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Bayesian Statistics and Data Analysis

Lecture 8b

Måns Magnusson

Department of Statistics, Uppsala University
Thanks to Aki Vehtari, Aalto University

- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
 - When is LOO applicable
 - PSIS LOO-CV
 - K-fold CV
 - Model Comparison
- Information criteria
- Model averaging and Stacking
- Summary



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- **Model assessment and selection**
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Section 1

Model assessment and selection



- Model assessment and selection

- Measures of predictive accuracy
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- Modeling complex phenomena with models that are simplified

All models are wrong... but some are useful.



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- Modeling complex phenomena with models that are simplified
All models are wrong... but some are useful.
- True predictive performance is found out by using it to *make predictions* and comparing predictions to *true observations*
 - external validation



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- Modeling complex phenomena with models that are simplified

All models are wrong... but some are useful.
- True predictive performance is found out by using it to make predictions and comparing predictions to true observations
 - external validation
- Expected predictive performance
 - approximates the external validation



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Goal of model evaluation

- **Model assessment and selection**

- Measures of predictive accuracy
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- **Model choice is a (model-)decision-theoretic problem**



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- Model choice is a (model-)decision-theoretic problem
- Evaluate the **utility** of a model M for new **unseen data** \tilde{y} :

$$U = \int u(\tilde{y}) p_{\text{true}}(\tilde{y}) d\tilde{y},$$

where \tilde{y} is an unseen observation generated from the true data generating process $p_{\text{text}}(\tilde{y})$, and y are observed data and $u(\tilde{y})$ is a utility function.



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- The expectation is with respect to p_{true} (f in BDA3)



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- The expectation is with respect to p_{true} (f in BDA3)
- Choose the model function to **maximize our utility**



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Model choice utility

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- Application specific utility/cost functions are important
 - eg. money, life years, quality adjusted life years, etc.



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- Application specific utility/cost functions are important
 - eg. money, life years, quality adjusted life years, etc.
- General utility: overall in the goodness of the predictive distribution
 - we don't know (yet) the application specific utility then good information theoretically justified choice is log-score for model M

$$\log p_M(y^{\text{rep}}|y)$$



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$$\log p_M(y^{\text{rep}}|y)$$

- We want the "best" model to explain the data



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Subsection 1

Measures of predictive accuracy



- Point residuals

$$e_i = y_i - E(\tilde{y}_i|y),$$

where

$$E(\tilde{y}|y) = \int \tilde{y} p(\tilde{y}|y) d\tilde{y},$$

i.e. the **expected predicted value**

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i.e. the **expected predicted value**

- Mean squared (prediction) error (MSE)

$$\text{MSE} = \frac{1}{n} \sum_i^n [y_i - E(\tilde{y}_i|y)]^2.$$

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Probabilistic predictions

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- The log score (a local and proper scoring rule)*

$$\log p(y|\theta)$$



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- The log score (a local and proper scoring rule)*

$$\log p(y|\theta)$$

- The log predictive density (lpd)

$$\begin{aligned}\text{lpd} &= \log p(y|y) \\ &= \log \int p(y|\theta)p(\theta|y)d\theta\end{aligned}$$



- The lpd is usually approximated with the log **point** predictive density (lppd or just lpd)

$$\begin{aligned}\text{lppd} &= \sum_i^n \log p(y_i|y) \\ &\approx \log p(y|y)\end{aligned}$$

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$$\begin{aligned}\text{lppd} &= \sum_i^n \log p(y_i|y) \\ &\approx \log p(y|y)\end{aligned}$$

- Estimation using MCMC

$$\text{lppd} = \sum_i^n \log \left(\frac{1}{S} \sum_s^S p(y_i|\theta_s) \right)$$



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Subsection 2

Model selection



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- Evaluate how model M **generalizes to unseen data** \tilde{y} (the *expected log predictive density*):

$$\text{elpd}_M = \int \log p_M(\tilde{y}|y) p_{\text{true}}(\tilde{y}) d\tilde{y},$$

where \tilde{y} is an unseen observation generated from the true data generating process $p_{\text{true}}(\tilde{y})$, and y are observed data.

- $\log p_M(\tilde{y}|y)$ is the log score (the utility of the model)



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- p_{true} is (almost always) **unknown**



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- p_{true} is (almost always) **unknown**
- The utility function is the log scoring rule.



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Section 2

Cross-validation



Leave-one-out cross-validation (LOO-CV)

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where \tilde{y}_i is an unseen observation generated from the true data generating process $p_{\text{true}}(\tilde{y}_i)$, and y are observed data.

- Can we approximate $p_{\text{true}}(\tilde{y}_i)$?



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Leave-one-out cross-validation (LOO-CV)

- Approximate $p_{\text{true}}(\tilde{y}_i)$ with data y

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Leave-one-out cross-validation (LOO-CV)

- Approximate $p_{\text{true}}(\tilde{y}_i)$ with data y
- Hold out observation i and try to predict y_i based on \mathbf{y}_{-i}
- Estimation of elpd_M using **leave-one-out cross-validation**

$$\begin{aligned}\text{elpd}_{\text{loo}} &= \sum_{i=1}^n \log p_M(y_i | \mathbf{y}_{-i}) \\ &= \sum_{i=1}^n \log \int p_M(y_i | \theta) p(\theta | \mathbf{y}_{-i}) d\theta\end{aligned}$$

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- **Analogy:** Monte Carlo approximation using our data
- Similar to **jack-knife resampling**

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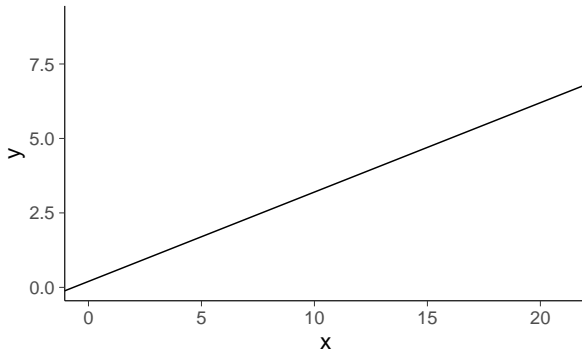
- **Analogy:** Monte Carlo approximation using our data
- Similar to **jack-knife resampling**
- The elpd , lpd and efficient number of parameters (p_{loo})

$$\text{elpd}_{\text{loo}} = \text{lpd} - p_{\text{loo}}$$



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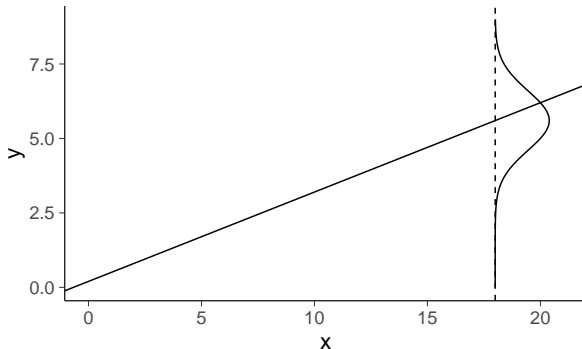
True mean $y = a + bx$





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True mean and sigma





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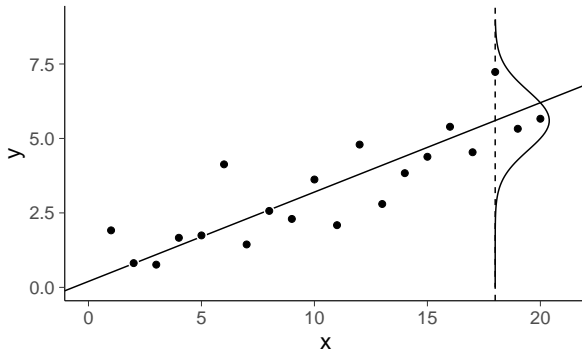
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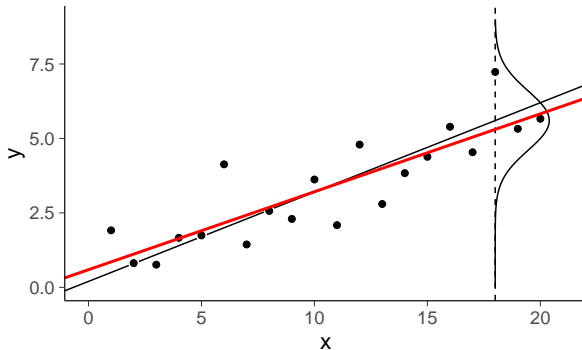
Data





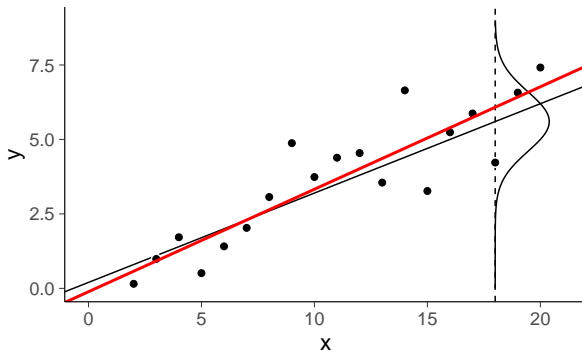
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Posterior mean





Posterior mean, alternative data realisation

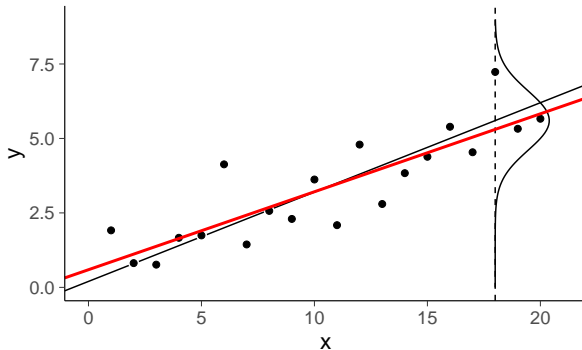


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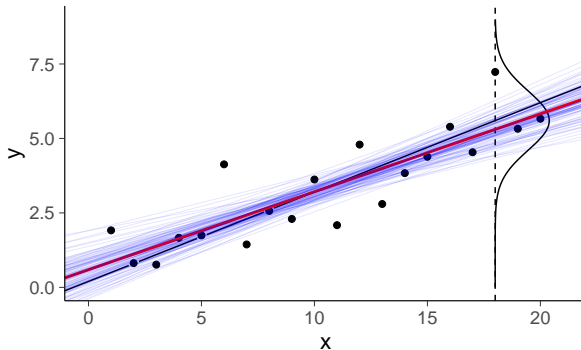
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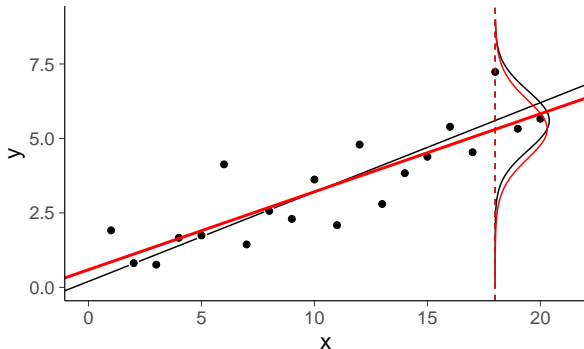
Posterior draws





