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Bayesian Statistics and Data Analysis
Lecture 8b

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Thanks to Aki Vehtari, Aalto University

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Section 1

Introduction

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Bayesian decision theory

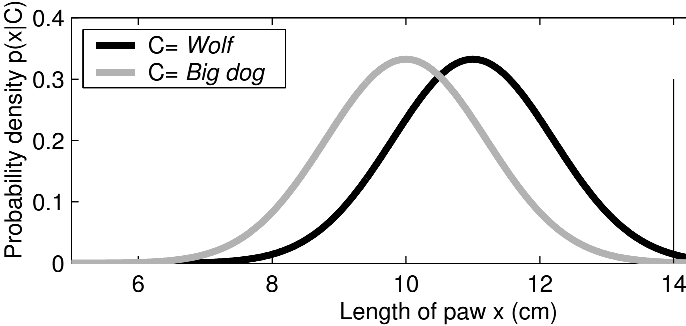
- Potential decisions d
  - or actions a
- Potential consequences x
  - x may be categorical, ordinal, real, scalar, vector, etc.
- Probability distributions of consequences given decisions $p(x|d)$
  - in decision making the decisions are controlled and thus $p(d)$ does not exist
- Utility function $U(x)$ maps consequences to real value
  - e.g. euro or expected lifetime
  - instead of utility sometimes cost or loss is defined
- Expected utility $E[U(x)|d] = \int U(x)p(x|d)dx$
- Choose decision d^*, which maximizes the expected utility

$$d^* = \arg\max_d E[U(x)|d]$$

Example of decision making: 2 choices

- Helen is going to pick mushrooms in a forest, while she notices a paw print which could made by a dog or a wolf
- Helen measures that the length of the paw print is 14 cm and goes home to Google how big paws dogs and wolves have,

  and tries then to infer which animal has made the paw print



  observed length has been marked with a horizontal line
- Likelihood of wolf is 0.92 (alternative being dog)

Example of decision making

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- Helen assumes also that in her living area there are about one hundred times more free running dogs than wolves, that is *a priori* probability for wolf, before observation is 1%.
- Likelihood and posterior

Animal	Likelihood	Posterior probability
Wolf	0.92	0.10
Dog	0.08	0.90

- Posterior probability of wolf is 10%

Example of decision making

- Helen has to make decision whether to go pick mushrooms
- If she doesn't go to pick mushrooms utility is zero
- Helen assigns positive utility 1 for getting fresh mushrooms
- Helen assigns negative utility -1000 for a event that she goes to the forest and wolf attacks (for some reason Helen assumes that wolf will always attack)

Decision d	Animal Wolf	 Dog
Stay home	0	0
Go to the forest	-1000	1

Utility matrix $U(x)$

Action d	Conditional utility $E[U(x)\|d]$
Stay home	0
Go to the forest	-100+0.9

Utilities for different actions

Example of decision making

- Maximum likelihood decision would be to assume that there is a wolf
- Maximum posterior decision would be to assume that there is a dog
- Maximum utility decision is to stay home, even if it is more likely that the animal is dog
- Example illustrates that the uncertainties (probabilities) related to all consequences need to be carried on until final decision making

Example of decision making: several choices

- Prof. Gelman has a jar of quarters
  - he first drew a line on the side of the jar and then filled the jar up to the line, and so the number coins was not chosen beforehand
  - Prof. Gelman does not know the number of coins in the jar
  - Prof. Gelman gives the class a chance to win the coins if they guess the number of coins correctly (someone else has counted the coins without telling Gelman)
  - How should the students make the decision?

Challenges in decision making

- Actual utility functions are rarely linear
- What is the cost of human life?
- Multipel parties having different utilities

Multi-stage decision making (Section 9.3)

- 95 year old has a tumor that is malignant with 90% probability
- Based on statistics
  - expected lifetime is 34.8 months if no cancer
  - expected lifetime is 16.7 months if cancer and radiation therapy is used
  - expected lifetime is 20.3 months if cancer and surgery, but the probability of dying in surgery is 35% (for 95 year old)
  - expected lifetime is 5.6 months is cancer and no treatment
- Which treatment to choose?
  - quality adjusted life time
  - 1 month is subtracted for the time spent in treatments
- Quality adjusted life time
  - Radiothreapy: 0.9*16.7 + 0.1*34.8 - 1 = 17.5mo
  - Surgery: 0.35*0 + 0.65*(0.9*20.3 + 0.1*34.8 - 1) = 13.5mo
  - No treatment: 0.9*5.6 + 0.1*34.8 = 8.5mo
- See the book for continuation of the example with additional test for cancer

