Imperial College
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Intelligent Business

Machine Learning in Finance

Lecture 6
Introduction to Sequence Models

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Outline:

- Introducing the concept of Memory
- The Embedding Layer
- Recurrent Neural Networks
- Programming Session

Part 1 : Introducing the concept of Memory

Review of the Sentiment Analysis Pipeline

- The Preprocessing steps:

Row Data

- Document 1 : « There were no wolves in the movie. »
- Document 2 : « This movie has one star and that star is Ryan Gosling. Great flick, highly recommend it. »

⋮ ⋮

- Document N : « How many times must Willy be freed before he's freed?. »

Preprocessed Data

- Document 1 : [0, 23, 43, 12, …, 2343, 9999]
- Document 2 : [0, 12, 1, 3453, …, 123, 9999]

⋮ ⋮

- Document N : [0, 1, 1252, …, 1232, 9999]

Vectorized Data

$$\begin{pmatrix} 0 & 1 & 1 & \dots & 0 & 1 \\ 0 & 0 & 1 & \dots & 0 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 1 & 1 & \dots & 1 & 1 \end{pmatrix}$$

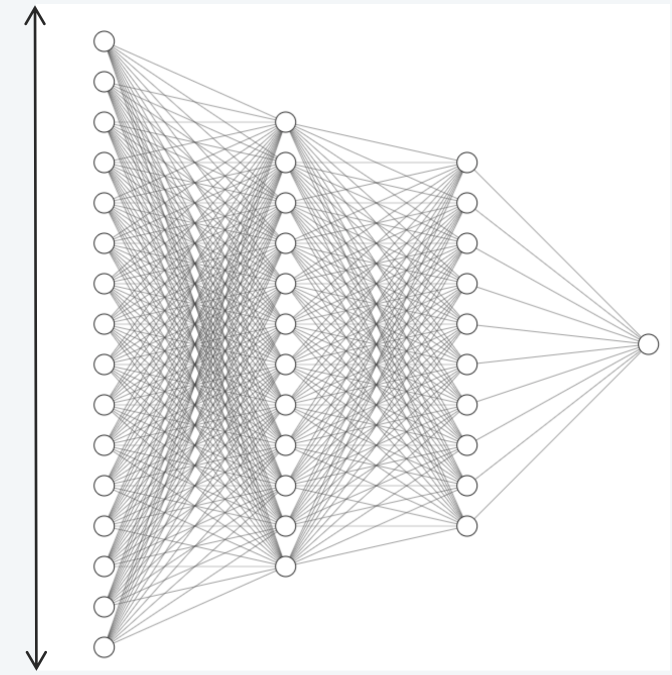
N

V

- The Tensor data is of shape (N, V) after the preprocessing steps.

- Subsequently, we supply the tensor data to either a conventional machine learning algorithm or a neural network composed of a series of densely connected layers..

V



Limitations of the approach

1. The primary limitation arises from the method we employ to encode the sentence into a V-dimensional vector :

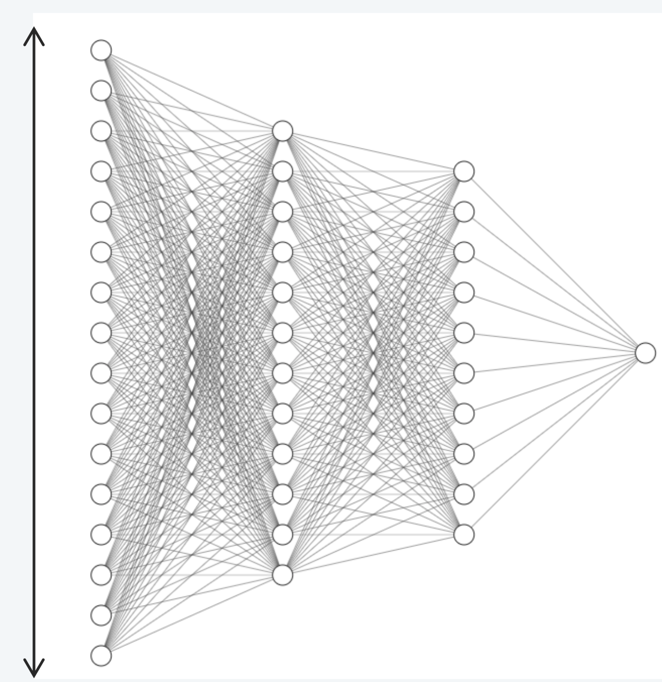
    - The encoding is performed regardless of the order in which the words come.

    - For instance, these two sentences will be encoded into the same vector:
        - « Never quit. Do your best. »
        - « Never do your best. Quit. »

1. The other limitation comes from the model itself. We feed the entire sequence (encoded into a V-dimensional vector) all at once.

$$\begin{array}{c} \overset{V}{\longleftrightarrow} \\ \begin{pmatrix} 0 & 1 & 1 & \dots & 0 & 1 \\ 0 & 0 & 1 & \dots & 0 & 1 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 1 & 1 & \dots & 1 & 1 \end{pmatrix} \end{array} \updownarrow N$$

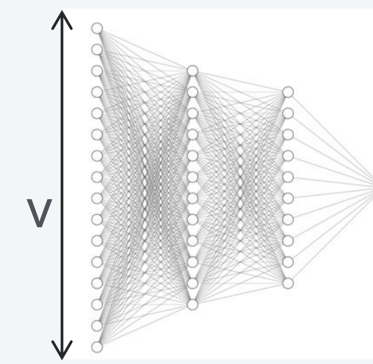
V



The concept of Memory in Neural Networks

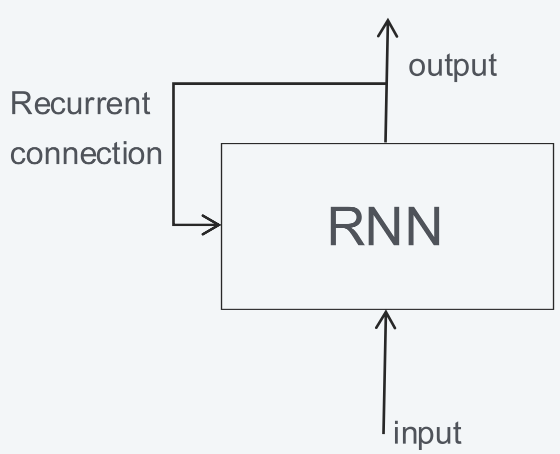
- When handling sequential data, the primary limitation of the neural networks we've encountered thus far is their treatment of each sequence as a singular large vector. These networks are commonly referred to as **feedforward neural networks**.

$$\underbrace{\text{What a great movie}}_{\boxed{\text{sequence}}} \longrightarrow \underbrace{[0, 1, 0, 1, \ldots, 1, 0]}_{\text{V}} \longrightarrow$$

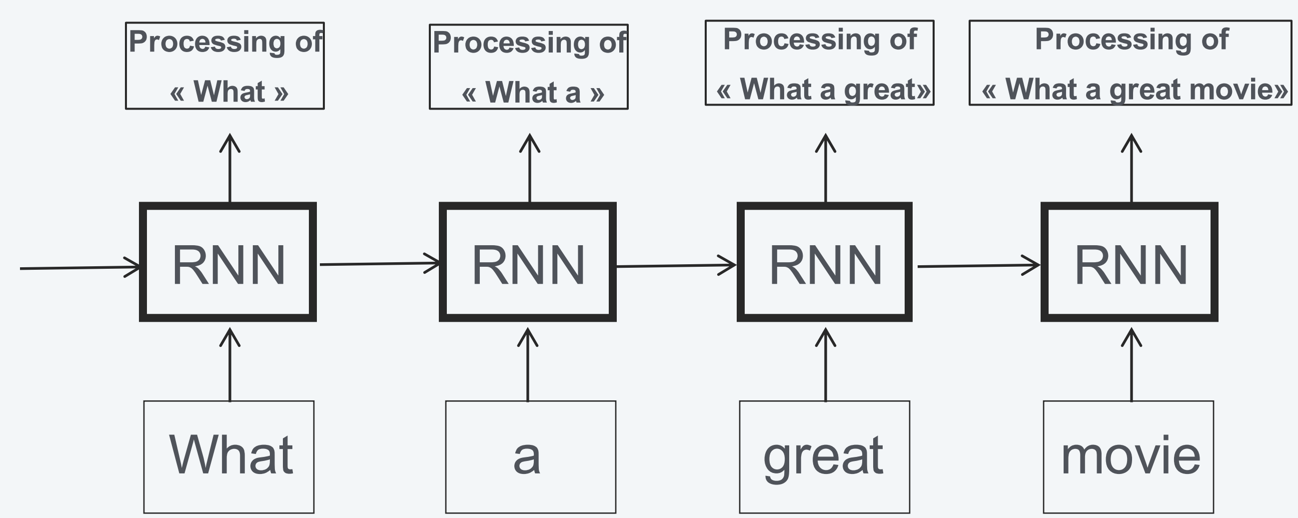


- On the contrary, a **recurrent neural network** (RNN) analyzes sequences by examining them element by element while maintaining a state that retains the information processed so far.