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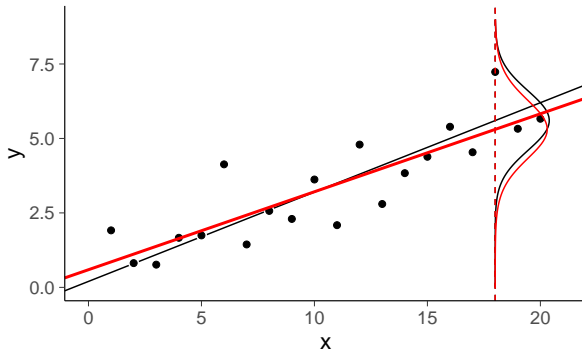
Posterior predictive distribution





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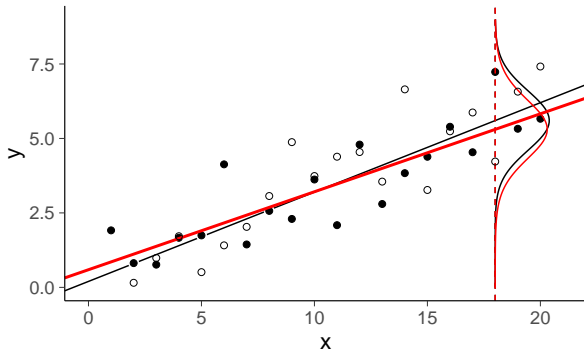
Posterior predictive distribution



$$p(\tilde{y}|\tilde{x} = 18, x, y) = \int p(\tilde{y}|\tilde{x} = 18, \theta)p(\theta|x, y)d\theta$$



New data

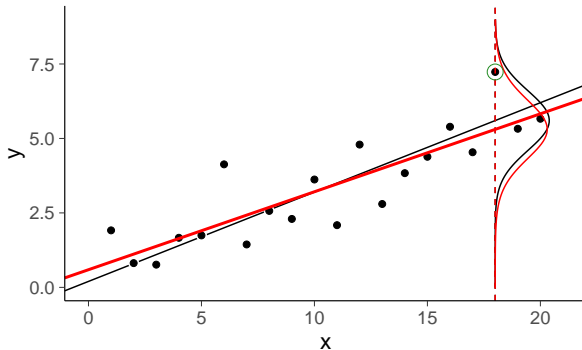


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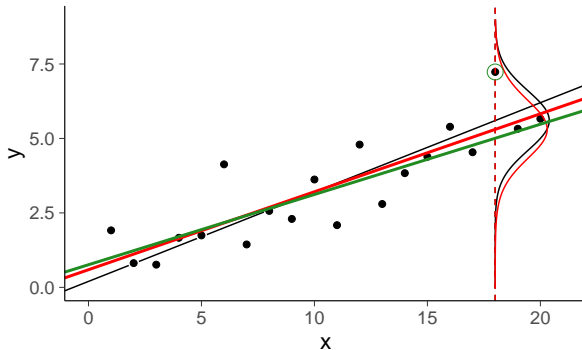
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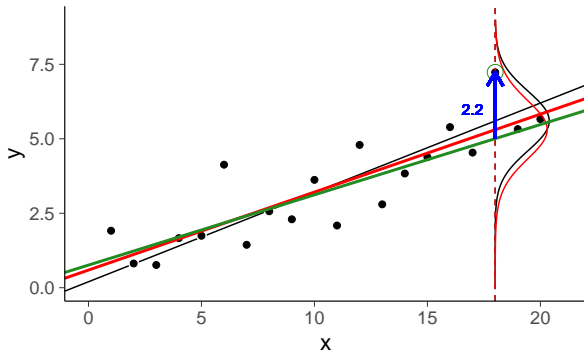
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Leave-one-out mean





Leave-one-out residual



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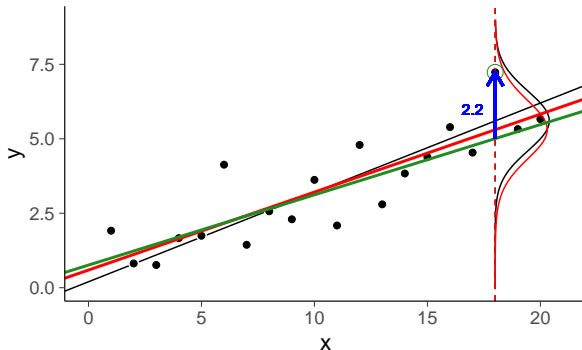
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Leave-one-out residual



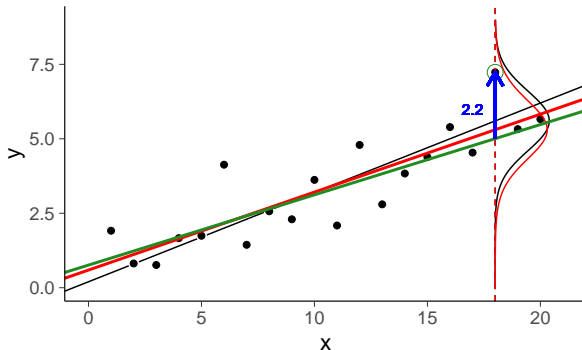
$$y_{18} - E[p(\tilde{y}|\tilde{x} = 18, x_{-18}, y_{-18})]$$

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Leave-one-out residual



$$y_{18} - E[p(\tilde{y}|\tilde{x} = 18, x_{-18}, y_{-18})]$$

Can be use to compute, e.g., RMSE, R^2 , 90% error



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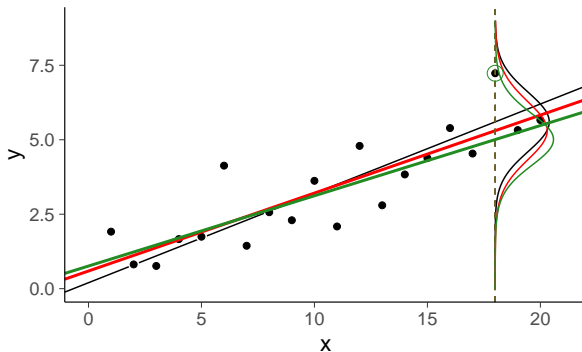
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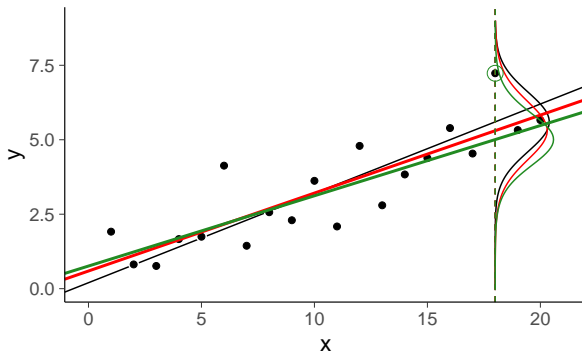
Leave-one-out predictive distribution





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- Information criteria
- Model averaging and Stacking
- Summary

Leave-one-out predictive distribution



$$p(\tilde{y}|\tilde{x} = 18, x_{-18}, y_{-18}) = \int p(\tilde{y}|\tilde{x} = 18, \theta)p(\theta|x_{-18}, y_{-18})d\theta$$



- Model assessment and selection

- Measures of predictive accuracy
- Model selection

- Cross-validation

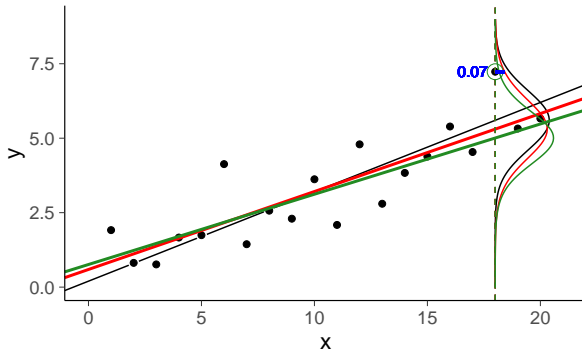
- When is LOO applicable
- PSIS LOO-CV
- K-fold CV
- Model Comparison

- Information criteria

- Model averaging and Stacking

- Summary

Posterior predictive density





- Model assessment and selection

- Measures of predictive accuracy
- Model selection

- Cross-validation

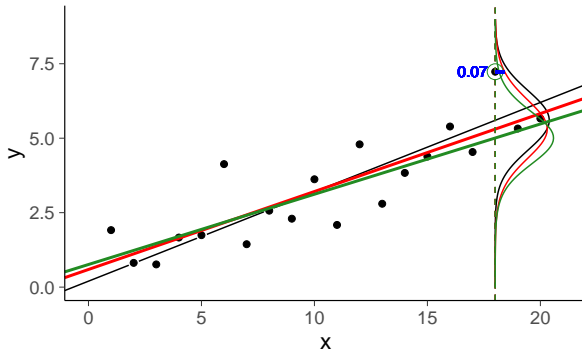
- When is LOO applicable
- PSIS LOO-CV
- K-fold CV
- Model Comparison

- Information criteria

- Model averaging and Stacking

- Summary

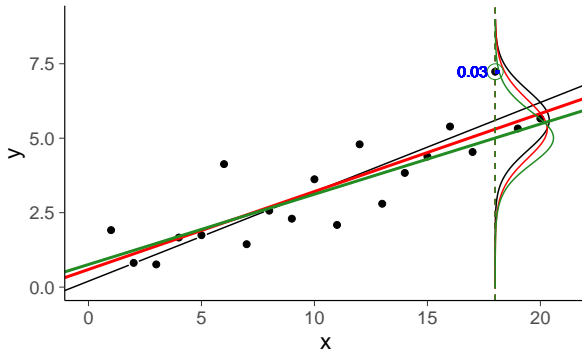
Posterior predictive density



$$p(\tilde{y} = y_{18} | \tilde{x} = 18, x, y) \approx 0.07$$



Leave-one-out predictive density



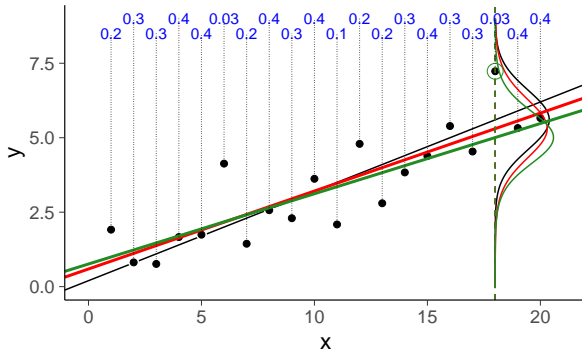
$$p(\tilde{y} = y_{18} | \tilde{x} = 18, x, y) \approx 0.07$$

$$p(\tilde{y} = y_{18} | \tilde{x} = 18, x_{-18}, y_{-18}) \approx 0.03$$

- Model assessment and selection
 - Measures of predictive accuracy
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Leave-one-out predictive densities