Design of experiment

- Which experiment would give most additional information
  - decide values x_{n+1} for the next experiment
  - which values of x_{n+1} would reduce the posterior uncertainty most
- Example
  - Imagine that in bioassay the posterior uncertainty of LD50 is too large
  - which dose should be used in the next experiment to reduce the variance of LD50 as much as possible ?
    - this way less experiments need to be made (and less animals need to be killed)

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Bayesian optimization

- Design of experiment
- Used to optimize, for example,
  - machine learning / deep learning model structures, regularization, and learning algorithm parameters
  - material science
  - engines
  - drug testing
  - part of Bayesian inference for stochastic simulators

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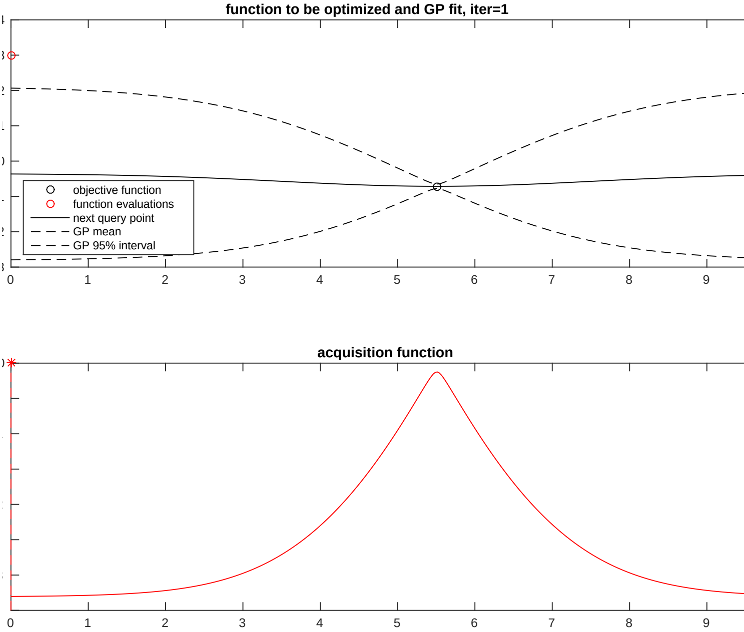
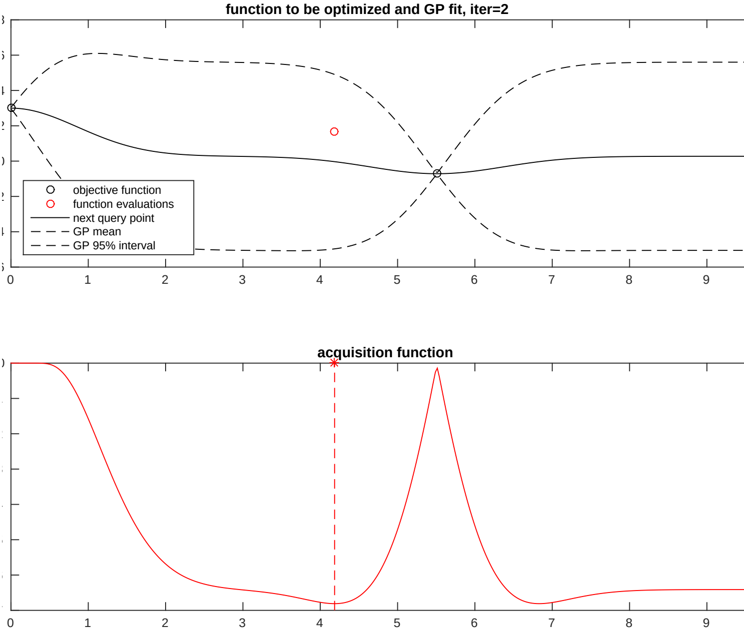
Bayesian optimization