Internal Loop



Preprocessing of « What a great movie »



A simple example – Part 1 –

- We want to predict whether the stock AAPL is going up or down for the next day, based on D = 4 characteristics of the stock during the last T days.

- At each time step t, let $x_t = [c_t^1, c_t^2, c_t^3, c_t^4]$ denote four features of the stock AAPL at time t.

- Let y_t denote the stock movement at time t:
  - $y_t = 1$ when the stock goes up between t-1 and t.
  - $y_t = 0$ when the stock goes down between t-1 and t.

- Let $x_1, \ldots, x_F$ be the whole sequence of characteristics from 2019 to 2024

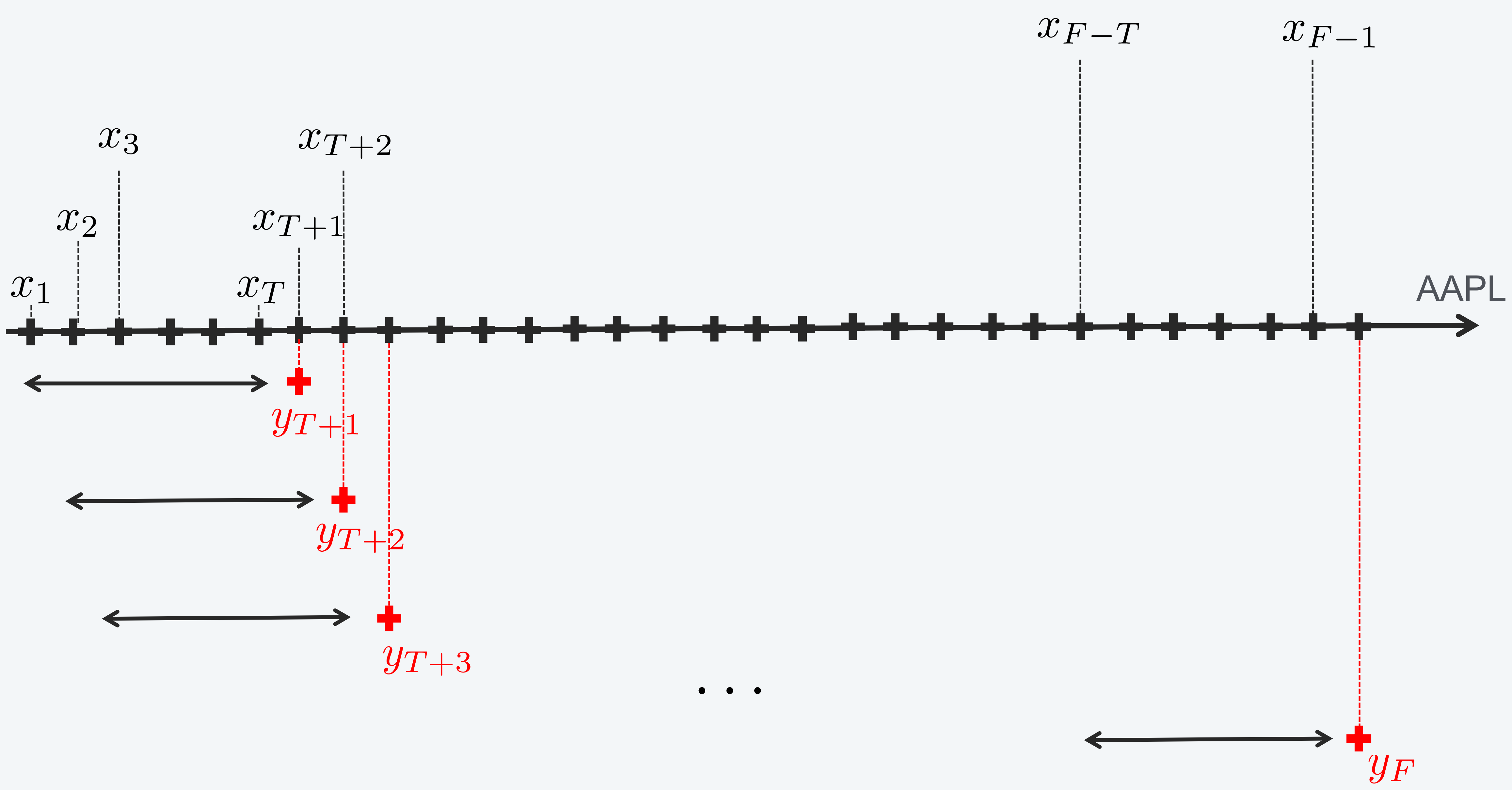
- We want to predict the stock movement as follows:

  - From the sequence $x_1, \ldots, x_T$, we want to predict y_{T+1}
  - From the sequence $x_2, \ldots, x_{T+1}$, we want to predict y_{T+2}

    $\vdots \qquad \vdots \qquad \vdots$

  - From the sequence $x_{F-T}, \ldots, x_{F-1}$, we want to predict y_F

A simple example – Part 2 –



A simple example – Part 3 –

- The input data tensor is a tensor of shape (N, T, D):
  - Each sample i among the N samples is a sequence s_i of length T with elements of dimension D.
  - We have N = F – T sequences.

- The target tensor data is of shape (N,):
  - Each sample i is associated with a target t_i.
  - Each target is a binary output : 1 for « up » and 0 for « down ».

