Collecting and analyzing data over multiple services under local differential privacy

Min Xu, Bolin Ding, Tianhao Wang, Jingren Zhou

Data Collection and Analysis in the Cloud

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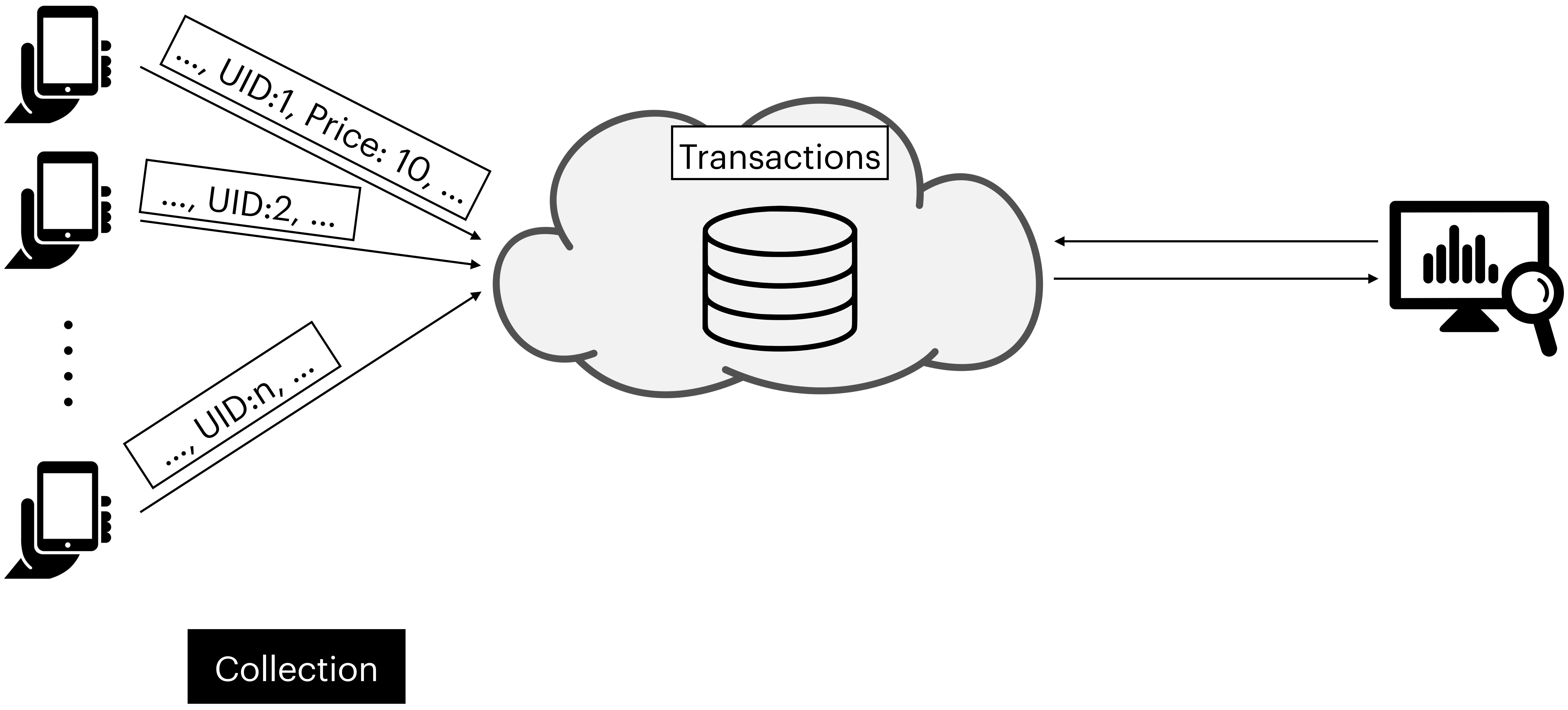

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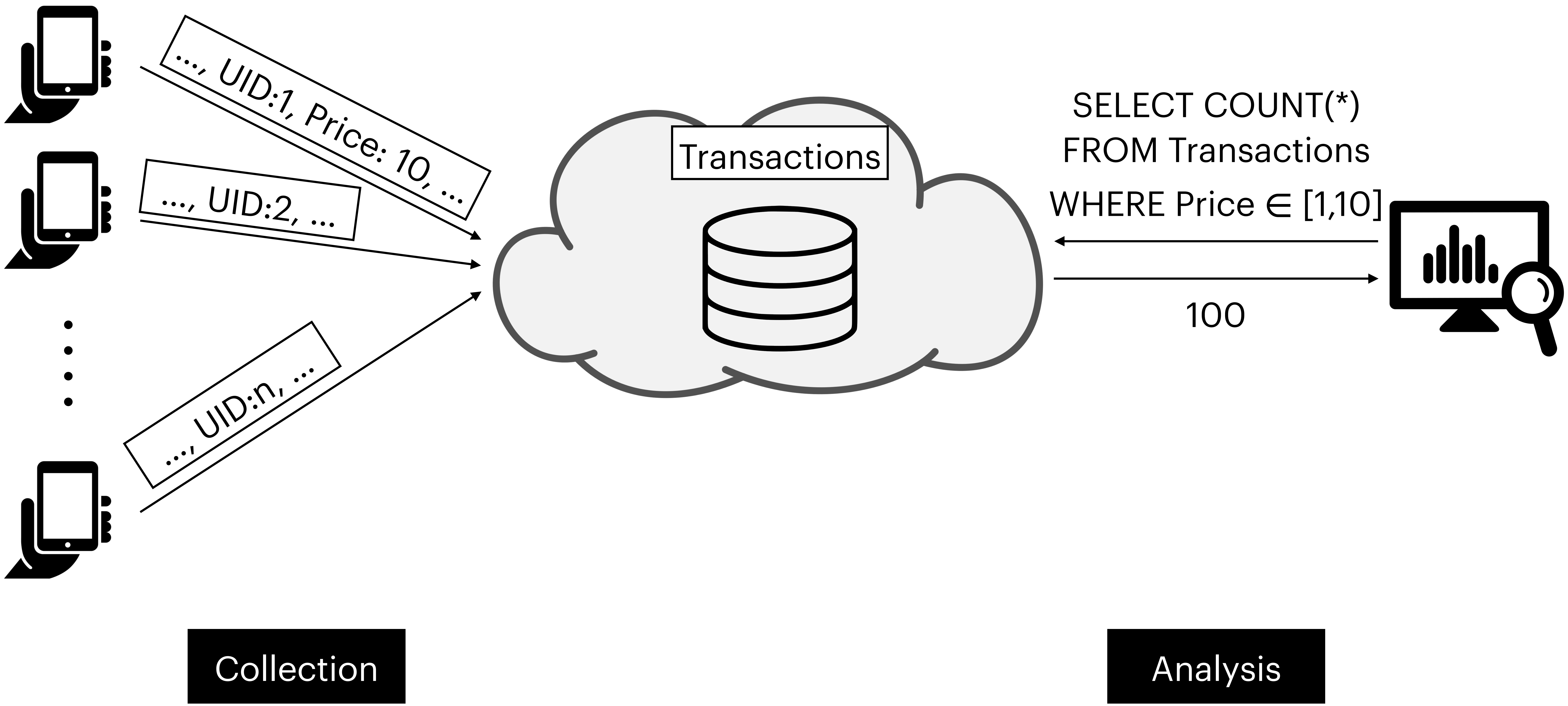

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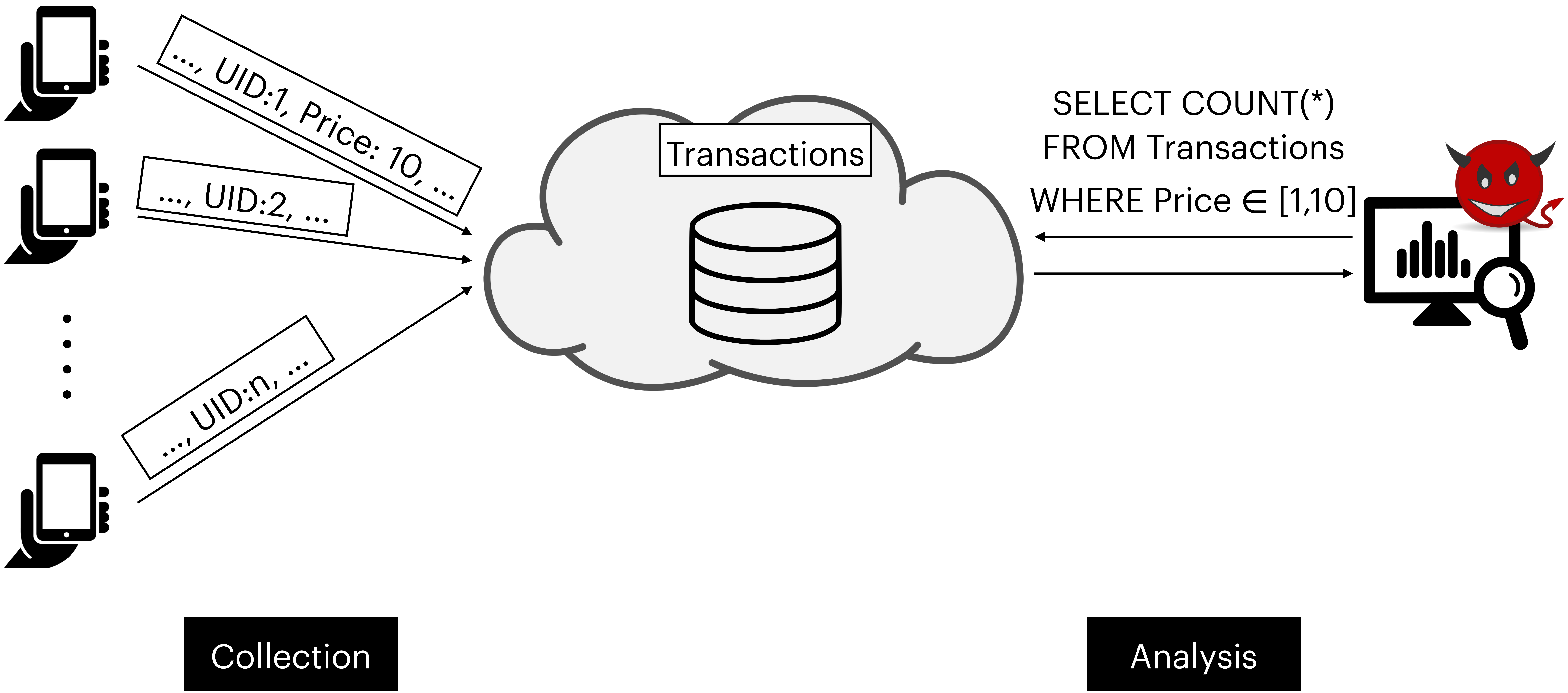

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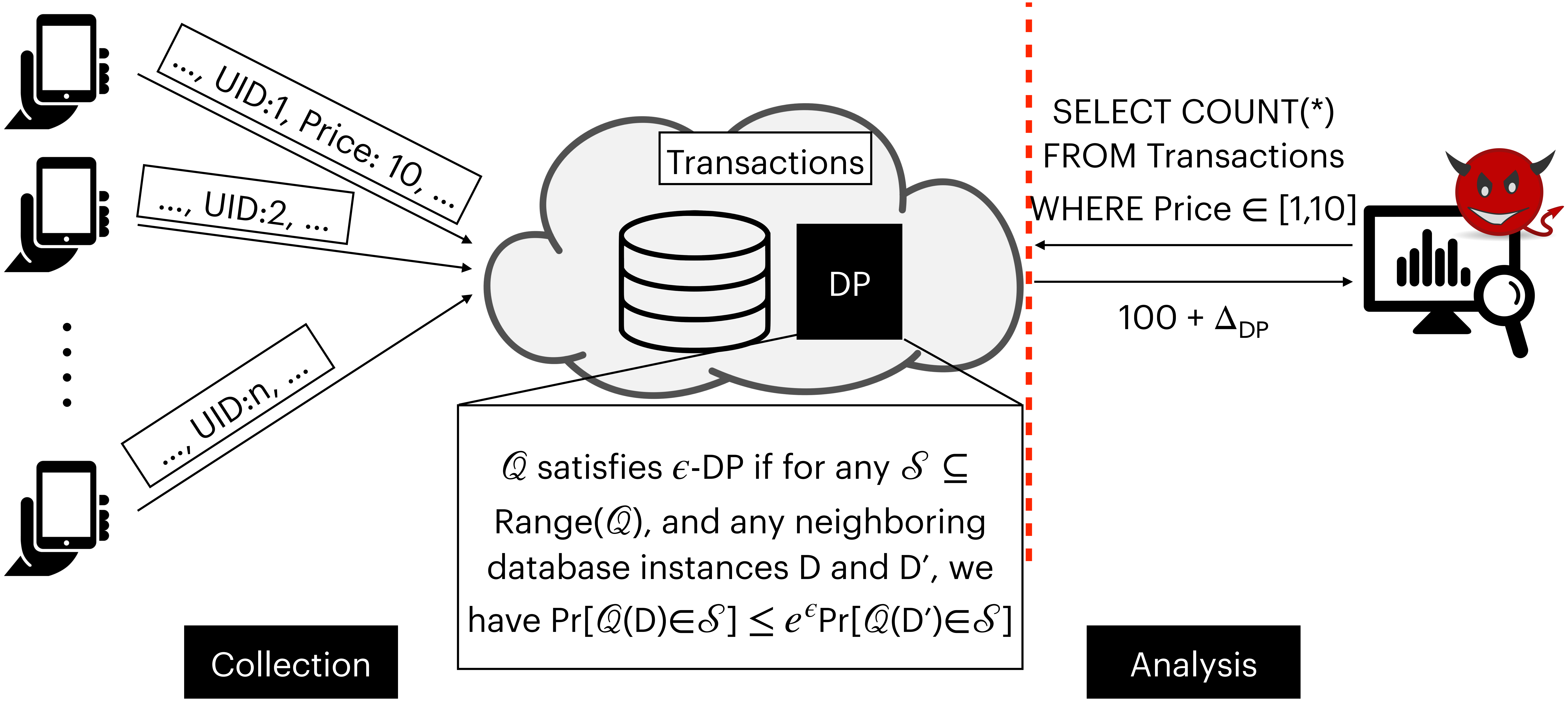

