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3.7

8) Tre myntkast. X = # kron siste kast, Y = # kron totalt. Finn $p_{XY}(x,y)$

Y \ X	0	1
0	1/8	0
1	1/4	1/8
2	1/8	1/4
3	0	1/8

$p_{XY}(0,0) = P(\{(0,0,0)\}) = 1/8$
$p_{XY}(0,1) = P(\{(1,0,0),(0,1,0)\}) = 1/4$
$p_{XY}(0,2) = P(\{(1,1,0)\}) = 1/8$
$p_{XY}(0,3) = P(\{\emptyset\}) = 0$
$p_{XY}(1,0) = P(\{\emptyset\}) = 0$
$p_{XY}(1,1) = P(\{(0,0,1)\}) = 1/8$
$p_{XY}(1,2) = P(\{(1,0,1),(0,1,1)\}) = 1/4$
$p_{XY}(1,3) = P(\{(1,1,1)\}) = 1/8$

22) $f_{X,Y}(x,y) = 2e^{-x}e^{-y}$ for $y \geq x \geq 0$. Finn $f_Y(y)$

$$f_Y(y) = \int_0^\infty 2e^{-x}e^{-y}\,dx = \left[-2e^{-x}e^{-y}\right]_{x=0}^{\infty} = \underline{\underline{2e^{-y}}}$$

30) $F_{XY}(x,y) = (1-e^{-\lambda y})(1-e^{-\lambda x})$, $x>0$, $y>0$. Finn $f_{X,Y}(x,y)$

$$f_{X,Y}(x,y) = \frac{\partial^2}{\partial x\,\partial y} F_{XY}(x,y)$$
$$= \frac{\partial^2}{\partial x\,\partial y}(1-e^{-\lambda y})(1-e^{-\lambda x})$$
$$= \frac{\partial}{\partial x}\lambda e^{-\lambda y}(1-e^{-\lambda x}) = \underline{\lambda^2 e^{-\lambda y}e^{-\lambda x}} = \underline{\underline{\lambda^2 e^{-\lambda(x+y)}}}$$

42) $f_{XY}(x,y) = \frac{2}{3}(x+2y)$, $x \in [0,1]$, $y \in [0,1]$ – X, Y uavh?

X, Y avh $\Leftrightarrow$ $f_{XY}(x,y) = g(x)h(y)$ for polynomer g og h
Ser at $f_{XY}(x,y) = \frac{2}{3}x + \frac{4}{3}y$ ikke kan skrives som produkt av noen polynomer $g(x)$ og $h(y)$, ergo

$\Rightarrow$ $\underline{\underline{X, Y \text{ ikke uavhengige variabler}}}$

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3.1.1

5) X, Y diskret, $p_{XY}(x,y) = k(x+y)$, $x = 1,2,3$ $y = 1,2,3$

a) Finn k:

$$\sum_{x=1}^{3}\sum_{y=1}^{3} k(x+y) = k \sum_{x=1}^{3}\sum_{y=1}^{3} x+y = k(2+3+4+3+4+5+4+5+6) = 36k$$

Summen av $p_{XY}(x,y)$ $\forall x,y = 1$ $\Rightarrow$ $\underline{\underline{k = \frac{1}{36}}}$

b) Finn $p_{Y|X}(1)$ for $x = 1,2,3$

$$p_X(x) = \sum_{y=1}^{3} \frac{1}{36}(x+y) = \frac{1}{36}(x+1+x+2+x+3) = \frac{1}{12}x + \frac{1}{6}$$

$$p_{Y|X}(1) = \frac{p_{XY}(x,1)}{p_X(x)} = \frac{\frac{1}{36}(x+1)}{\frac{1}{12}(x+2)} = \underline{\underline{\frac{x+1}{3x+6}, \quad x = 1,2,3}}$$

x			$p_{Y\vert X}(1)$
1	2/9	=	2/9
2	3/12	=	1/4
3	4/15	=	4/15

6) La X = tall på kule fra urne, $X = 1,2,3$

Y = #kron på X terningkast

a) Finn $p_{XY}(x,y)$

Muligheter (x,y): $(1,0), (1,1)$
$(2,0), (2,1), (2,2)$
$(3,0), (3,1), (3,2), (3,3)$

$$p_{Y|X}(y) = P(Y=y \mid X=x) = \binom{x}{y}\left(\frac{1}{2}\right)^y\left(\frac{1}{2}\right)^{x-y} = \binom{x}{y}\left(\frac{1}{2}\right)^x$$