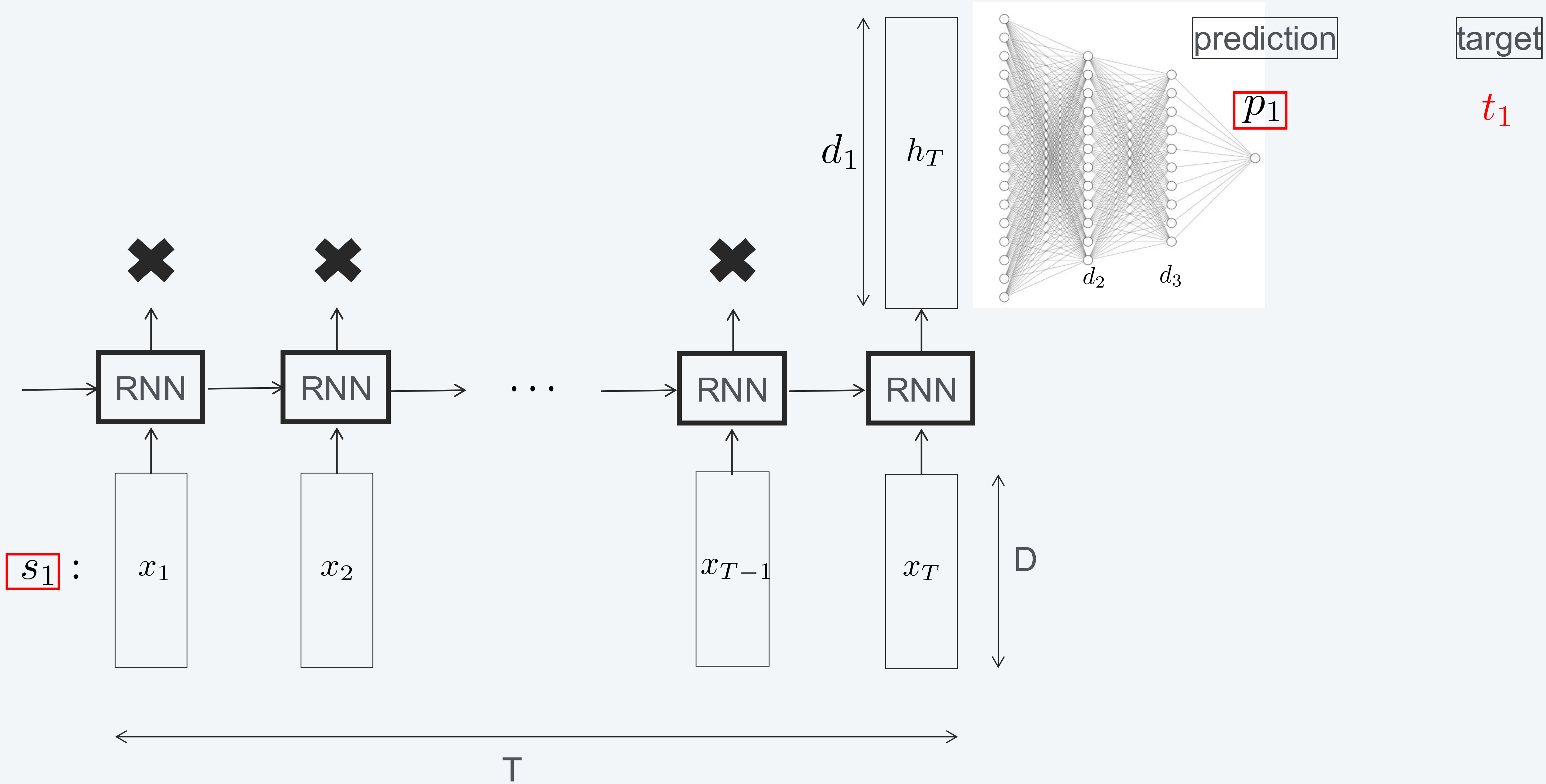
Input Data Tensor

$$\begin{array}{c|cccc}
 & & \xleftrightarrow{\qquad\quad T \quad\qquad} & \\
 & \boxed{s_1} : & x_1 & \cdots & x_T \\
 & \boxed{s_2} : & x_2 & \cdots & x_{T+1} \\
N & & \vdots & \vdots & \vdots \\
 & \boxed{s_N} : & x_{F-T} & \cdots & x_{F-1}
\end{array}$$

Target Tensor

$$\begin{array}{cc|c}
y_{T+1} & : \boxed{t_1} & \\
y_{T+2} & : \boxed{t_2} & \\
 & & N \\
y_F & : \boxed{t_N} &
\end{array}$$

A simple example – Part 4 – The Forward Propagation:



- The loss function associated with the N samples is the following binary cross entropy loss function:

$$\boxed{J = -\frac{1}{N}\sum_{i=1}^{N}(t_i \log(p_i) + (1 - t_i)\log(1 - p_i))}$$

A simple example – Part 4 – The Forward Propagation:

- Let's focus on the evolution of the tensor shape at each layer transformation for the stock movement prediction problem:

Input Data | | | | Predictions

$$(N,T,D) \xrightarrow[\textbf{RNN layer}]{} (N,d_1) \xrightarrow[\textbf{Dense layer}]{} (N,d_2) \xrightarrow[\textbf{Dense layer}]{} (N,d_3) \xrightarrow[\textbf{Dense layer}]{} (N,\,)$$

- In the previous example, the input data is a 3D tensor representing N sequences of length T. Each sequence is composed of D-dimensional continuous vectors.

- In the Sentiment Analysis problem, the input data comes as sequences of integers.

- In order to use RNN models for the Sentiment Analysis problem, we will need an intermediate layer called the **Embedding layer.**

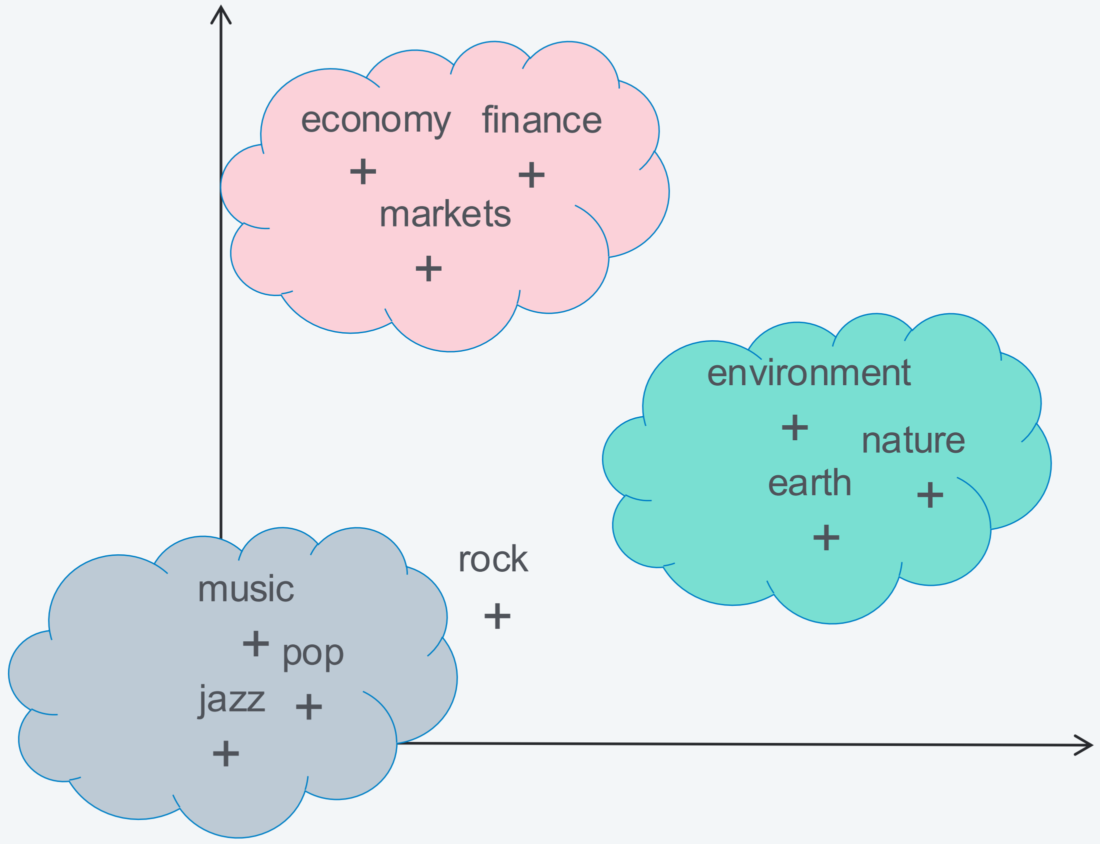
Interactive Session

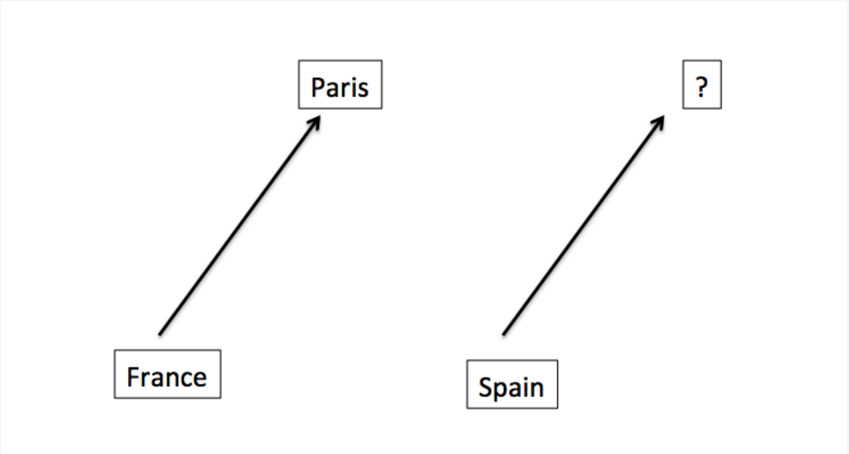
Part 2 : The Embedding Layer

The Embedding Space

- The embedding layer aims at mapping each word into a geometric space, called **the embedding space.**
- Each word is encoded into a D-dimensional vector (D is around 50 or 100).
- In the embedding space, words with similar meaning are encoded into similar word vectors.
- Since we use unsupervised learning to obtain these word vectors, they don't necessarily need to be meaningful to us. They only have to make sense geometrically.
- Commun example of meaningul geometric transformations are « gender ».

