ØVING 11 side 1 Andreas B. Berg

3.3

6) Bruk de Moivres formel, $\cos 2\theta$ og $\sin 2\theta$ ved $\cos\theta$, $\sin\theta$

$$\cos 2\theta + i\sin 2\theta = (\cos\theta + i\sin\theta)^2$$
$$= \cos^2\theta + 2i\cos\theta\sin\theta - \sin^2\theta$$

$$\Rightarrow \underline{\underline{\cos 2\theta = \cos^2\theta - \sin^2\theta}}$$

$$\Rightarrow \underline{\underline{\sin 2\theta = 2\sin\theta\cos\theta}}$$

10.5

1) Finn de generelle løsningene:

a) $y'' + y' - 6y = 0$

Ser på $r^2 + r - 6 = (r+3)(r-2) = 0$

$\Rightarrow r = -3, \quad r = 2$

$$\underline{\underline{y = Ce^{-3x} + De^{2x}}}, \quad C, D \in \mathbb{R}$$

f) $y'' - y' + \frac{y}{4} = 0$

$r^2 - r + \frac{1}{4} = (r - \frac{1}{2})^2 = 0 \Rightarrow r = \frac{1}{2}$

$$\underline{\underline{y = Ce^{\frac{1}{2}x} + Dx e^{\frac{1}{2}x}}}, \quad C, D \in \mathbb{R}$$

2) Finn de generelle løsningene:

c) $y'' + 16y = 0$

$r^2 + 16 = (r + 4i)(r - 4i) = 0 \Rightarrow r = \pm 4i$

$$\underline{\underline{y = C\cos(4x) + D\sin(4x)}}, \quad C, D \in \mathbb{R}$$

d) $y'' - 8y' + 20y = 0$

$r^2 - 8r + 20: \quad r = \frac{8 \pm \sqrt{64 - 80}}{2} = \frac{8 \pm \sqrt{-16}}{2} = 4 \pm 2i$

$$\underline{\underline{y = e^{4x}\left(C\cos(2x) + D\sin(2x)\right)}}, \quad C, D \in \mathbb{R}$$

ØVING 11 side 2 Andreas B. Berg

10.6

1) a) Finn gen. løsning $y'' - y' - 2y = 0$

$$r^2 - r - 2 = (r-2)(r+1) = 0 \Rightarrow r_1 = 2,\ r_2 = -1$$

$$\underline{\underline{y = Ce^{2x} + De^{-x}}} \quad , \quad C, D \in \mathbb{R}$$

b) Finn en part. løsning $y'' - y' - 2y = e^x = e^{1x} \cdot 1$

1 er ikke rot til kar.likn, så prøver $y_p = e^x \cdot a$

$$ae^x - ae^x - 2ae^x = -2ae^x = e^x$$

$$\Rightarrow a = -\frac{1}{2}$$

$$\Rightarrow \underline{\underline{y_p = -\frac{1}{2}e^x}}$$

c) Finn løsn. så $y(0) = y'(0) = 2$

$$y = -\frac{1}{2}e^x + Ce^{2x} + De^{-x}$$

$$y(0) = -\frac{1}{2} + C + D = 2 \Rightarrow C + D = \frac{5}{2}$$

$$y'(0) = -\frac{1}{2} + 2C - D = 2 \Rightarrow 2C - D = \frac{5}{2}$$

$$C + D = 2C - D \Rightarrow 0 = C - 2D \Rightarrow C = 2D$$

$$2D + D = \frac{5}{2} \Rightarrow 3D = \frac{5}{2} \Rightarrow D = \frac{5}{6} \Rightarrow C = \frac{5}{3}$$

$$\underline{\underline{y = -\frac{1}{2}e^x + \frac{5}{3}e^{2x} + \frac{5}{6}e^{-x}}}$$

Andreas B. Berg

10.6

11) a) Finn gen. løsn. $y'' + 2y' + 2y = 0$

$r^2 + 2r + 2 \quad : \quad r = \frac{-2 \pm \sqrt{4-8}}{2} = \frac{-2 \pm \sqrt{-4}}{2} = -1 \pm i$