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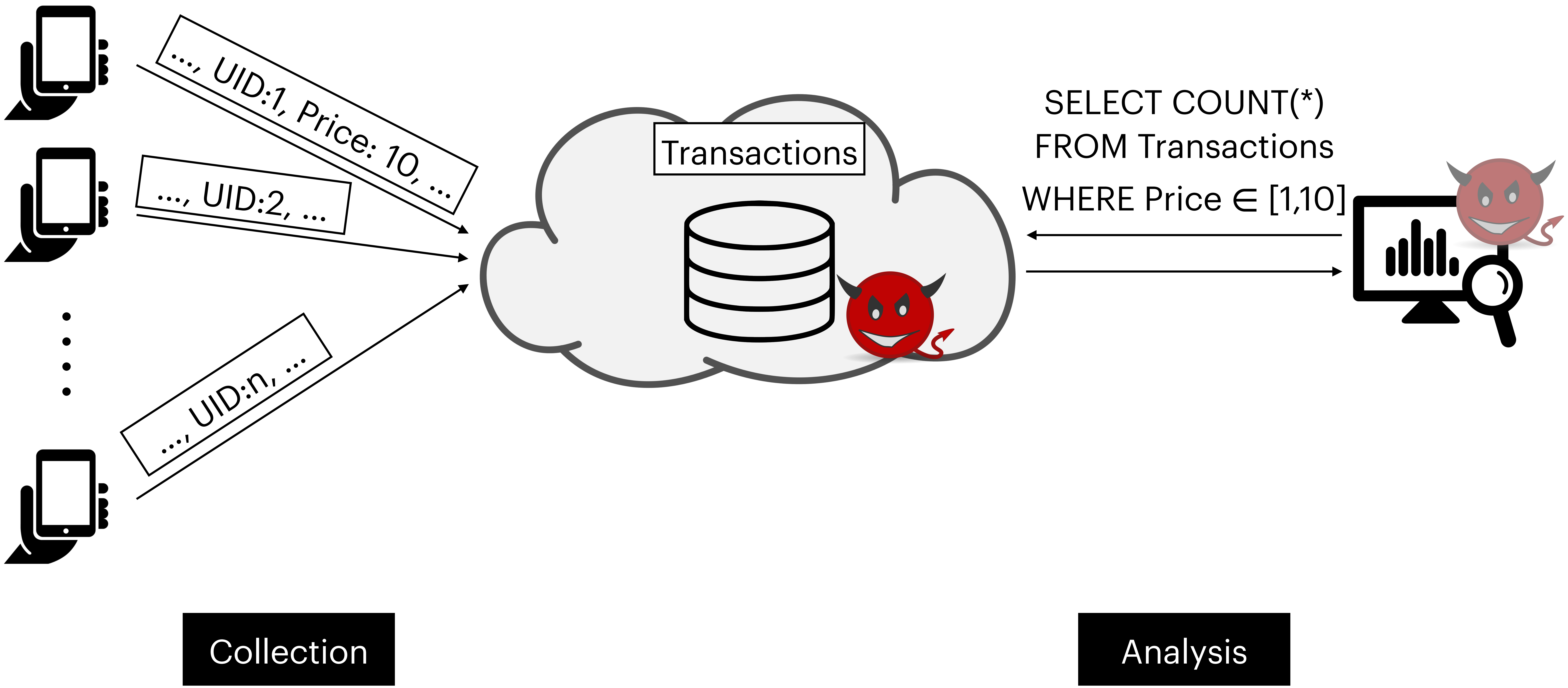

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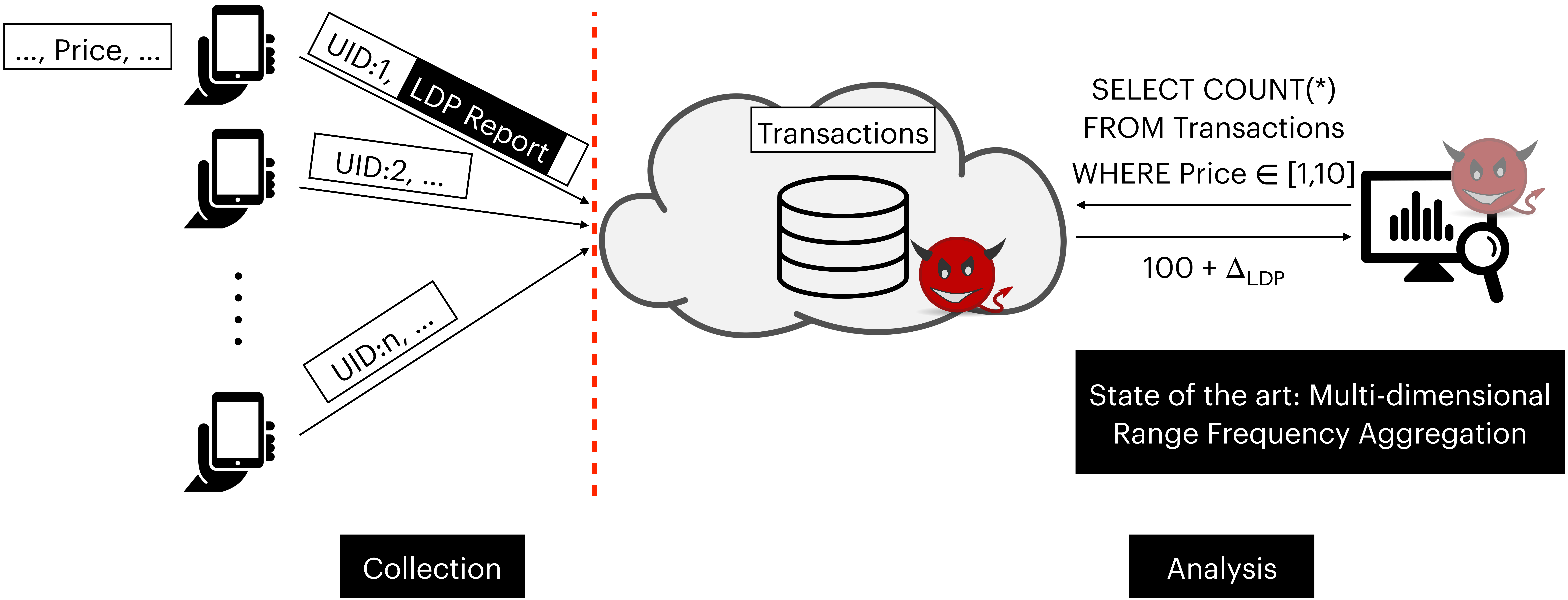

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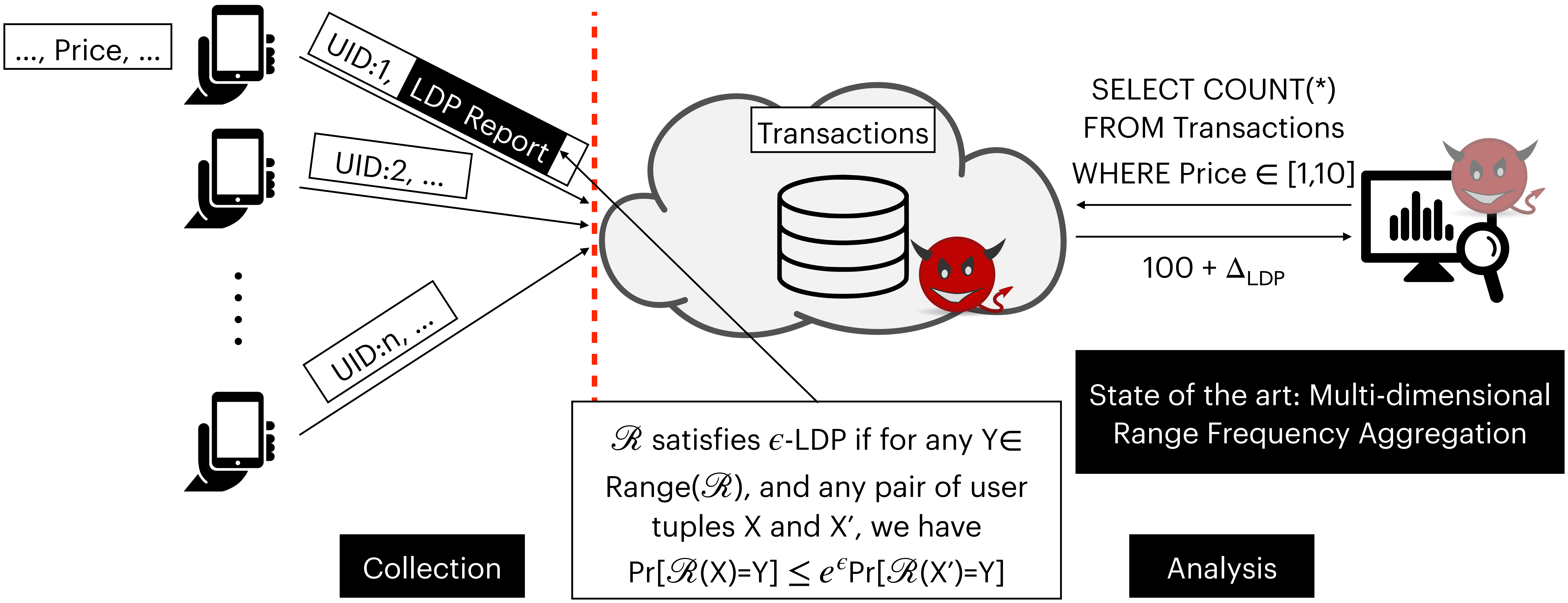
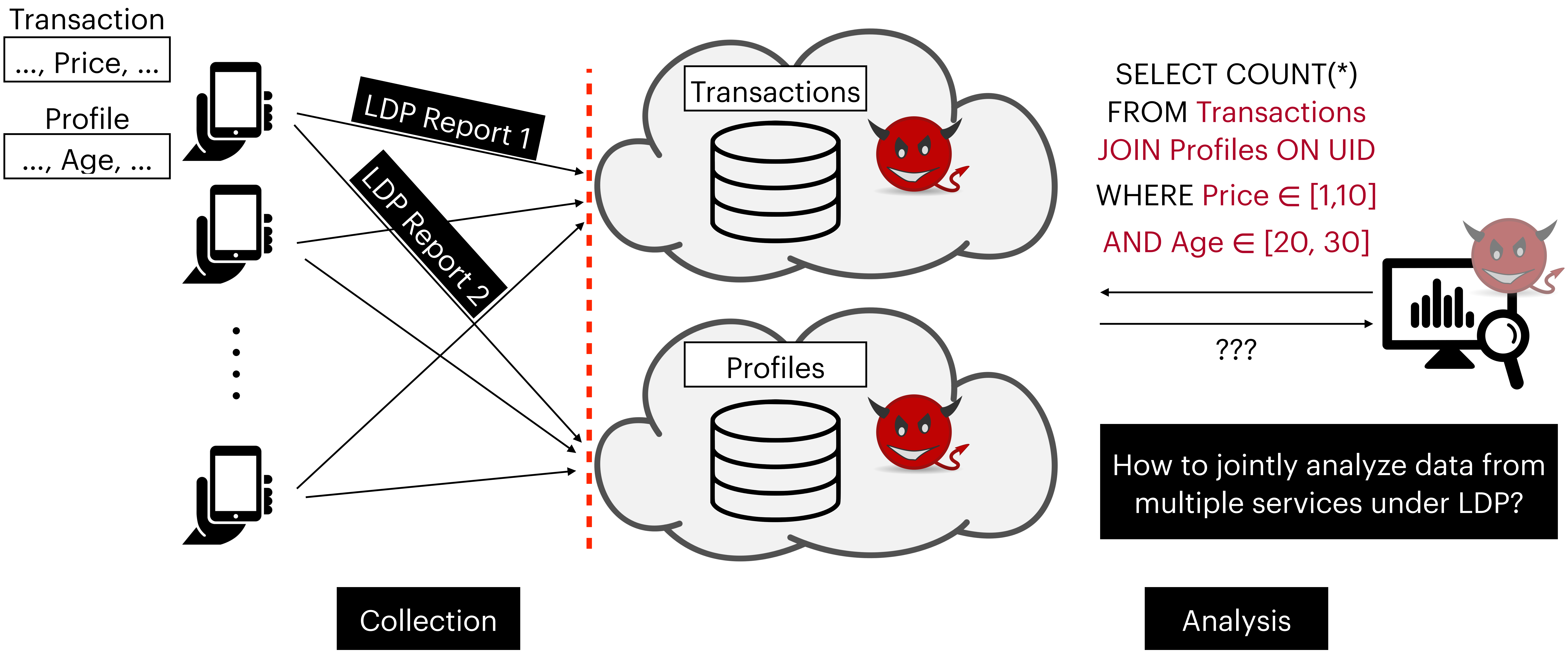