$$w_{\text{King}} - w_{\text{Man}} \approx w_{\text{Queen}} - w_{\text{Woman}}$$

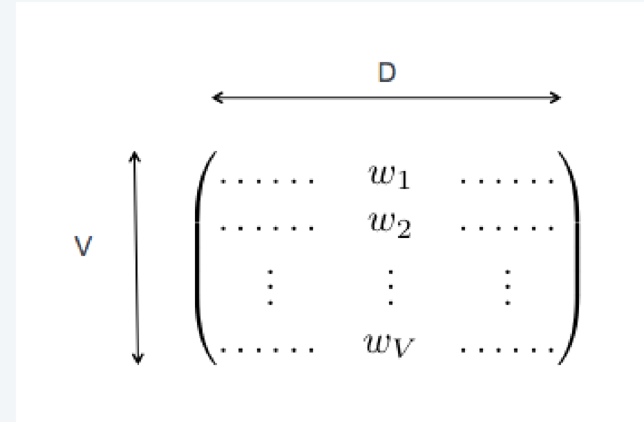




$$w_{\text{France}} - w_{\text{Paris}} \approx w_{\text{Spain}} - w_{\text{Madrid}}$$

Different Word Embeddings

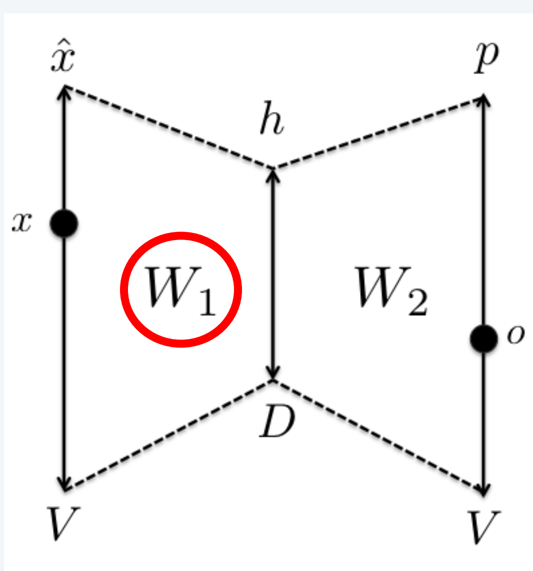
- We can store all the word vectors into a matrix called : **The Embedding Matrix.**



- The idea of a low dimensional embedding space for words, computed using unsupervised learning was initially explored by Bengio et al. in the early 2000s.

- It started to take off in the industry after the release of the **Word2vec** algorithm, explored by Tomas Mikolov at Google in 2013.

- In the previous lecture, we explored the **Word2vec** algorithm.



« *You shall know a word by the company it keeps* » (J.R. Firth 1957:11)

…dire consequences for the UK **economy**, even as markets were rocked…
…High pay for bosses hurting **economy** says senior Bank of England…
…Mervyn King believes the world **economy** will soon face another crash…

Context

- In addition to the Word2vec vectors, there are various pretrained word embeddings that can be downloaded and used, like **GloVe** and **FastText**.

The Embedding Layer

- The **Embedding Layer** takes as input the sequences of integers. But all the sequences should be of the same length T, so that we can pack them into the same tensor :
  - Sequences that are shorter than T are padded with zeros.
  - Sequences that are longer that T are truncated.

Row Data

- Document 1 : « There were no wolves in the movie. »
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⋮ ⋮

- Document N : « How many times must Willy be freed before he's freed?. »

Preprocessed Data

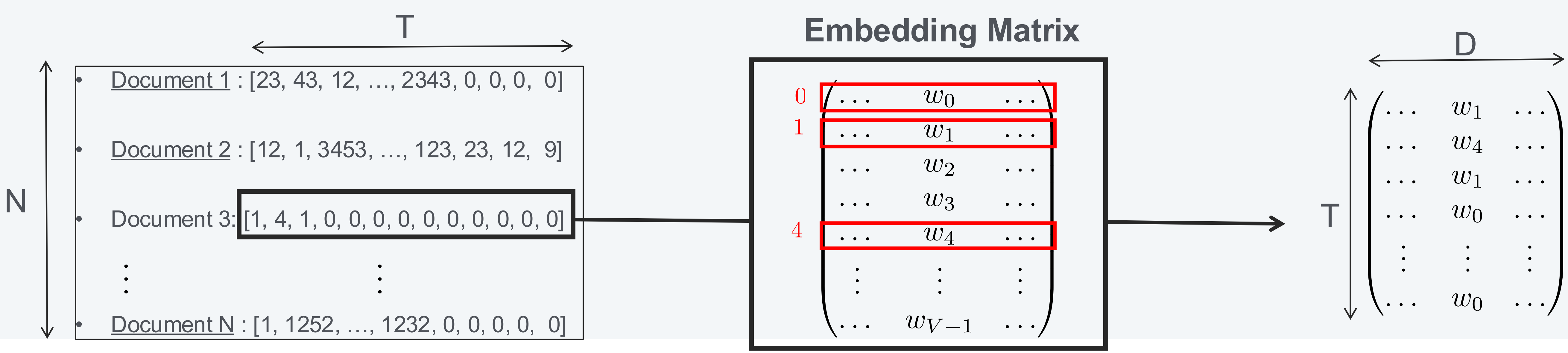
- Document 1 : [23, 43, 12, …, 2343, 0, 0, 0, 0]
- Document 2 : [12, 1, 3453, …, 123, 23, 12, 9]

⋮ ⋮

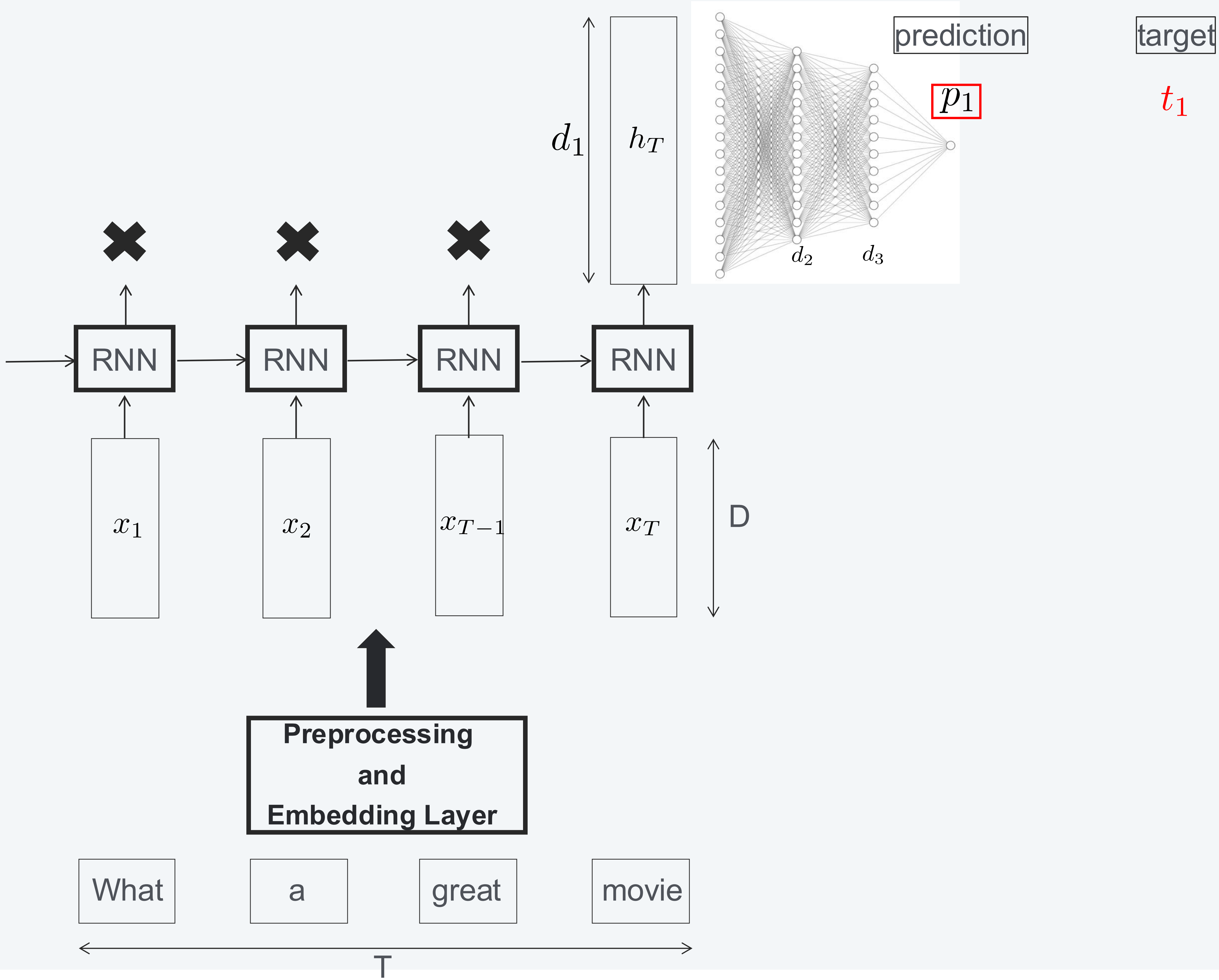
- Document N : [1, 1252, …, 1232, 0, 0, 0, 0, 0]