$$\underline{y = e^{-x}\left(C\cos(x) + D\sin(x)\right)}$$

b) Finn løsn. $y'' + 2y' + 2y = 1 + x + 2e^{2x} \quad y(0) = 0, \; y'(0) = 1$

La $y_1 = e^{-x}\sin x, \quad y_2 = e^{-x}\cos x$

$$y(x) = c(x)\,y_1(x) + d(x)\,y_2(x)$$

$$c(x) = -\int \frac{y_2(x)\,f(x)}{w(y_1, y_2)}\,dx$$

$$w(y_1, y_2) = y_1 y_2' - y_2 y_1' = e^{-x}\sin x\left(-e^{-x}\cos x - e^{-x}\sin x\right)$$
$$- e^{-x}\cos x\left(-e^{-x}\sin x + e^{-x}\cos x\right)$$
$$= -e^{-2x}\sin x\cos x - e^{-2x}\sin^2 x + e^{-2x}\sin x\cos x - e^{-2x}\cos^2 x$$
$$= -e^{-2x}\left(\sin^2 x + \cos^2 x\right) = -e^{-2x}$$

$$c(x) = -\int \frac{e^{-x}\cos x\left(1 + x + 2e^{2x}\right)}{-e^{-2x}}\,dx$$
$$= \int e^{x}\cos x + xe^{x}\cos x + 2e^{2x}e^{x}\cos x\,dx$$
$$= \int e^{x}\cos x\,dx + \int xe^{x}\cos x\,dx + \int 2e^{3x}\cos x\,dx$$
$$= \frac{1}{2}e^{x}(\sin x + \cos x) + \frac{1}{2}xe^{x}(\sin x + \cos x) - \frac{1}{2}e^{x}\sin x +$$
$$\frac{1}{5}e^{3x}(\sin x + 3\cos x) + C$$
$$= \frac{1}{2}e^{x}\cos x + \frac{1}{2}xe^{x}(\sin x + \cos x) + \frac{1}{5}e^{3x}(\sin x + 3\cos x) + C$$

$$d(x) = \int \frac{e^{-x}\sin x\left(1 + x + 2e^{2x}\right)}{-e^{-2x}}\,dx = -\int e^{x}\sin x + xe^{x}\sin x + 2e^{3x}\sin x$$
$$= \frac{1}{2}e^{x}(\cos x - \sin x) + \frac{1}{2}e^{x}x(\cos x - \sin x) - \frac{1}{2}e^{x}\cos x$$
$$+ \frac{1}{5}e^{3x}(\cos x - 3\sin x) + D$$
$$= \frac{1}{2}e^{x}(-\sin x) + \frac{1}{2}e^{x}x(\cos x - \sin x) + \frac{1}{5}e^{3x}(\cos x - 3\sin x) + D$$

10.6 ØVING 11 side 4 Andreas B. Berg

11) b) $y(0) = c(0)\, y_1(0) + d(0)\, y_2(0)$

$= 0 = \frac{1}{5} + D \Rightarrow D = -\frac{1}{5}$

JEG SER AT DETTE BARE BLIR SURR.

Hvordan skal jeg finne y her?

Jeg prøvde variasjon av parametre, men ser ikke ut som jeg er på vei riktig retning...

10.8

2) Bruk Eul. met. og mpmet. finn $y(1)$, $h = \frac{1}{4}$

a) $y' = \sin y + x$, $y(0) = 2$

$x_n = x_0 + nh = 0 + n/4$

$y_n = y_{n-1} + f(x_{n-1}, y_{n-1})h = y_{n-1} + \sin(y_{n-1}) + x_{n-1}$

$x_n = \frac{n}{4}$: $x = 1 \Rightarrow n = 4$

$y_0 = 2$

$y_1 = 2 + \sin(2) + x_1 = 2 + \sin(2) + \frac{1}{4} \approx 3,1593$

$y_2 = 3.1593 + \sin(3.1593) + \frac{2}{4} \approx 3,6416$

$y_3 = 3,6416 + \sin(3.6416) + \frac{3}{4} \approx 3,9122$

$y_4 = 3,9122 + \sin(3,9122) + 1 \approx \underline{\underline{4,2156}}$

b) $y_n = y_{n-1} + f\left(x_{n-1} + \frac{h}{2},\ y_{n-1} + f(x_{n-1}, y_{n-1})\frac{h}{2}\right) \cdot h$

$= y_{n-1} + f\left(x_{n-1} + \frac{1}{8},\ y_{n-1} + \frac{1}{8}(\sin(y_{n-1}) + x_{n-1})\right)$

$= y_{n-1} + \sin\left(y_{n-1} + \frac{1}{8}(\sin(y_{n-1}) + x_{n-1})\right) + x_{n-1} + \frac{1}{8}$

$y_1 \approx 2,9812$ $y_2 \approx 3.4652$

$y_3 \approx 3,5303$

$y(1) = y_4 \approx \underline{\underline{3,5413}}$