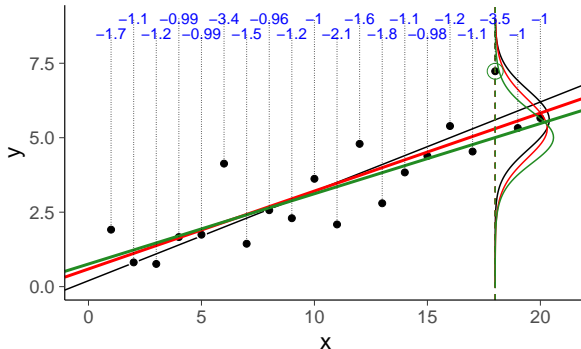


$$p(y_i|x_i, x_{-i}, y_{-i}), \quad i = 1, \dots, 20$$

- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
 - When is LOO applicable
 - PSIS LOO-CV
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- Information criteria
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Leave-one-out log predictive densities

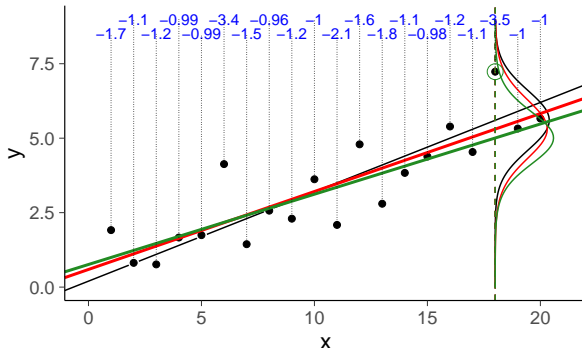


$$\log p(y_i | x_i, x_{-i}, y_{-i}), \quad i = 1, \dots, 20$$

- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
 - When is LOO applicable
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Leave-one-out log predictive densities

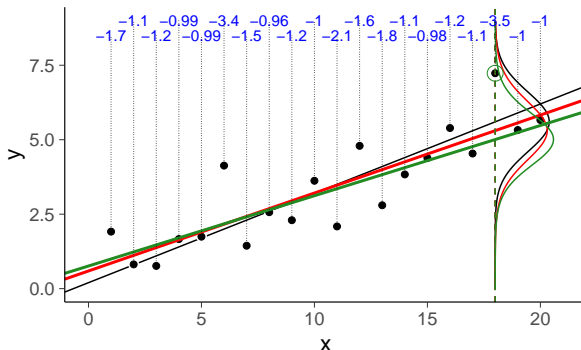


$$\sum_{i=1}^{20} \log p(y_i | x_i, x_{-i}, y_{-i}) \approx -29.5$$

- Model assessment and selection
 - Measures of predictive accuracy
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- Cross-validation
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Leave-one-out log predictive densities

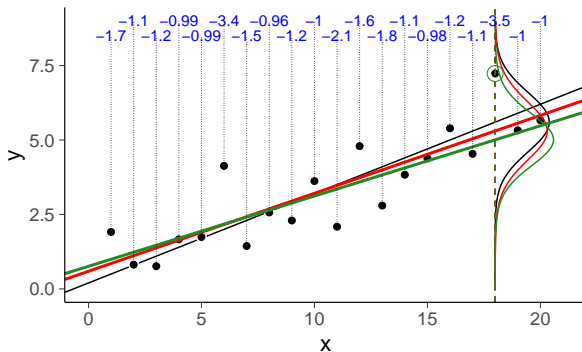


$$\text{elpd}_{\text{loo}} = \sum_{i=1}^{20} \log p(y_i | x_i, x_{-i}, y_{-i}) \approx -29.5$$

- Model assessment and selection
 - Measures of predictive accuracy
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Leave-one-out log predictive densities



$$\text{elpd}_{\text{loo}} = \sum_{i=1}^{20} \log p(y_i | x_i, x_{-i}, y_{-i}) \approx -29.5$$

unbiased estimate of log posterior pred. density for new data

- Model assessment and selection

- Measures of predictive accuracy
- Model selection

- Cross-validation

- When is LOO applicable
- PSIS LOO-CV
- K-fold CV
- Model Comparison

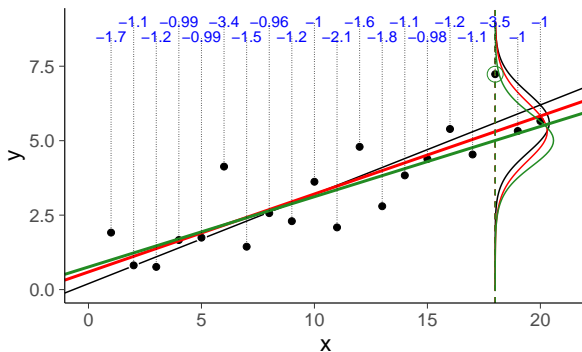
- Information criteria

- Model averaging and Stacking

- Summary



Leave-one-out log predictive densities



$$\text{elpd}_{\text{loo}} = \sum_{i=1}^{20} \log p(y_i | x_i, x_{-i}, y_{-i}) \approx -29.5$$

$$\text{lpd} = \sum_{i=1}^{20} \log p(y_i | x_i, x, y) \approx -26.8$$

- Model assessment and selection

- Measures of predictive accuracy
- Model selection

- Cross-validation

- When is LOO applicable
- PSIS LOO-CV
- K-fold CV
- Model Comparison

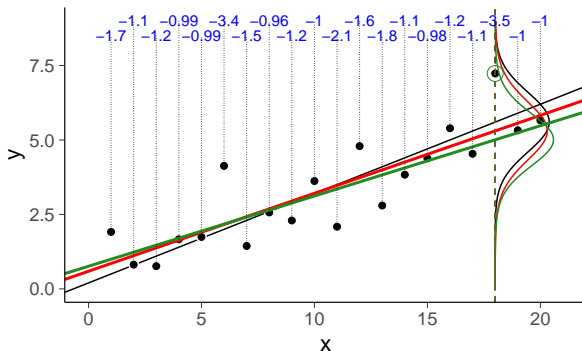
- Information criteria

- Model averaging and Stacking

- Summary



Leave-one-out log predictive densities



$$\text{elpd_loo} = \sum_{i=1}^{20} \log p(y_i | x_i, x_{-i}, y_{-i}) \approx -29.5$$

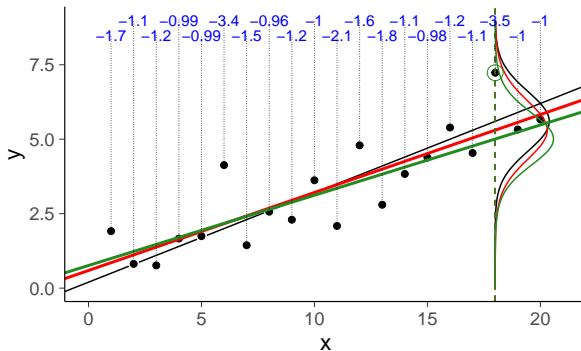
$$\text{lpd} = \sum_{i=1}^{20} \log p(y_i | x_i, x, y) \approx -26.8$$

$$\text{p_loo} = \text{lpd} - \text{elpd_loo} \approx 2.7$$

- Model assessment and selection
 - Measures of predictive accuracy
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Leave-one-out log predictive densities



$$\text{elpd_loo} = \sum_{i=1}^{20} \log p(y_i | x_i, x_{-i}, y_{-i}) \approx -29.5$$

$$\text{SE} = \text{sd}(\log p(y_i | x_i, x_{-i}, y_{-i})) \cdot \sqrt{20} \approx 3.3$$

- Model assessment and selection

- Measures of predictive accuracy
- Model selection

- Cross-validation

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Computed from 4000 by 20 log-likelihood matrix

	Estimate	SE
elpd_loo	-29.5	3.3
p_loo	2.7	1.0

Monte Carlo SE of elpd_loo is 0.1.

Pareto k diagnostic values:

		Count	Pct.	Min.	n_eff
(-Inf, 0.5]	(good)	18	90.0%	899	
(0.5, 0.7]	(ok)	2	10.0%	459	
(0.7, 1]	(bad)	0	0.0%	<NA>	
(1, Inf)	(very bad)	0	0.0%	<NA>	

All Pareto k estimates are ok ($k < 0.7$).
See `help('pareto-k-diagnostic')` for details.

- Model assessment and selection
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- Model assessment and selection
 - Measures of predictive accuracy
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Subsection 1

When is LOO applicable



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Pro and cons with LOO-CV

- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
 - When is LOO applicable
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 - K-fold CV
 - Model Comparison
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- Model averaging and Stacking
- Summary

- + Intuitive
- + Robust
- + Good theoretical properties



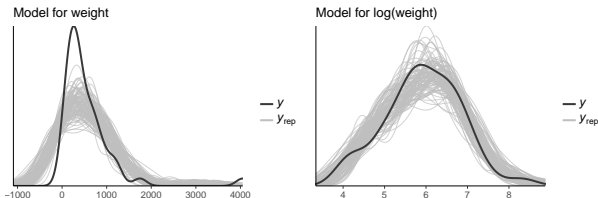
- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
 - When is LOO applicable
 - PSIS LOO-CV
 - K-fold CV
 - Model Comparison
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- Model averaging and Stacking
- Summary

- + Intuitive
- + Robust
- + Good theoretical properties
 - Can be costly (naive LOO-CV mean n posterior computations)



Sometimes cross-validation is not needed

- Posterior predictive checking can be sufficient



Predicting the yields of mesquite bushes.

Gelman, Hill & Vehtari (2020, Ch. 11)

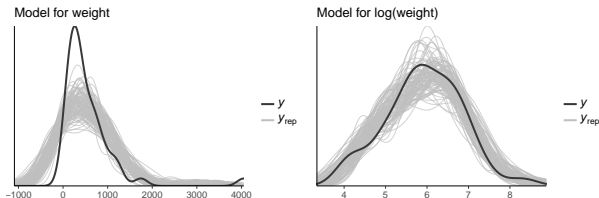
- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
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Sometimes cross-validation is not needed

- Model assessment and selection
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- Posterior predictive checking can be sufficient



Predicting the yields of mesquite bushes.