2D tensor of integers, of shape (N, T)

- The Embedding Layer transforms the 2D input tensor of shape (N, T) into a tensor of shape (N, T, D).



The Sentiment Analysis new Pipeline – Part 1 –



The Sentiment Analysis new Pipeline – Part 2 –

- Let’s keep track of the evolution of the tensor shape at each layer transformation:

Sequences of Integers

Input Data

Predictions

$$\underbrace{(N,T)}_{\text{Input Data}} \xrightarrow[\text{layer}]{\text{Embedding}} (N,T,D) \xrightarrow[\text{layer}]{\text{RNN}} (N,d_1) \xrightarrow[\text{layer}]{\text{Dense}} (N,d_2) \xrightarrow[\text{layer}]{\text{Dense}} (N,d_3) \xrightarrow[\text{layer}]{\text{Dense}} (N,)$$

The Forward Propagation

- The next sections will detail the RNN transformation
  - We will first start with a simple RNN model.
  - But the simple RNN model suffers from the vanishing gradient problem
  - We will then explain how to solve the vanishing gradient problem by using a better transformation called the Long Short Term Memory model.

Part 3 : Recurrent Neural Networks

A Simple RNN layer – Part 1–

- **The input:** A sequence of length T, composed of D-dimensional vectors.

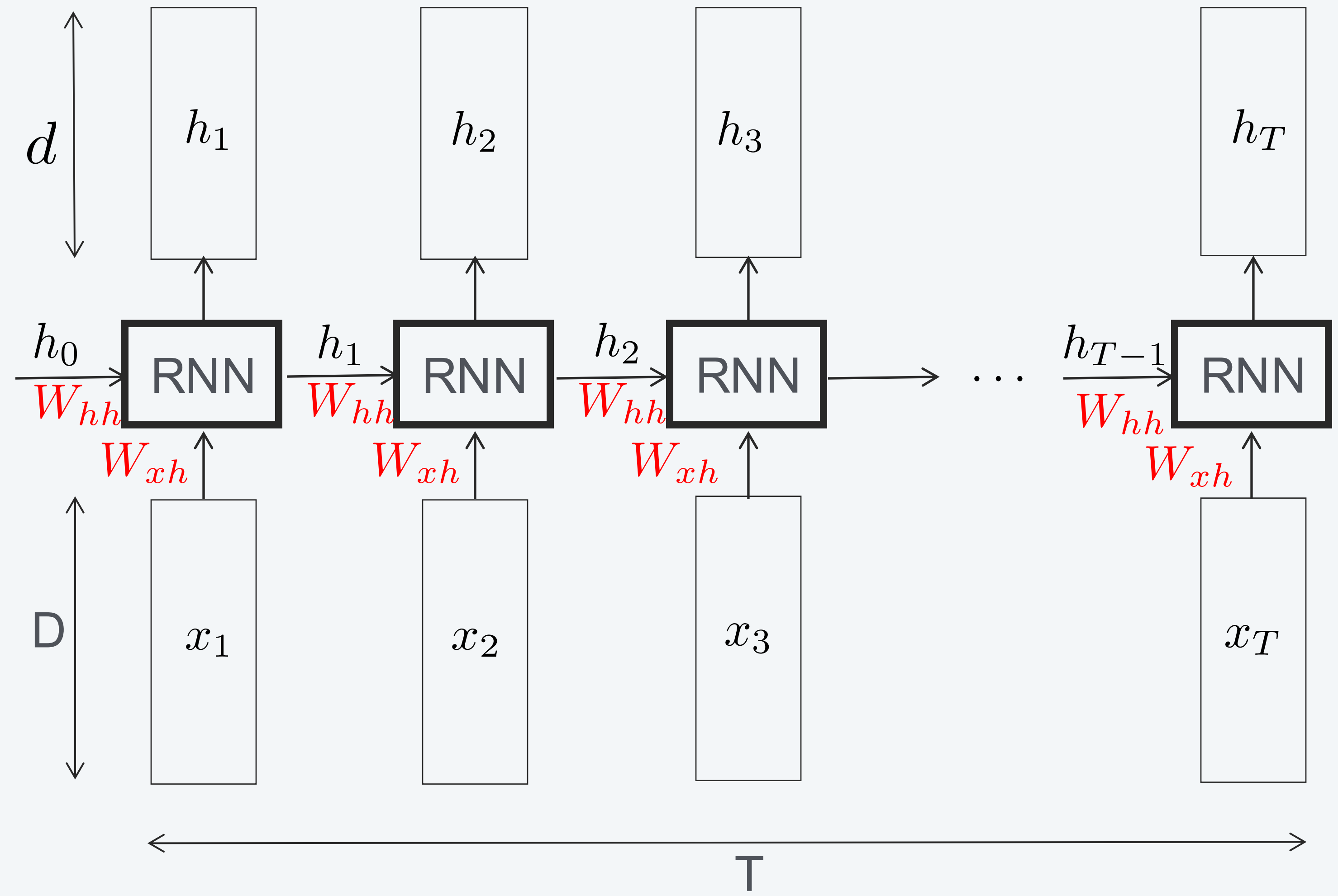
$$x_1, \dots, x_T$$

- **The output:** A sequence of length T, composed of d-dimensional vectors.

$$h_1, \dots, h_T$$

- **The weights:**

$$W_{xh} \in \mathbb{R}^{D\times d}\ ,\ W_{hh} \in \mathbb{R}^{d\times d} \quad \text{and} \quad b_h \in \mathbb{R}^d$$