Data Collection and Analysis in the Cloud - This Work



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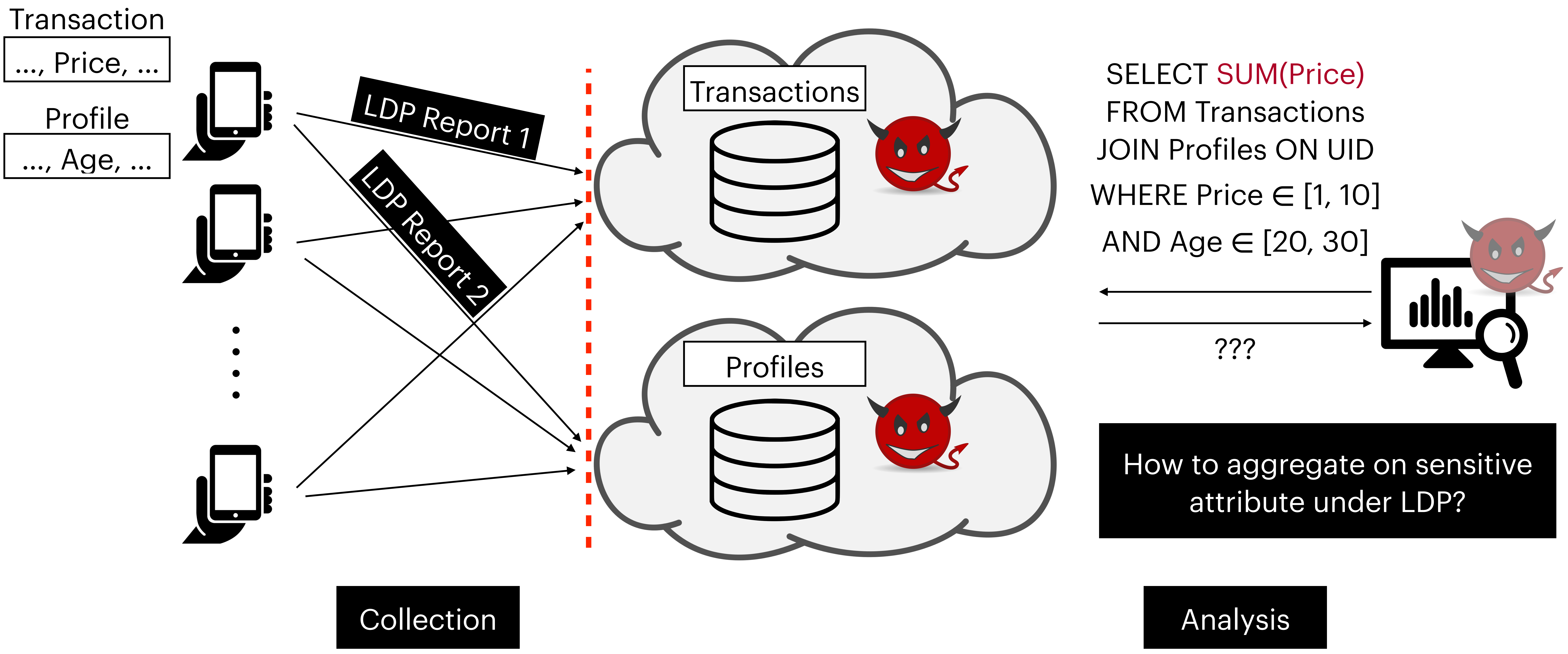
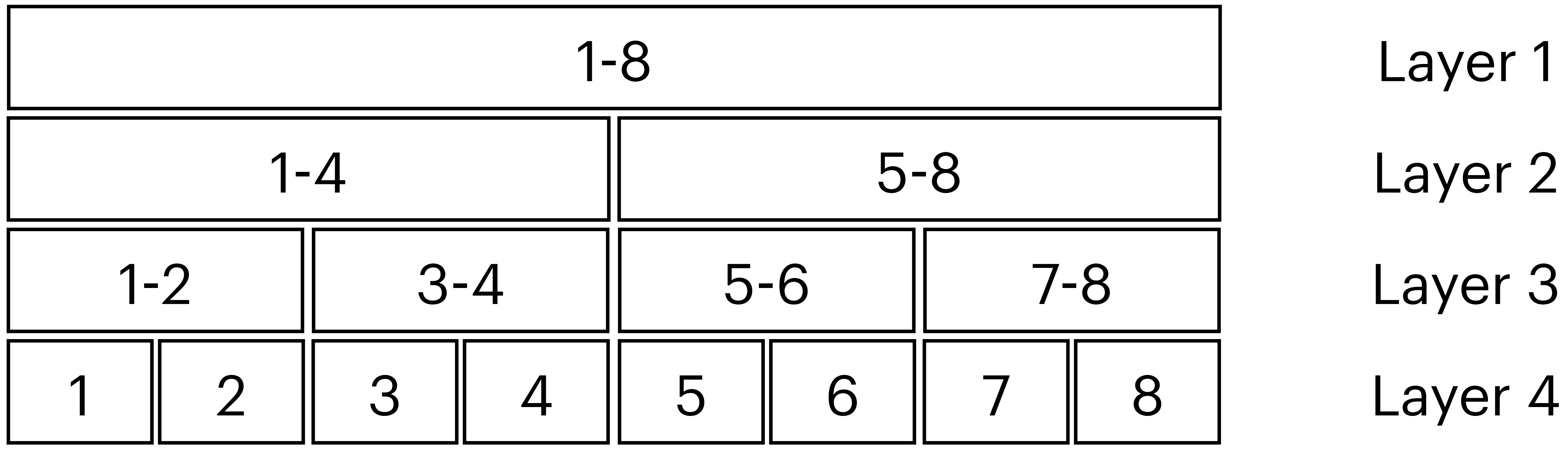


Recap: Hierarchical Interval Optimized (HIO)

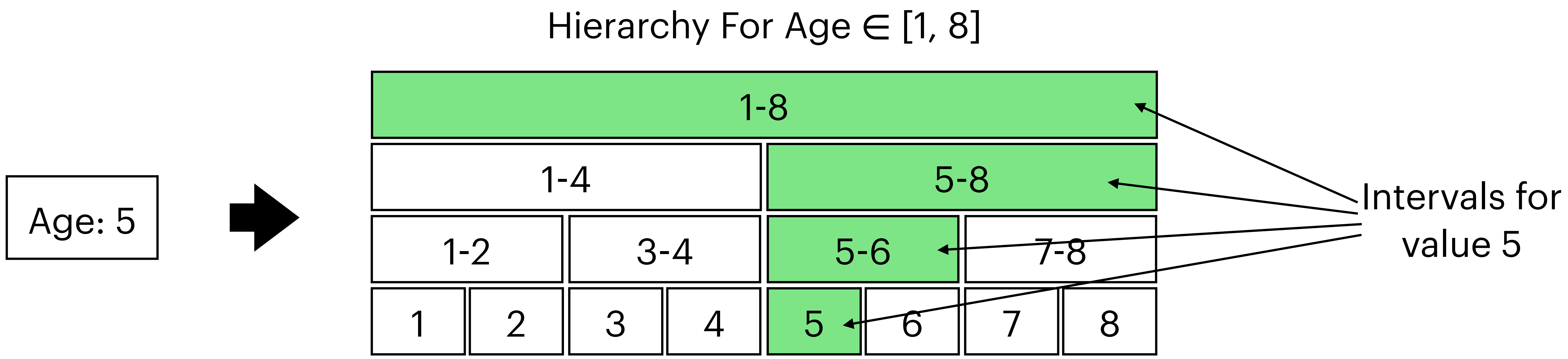
Collection

Hierarchy For Age ∈ [1, 8]



Recap: Multi-dimensional Range Frequency Oracles using Hierarchical Interval Optimized (HIO)

Collection



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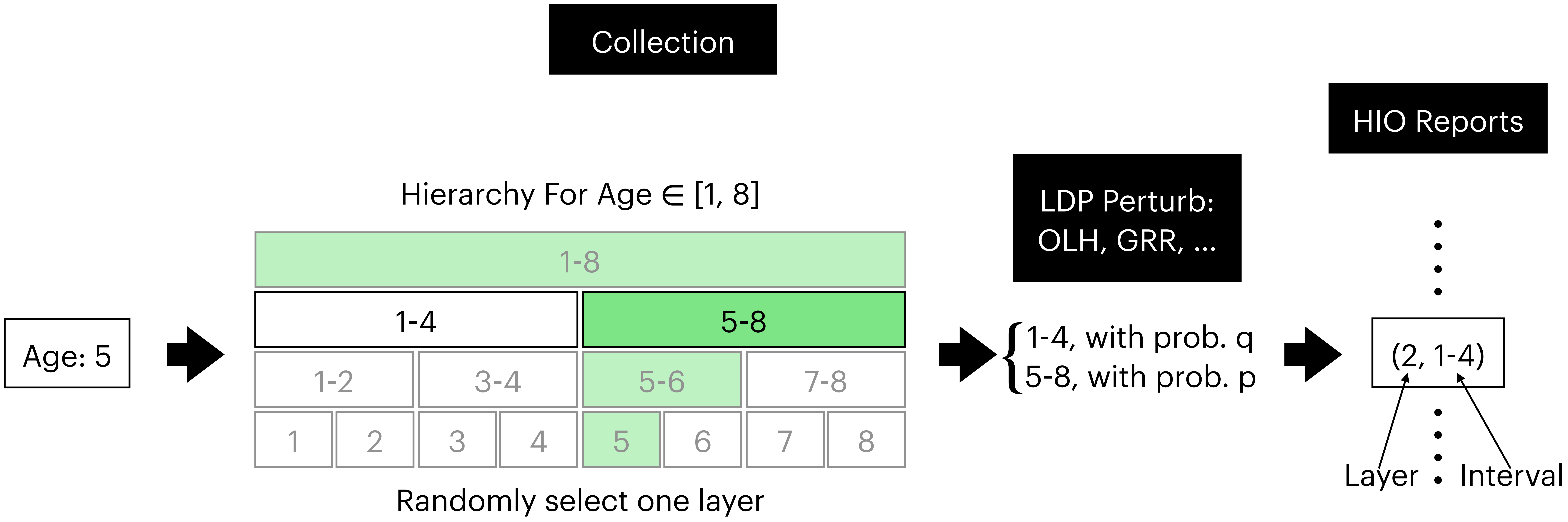
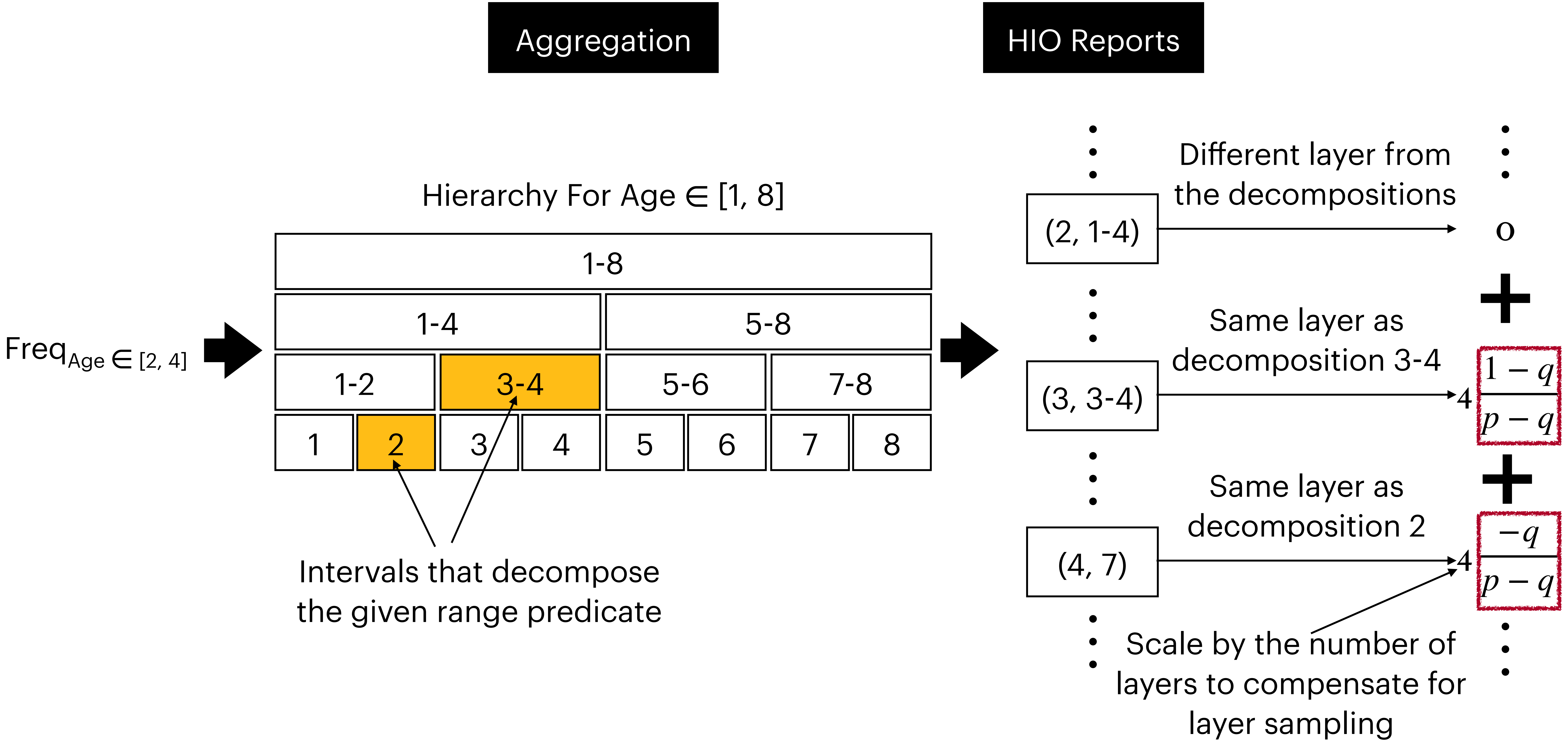


