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• Introduction

# Bayesian Statistics and Data Analysis

## Lecture 1

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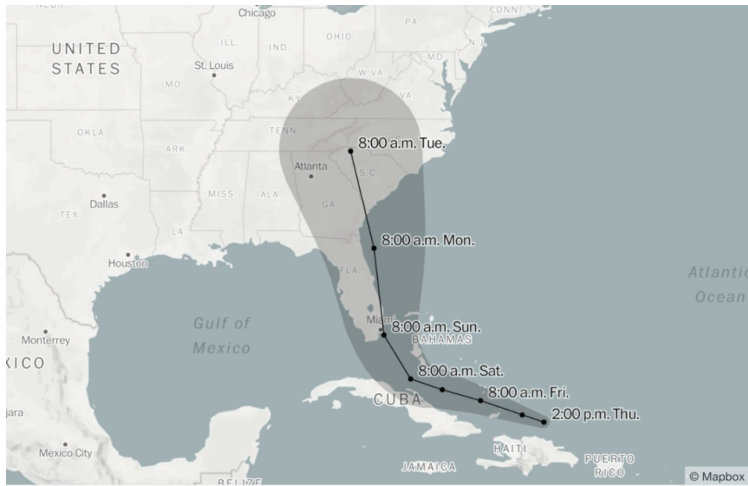
● Introduction

# Section 1

## Introduction



# Decision making in case of uncertainties





• Introduction

- Based on Bayesian probability theory
  - uncertainty is presented with probabilities
  - probabilities are updated based on new information
- Thomas Bayes (170?–1761)
  - English nonconformist, Presbyterian minister, mathematician
  - considered the problem of *inverse probability*
    - significant part of the Bayesian theory
- Bayes did not invent all, but was first to solve problem of inverse probability in special case
- Modern Bayesian theory with rigorous proofs developed in 20th century



# Term Bayesian used first time in mid 20th century

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- Earlier there was just "probability theory"
  - concept of the probability was not strictly defined, although it was close to modern Bayesian interpretation
  - in the end of 19th century there were increasing demand for more strict definition of probability (mathematical and philosophical problem)
- In the beginning of 20th century frequentist view gained popularity
  - accepts definition of probabilities only through frequencies
  - does not accept inverse probability or use of prior
  - gained popularity due to apparent objectivity and "cook book" like reference books
- R. A. Fisher used in 1950 first time term "Bayesian" to emphasize the difference to general term "probability theory"
  - term became quickly popular, because alternative descriptions were longer



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# Uncertainty and probabilistic modeling

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- Two types of uncertainty: aleatoric and epistemic
- Representing uncertainty with probabilities
- Updating uncertainty



# Two types of uncertainty

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- Aleatoric uncertainty due to randomness
  - we are not able to obtain observations which could reduce this uncertainty
- Epistemic uncertainty due to lack of knowledge
  - we are able to obtain observations which can reduce this uncertainty
  - two observers may have different epistemic uncertainty



# Updating uncertainty

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- Probability of red  $\frac{\#red}{\#red + \#yellow} = \theta$
- $p(y = \text{red}|\theta) = \theta$  aleatoric uncertainty
- $p(\theta)$  epistemic uncertainty
- Picking many chips updates our uncertainty about the proportion
- $p(\theta|y = \text{red, yellow, red, red, } \dots) = ?$
- Bayes rule  $p(\theta|y) = \frac{p(y|\theta)p(\theta)}{\int p(y|\theta)p(\theta)d\theta}$



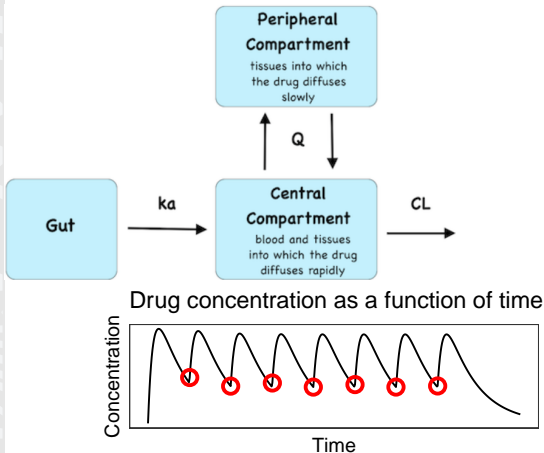


- Bayes rule  $p(\theta|y) \propto p(y|\theta)p(\theta)$
- Model:  $p(y|\theta)$  as a function of  $y$  given fixed  $\theta$  describes the aleatoric uncertainty
- Likelihood:  $p(y|\theta)$  as a function of  $\theta$  given fixed  $y$  provides information about epistemic uncertainty, but is not a probability distribution
- Bayes rule combines the likelihood with prior uncertainty  $p(\theta)$  and transforms them to updated posterior uncertainty



## Example application: Drug dosage for liver transplant<sup>1</sup>

- Everolimus is immunosuppressant to prevent rejection of organ transplants
- Pharmacokinetic model of drug and body, optimal dosage depends on weight





# The art of probabilistic modeling

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- The art of probabilistic modeling is to describe in a mathematical form (model and prior distributions) what we already know and what we don't know
- “Easy” part is to use Bayes rule to update the uncertainties
  - computational challenges
- Other parts of the art of probabilistic modeling are, for example,
  - model checking: is data in conflict with our prior knowledge?
  - presentation: presenting the model and the results to the application experts



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- Galaxy clusters for cosmology
- Coagulation of blood
- Gene regulation
- Pharmacokinetics and -dynamics
- Decision support
- Effects of nutrition for diabetes
- Evolutionary anthropology
- Clinical trial designs
- Daily demand for gas
- Brain structure trees
- School enrollment
- Sports
- Product demand
- Cocoa bean fermentation
- Marine propulsion power
- Alcohol consumption trends
- Flood probability
- Instantaneous heart rate distributions
- Drug dosing regimens in pediatrics
- Human T stem cell memory cells
- Fairness in university admission policies
- Destruction of bacteria and bacterial spores under heat



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- Treatment/control
  - randomize patients to treatment or control
  - is the treatment effective?
- Continuous valued treatment
  - randomize patients with different dosages
  - which dosage is sufficient without too many side effects?
- Different effects for different patients?
  - Is the treatment effect different for male/female, child/adult, light/heavy, ...



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# Bayesian approach

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- Benefits of Bayesian approach
  - integrate over uncertainties to focus to interesting parts
  - use relevant prior information
  - hierarchical models
  - model checking and evaluation



# Computation

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We need to be able to compute expectations with respect to posterior distribution  $p(\theta|y)$

$$E_{\theta|y} [g(\theta)] = \int p(\theta|y)g(\theta)d\theta$$

- Analytic
  - only for very simple models
- Monte Carlo, Markov chain Monte Carlo
  - generic
- Distributional approximations
  - e.g. Laplace, variational, expectation propagation
  - less generic, but can be much faster with sufficient accuracy