- **The Transformation:**

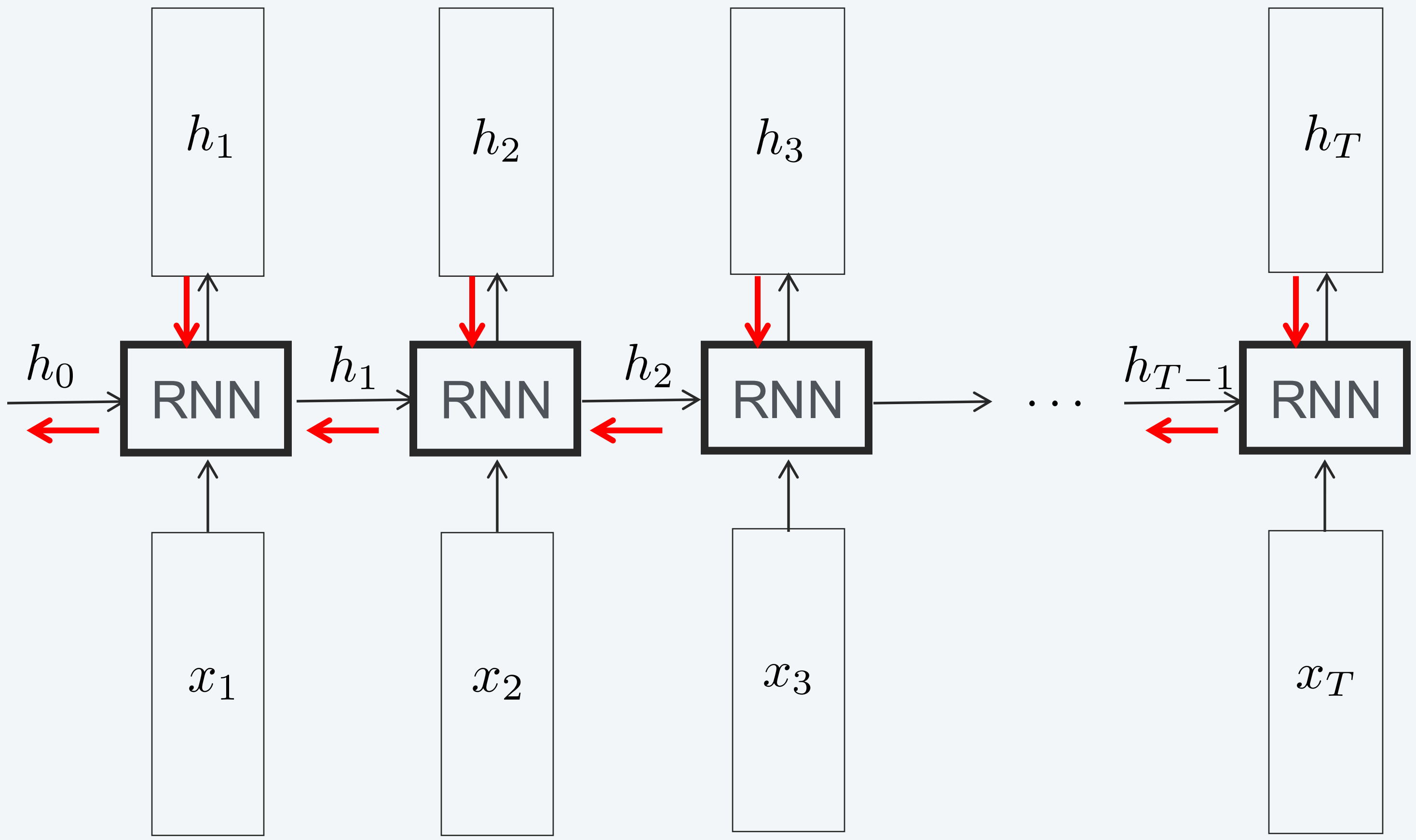
$$\left.\begin{aligned} h_1 &= \tanh(W_{hh}^T h_0 + W_{xh}^T x_1 + b_h) \\ h_2 &= \tanh(W_{hh}^T h_1 + W_{xh}^T x_2 + b_h) \\ &\vdots \\ h_T &= \tanh(W_{hh}^T h_{T-1} + W_{xh}^T x_T + b_h) \end{aligned}\right\}$$

$$\boxed{\begin{gathered} \forall t \in \{1, \dots, T\} \\ h_t = \tanh(W_{hh}^T h_{t-1} + W_{xh}^T x_t + b_h) \end{gathered}}$$

Interactive Session

A Simple RNN layer – Part 2–

- As usual, we use the Gradient Descent Algorithm to update the weigths.

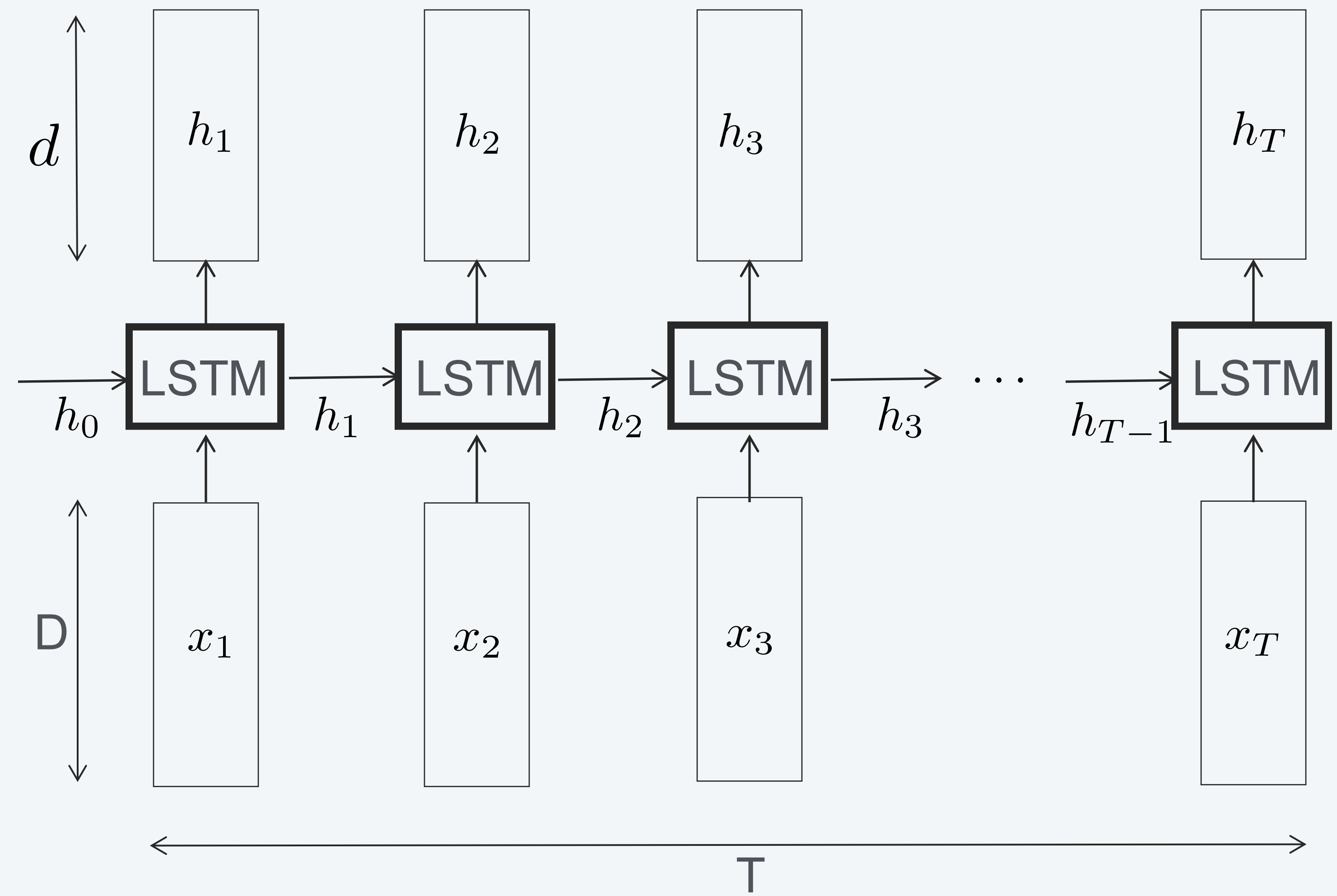


- Unfortunately, simple RNNs aren't capable of learning « long term dependencies » and it's hard to make h_T influenced by x_1, x_2, x_3 due to the vanishing gradient problem, as explained in:
  - [Hochreiter 1991]: *Untersuchungen zu dynamischen neuronalen Netzen*
  - [Bengio, et al. 1994]: *Learning Long-Term Dependencies with Gradient Descent is Difficult*
  - [Bengio et Mikolov 2013]: *On the difficulty of training recurrent neural networks*