$$p_{XY}(x,y) = p_X(x)\cdot p_{Y|X}(y) = \underline{\underline{\frac{1}{3}\binom{x}{y}\left(\frac{1}{2}\right)^x}}, \quad x = 1,2,3, \quad y \in \mathbb{N} \le x$$

b) Finn $p_Y(y) = \sum_{x=1}^{3} \frac{1}{3}\binom{x}{y}\left(\frac{1}{2}\right)^x = \frac{1}{3}\left(\binom{1}{y}\left(\frac{1}{2}\right) + \binom{2}{y}\left(\frac{1}{2}\right)^2 + \binom{3}{y}\left(\frac{1}{2}\right)^3\right)$

$$= \frac{1}{3}\left(\frac{1!}{2y!(1-y)!} + \frac{2!}{4y!(2-y)!} + \frac{3!}{8y!(3-y)!}\right) = \frac{1}{3}\left(\frac{1}{2y!} + \frac{1}{2y!(2-y)!} + \frac{3}{4y!(3-y)!}\right)$$

$$= \underline{\underline{\frac{2(2-y)(3-y) + 2(3-y) + 3}{12(3-y)!\,y!}}}$$

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3.11

15) $f_{Y|X}(y) = \frac{2y+4x}{1+4x}$, $f_X(x) = \frac{1}{3}(1+4x)$, $0<x<1$, $0<y<1$. Finn $f_Y(y)$

$$f_{XY}(x,y) = f_{Y|X}(y) \cdot f_X(x) = \frac{1}{3}(2y+4x)$$

$$f_Y(y) = \int_{-\infty}^{\infty} f_{XY}(x,y)\,dx = \int_0^1 \frac{2}{3}y + \frac{4}{3}x\,dx = \left[\frac{2}{3}xy + \frac{2}{3}x^2\right]_0^1$$

$$= \frac{2}{3}y + \frac{2}{3} = \underline{\underline{\frac{2}{3}(y+1),\quad 0<y<1}}$$

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2)b) A = 2 fisk på 0,5t   B = 2 fisk på 2.5t – uavh. grunnet ingen overlapp i tidsrom

$$P(A) = P(X=2 \mid t=0{,}5) = \frac{(2\cdot 0{,}5)^2 e^{-2\cdot 0{,}5}}{2!} = \frac{1}{2e}$$

$$P(B) = P(X=2 \mid t=1.5) = \frac{3^2 e^{-3}}{2!} = \frac{9}{2e^3}$$

Uavh $\Rightarrow P(A\cap B) = P(A)P(B) = \frac{1}{2e}\cdot\frac{9}{2e^3} = \frac{9}{4e^4} \approx \underline{\underline{0{,}041}}$

La Y = 4 fisk på 2 timer

$$P(2 \text{ av } 4 \text{ fisk på } 30\text{ min}) = P(A\mid X) = \frac{P(A\cap Y)}{P(Y)}$$

Ser at $P(A\cap Y) = P(2 \text{ fisk på } 30 \text{ min}, 2 \text{ fisk de neste } 90 \text{ min})$

$$= P(A\cap B) = 0{,}041$$

$$P(Y) = P(X=4, t=2) = \frac{4^4 e^{-4}}{4!} = \frac{32}{3e^4}$$

$$P(A\mid X) = \frac{9}{4e^4}\cdot\frac{3e^4}{32} = \underline{\underline{0{,}211}}$$

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2) c) Uavhengig av fordeling vil $E(T_i) = 2$.

T_i er tid til hendelse 4: Poisson fordeling, og er dermed

<u>gammafordelt</u>

$$E(T_i) = \frac{4}{\lambda} = 2 \qquad Var(T_i) = \frac{4}{\lambda^2} = \frac{4}{4} = \underline{1}$$

La V = summen av timer brukt $= \sum_{i=1}^{21} T_i$

$$P(V \geq 50) = 1 - P\left(\frac{V - 21 \cdot 2}{\sqrt{21}} \leq \frac{50 - 21 \cdot 2}{\sqrt{21}}\right)$$

[La $Z \sim N(21 \cdot 2, 21)$ - sentralgrenseteoremet:]

$$= 1 - P\left(Z \leq \frac{50 - 42}{\sqrt{21}} = \frac{8}{\sqrt{21}} = 1{,}75\right) = 1 - 0{,}95994 = \underline{\underline{0{,}040}}$$