Gelman, Hill & Vehtari (2020, Ch. 11)

- In nested case, often easier and more accurate to analyse posterior distribution of more complex model directly



Data generating mechanisms and prediction tasks

- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
 - When is LOO applicable
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 - K-fold CV
 - Model Comparison
- Information criteria
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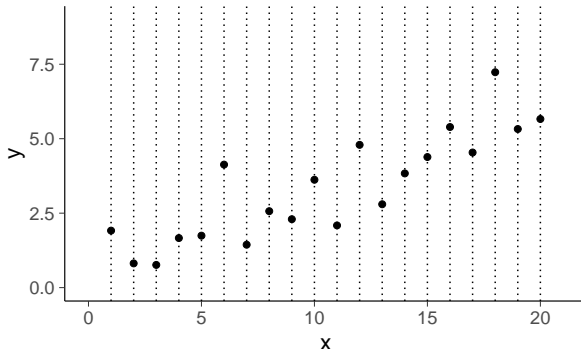
1. You have to make some assumptions on data generating mechanism p_{true}

$$\text{elpd}_M = \int \log p_M(\tilde{y}|y) p_{\text{true}}(\tilde{y}) d\tilde{y},$$

2. Use the knowledge of the prediction task if available
3. Cross-validation can be used to analyse different parts, even if there is no clear prediction task



Fixed / designed x

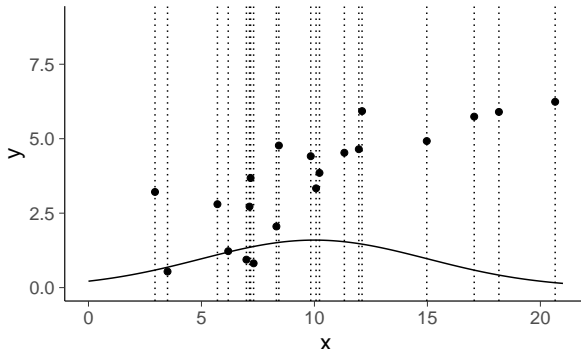


LOO is ok for fixed / designed x : $p_{\text{true}}(y|x)$

- Model assessment and selection
 - Measures of predictive accuracy
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Distribution for x



LOO is ok for random x . $p_{\text{true}}(y, x)$

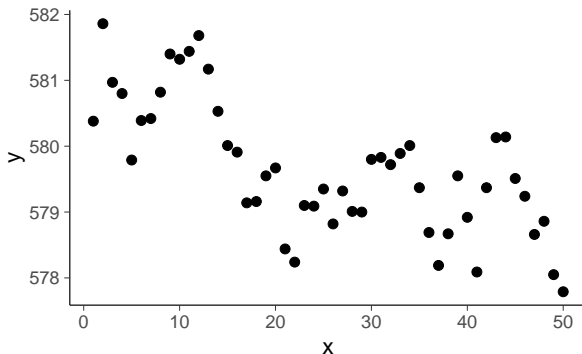
- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
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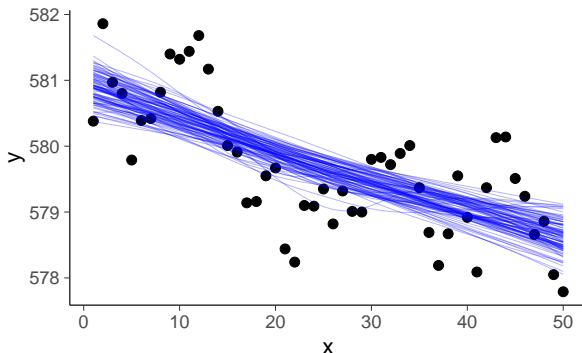
p_{true} extrapolation

- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
 - When is LOO applicable
 - PSIS LOO-CV
 - K-fold CV
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- Summary





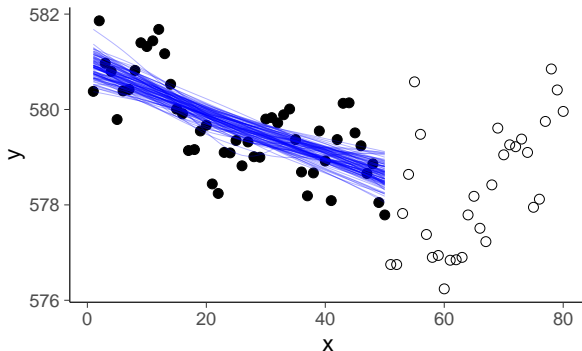
Nonlinear model fit



- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
 - *When is LOO applicable*
 - PSIS LOO-CV
 - K-fold CV
 - Model Comparison
- Information criteria
- Model averaging and Stacking
- Summary



Nonlinear model fit + new data



Extrapolation is more difficult

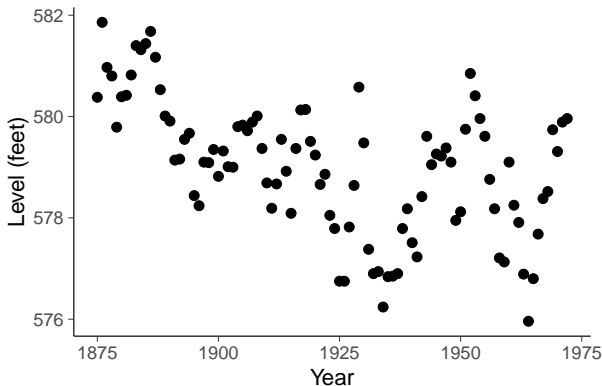
- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
 - When is LOO applicable
 - PSIS LOO-CV
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- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
 - When is LOO applicable
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LOO for time series data



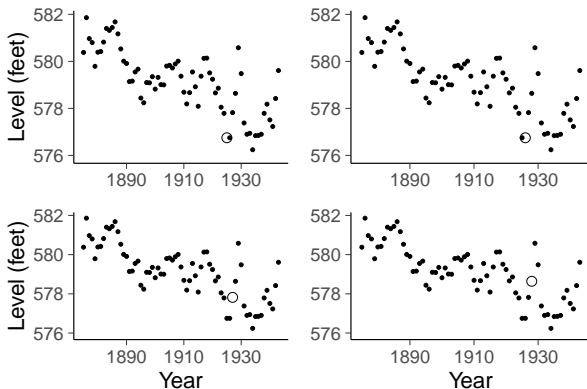
p_{true} for time series?



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- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
 - When is LOO applicable
 - PSIS LOO-CV
 - K-fold CV
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LOO for time series

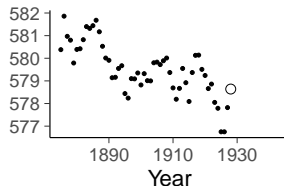
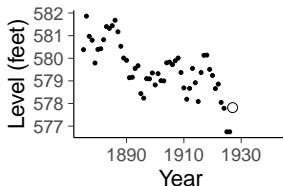
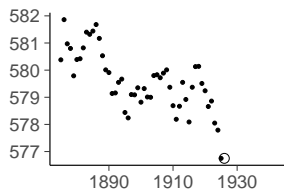
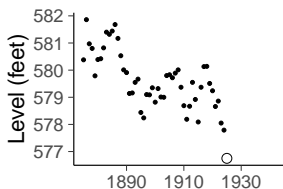


Leave-one-out cross-validation is ok for assessing conditional model



- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
 - When is LOO applicable
 - PSIS LOO-CV
 - K-fold CV
 - Model Comparison
- Information criteria
- Model averaging and Stacking
- Summary

LOO for time series

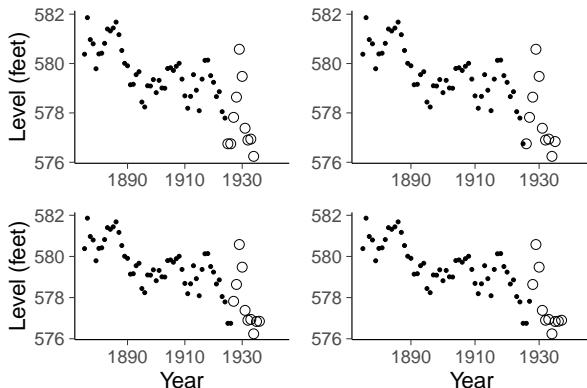


Leave-future-out cross-validation is better for predicting future



- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
 - When is LOO applicable
 - PSIS LOO-CV
 - K-fold CV
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LOO for time series

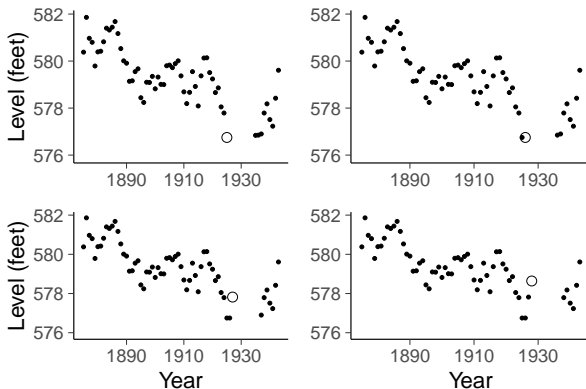


m-step-ahead cross-validation is better for predicting further future



- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
 - When is LOO applicable
 - PSIS LOO-CV
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- Information criteria
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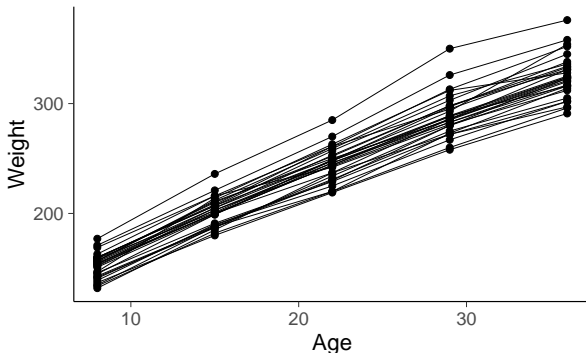
LOO for time series



m-step-ahead leave-a-block-out cross-validation



Rats data

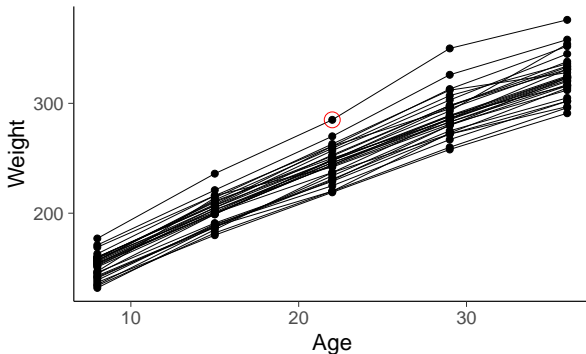


Can LOO or other cross-validation be used with hierarchical data?

- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
 - When is LOO applicable
 - PSIS LOO-CV
 - K-fold CV
 - Model Comparison
- Information criteria
- Model averaging and Stacking
- Summary



Leave-one-out?

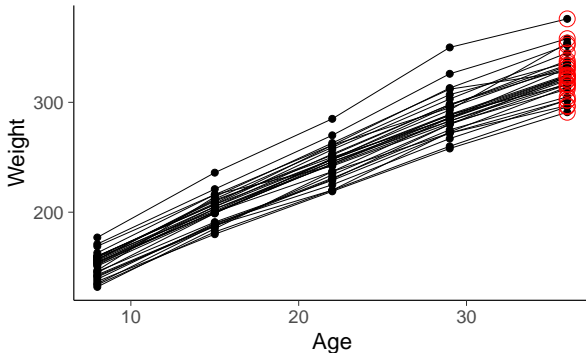


Yes!

- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
 - When is LOO applicable
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1-step-ahead?