Bayesian optimization



Bayesian optimization

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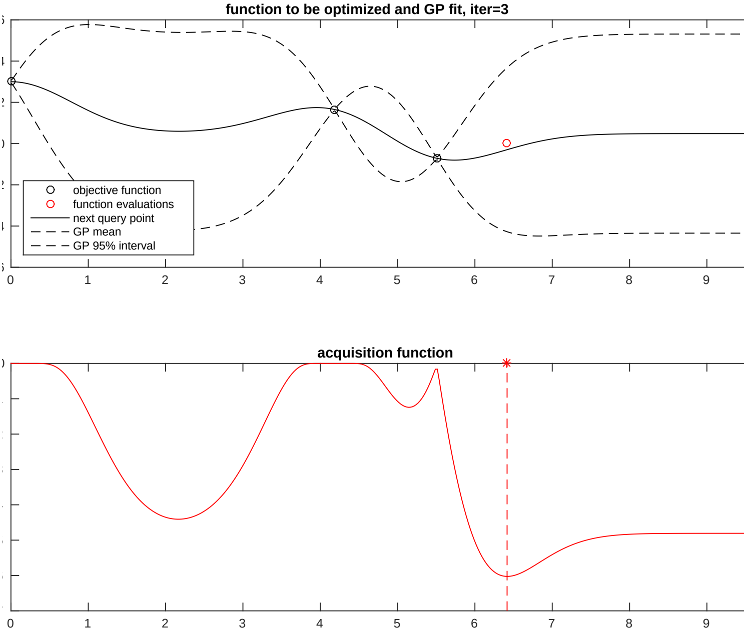

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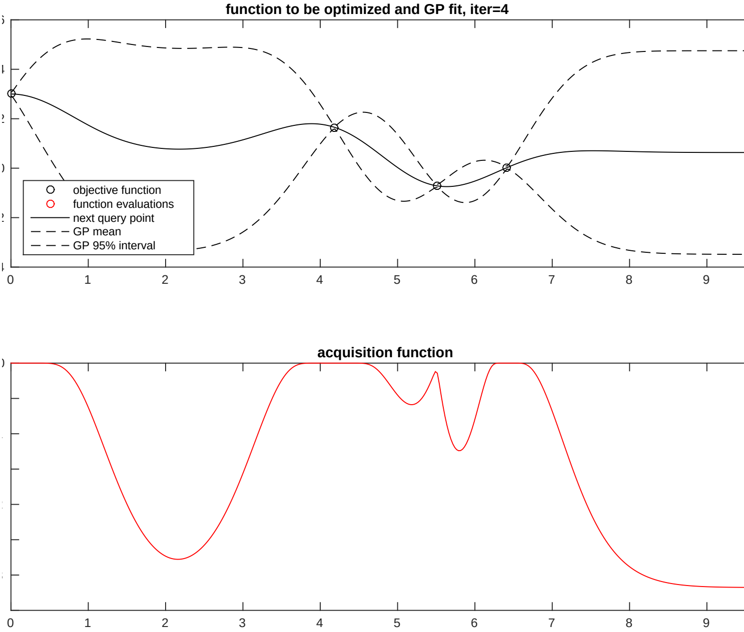
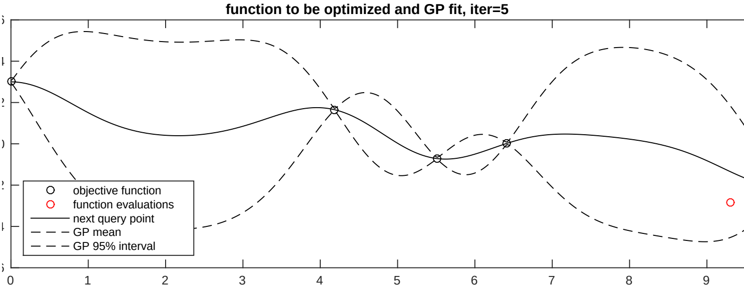
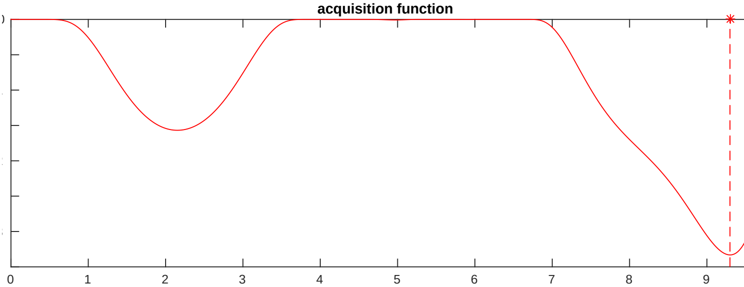