For GRR, $p = \frac{e^{\epsilon}}{e^{\epsilon} + m - 1}$, $q = \frac{1}{e^{\epsilon} + m - 1}$, where m is the cardinality of the domain

Recap: Multi-dimensional Range Frequency Oracles using Hierarchical Interval Optimized (HIO)



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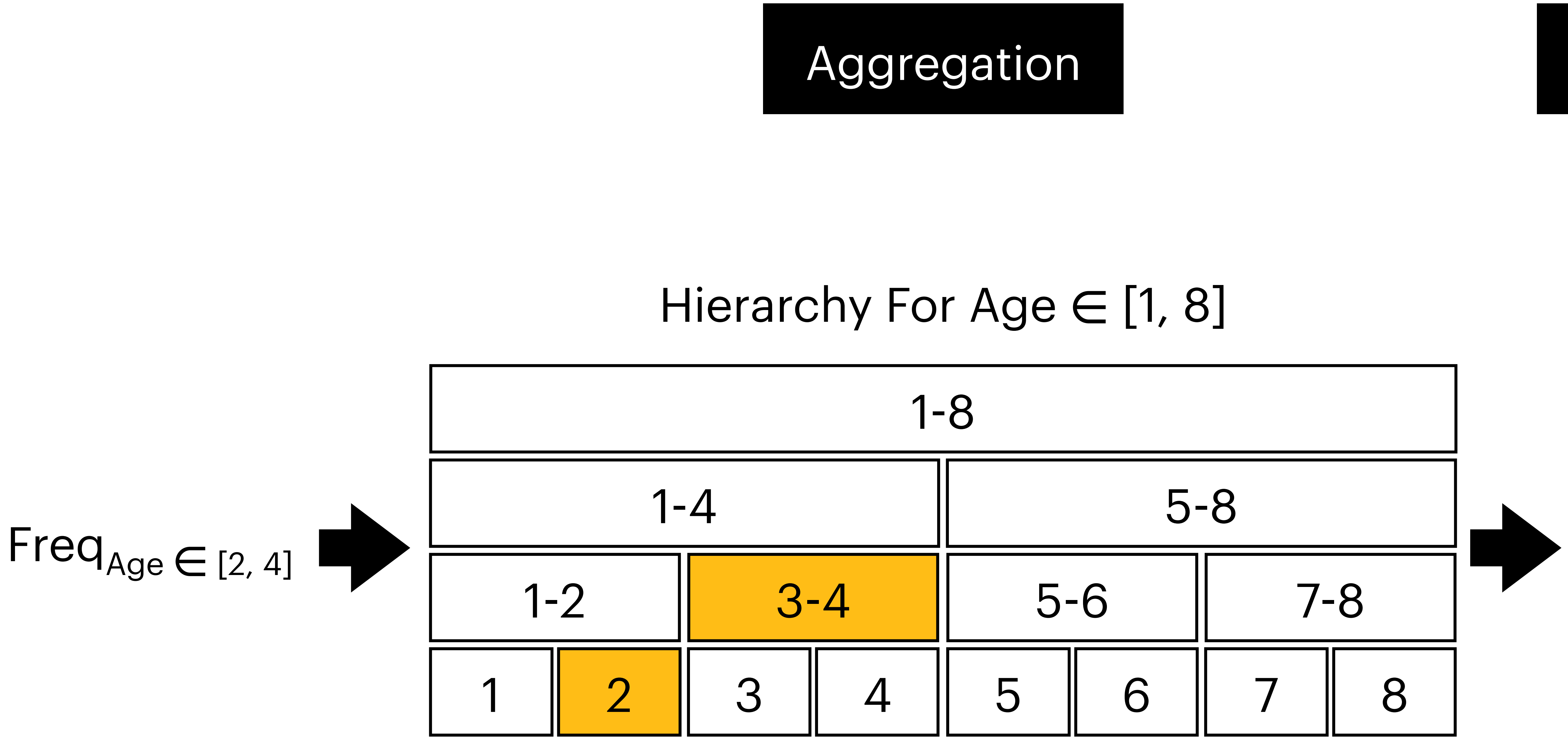

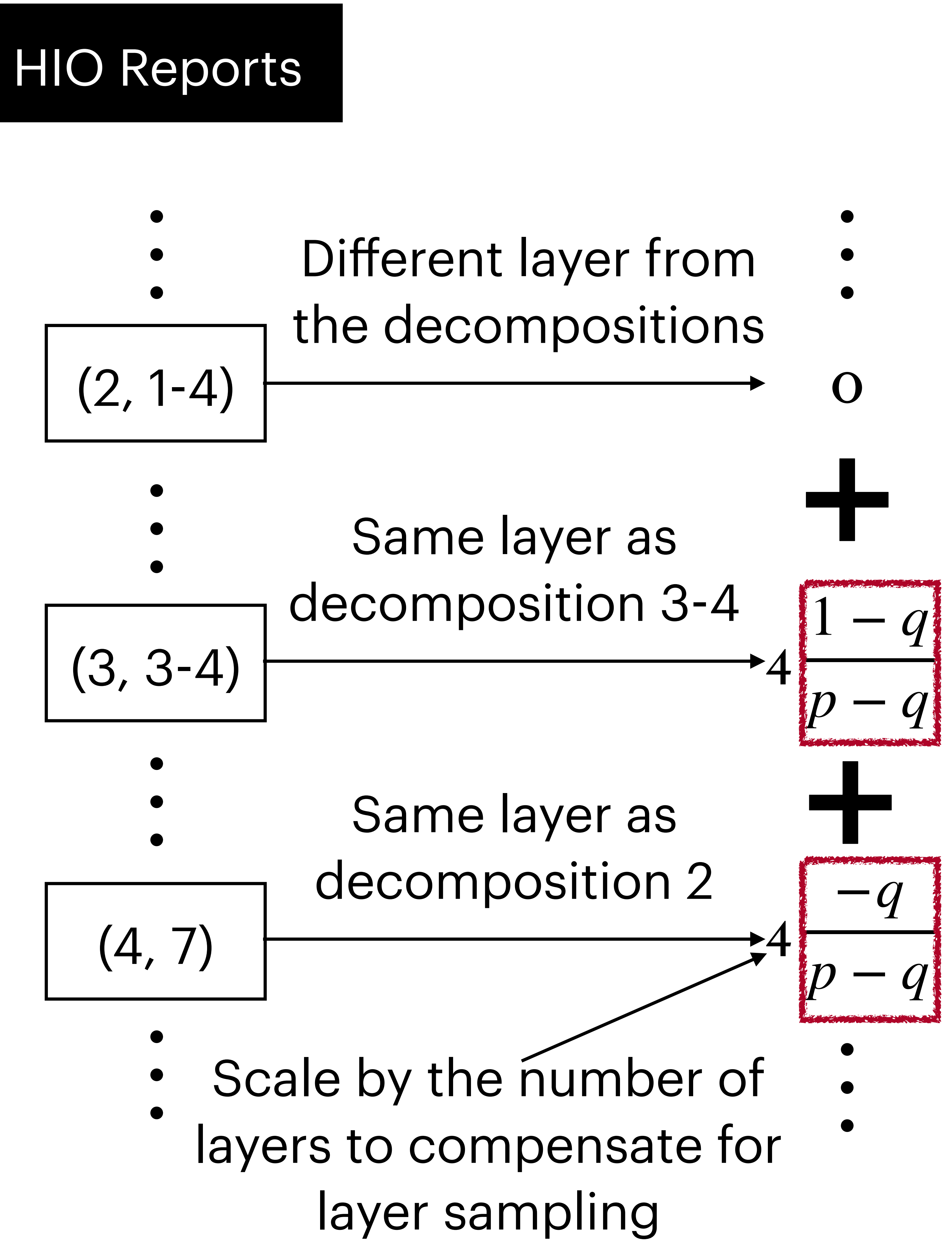

Mean Squared Error: $O\left(\frac{n \log^{d+d_q} m}{\epsilon^2}\right)$, where n is the number of users, d and d_q are the numbers of attributes in the schema and predicate

Data Collection with Two Independent Services using HIO

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$\text{Freq}_{\text{Age} \in [5, 8]} = \dfrac{-q}{p-q}$

$\text{Freq}_{\text{Price} \in [1, 2]} = \dfrac{1-q}{p-q}$

Data Collection with Two Independent Services

$\mathrm{Freq}_{\mathrm{Age}\in[5,\,8]} = \dfrac{-q}{p-q}$

$\mathrm{Freq}_{\mathrm{Age}\in[5,\,8]\,\wedge\,\mathrm{Price}\in[1,\,2]}?$

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State Inversion with Two Independent Hierarchies

Target frequency estimation

$\mathbf{X[11] := Freq_{X1 \in [5,8] \wedge X2 \in [1,2]}}$

$X[10] := Freq_{X1 \in [5,8] \wedge X2 \notin [1,2]}$

$X[01] := Freq_{X1 \notin [5,8] \wedge X2 \in [1,2]}$

$X[00] := Freq_{X1 \notin [5,8] \wedge X2 \notin [1,2]}$

$Y[11] := Freq_{Y1 \in [5,8] \wedge Y2 \in [1,2]}$

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$$\mathbf{E}\begin{bmatrix} Y[11] \\ Y[10] \\ Y[01] \\ Y[00] \end{bmatrix} = \begin{bmatrix} Pr[Y11|X11] & Pr[Y11|X10] & Pr[Y11|X01] & Pr[Y11|X00] \\ Pr[Y10|X11] & Pr[Y10|X10] & Pr[Y10|X01] & Pr[Y10|X00] \\ Pr[Y01|X11] & Pr[Y01|X10] & Pr[Y01|X01] & Pr[Y01|X00] \\ Pr[Y00|X11] & Pr[Y00|X10] & Pr[Y00|X01] & Pr[Y00|X00] \end{bmatrix} * \begin{bmatrix} \mathbf{X[11]} \\ X[10] \\ X[01] \\ X[00] \end{bmatrix}$$

State Inversion with Two Independent Hierarchies

$X[11] := \mathrm{Freq}_{X1\in[5,8]\,\wedge\,X2\in[1,2]}$

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$$\mathbf{E}\underbrace{\begin{bmatrix} Y[11] \\ Y[10] \\ Y[01] \\ Y[00] \end{bmatrix}}_{\mathbf{Y}} = \underbrace{\begin{bmatrix} \Pr[Y11|X11] & \Pr[Y11|X10] & \Pr[Y11|X01] & \Pr[Y11|X00] \\ \Pr[Y10|X11] & \Pr[Y10|X10] & \Pr[Y10|X01] & \Pr[Y10|X00] \\ \Pr[Y01|X11] & \Pr[Y01|X10] & \Pr[Y01|X01] & \Pr[Y01|X00] \\ \Pr[Y00|X11] & \Pr[Y00|X10] & \Pr[Y00|X01] & \Pr[Y00|X00] \end{bmatrix}}_{\mathbf{M}} * \underbrace{\begin{bmatrix} \mathbf{X[11]} \\ X[10] \\ X[01] \\ X[00] \end{bmatrix}}_{\mathbf{X}} \Rightarrow \mathbf{X} \leftarrow \mathbf{M}^{-1}\mathbf{Y}$$

Unbiased estimation

- How to calculate Pr[Y11|X11]?