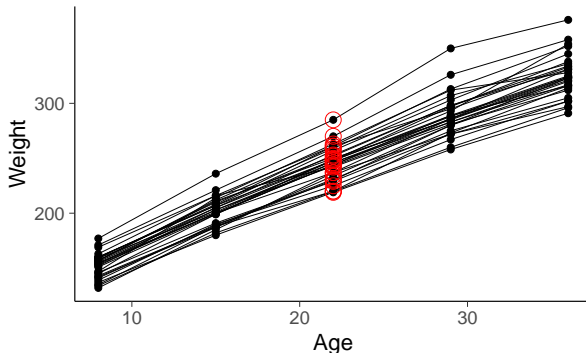


Yes!

- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
 - When is LOO applicable
 - PSIS LOO-CV
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Leave-one-time-point-out?



Yes!

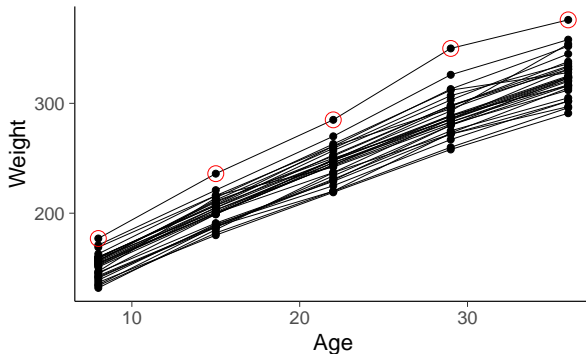
- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
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 - When is LOO applicable
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- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
 - When is LOO applicable
 - PSIS LOO-CV
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LOO for hierarchical data

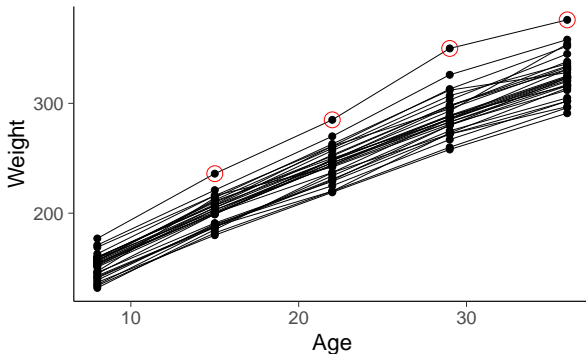
Leave-one-rat-out?



Yes!



Predict given initial weight?



Yes!

- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
 - When is LOO applicable
 - PSIS LOO-CV
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Fast cross-validation

1. Pareto smoothed importance sampling LOO (PSIS-LOO)

- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
 - When is LOO applicable
 - PSIS LOO-CV
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1. Pareto smoothed importance sampling LOO (PSIS-LOO)
2. K-fold cross-validation

- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
 - When is LOO applicable
 - PSIS LOO-CV
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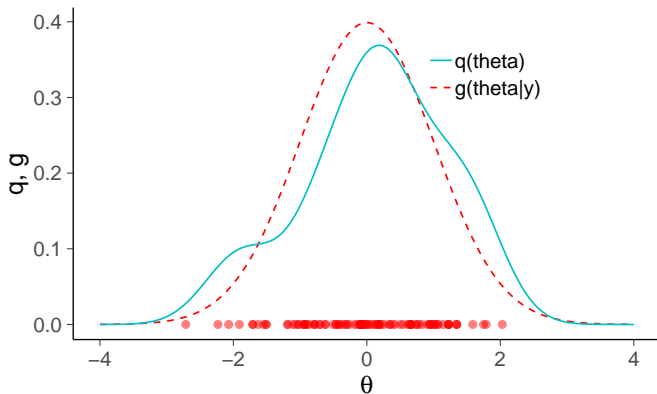
- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
 - When is LOO applicable
 - **PSIS LOO-CV**
 - K-fold CV
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Subsection 2

PSIS LOO-CV



Target, proposal, and draws



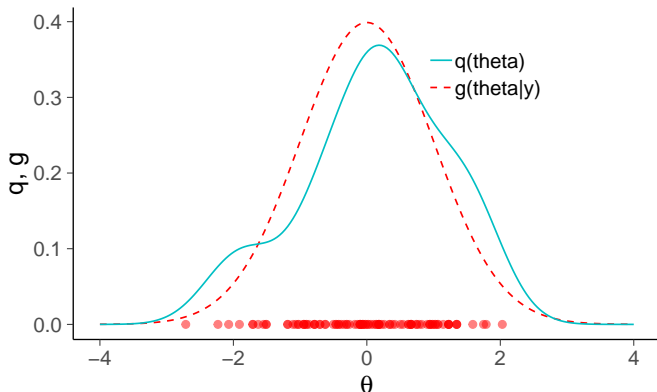
- Model assessment and selection
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- Model assessment and selection
 - Measures of predictive accuracy
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Recap: Importance sampling

Target, proposal, and draws



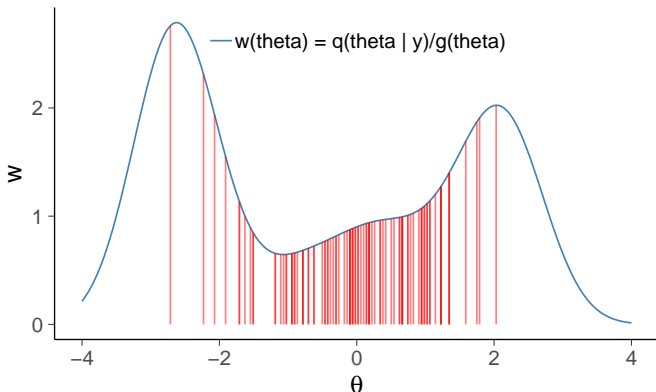
$$E[f(\theta)] \approx \frac{\sum_s w_s f(\theta^{(s)})}{\sum_s w_s}, \quad \text{where} \quad w_s = \frac{q(\theta^{(s)})}{g(\theta^{(s)})}$$



- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
 - When is LOO applicable
 - **PSIS LOO-CV**
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Recap: Importance sampling

Draws and importance weights



$$E[f(\theta)] \approx \frac{\sum_s w_s f(\theta^{(s)})}{\sum_s w_s}, \quad \text{where } w_s = \frac{q(\theta^{(s)})}{g(\theta^{(s)})}$$



- Model assessment and selection
 - Measures of predictive accuracy
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- Let $f = p(\tilde{y}_i|y_{-i})$, $g = p(\theta|y)$, and $q = p(\theta|y_{-i})$



- Model assessment and selection
 - Measures of predictive accuracy
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- Let $f = p(\tilde{y}_i|y_{-i})$, $g = p(\theta|y)$, and $q = p(\theta|y_{-i})$
- Having samples θ^s from $p(\theta^s|y)$

$$p(\tilde{y}_i|y_{-i}) \approx \frac{\sum_{s=1}^S p(\tilde{y}_i|\theta^s) w_i^s}{\sum_{s=1}^S w_i^s},$$

where w_i^s are importance weights and

$$w_i^s = \frac{p(\theta^s|y_{-i})}{p(\theta^s|y)}.$$



- We get a nice solution

$$w_i^s = \frac{p(\theta^s | y_{-i})}{p(\theta^s | y)} = \frac{\frac{p(\theta^s) \prod_{j \neq i} p(y_j | \theta^s)}{p(y_{-i})}}{\frac{p(\theta^s) \prod_j p(y_j | \theta^s)}{p(y)}} \propto \frac{1}{p(y_i | \theta^s)}.$$

- Model assessment and selection
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- We get a nice solution

$$w_i^s = \frac{p(\theta^s | y_{-i})}{p(\theta^s | y)} = \frac{\frac{p(\theta^s) \prod_{j \neq i} p(y_j | \theta^s)}{p(y_{-i})}}{\frac{p(\theta^s) \prod_j p(y_j | \theta^s)}{p(y)}} \propto \frac{1}{p(y_i | \theta^s)}.$$

- If evaluated with $\tilde{y}_i = y_i$

$$p(\tilde{y}_i | y_{-i}) \approx \frac{\sum_{s=1}^S p(\tilde{y}_i | \theta^s) w_i^s}{\sum_{s=1}^S w_i^s} = \frac{1}{\sum_{s=1}^S \frac{1}{p(y_i | \theta^s)}},$$

and

$$\log w_i^{(s)} = -\log p(y_i | \theta^{(s)})$$

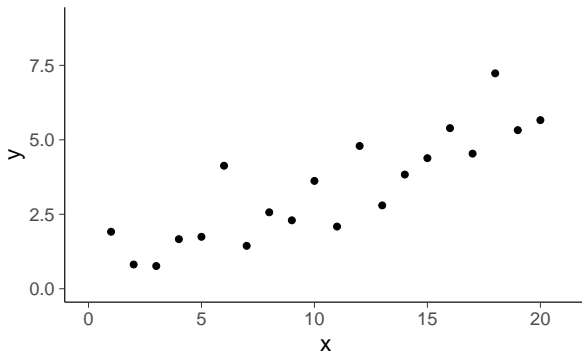
- Model assessment and selection
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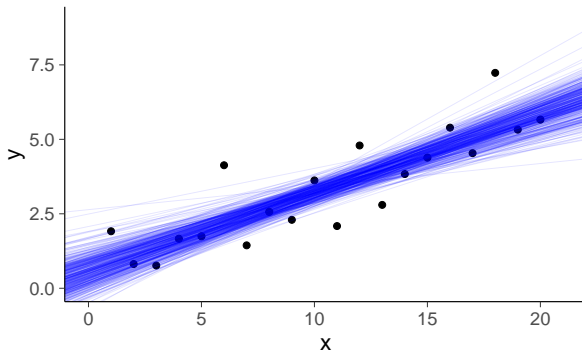
Data





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Posterior draws

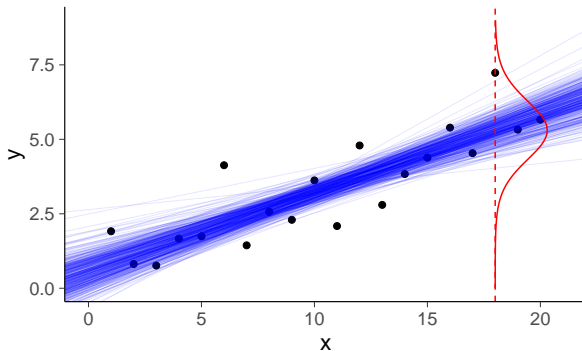


$$\theta^{(s)} \sim p(\theta|x, y)$$



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Posterior predictive distribution

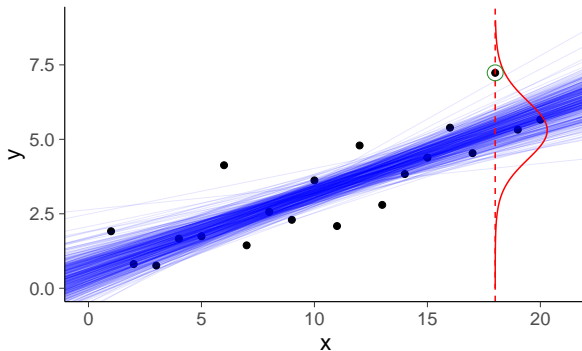


$$\theta^{(s)} \sim p(\theta|x, y), \quad p(\tilde{y}|\tilde{x}, x, y) \approx \frac{1}{S} \sum_{s=1}^S p(\tilde{y}|\tilde{x}, \theta^{(s)})$$



