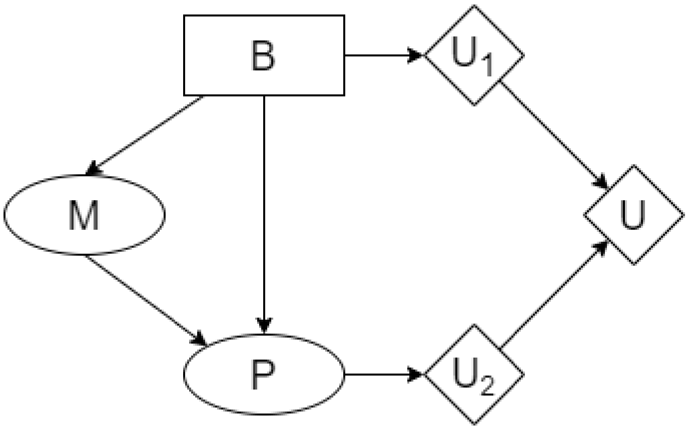
# TDT4171 - Exercise 3

Andreas B. Berg

## 1 – Decision Network

### *a) Draw the decision network for this problem.*




### *b) Compute the expected utility of the two decision alternatives B = true (buying the book) and B = false (not buying it) (show the calculations). What should Geir do?*

| **B =** | **Expected utility** |
|---|---|
| TRUE | 1635 |
| FALSE | 1102.5 |

**Geir should buy the book, as that gives the highest expected utility.**

We know that expected utility is given by

$$EU(a \mid e) = \sum_{s'} P(RESULT(a) = s' \mid a, e) * U(s')$$

We thus need to calculate the probability of the different results given $B$, and their utility. Let lower-case letters indicate a true variable. As the utility rely on boolean nodes B and P, we have the following four possible results:

| | $\boldsymbol{U_1}$ | $\boldsymbol{+U_2}$ | $\boldsymbol{= U}$ |
|---|---|---|---|
| $b, p$ | $-150$ | 2100 | 1950 |
| $b, \neg p$ | $-150$ | 0 | $-150$ |
| $\neg b, p$ | 0 | 2100 | 2100 |
| $\neg b, \neg p$ | 0 | 0 | 0 |

We then need to calculate their related probabilities. We know the following probabilities:

$$P(m \mid b) = 0.9$$

$$P(m \mid \neg b) = 0.65$$

$$P(p \mid m, b) = 0.9$$

$$P(p \mid \neg m, b) = 0.4$$

$$P(p \mid m, \neg b) = 0.7$$

$$P(p \mid \neg m, \neg b) = 0.2$$

We see that if he buys the book:

$$P(p \mid b) = \sum_{M} P(p \mid M, b) * P(M \mid b)$$

$$= P(p \mid m, b) * P(m \mid b) + P(p \mid \neg m, b) * P(\neg m \mid b)$$

$$= 0.9 * 0.9 + 0.4 * 0.1 = 0.85$$

If he does not buy the book:

$$P(p \mid \neg b) = \sum_{M} P(p \mid M, \neg b) * P(M \mid \neg b)$$

$$= P(p \mid m, \neg b) * P(m \mid \neg b) + P(p \mid \neg m, \neg b) * P(\neg m \mid \neg b)$$

$$= 0.7 * 0.65 + 0.2 * 0.35 = 0.525$$

We can see from this that

$$P(\neg p \mid b) = 1 - 0.85 = 0.15$$

$$P(\neg p \mid \neg b) = 1 - 0.525 = 0.475$$

We can then compute the expected utilities. If Geir buys the book:

$$EU(b) = P(p \mid b) * U(b, p) + P(\neg p \mid b) * U(b, \neg p)$$

$$= 0.85 * 1950 + 0.15 * (-150) = 1635$$

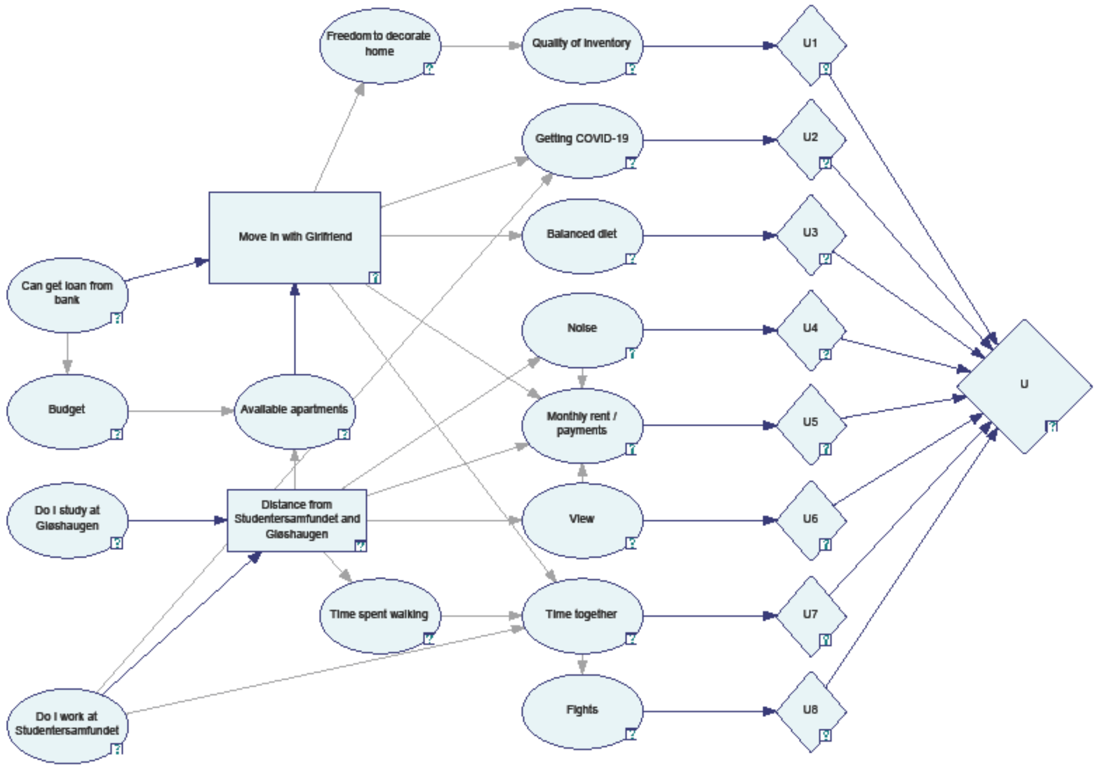
If he does not buy the book:

$$EU(\neg b) = P(p \mid \neg b) * U(\neg b, p) + P(\neg p \mid \neg b) * U(\neg b, \neg p)$$

$$= 0.525 * 2100 + 0.475 * 0 = 1102.5$$

# 2 – Decision Support System

I have decided to model a decision support system to help me decide how to live while studying in Trondheim, as this is a decision I am considering at the moment. I have chosen two decision nodes, with three options each. Node “Move in with Girlfriend” is used to decide if I want to live alone or rent or buy an apartment with my girlfriend. Node “Distance from Studentersamfundet and Gløshaugen” is used to decide how far away from the two areas of interest I want to live, and has the options “< 10 minutes”, “10-20 minutes” and “> 20 minutes”. Note that for simplicity’s sake, all nodes with a continuous range of options will be divided into two or three subranges, like “Distance from Studentersamfundet to Gløshaugen” above. The complete system can be seen in the figure below.




*System structure*

There are several things one would consider before making such decisions, which I have tried to model as certain nodes above. These include whether I study at Gløshaugen and work at Studentersamfundet, which will affect the decision on how far from those places I live; if I can get a loan, which will affect whether I am able to buy an apartment and our budget, which affects available apartments.

The uncertain nodes are based on considerations I have looked at when making this decision in the past and now, as well as certain elements that is impacting my current living situation. As I currently live in a shared house with six housemates, the quality of inventory (such as kitchen utensils) is poor, the time I get to spend with my girlfriend is limited as she lives some distance away and my chances of getting COVID-19 is higher than otherwise as we each have our own friend groups outside of the house. After temporarily living with my girlfriend during lockdown and vacations, I have discovered that I eat better when I am with her, as she is better than I am at being organized and eating healthy.

The node with the most edges is "Monthly rent / payments". While it only affects the utility, it is affected by several other nodes. The localization (represented by distance from Studentersamfundet and Gløshaugen), noise levels around the apartment and the view from the apartment all impact how much an apartment costs to rent or buy. The node is also affected by the choice to live with my girlfriend or not. Based on the current market, it is reasonable to assume that renting an apartment together will be somewhat cheaper than renting alone, while buying an apartment together will be somewhat more expensive, per month.

One node of special interest in this system is "Available apartments". This indicates how many apartments are available in the chosen distance from Studentersamfundet and Gløshaugen and affects my choice of living situation – as an example, if there are no available apartments for sale in our price range and distance from Studentersamfundet and Gløshaugen, then it is not possible to buy and apartment. Because this node relies on the chosen distance from Studentersamfundet and Gløshaugen and points towards whether I move in with my girlfriend or not, then this means that I would first have to decide on where to live, before choosing if I want to live with my girlfriend.

*Assumptions*

In order to model such a complicated network, several assumptions are needed, especially regarding conditional independence. Most of these are obvious, such as the quality of inventory in an apartment not affecting the chance of getting COVID-19 and will not be discussed.