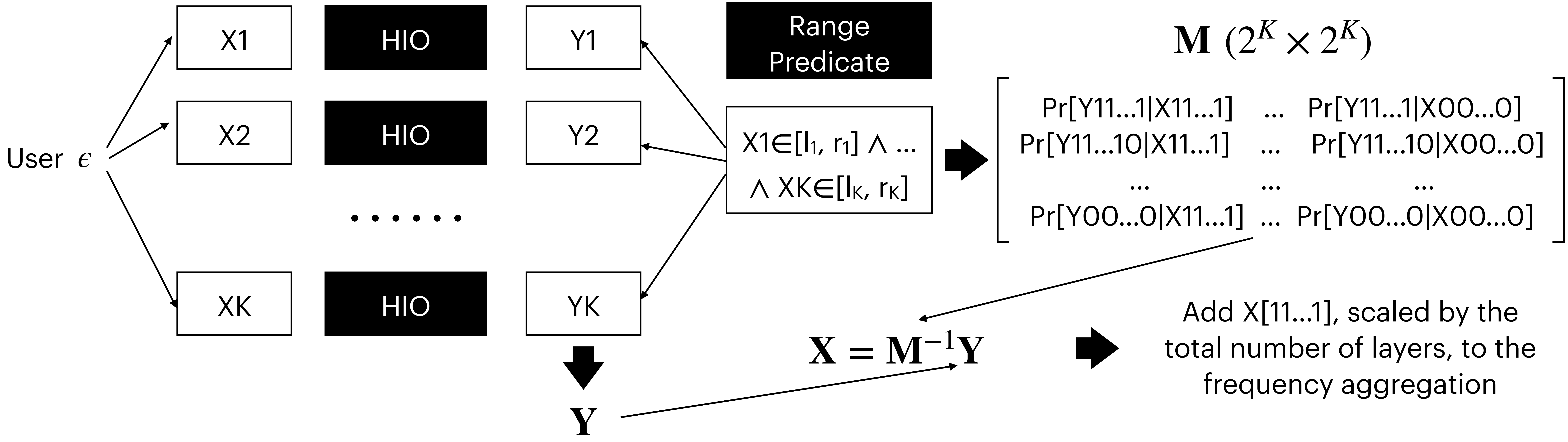
  Because X1 and X2 are independently perturbed into Y1 and Y2, Pr[Y11|X11] = Pr[Y1∈[2, 4]|X1∈[2, 4]]*Pr[Y2∈[1, 4]|X2∈[1, 4]] = p*p. We can derive other conditional prob. similarly

Joint Frequency Oracles with Two Independent Services

$$\text{Freq}_{\text{Age}\in[5,\,8]\,\wedge\,\text{Price}\in[1,\,2]} \leftarrow (\mathbf{M}^{-1}\mathbf{Y})[11\ldots1]$$

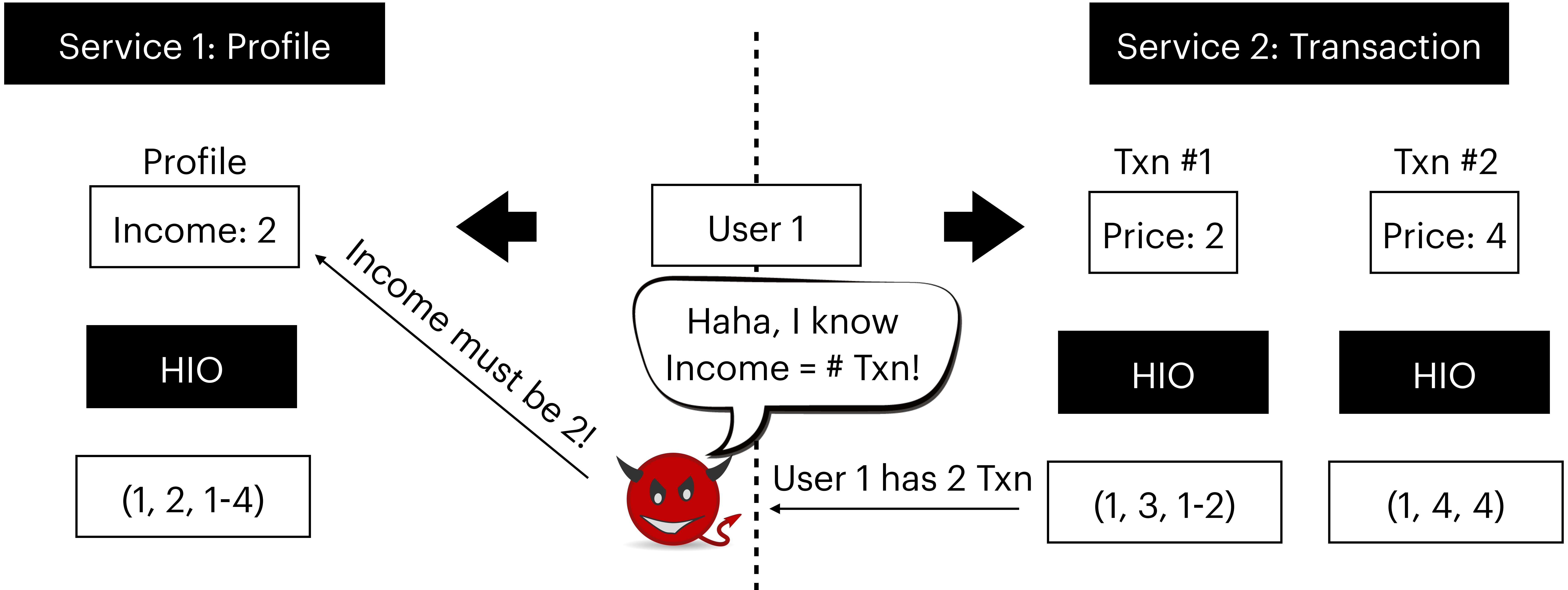
Joint Frequency Oracles with Two Independent Services

General Joint Frequency Oracles with K Services (HIO-JOIN)



- Mean Squared Error: $O\left(\frac{n \log^{K(d+d_q)} m}{(\epsilon/K)^{2K}}\right)$, where K is the number of services, d and d_q are the numbers of attributes in the schema and predicate for each service
- In fact, it can handle range predicate on attributes of arbitrary subset of the K services

1-Many join: Primary-Foreign-Key JOIN with Two Services



Frequency-Based Attack: Infer sensitive information based on the leaked number of tuples of a user collected by each service

- The information that user 1 made 2 transactions with service 2 is leaked at join
- If such information is correlated with the sensitive information, say, the income, attacker can infer the income of the user based on the number of transactions she made

User-Level LDP (uLDP)

ϵ-uLDP: Intuitively, for any two users, the difference between the distributions of their collected data is bounded by e^{ϵ}

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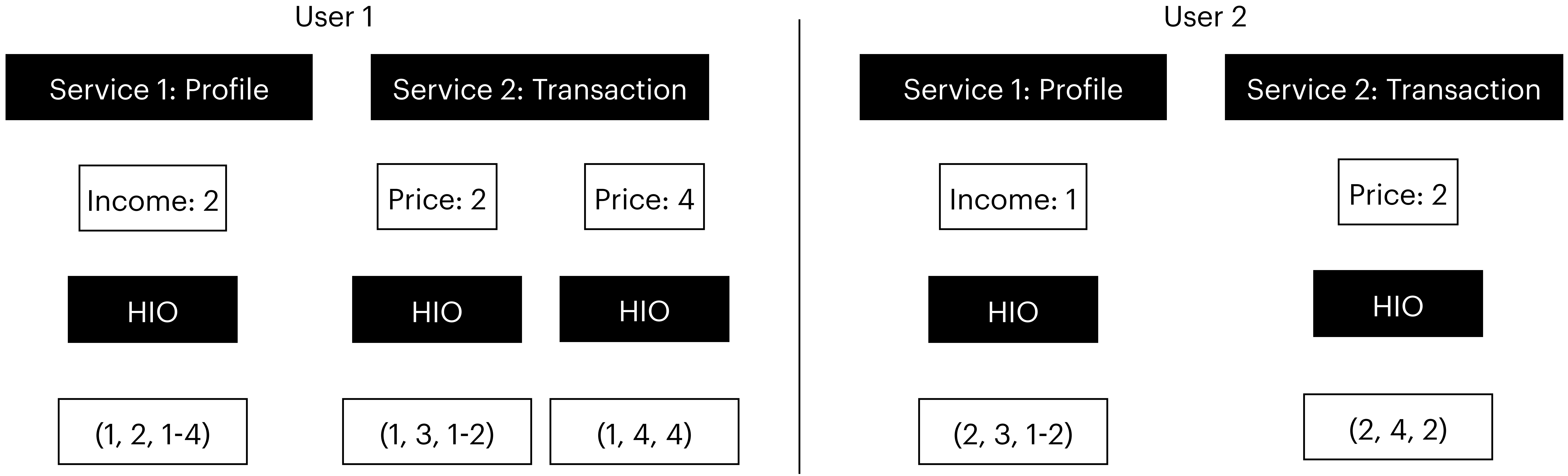

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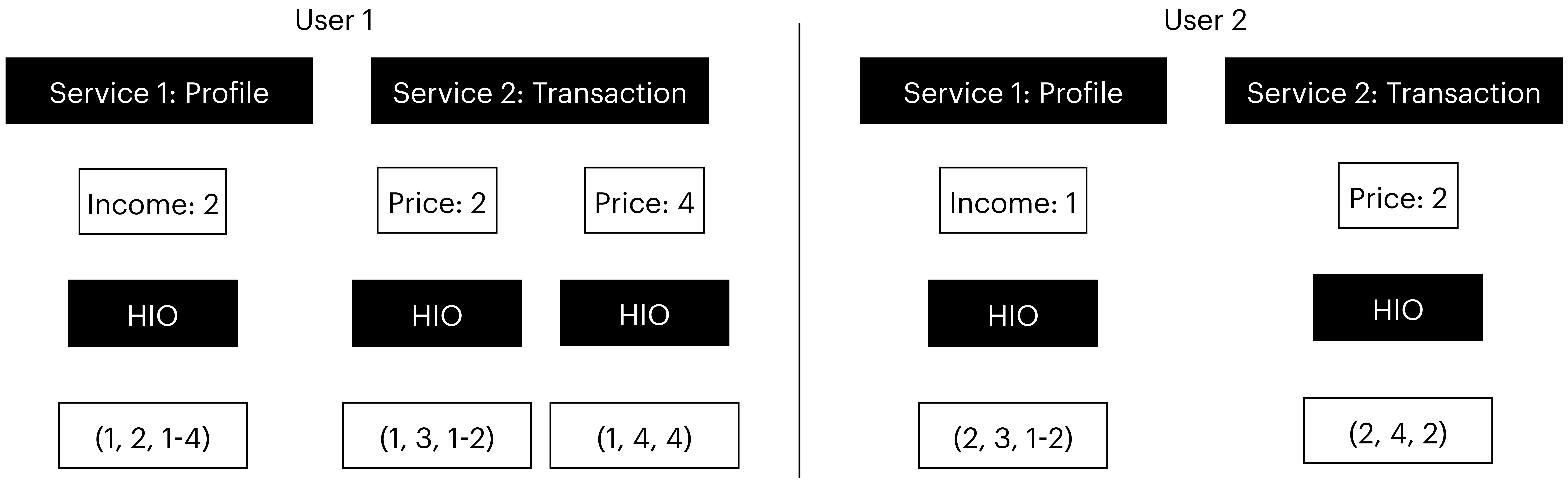
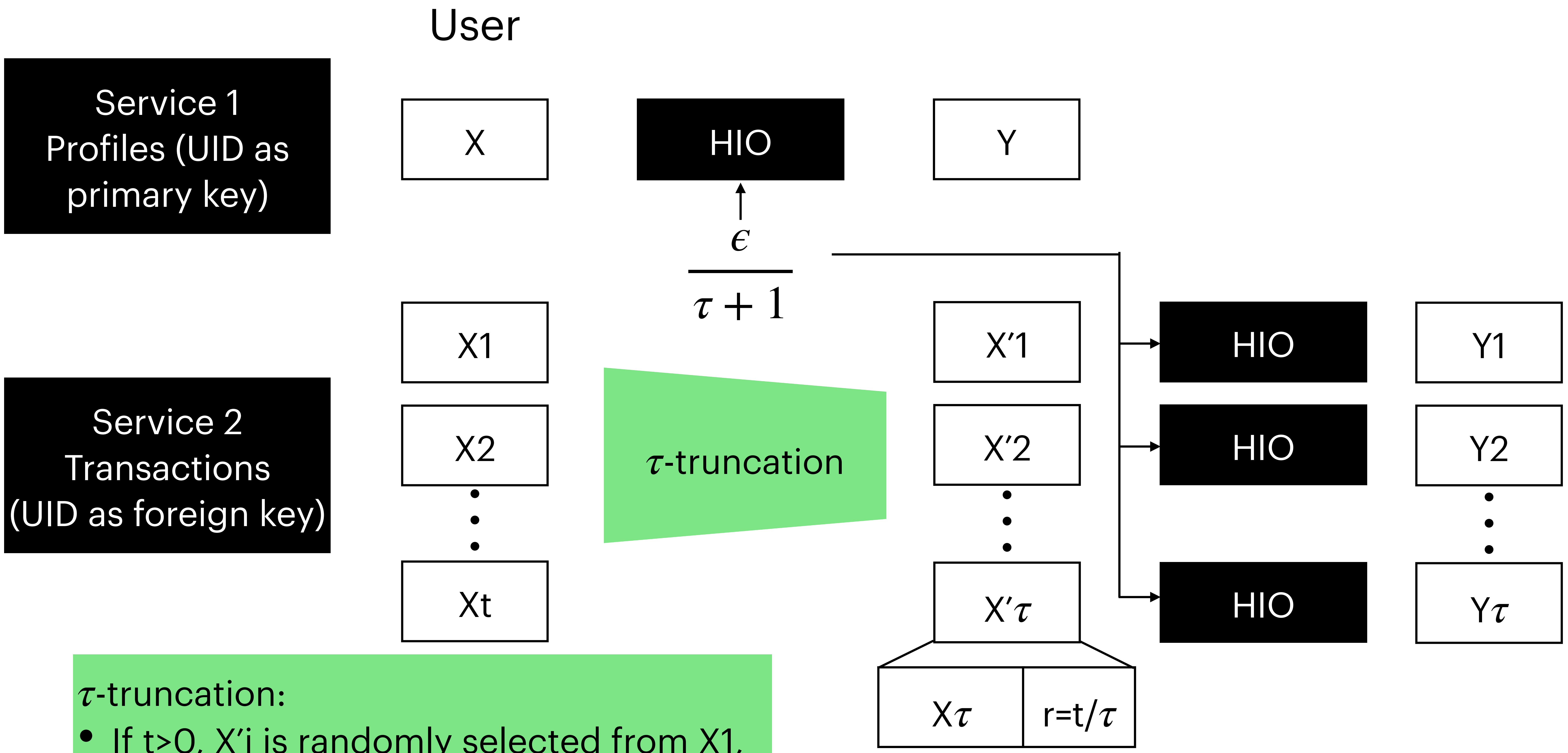


Impossible to have the same collected data for user 1 and 2! => ∞-uLDP => no privacy at all!