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Posterior predictive distribution

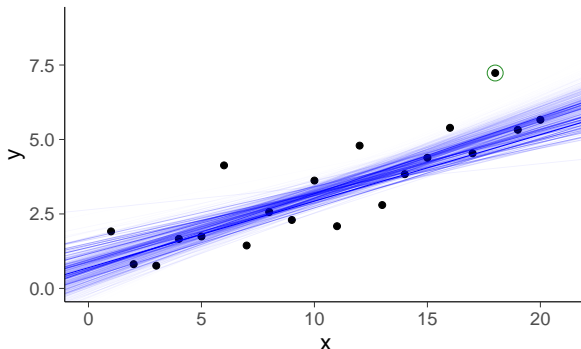


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PSIS-LOO weighted draws



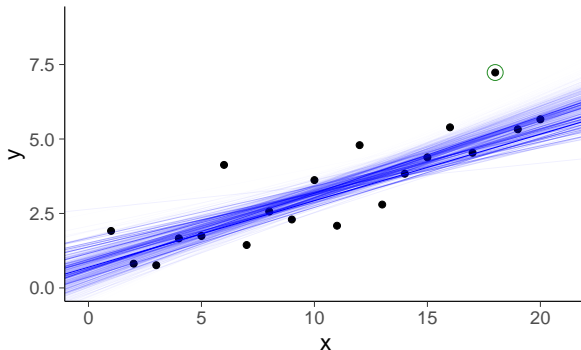
$$\theta^{(s)} \sim p(\theta|x, y)$$

$$w_i^{(s)} = p(\theta^{(s)}|x_{-i}, y_{-i})/p(\theta^{(s)}|x, y)$$



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PSIS-LOO weighted draws



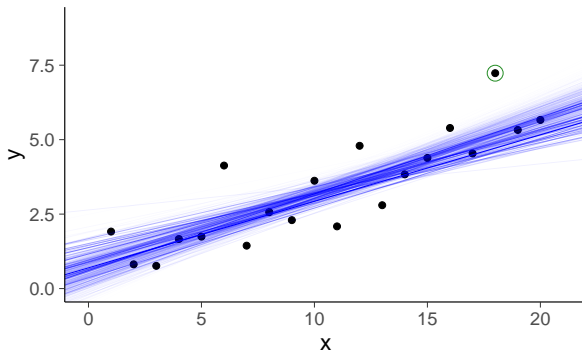
$$\theta^{(s)} \sim p(\theta|x, y)$$

$$w_i^{(s)} = p(\theta^{(s)}|x_{-i}, y_{-i})/p(\theta^{(s)}|x, y) \propto 1/p(y_i|x_i, \theta^{(s)})$$



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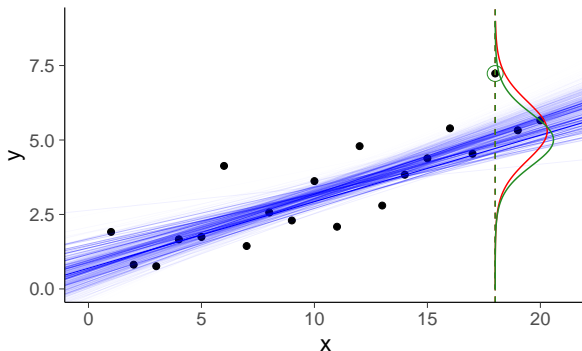
$$w_i^{(s)} = p(\theta^{(s)}|x_{-i}, y_{-i})/p(\theta^{(s)}|x, y) \propto 1/p(y_i|x_i, \theta^{(s)})$$

$$\log(1/p(y_i|x_i, \theta^{(s)})) = -\log_lik[i]$$



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PSIS-LOO weighted predictive distribution



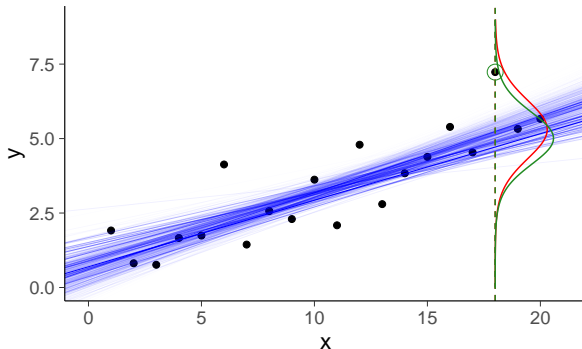
$$\theta^{(s)} \sim p(\theta|x, y)$$

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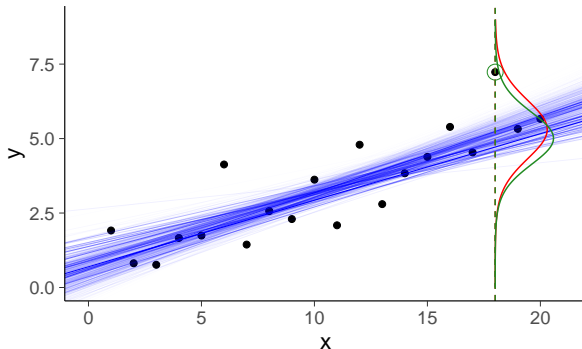
$$w_i^{(s)} = p(\theta^{(s)}|x_{-i}, y_{-i})/p(\theta^{(s)}|x, y) \propto 1/p(y_i|x_i, \theta^{(s)})$$

$$p(y_i|x_i, x_{-i}, y_{-i}) \approx \sum_{s=1}^S [w_i^{(s)} p(y_i|x_i, \theta^{(s)})]$$



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Recap: Pareto smoothed importance sampling

- Model assessment and selection
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- Pareto-Smoothed Importance sampling smooth the weights according to a Generalized Pareto(k) distribution
 - Pareto- k diagnostic estimate the number of existing moments ($\lfloor 1/k \rfloor$)



Recap: Pareto smoothed importance sampling

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 - Finite variance and central limit theorem for $k < 1/2$



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- Pareto-Smoothed Importance sampling smooth the weights according to a Generalized Pareto(k) distribution
- Pareto- k diagnostic estimate the number of existing moments ($\lfloor 1/k \rfloor$)
- Finite variance and central limit theorem for $k < 1/2$
- Finite mean and generalized central limit theorem for $k < 1$, but pre-asymptotic constant grows impractically large for $k > 0.7$



Recap: Pareto smoothed importance sampling

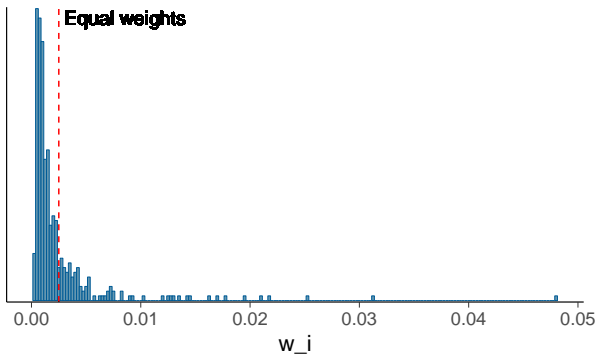
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- Pareto-Smoothed Importance sampling smooth the weights according to a Generalized Pareto(k) distribution
- Pareto- k diagnostic estimate the number of existing moments ($\lfloor 1/k \rfloor$)
- Finite variance and central limit theorem for $k < 1/2$
- Finite mean and generalized central limit theorem for $k < 1$, but pre-asymptotic constant grows impractically large for $k > 0.7$
- Hence $k > 0.7$ can be used to indicate what observation that can "hold-out" using IS



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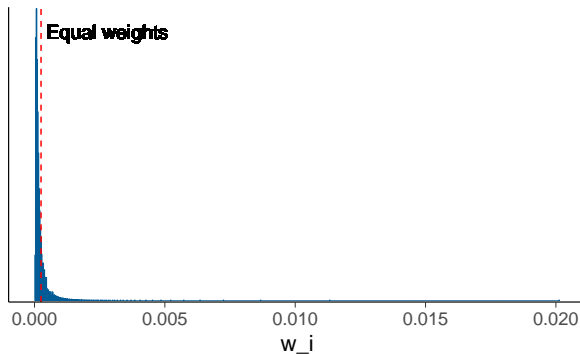
400 importance weights for leave-18th-out





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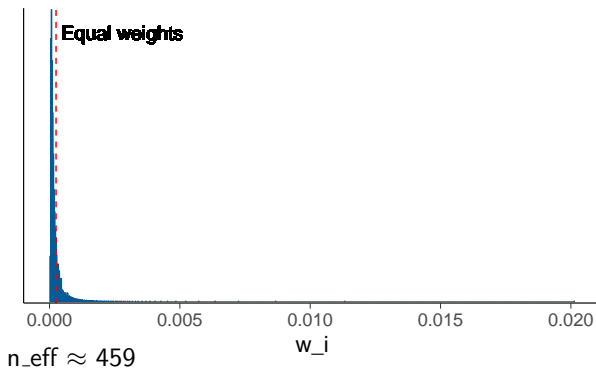
4000 importance weights for leave-18th-out





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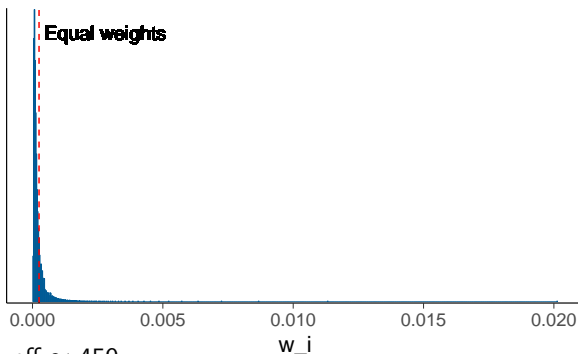
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4000 importance weights for leave-18th-out



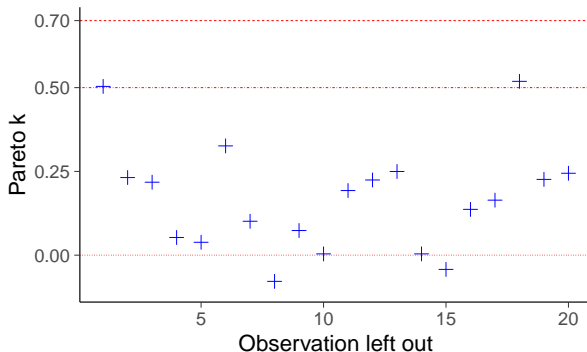
$n_{\text{eff}} \approx 459$

Pareto: $\hat{k} \approx 0.52$ Pareto \hat{k} estimates the tail shape which determines the convergence rate of PSIS. Less than 0.7 is ok.