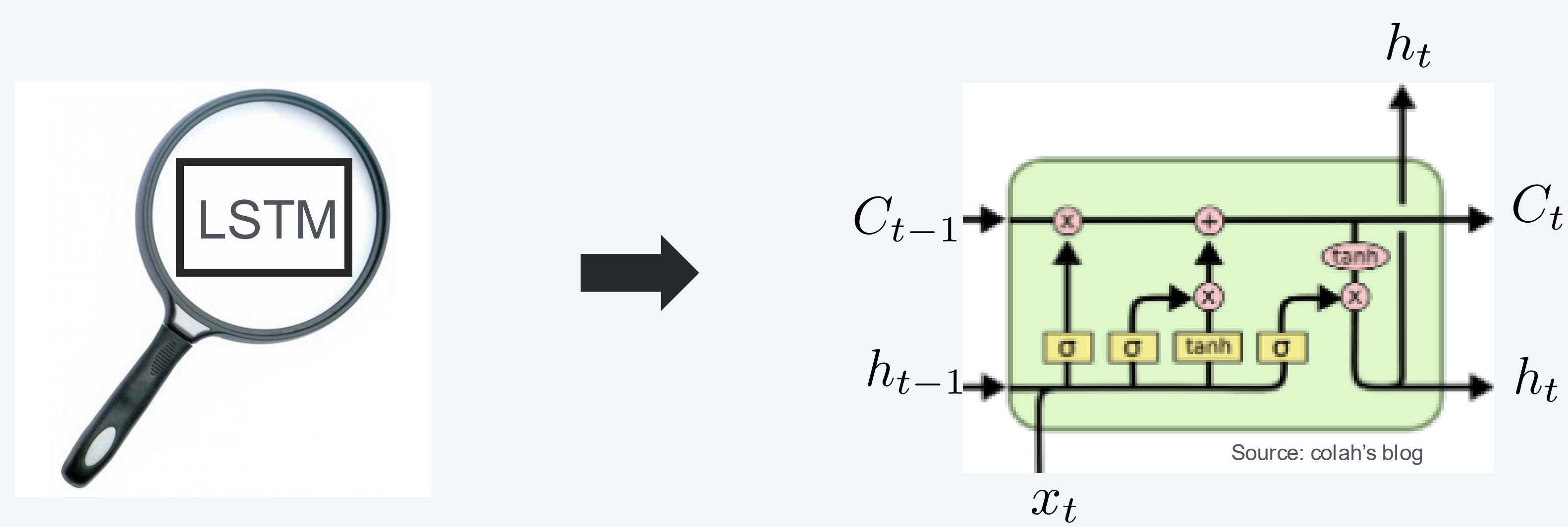
LSTM networks – Part 1 –

- Long Short Term Memory networks (LSTMs) are a special type of RNN explicitly designed to avoid long term dependency problem

- Like the standard RNNs, LSTMs also have the form of a chain of repeating modules of neural networks.



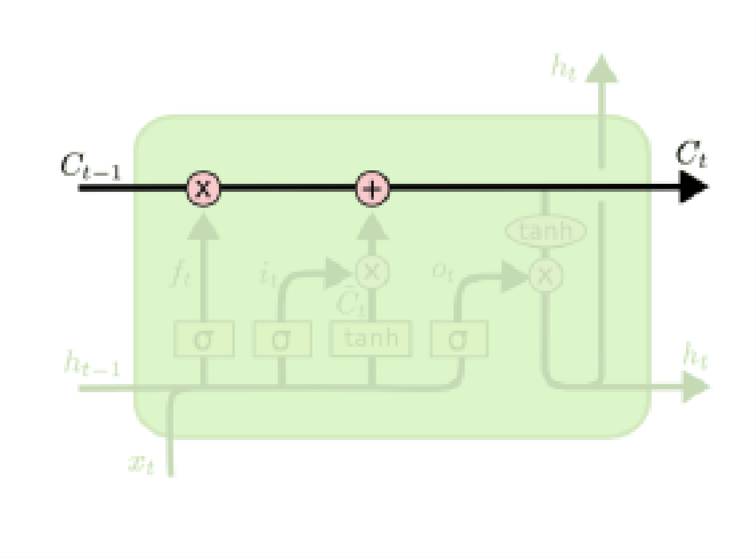
- Unlike the standard RNNs, the repeating module has four single neural network layers, interacting in a special way.



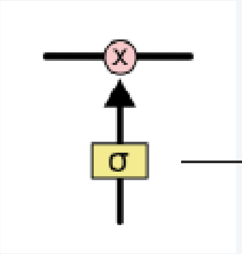
LSTM networks – Part 2 –

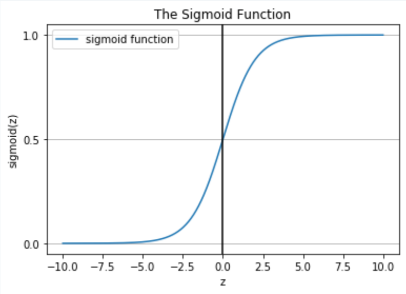
- There are two main concepts in LSTMs:

1. **The cell state** :
    - It's represented by the sequence C_t for $t \in \{1, \ldots, T\}$
    - The **cell state** represents the memory.
    - At each step $t-1 \longrightarrow t$, the LSTM will remove some information from the cell state and add some other information using the concept of **gates**.
    - The LSTM has 3 gates to protect the cell state.



2. **The gates:**
    - The gate is a way to controle the amount of information we want to keep or change.
    - It's composed of a sigmoid neural network and an element wise multiplication.
    - For each dimension, the sigmoid layer outputs a value between zero and 1, describing how much we want to let through: « close to zero » means « let nothing through !» and « close to one » means « let everything through! ».

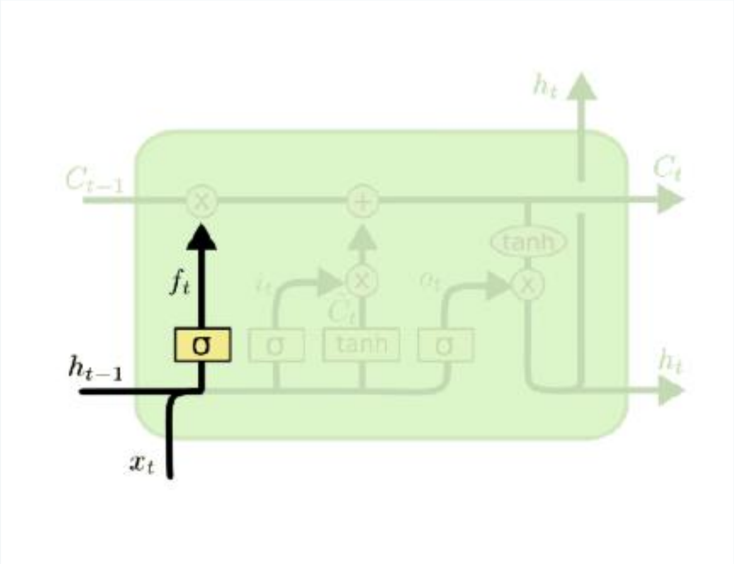




LSTM networks – Part 3 –

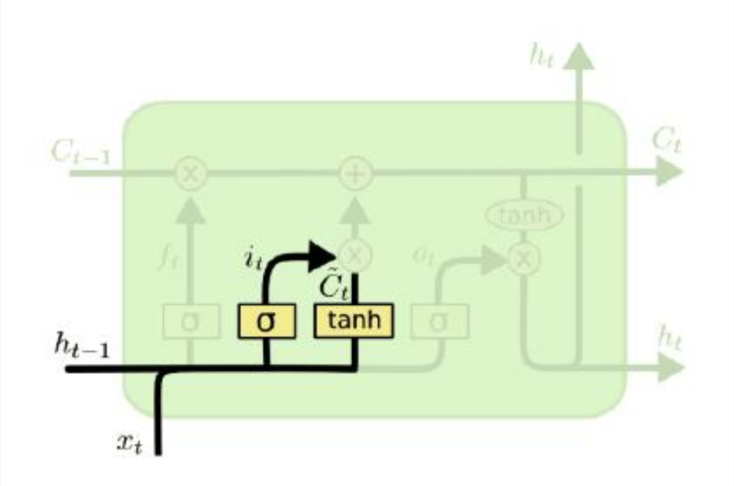
- To transition from t-1 to t using LSTMs, there are 4 steps:

- **STEP 1: The forget gate layer:**
  - This step concerns the information we want to throw away from the cell state.
  - For that, we use the forget gate layer: A sigmoid applied to the concatenation of h_{t-1} and x_t outputs the forget vector f_t
  - The parameters are: (W_f, b_f)



$$f_t = \sigma(W_f^T[h_{t-1}, x_t] + b_f)$$

- **STEP 2: The input gate layer:**
  - Conversly, this step concerns the information we want to store in the cell state.
  - For that, we use the input gate layer: Again, a sigmoid applied to the concatenation of h_{t-1} and x_t outputs the input vector i_t
  - Then, we create a new candidate for the cell state $\tilde{C}_t$ thanks to a $\tanh$ layer.
  - The parameters are: (W_i, b_i) and (W_C, b_C)