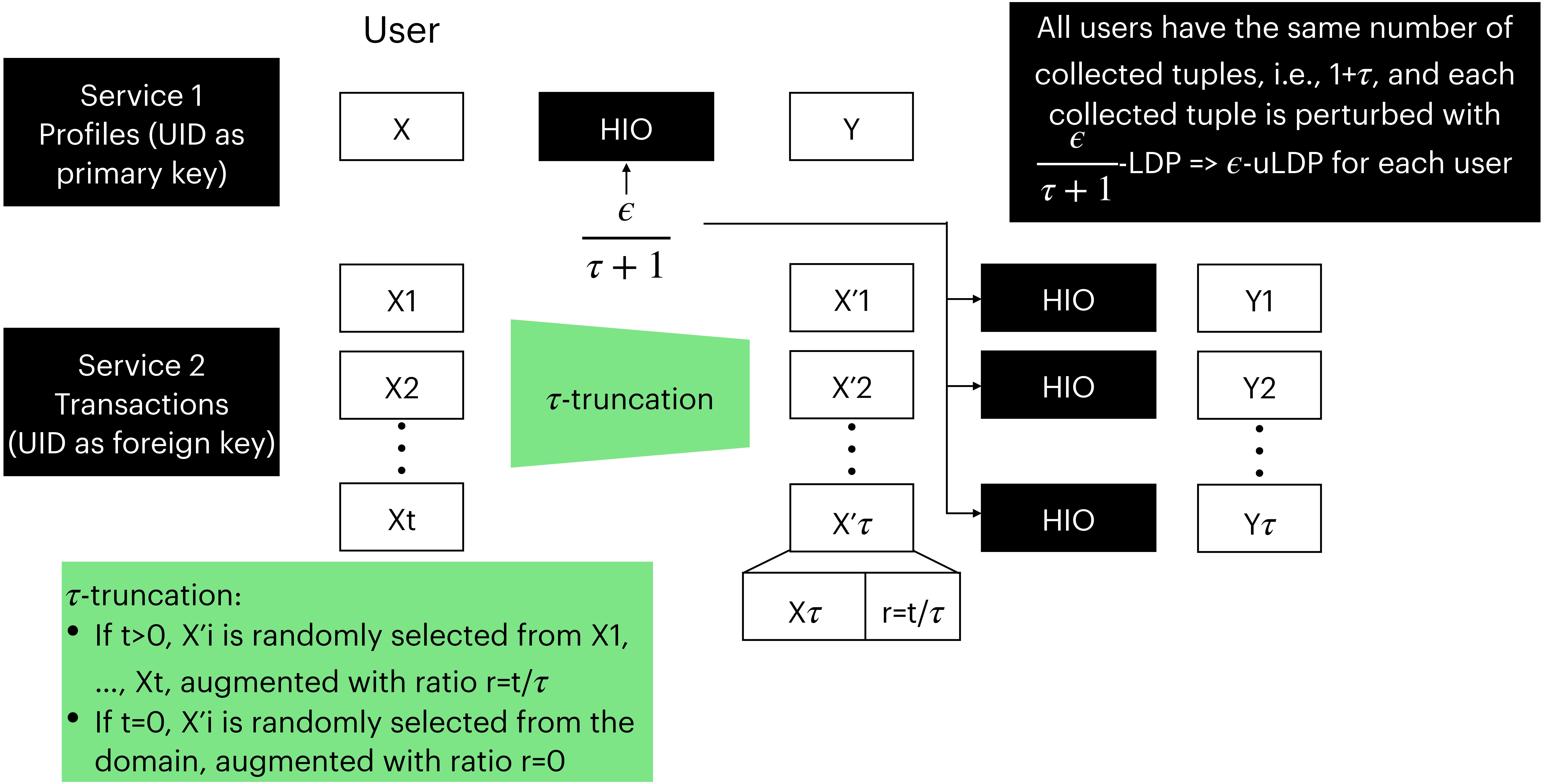
1-Many JOIN under ϵ-uLDP



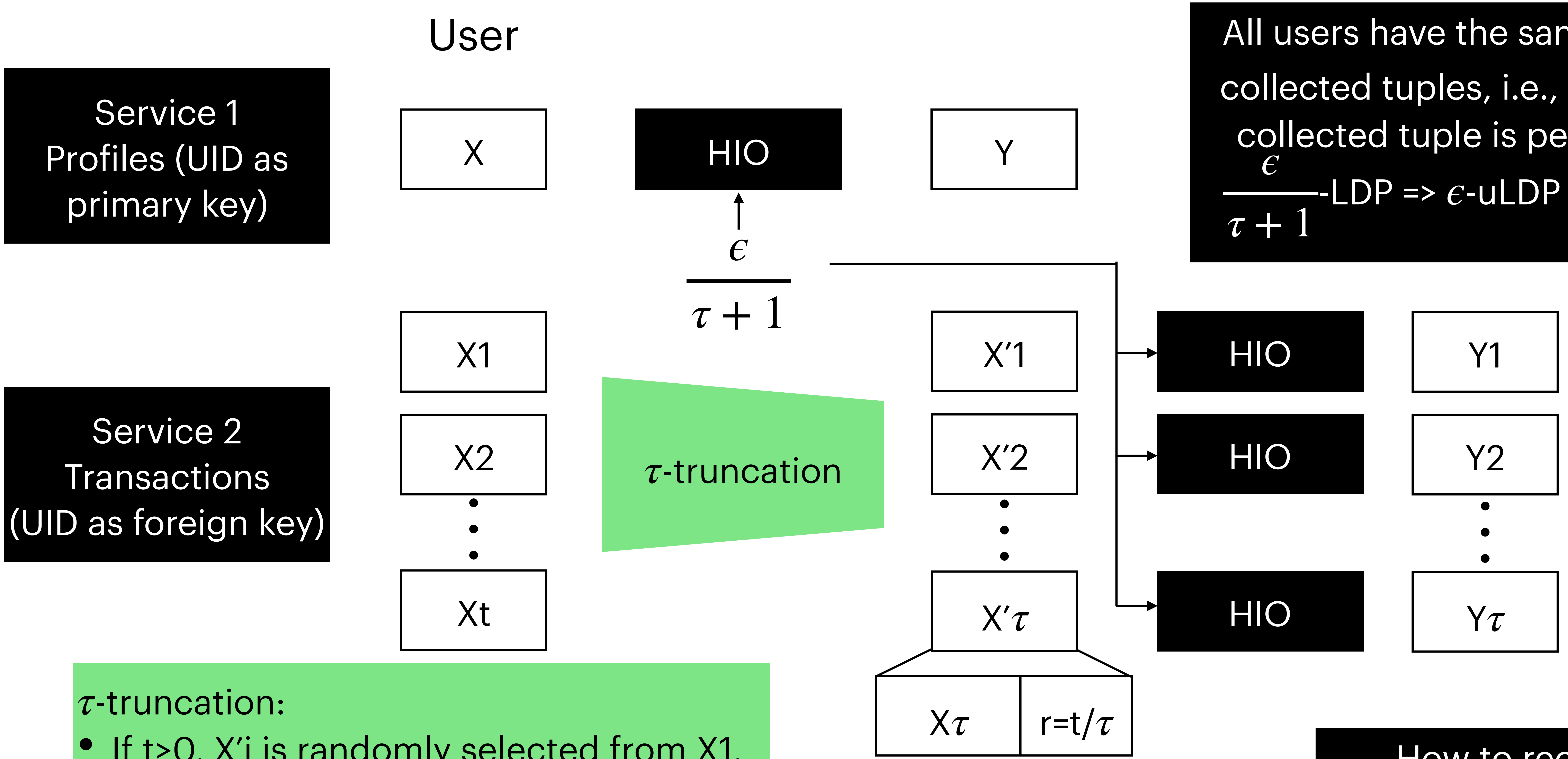
τ-truncation:

- If t>0, X'i is randomly selected from X1, …, Xt, augmented with ratio r=t/τ
- If t=0, X'i is randomly selected from the domain, augmented with ratio r=0

1-Many JOIN under ϵ-uLDP



1-Many JOIN under ϵ-uLDP



All users have the same number of collected tuples, i.e., 1+τ, and each collected tuple is perturbed with $\frac{\epsilon}{\tau+1}$-LDP => ϵ-uLDP for each user

τ-truncation:

- If t>0, X'i is randomly selected from X1, …, Xt, augmented with ratio r=t/τ
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How to recover the unbiased aggregations from the truncated tuples?

Sensitive Weight Frequency Oracles (SWFO)

- So far, we can handle frequency oracles on the collected data with range predicate over arbitrary subset of attributes from the multiple services
- It is necessary to further support oracles for the aggregated frequency, weighted by some sensitive attribute of each tuple. We call such oracles the *sensitive weight frequency oracles*:
  - **Attribute Aggregation**

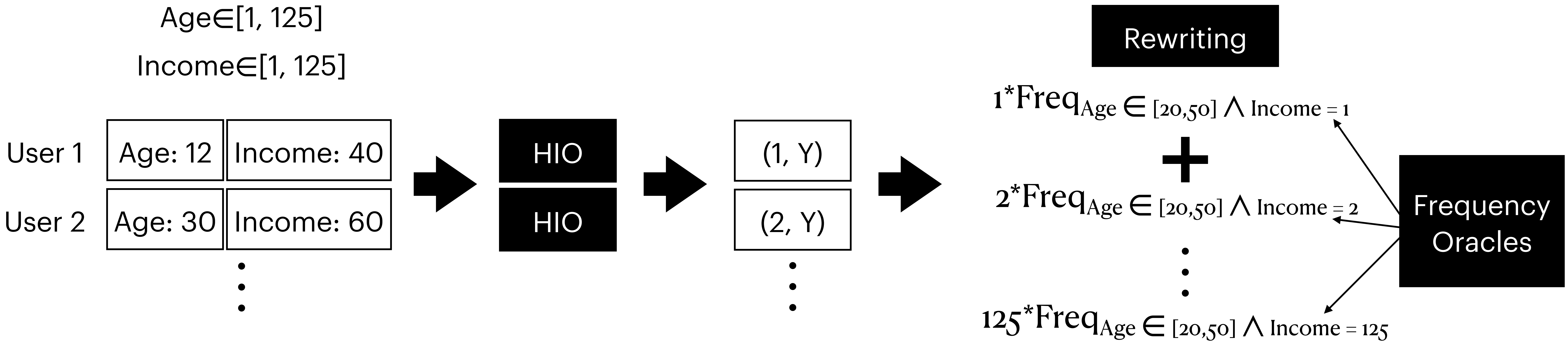
    SELECT SUM(Income) FROM Profiles WHERE Age $\in$ [20,50]
  - **Recover Unbiased Aggregation with τ-Truncation** (weighing each tuple by r=t/τ)

    SELECT SUM(r) FROM Profiles JOIN Transactions ON UID WHERE Age $\in$ [20,50]

    SELECT SUM(r*Price) FROM Profiles JOIN Transactions ON UID WHERE Age $\in$ [20,50]
  - Previous works leave such aggregation analysis as open problems

(Joint) Frequency Oracles to SWFO (Straw-man)

SELECT SUM(Income) FROM Profiles WHERE Age ∈ [20,50]



Mean Squared Error: $O\left(\boxed{\sum_{v \in A} v^2} \frac{n \log^{d+d_q} m}{\epsilon^2}\right)$

The multiplicative factor of sum of squares of the domain values in the error bound is prohibitive for aggregation on large domains

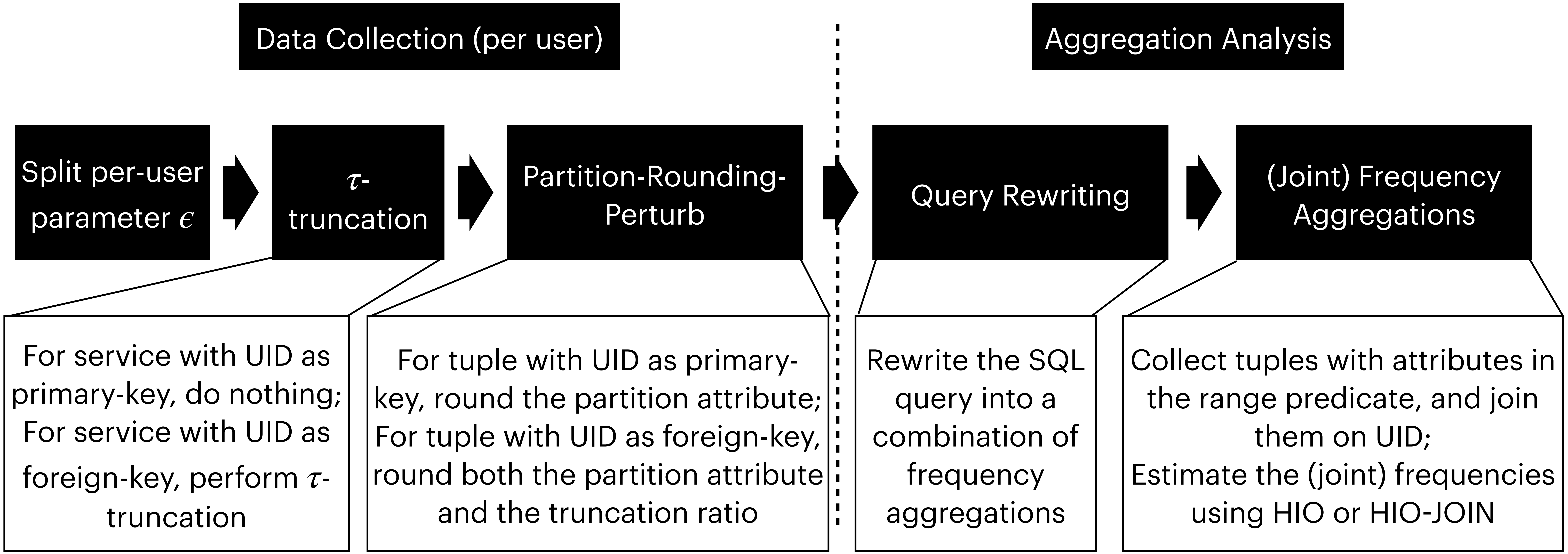
Partition-Rounding-Perturb Framework for SWFO