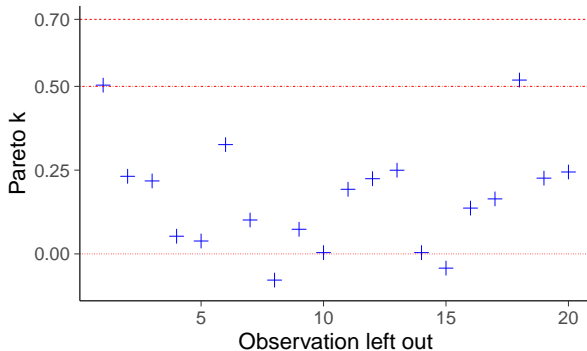
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PSIS-LOO diagnostics





PSIS-LOO diagnostics

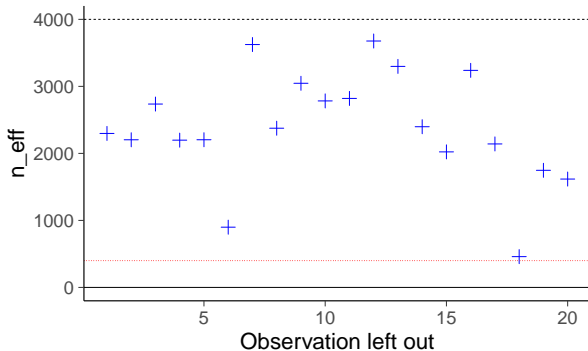


Pareto k diagnostic values:

		Count	Pct.	Min.	n_eff
(-Inf, 0.5]	(good)	18	90.0%	899	
(0.5, 0.7]	(ok)	2	10.0%	459	
(0.7, 1]	(bad)	0	0.0%	<NA>	
(1, Inf)	(very bad)	0	0.0%	<NA>	



PSIS-LOO diagnostics



Pareto k diagnostic values:

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Stan code

$$\log(w_i^{(s)}) = \log(1/p(y_i|x_i, \theta^{(s)})) = -\text{log_lik}[i]$$

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$$\log(w_i^{(s)}) = \log(1/p(y_i|x_i, \theta^{(s)})) = -\text{log_lik}[i]$$

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```
...  
model {  
  alpha ~ normal(pmualpha, psalpha);  
  beta ~ normal(pmubeta, psbeta);  
  y ~ normal(mu, sigma);  
}  
generated quantities {  
  vector[N] log_lik;  
  for (i in 1:N)  
    log_lik[i] = normal_lpdf(y[i] | mu[i], sigma);  
}
```



Computed from 4000 by 20 log-likelihood matrix

	Estimate	SE
elpd_loo	-29.5	3.3
p_loo	2.7	1.0

Monte Carlo SE of elpd_loo is 0.1.

Pareto k diagnostic values:

		Count	Pct.	Min.	n_eff
(-Inf, 0.5]	(good)	18	90.0%	899	
(0.5, 0.7]	(ok)	2	10.0%	459	
(0.7, 1]	(bad)	0	0.0%	<NA>	
(1, Inf)	(very bad)	0	0.0%	<NA>	

All Pareto k estimates are ok ($k < 0.7$).
See `help('pareto-k-diagnostic')` for details.

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1. PSIS-LOO for hierarchical models

1.1 leave-one-group out is challenging for PSIS-LOO



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1. PSIS-LOO for hierarchical models
 - 1.1 leave-one-group out is challenging for PSIS-LOO
2. PSIS-LOO for non-factorizable models

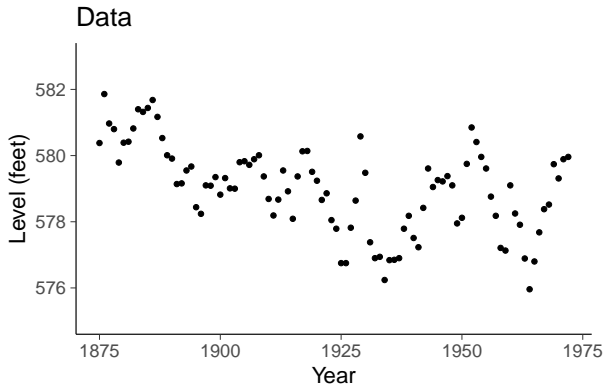


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1. PSIS-LOO for hierarchical models
 - 1.1 leave-one-group out is challenging for PSIS-LOO
2. PSIS-LOO for non-factorizable models
3. PSIS-LOO for time series
 - 3.1 Approximate leave-future-out cross-validation

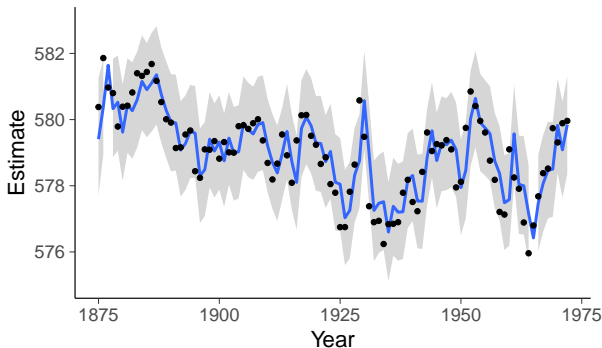


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AR-4 prediction with 95% interval



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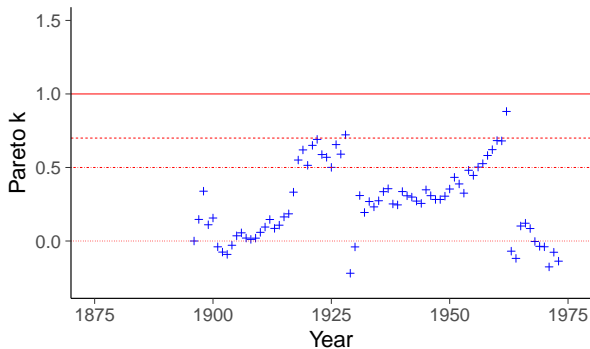


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PSIS-LOO for time series

PSIS-1-step-ahead with refits



mc-stan.org/loo/articles/loo2-lfo.html



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Subsection 3

K-fold CV



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K-fold cross-validation

- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
- Cross-validation
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- We can approximate $elpd_{loo}$ using K-fold CV



- Model assessment and selection
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 - Model selection
- Cross-validation
 - When is LOO applicable
 - PSIS LOO-CV
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- We can approximate $elpd_{loo}$ using K-fold CV
 1. for k in K
 - 1.1 hold $1/K$ observations out (i.e. $y_{-y(k)}$)
 - 1.2 compute $p(\theta|y_{-y(k)})$
 - 1.3 compute lpd for $y_{-y(k)}$
 2. sum the lpd over K folds



- Model assessment and selection
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- We can approximate $elpd_{\text{loo}}$ using K-fold CV
 1. for k in K
 - 1.1 hold $1/K$ observations out (i.e. $y_{-y(k)}$)
 - 1.2 compute $p(\theta|y_{-y(k)})$
 - 1.3 compute lpd for $y_{-y(k)}$
 2. sum the lpd over K folds
- We only need to estimate K models



K-fold cross-validation

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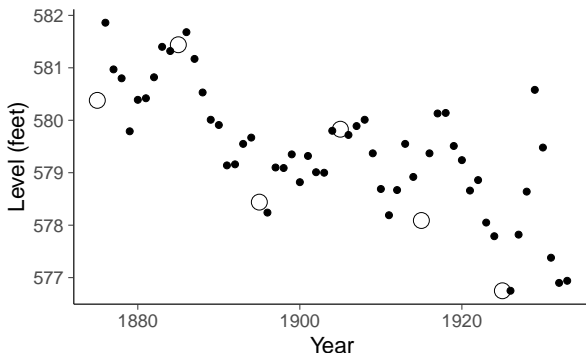
1. K-fold cross-validation can approximate LOO
 - 1.1 can be used all uses for LOO
2. K-fold cross-validation can be used for hierarchical models
 - 2.1 good for leave-one-group-out
3. K-fold cross-validation can be used for time series
 - 3.1 with leave-block-out



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- Model assessment and selection
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Balance k-fold approximation of LOO

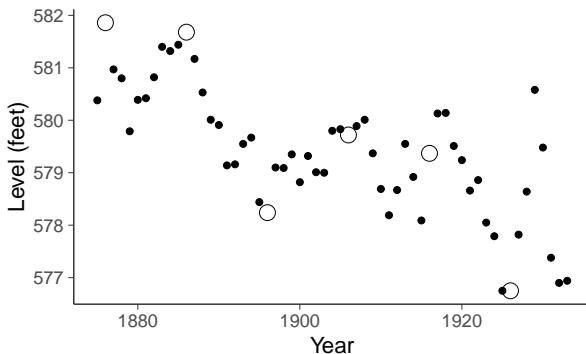




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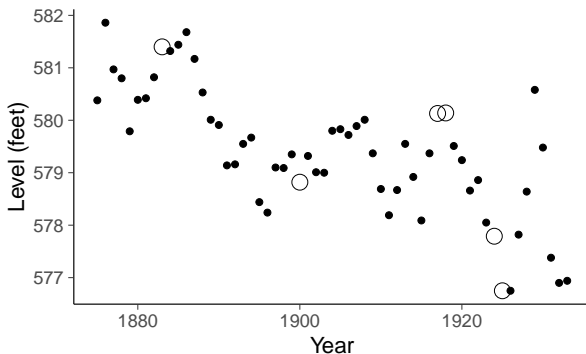




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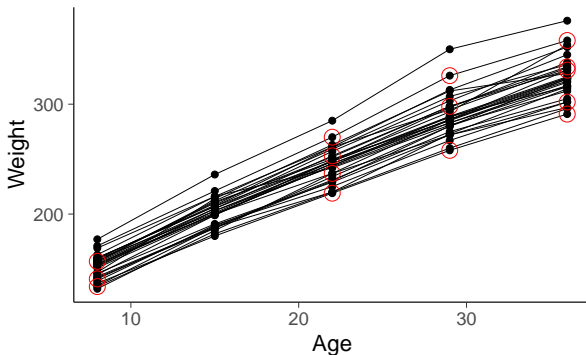
- Model assessment and selection
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Random k-fold approximation of LOO





Random kfold approximation of LOO



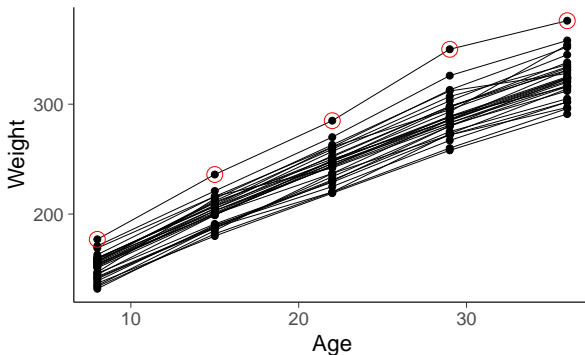
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 - Model Comparison
- Information criteria
- Model averaging and Stacking
- Summary

Leave-one-rat-out





- Model assessment and selection
 - Measures of predictive accuracy
 - Model selection
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1. CV is good for model assessment when application specific utility/cost functions are used



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1. CV is good for model assessment when application specific utility/cost functions are used
2. Also useful in model checking in similar way as posterior predictive checking (PPC)
 - 2.1 model misspecification diagnostics (e.g. Pareto- k and p_{loo})
 - 2.2 checking calibration of leave-one-out predictive posteriors (`ppc_loo_pit` in `bayesplot`)



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Subsection 4

Model Comparison



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1. “A popular hypothesis has it that primates with larger brains produce more energetic milk, so that brains can grow quickly” (from Statistical Rethinking)

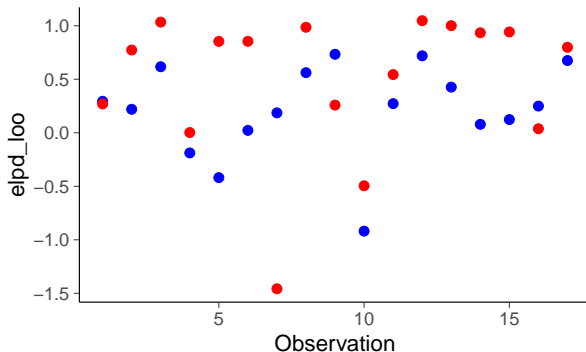
1.1 Model 1: $\text{formula} = \text{kcal.per.g} \sim \text{neocortex}$

1.2 Model 2: $\text{formula} = \text{kcal.per.g} \sim \text{neocortex} + \log(\text{mass})$



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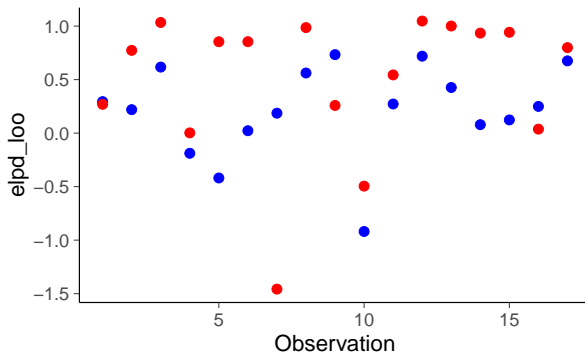
Pointwise comparison LOO models: Model 1





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Pointwise comparison LOO models: Model 1



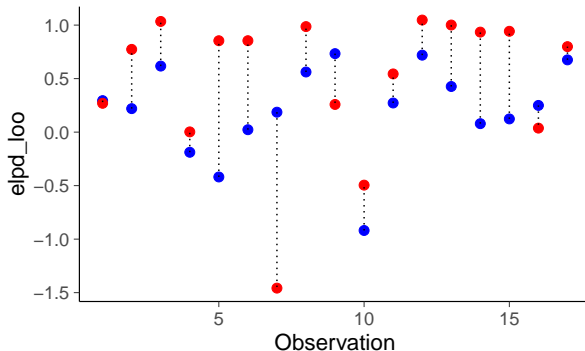
Model 1 $\text{elpd_loo} \approx 3.7$, $\text{SE}=1.8$

Model 2 $\text{elpd_loo} \approx 8.4$, $\text{SE}=2.8$



- Model assessment and selection
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Pointwise comparison LOO models: Model 1

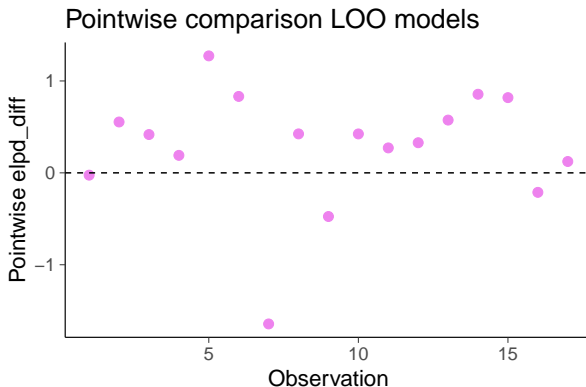


Model 1 $\text{elpd_loo} \approx 3.7$, $\text{SE}=1.8$

Model 2 $\text{elpd_loo} \approx 8.4$, $\text{SE}=2.8$



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Model comparison:
(negative 'elpd_diff' favors 1st model, positive favors 2nd)

elpd_diff	se
4.7	2.7



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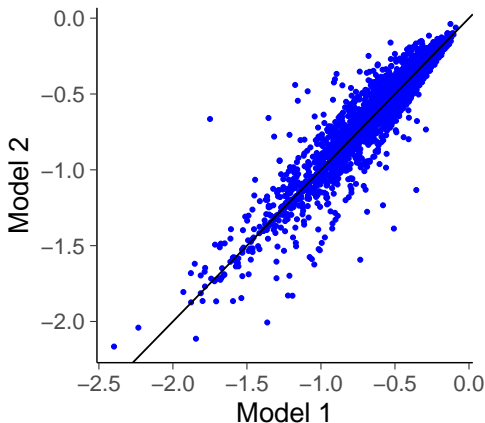
Arsenic well example – Model comparison

1. Probability of switching well with high arsenic level in rural Bangladesh
 - 1.1 Model 1 covariates: $\log(\text{arsenic})$ and distance
 - 1.2 Model 2 covariates: $\log(\text{arsenic})$, distance and education level

Gelman, Hill & Vehtari (2020): Regression and Other Stories, Chapter 13.



Model 1 vs Model 2



Model 1 elpd_loo \approx -1952, SE=16

Model 2 elpd_loo \approx -1938, SE=17

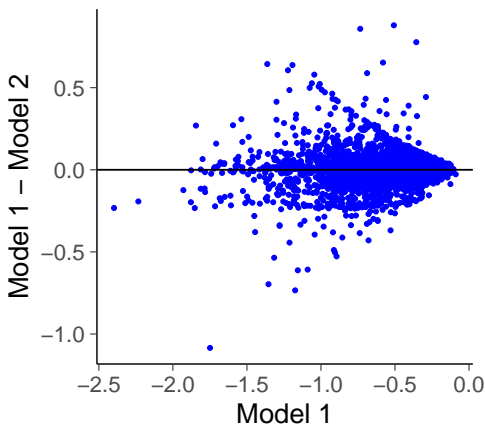
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Arsenic well example – Model comparison

Model 1 vs Model 2