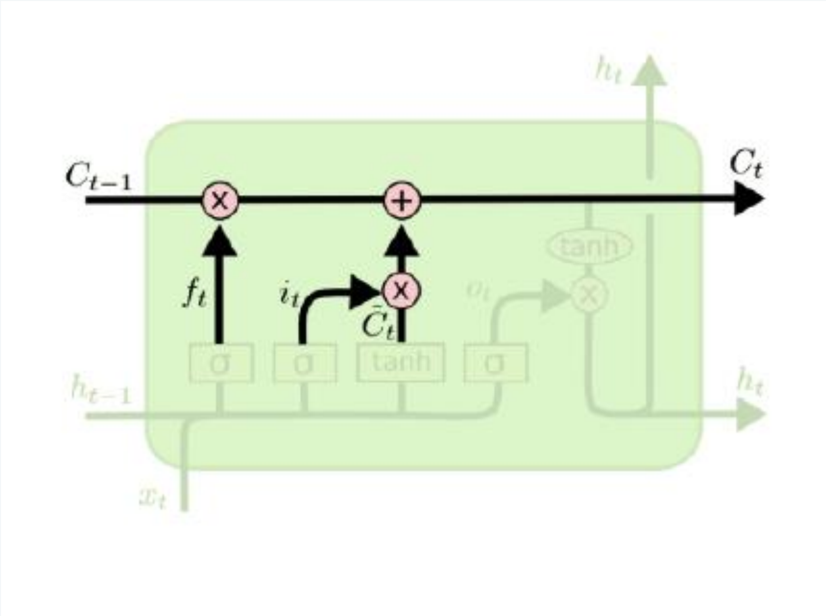


$$i_t = \sigma(W_i^T[h_{t-1}, x_t] + b_i)$$

$$\tilde{C}_t = \tanh(W_C^T[h_{t-1}, x_t] + b_C)$$

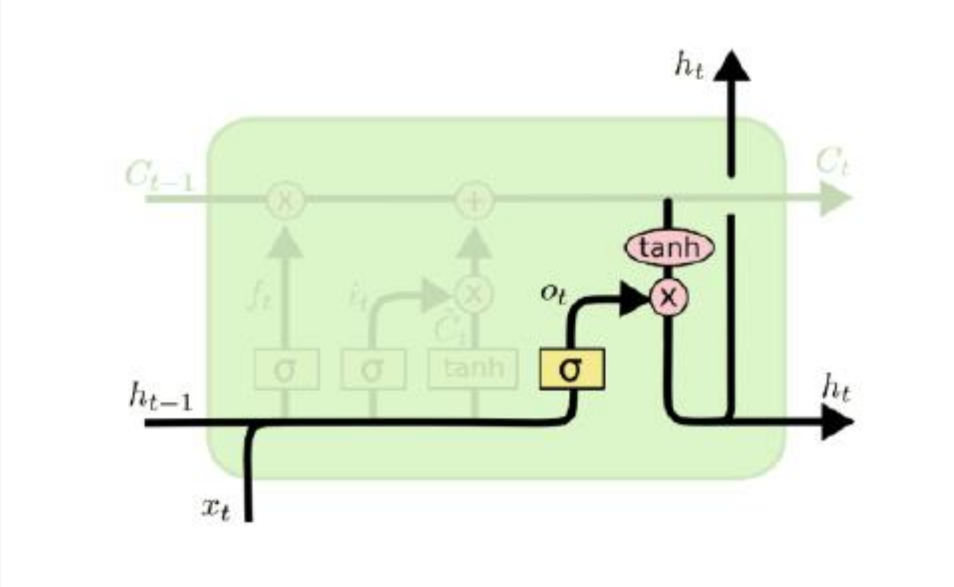
LSTM networks – Part 4 –

- **STEP 3: Updating the cell state:**
  - We know from the previous steps that by multiplying (element wise)the forget vector (f_t) by the old cell state (C_{t-1}), we obtain the first part of the uptated cell state: $f_t * C_{t-1}$ corresponding to what we want to forget.
  - We also know that $i_t * \tilde{C}_t$ represents the new candidate vector, scaled by how much we want to uptade each dimension of the cell state.



$$C_t = f_t * C_{t-1} + i_t * C_t$$

- **STEP 4: The output gate layer:**
  - We want to output a filtered version of the updated cell state.
  - For that, we use the output gate layer to decide what dimensions of the cell state we want to output: A simple sigmoid function applied to the concatenation of h_{t-1} and x_t gives the output vector o_t
  - The output at time t is just the element wise multiplication of the output vector and the updated cell state (after a $\tanh$ transformation).
  - The parameters are: (W_o, b_o)

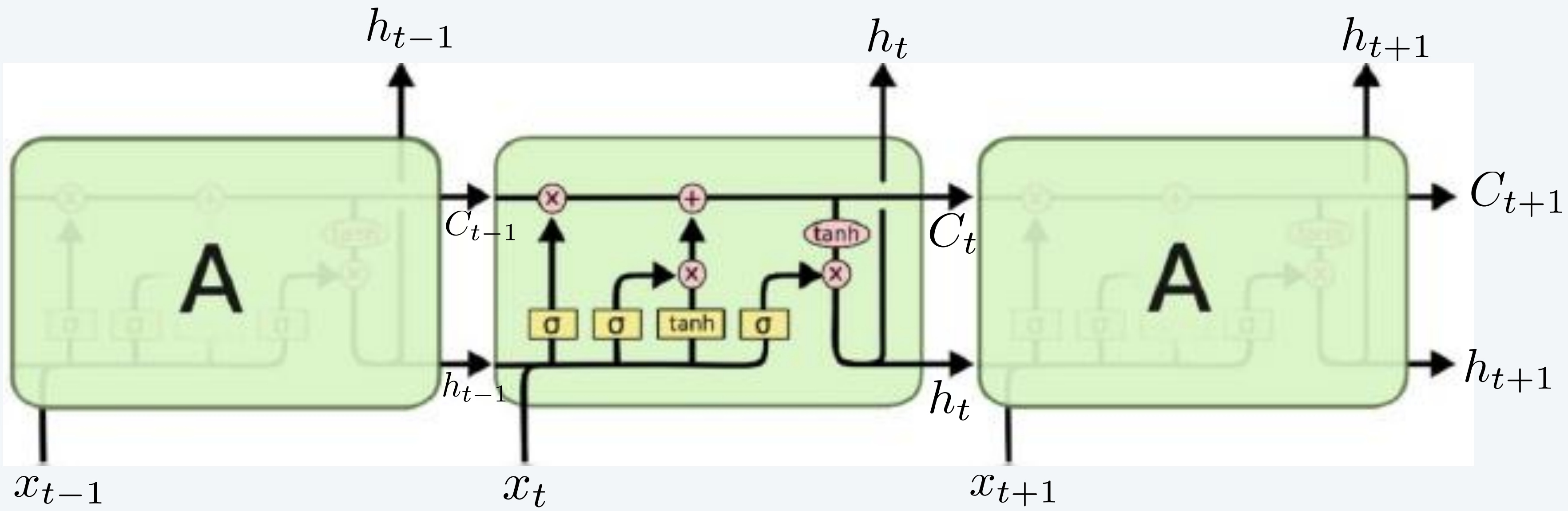


$$o_t = \sigma(W_o^T[h_{t-1}, x_t] + b_o)$$

$$h_t = o_t * \tanh(C_t)$$

LSTM networks – Part 5 –

- **Summary:**



Equations of the LSTM

The gates:

$$f_t = \sigma(W_f^T[h_{t-1}, x_t] + b_f)$$

$$i_t = \sigma(W_i^T[h_{t-1}, x_t] + b_i)$$

$$o_t = \sigma(W_o^T[h_{t-1}, x_t] + b_o)$$

The updates:

$$\tilde{C}_t = \tanh(W_C^T[h_{t-1}, x_t] + b_C)$$

$$C_t = f_t * C_{t-1} + i_t * \tilde{C}_t$$

$$h_t = o_t * \tanh(C_t)$$

Part 4 : Programming Session

Programming Session