SELECT SUM(Income) FROM Profiles WHERE Age ∈ [20,50]

Mean Squared Error: $O\left(\frac{\boxed{2(A_{\min}^2 + A_{\max}^2)d} \cdot n \log^{d+d_q} m}{\epsilon^2}\right)$

The multiplicative factor is linear to the number of attributes times the sum of squares of the min/max

End-to-End Data Collection and Analysis Pipeline



- Query independent - can handle arbitrary frequency and attribute aggregation with range predicate over any subset of attributes
- Minimal coordination among services: i) value of τ; and ii) the total number of tuples to be collected for each user

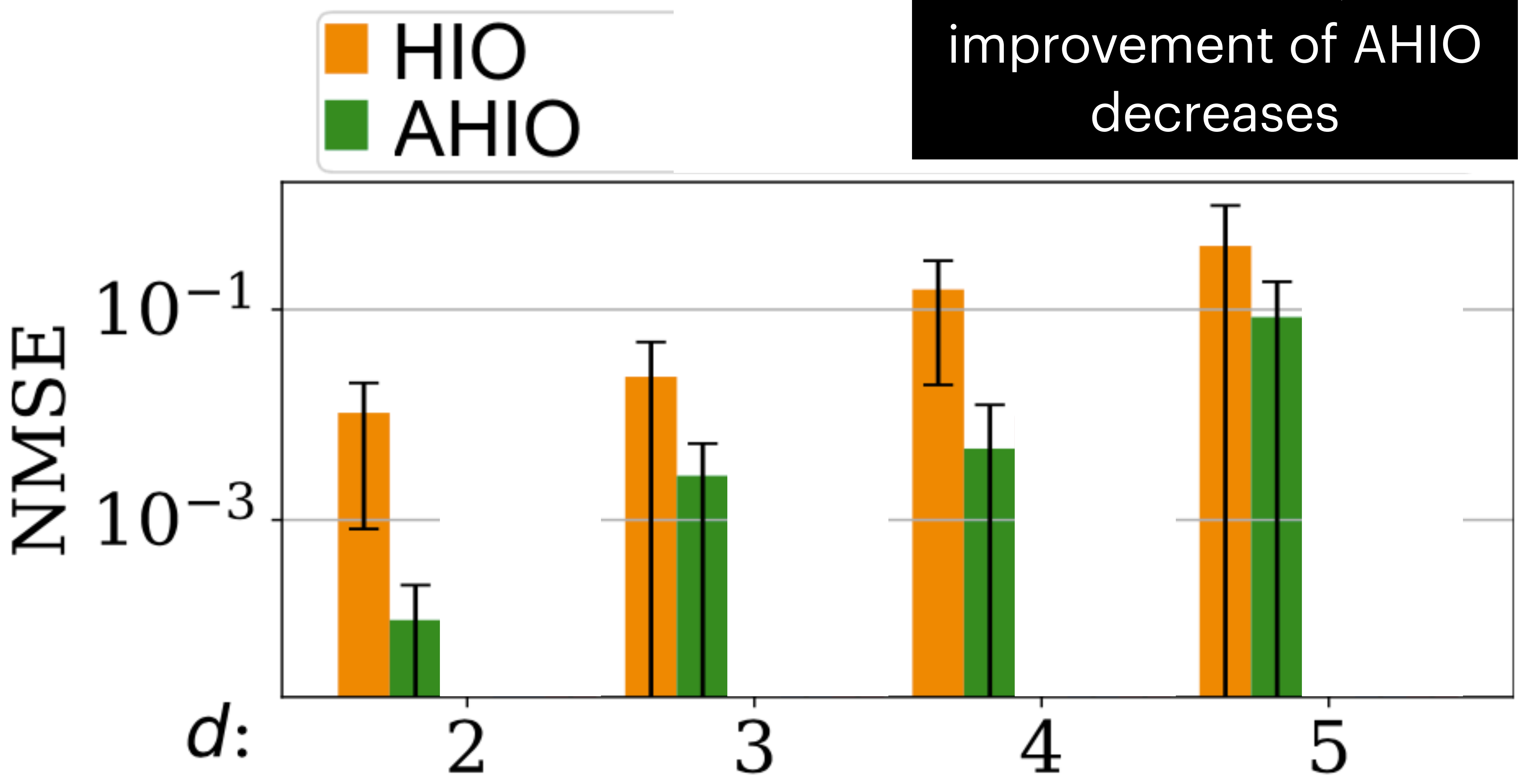
Experimental Results

- We setup a single-node SparkSQL cluster for the experiments, and register the LDP perturb and aggregation estimation primitives as UDFs of SparkSQL
- Datasets
  - SYN-1: Single-table synthetic data , with 1M records and configurable number of attributes d and attribute cardinality m
  - SYN-2: Two-table synthetic data, with two join configurations: 1-1; and 1-many with join degree in range [1, 10]
- We focus on the utilities for COUNT, SUM and AVG aggregations of the following schemes
  - AHIO: Partition-Rounding-Perturb with the rounding value stored in separate attribute
  - HIO-JOIN: no truncation for 1-1 join; with τ-truncation for 1-many join

Attribute Aggregation

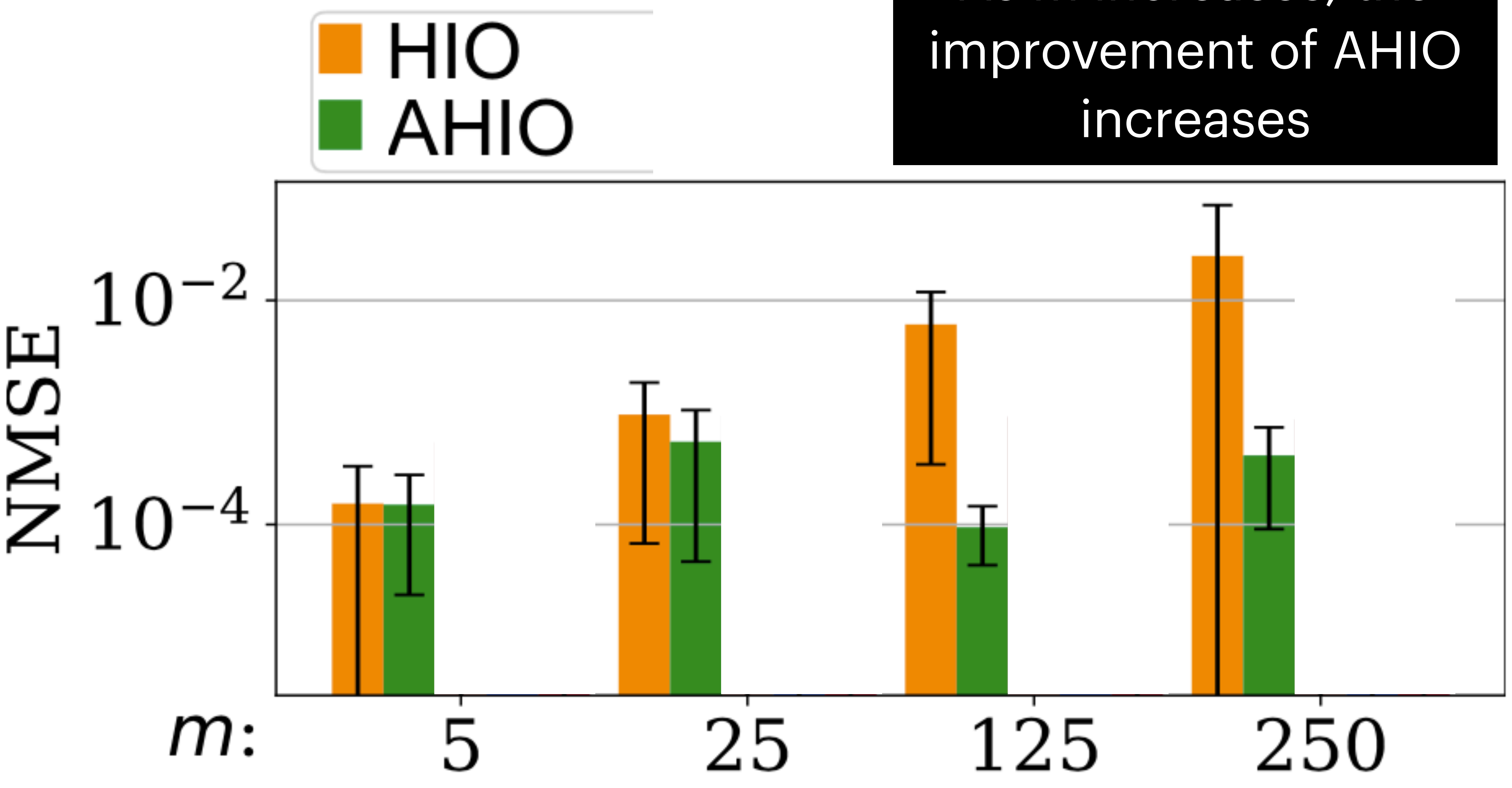
SELECT SUM(A) FROM SYN-1 WHERE C

As d increases, the improvement of AHIO decreases



ϵ=2, m=125, d_q=1

As m increases, the improvement of AHIO increases



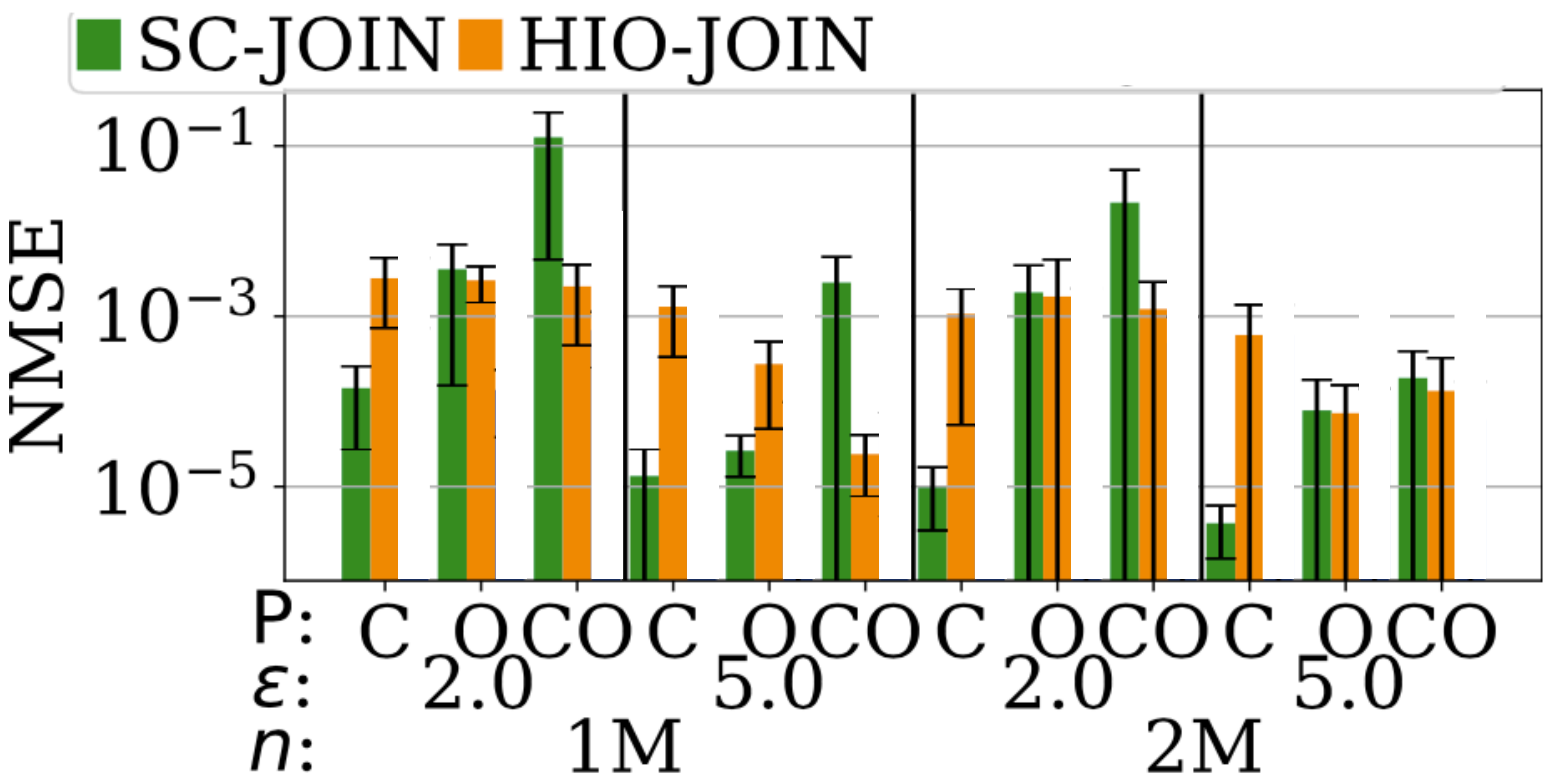
ϵ=2, d=2, d_q=1

Asymptotic Mean Squared Error

HIO: $O\left(\sum_{v\in A} v^2 \frac{n \log^{d+d_q} m}{\epsilon^2}\right)$, AHIO: $O\left(\frac{2(A_{\min}^2 + A_{\max}^2)d \cdot n \log^{d+d_q} m}{\epsilon^2}\right)$