Bayesian optimization





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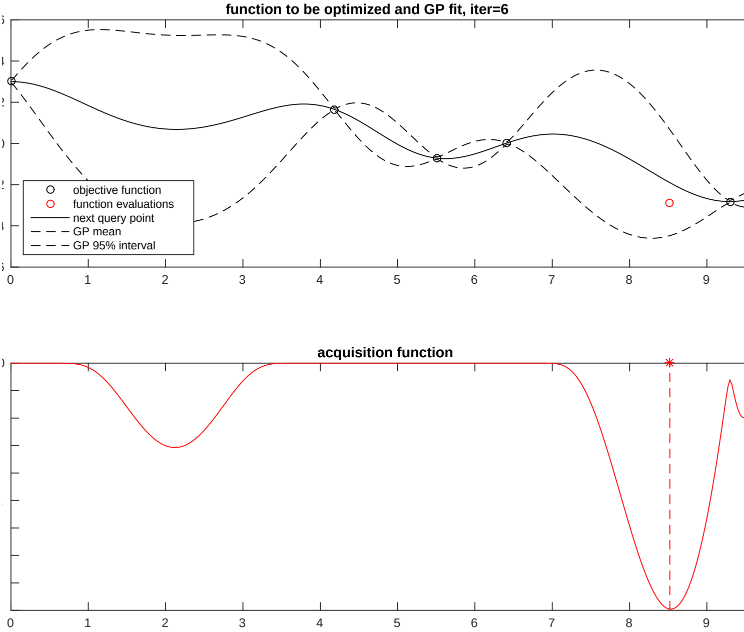

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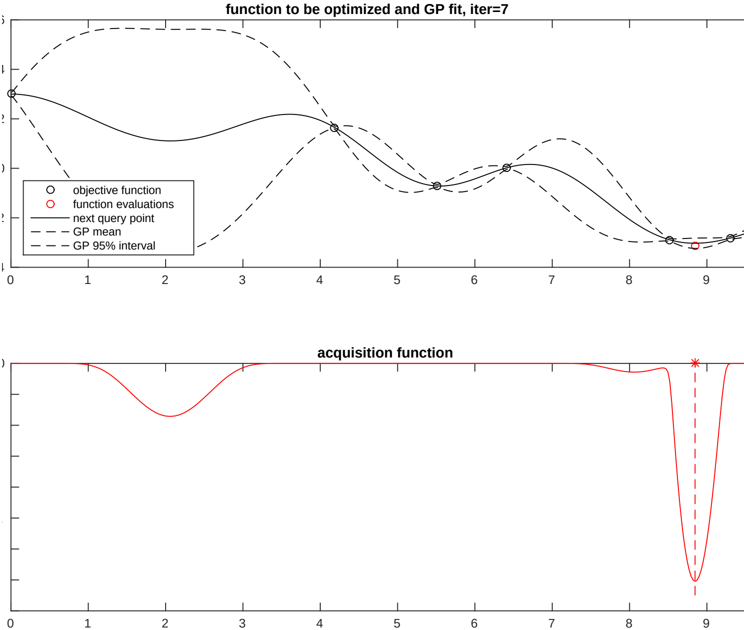

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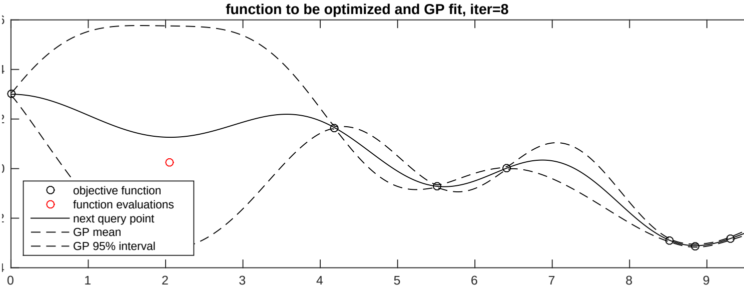