```
> loo_compare(model1, model2)
      elpd_diff se_diff
model2    0.0     0.0
model1 -14.4     6.1
```



Arsenic well example – Model comparison

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```
> loo_compare(model1, model2)
      elpd_diff se_diff
model2      0.0      0.0
model1 -14.4      6.1
```

`se_diff` and normal approximation for the uncertainty in the difference is good only if models are well specified and the number of observations is relatively big.



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What if one is not clearly better than others?

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1. Continuous expansion including all models?

1.1 and then analyse the posterior distribution directly

1.2 sparse priors like regularized horseshoe prior instead of variable selection



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1. Continuous expansion including all models?
 - 1.1 and then analyse the posterior distribution directly
 - 1.2 sparse priors like regularized horseshoe prior instead of variable selection
2. Model averaging with BMA or Bayesian stacking?



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3. In a nested case choose simpler if assuming some cost for extra parts?



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1. Continuous expansion including all models?
 - 1.1 and then analyse the posterior distribution directly
 - 1.2 sparse priors like regularized horseshoe prior instead of variable selection
2. Model averaging with BMA or Bayesian stacking?
3. In a nested case choose simpler if assuming some cost for extra parts?
4. In a nested case choose more complex if you want to take into account all the uncertainties.



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1. Cross-validation can be used for model selection if
 - 1.1 small number of models
 - 1.2 the difference between models is clear



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1. Cross-validation can be used for model selection if
 - 1.1 small number of models
 - 1.2 the difference between models is clear
2. Do not use cross-validation to choose from a large set of models
 - 2.1 selection process leads to overfitting



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1. Cross-validation can be used for model selection if
 - 1.1 small number of models
 - 1.2 the difference between models is clear
2. Do not use cross-validation to choose from a large set of models
 - 2.1 selection process leads to overfitting
3. Overfitting in selection process is not unique for cross-validation



Selection induced bias and overfitting

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- Selection induced bias in cross-validation
 - same data is used to assess the performance and make the selection
 - the selected model fits more to the data
 - the CV estimate for the selected model is biased



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- Selection induced bias in cross-validation
 - same data is used to assess the performance and make the selection
 - the selected model fits more to the data
 - the CV estimate for the selected model is biased
 - Performance of the selection process itself can be assessed using two level cross-validation, but it does not help choosing better models



Selection induced bias and overfitting

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- Selection induced bias in cross-validation
 - same data is used to assess the performance and make the selection
 - the selected model fits more to the data
 - the CV estimate for the selected model is biased
- Performance of the selection process itself can be assessed using two level cross-validation, but it does not help choosing better models
- Bigger problem if there is a large number of models as in covariate selection



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Section 3

Information criteria



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1. Remember the relationship between the elpd and the lpd

$$\text{elpd}_{\text{loo}} = \sum_i \log p(y_i | y) - p_{\text{loo}}$$



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1. Remember the relationship between the elpd and the lpd

$$\text{elpd}_{\text{loo}} = \sum_i \log p(y_i|y) - p_{\text{loo}}$$

2. Akaike information criteria

$$\text{elpd}_{\text{AIC}} = \sum_i \log p(y_i|\hat{\theta}_{\text{MLE}}) - k$$



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1. Deviance information criteria

$$\text{elpd}_{\text{AIC}} = \sum_i \log p(y_i | \hat{\theta}_{\text{Bayes}}) - p_{\text{DIC}}$$

2. We estimate the number of effeicient parameters with

$$p_{\text{DIC}} =$$



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WAIC vs PSIS-LOO

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1. WAIC has same assumptions as LOO



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1. WAIC has same assumptions as LOO
2. PSIS-LOO is more accurate



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1. WAIC has same assumptions as LOO
2. PSIS-LOO is more accurate
3. PSIS-LOO has much better diagnostics



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1. WAIC has same assumptions as LOO
2. PSIS-LOO is more accurate
3. PSIS-LOO has much better diagnostics
4. LOO makes the prediction assumption more clear, which helps if K-fold-CV is needed instead



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1. WAIC has same assumptions as LOO
2. PSIS-LOO is more accurate
3. PSIS-LOO has much better diagnostics
4. LOO makes the prediction assumption more clear, which helps if K-fold-CV is needed instead
5. Multiplying by -2 doesn't give any benefit (Watanabe didn't multiply by -2)



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1. AIC uses maximum likelihood estimate for prediction
2. DIC uses posterior mean for prediction
3. BIC is an approximation for marginal likelihood
4. TIC, NIC, RIC, PIC, BPIC, QIC, AIC_c, ...



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Section 4

Model averaging and Stacking



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Special lecture!

- Model assessment and selection
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1. Special lecture on model averaging and stacking next week!



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Section 5

Summary



Take-home messages

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1. It's good to think predictions of observables, because observables are the only ones we can observe
2. Cross-validation can simulate predicting and observing new data
3. Cross-validation is good if you don't trust your model
4. Different variants of cross-validation are useful in different scenarios
5. Cross-validation has high variance, and **if** you trust your model you can beat cross-validation in accuracy



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