1-1 Joint Aggregation

SELECT COUNT(*) FROM T1 JOIN T2 ON UID WHERE C



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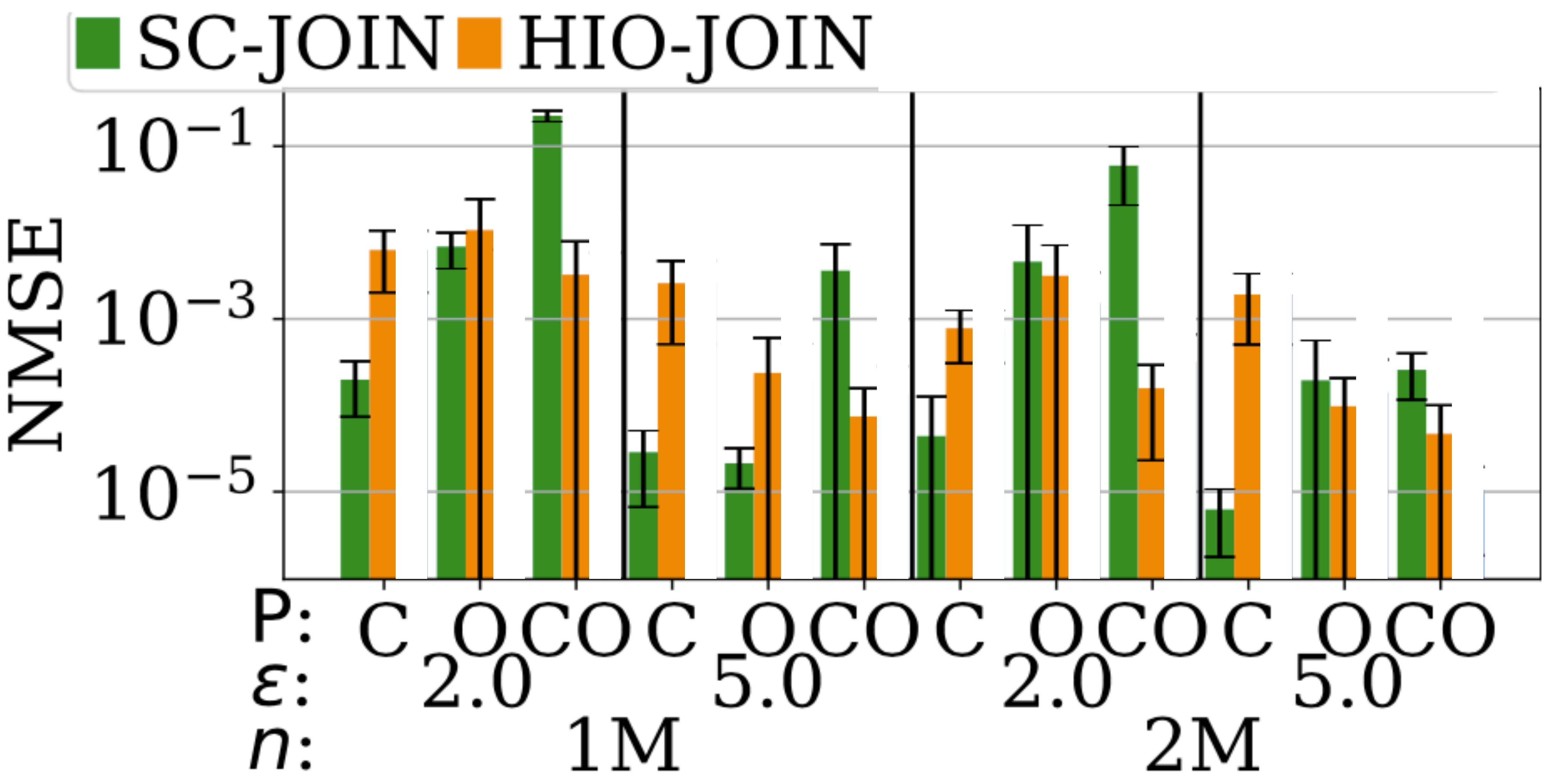


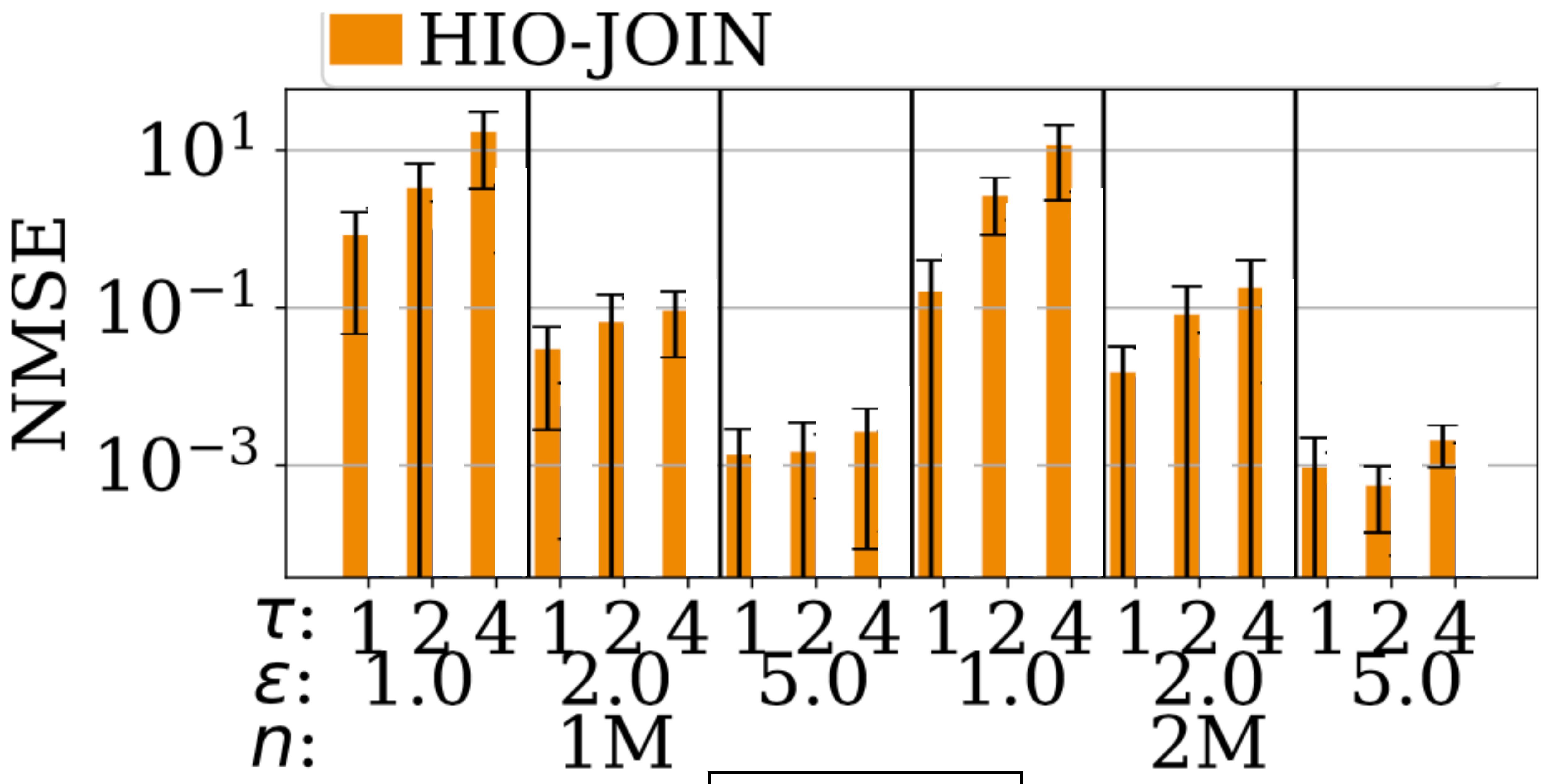
Asymptotic Mean Squared Error

SC-JOIN: $O\left(\frac{n \log^{3Kd_q} m}{(\epsilon/Kd)^{2Kd_q}}\right)$, HIO-JOIN: $O\left(\frac{n \log^{K(d+d_q)} m}{(\epsilon/K)^{2K}}\right)$

As range predicate gets complicated, i.e., d_q increases, error of SC-JOIN increases faster than that of HIO-JOIN

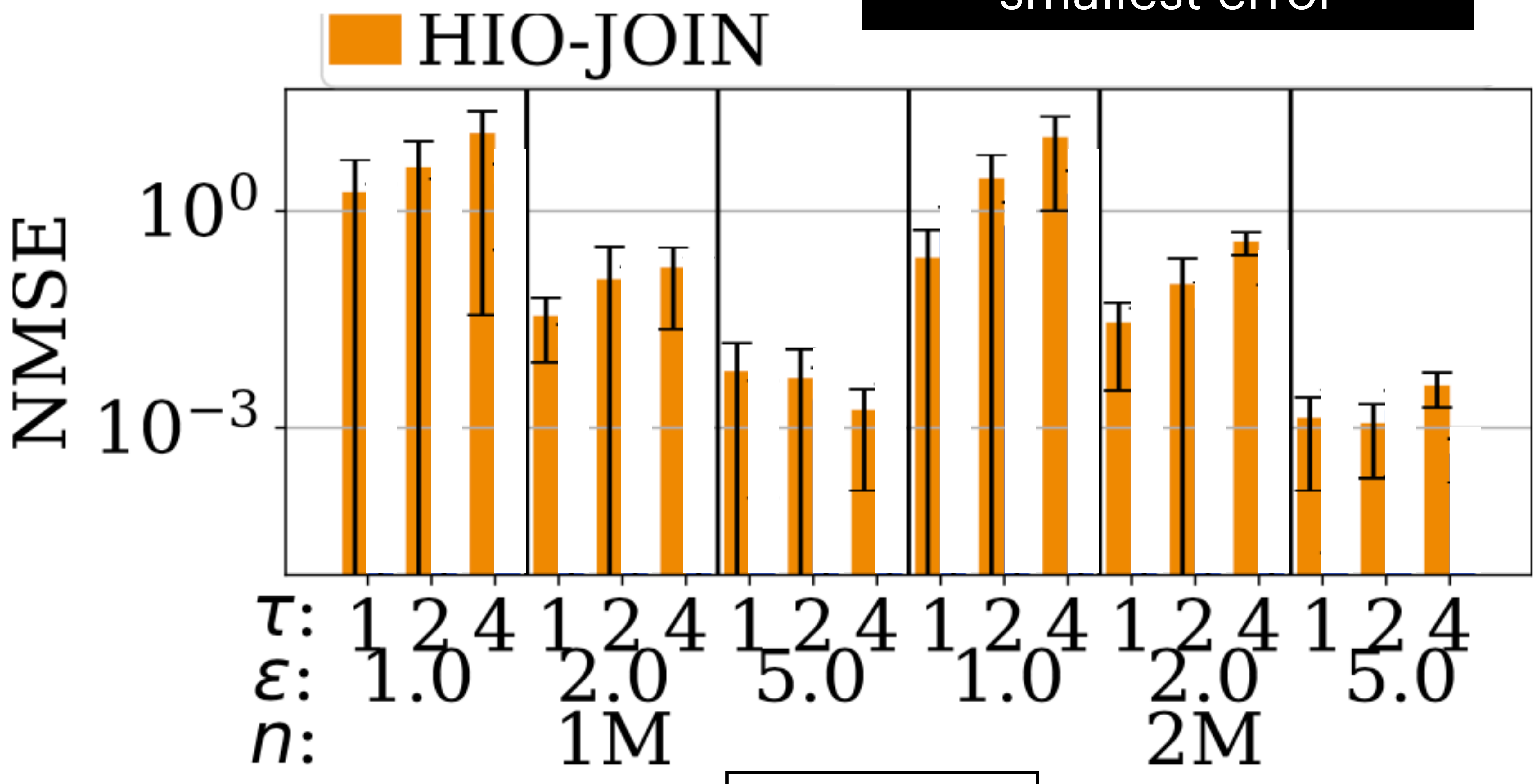
1-Many Joint Aggregation

τ = 1 leads to the smallest error



COUNT

d_q=2



SUM

Asymptotic Mean Squared Error of HIO-JOIN (with τ-truncation):

COUNT: $O\left(\frac{nd\tau(1+\tau)^4\log^{2(d+d_q)} m}{\epsilon^4}(r_{\min}^2+r_{\max}^2)\right)$, SUM: $O\left(\frac{nd\tau(1+\tau)^4\log^{2(d+d_q)} m}{\epsilon^4}(r_{\min}^2+r_{\max}^2)(A_{\min}^2+A_{\max}^2)\right)$

More in the Paper

- Efficient range utility optimization based on range decomposition and consistency
- Support for group-by aggregations
- More experiments using real-world PUMS datasets and other tested schemes
- Alternative instantiation of the Partition-Rounding-Perturb framework by embedding the rounding value in the partition attribute

Thank you!
Q & A