = Ar(\Delta APD) + Ar(\Delta PCB)$



Solution:

Given a Parallelogram 'ABCD' with a interior point 'P'
We Know That,

$$Ar(\Delta le) = \frac{1}{2} * Base * Height \qquad (eq1)$$

$$\text{Ar}(||\text{gm}) = \text{Base*Height} \qquad \text{(eq 2)}$$

(i) $Ar(\Delta APB) + Ar(\Delta PCD) = \frac{1}{2}Ar(ABCD)$:

Let us consider a line 'EF' drawn parallel to 'AB' and 'CD' that is passing through the point 'P' as shown in the Fig.1.

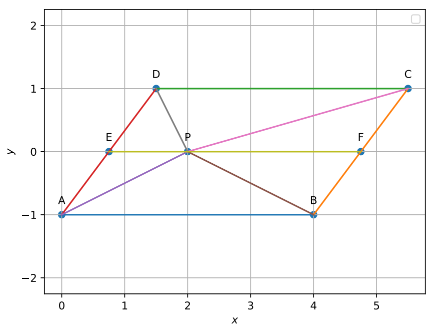


Fig.1

Now 'EFCD' becomes a parallelogram with Triangle PCD inside it.
Parallelogram EFCD and Triangle PCD have the same base 'CD'.
If we consider a perpendicular from point 'P' to line 'CD', it becomes the height h for both ||gm EFCD and $\Delta le PCD$.
From "eq 1" and "eq 2",

$$\text{Ar}(\Delta PCD) = \frac{1}{2} * CD * h$$

$$\text{Ar}(||\text{gm EFCD}) = \text{CD*h}$$

$$\Rightarrow Ar(\Delta PCD) = \frac{1}{2} * Ar(||gmEFCD) \qquad (eq3)$$

Similarly,

$$\Rightarrow Ar(\Delta APB) = \frac{1}{2} * Ar(||gmABFE) \qquad (eq4)$$

On adding RHS of "eq3" and "eq4" we get total area of Paralleogram ABCD,

$$\therefore Ar(\Delta APB) + Ar(\Delta PCD) = \frac{1}{2}Ar(||gmABCD) \qquad (eq5)$$

(ii) $Ar(\Delta APB) + Ar(\Delta PCD) = Ar(\Delta APD) + Ar(\Delta PCB)$:

Now let's consider a line 'GH' drawn parallel to 'AD' and 'BC' that is passing through the point 'P' as shown in the Fig.2.

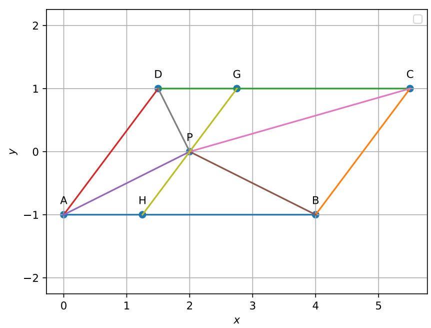


Fig.2

Similarly, from the above proof we can state that,

$$\Rightarrow Ar(\Delta APD) + Ar(\Delta PCB) = \frac{1}{2}Ar(||gmABCD) \qquad (eq6)$$

On comparing "eq 5" and "eq 6",

$$\therefore Ar(\Delta APB) + Ar(\Delta PCD) = Ar(\Delta APD) + Ar(\Delta PCB)$$

"Hence Proved"