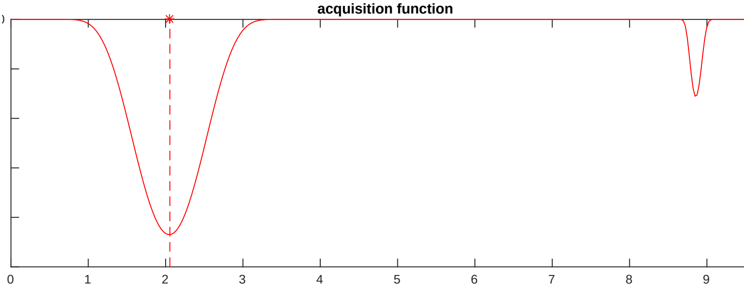

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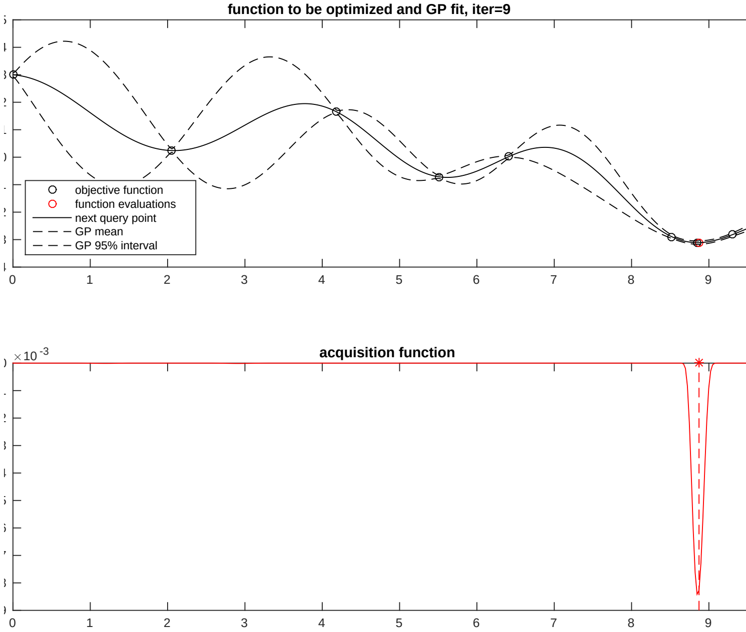
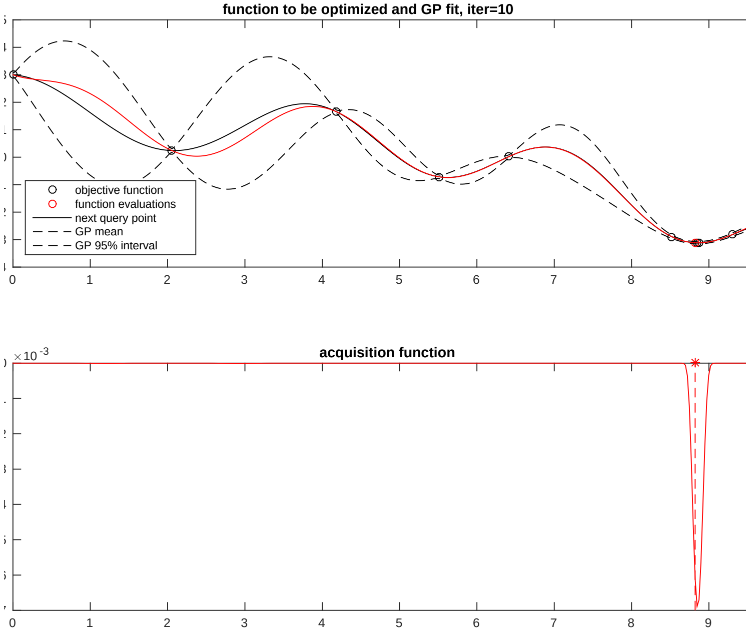


Bayesian optimization



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Model selection as decision problem

- Expected utility of using the model in the future