One interesting assumption has been made regarding independencies of "Fights", which indicates the probability of my girlfriend and I fighting. Based on my personal experience, our fights usually stem from too little time spent together, which makes us both frustrated. While other couples might observe that noise in the apartment, balanced diets or monthly payments increase the chances of them fighting, this has not been the case in my relationship, making it safe to assume that these factors do not impact the probability of fights.

Several minor assumptions have been made as well. I have assumed that time spent walking and having a balanced diet does not impact the probability of getting COVID-19. This seems reasonable, as there is very little evidence for outdoor transmission[1]. While eating a well-balanced diet tends to lower the risk of chronic illness and infectious diseases[2], what I consider an unbalanced diet for myself is still far outside of the danger zone, so it does not have any realistic effect on COVID-probability. I have also assumed that the quality of inventory such as kitchen utensils does not impact my diet. This seems reasonable based on my own experience, as my diet stays somewhat the same even though we replace inventory in my apartment now and then. Finally, I have assumed that whether I get a loan or not does not impact how much I have to pay monthly. Based on the market, the monthly price for one apartment is around the same if one rents or buys with a loan. The monthly cost is therefore independent of whether I can get a loan, given distance from Studentersamfundet and Gløshaugen and choice of living situation.

*Quantification of uncertainties*

Due to the number of nodes in the system, it is unreasonable to go over all probabilities. I will, however, show and describe some probabilities, as well as ways to find or assume the others.

As a previously mentioned node, “Monthly rent / payment” is an interesting node to look at the probability table for. Based on searches on Finn.no, minor assumptions about noisy areas and walking speeds and assumptions for lacking apartments in some segments, I came up with the following excerpt from the probability table. Note that this is only one third of the total table, as it is too wide to display in the document. Below are parts of the rest of the table, to display the change when renting or buying with my girlfriend.

| Move in with Girlfriend | live_separate | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| View | nice_view | | | | | | no_view | | | | | |
| Noise | low_noise | | | high_noise | | | low_noise | | | high_noise | | |
| Distance from Studentersamf... | less_than_1... | between_1... | more_than_... | less_than_1... | between_1... | more_than_... | less_than_1... | between_1... | more_than_... | less_than_1... | between_1... | more_than_... |
| low_lt_4500 | 0.05 | 0.1 | 0.3 | 0.1 | 0.3 | 0.5 | 0.1 | 0.3 | 0.5 | 0.3 | 0.5 | 0.6 |
| moderate_4500_to_5500 | 0.7 | 0.7 | 0.6 | 0.7 | 0.6 | 0.45 | 0.7 | 0.6 | 0.45 | 0.6 | 0.45 | 0.39 |
| high_over_5500 | 0.25 | 0.2 | 0.1 | 0.2 | 0.1 | 0.05 | 0.2 | 0.1 | 0.05 | 0.1 | 0.05 | 0.01 |

| Move in with Girlfriend | live_s... | rent_together | | |
|---|---|---|---|---|
| View | ... | nice_view | | |
| Noise | ... | low_noise | | |
| Distance from Studentersamf... | ... | less_than_1... | between_1... | more_than_... |
| low_lt_4500 | ... | 0.1 | 0.3 | 0.5 |
| moderate_4500_to_5500 | ... | 0.7 | 0.6 | 0.45 |
| high_over_5500 | ... | 0.2 | 0.1 | 0.05 |

| Move in with Girlfriend | live_s... | rent_t... | buy_together | | |
|---|---|---|---|---|---|
| View | ... | ... | nice_view | | |
| Noise | ... | ... | low_noise | | |
| Distance from Studentersamf... | ... | ... | less_than_1... | between_1... | more_than_... |
| low_lt_4500 | ... | ... | 0.01 | 0.05 | 0.1 |
| moderate_4500_to_5500 | ... | ... | 0.29 | 0.35 | 0.4 |
| high_over_5500 | ... | ... | 0.7 | 0.6 | 0.5 |

1 https://assets.publishing.service.gov.uk/government/uploads/system/uploads/attachment_data/file/945978/S0921_Factors_contributing_to_risk_of_SARS_18122020.pdf

2 http://www.emro.who.int/nutrition/nutrition-infocus/nutrition-advice-for-adults-during-the-covid-19-outbreak.html

Some probability tables can be found by looking at the current market, similar to what is done above. By looking at the distribution found in the market or real world, one can assume that future endeavors maintain the same distribution. “Available apartments”, “View”, “Noise” and “Getting COVID-19” are examples of such tables.

Some probability tables can be found by looking at current behavior and making assumptions regarding how the behavior will change in the future. An example of this can be seen in “Time spent walking” below. I assume that I stay at home 5% of my days, walk twice to and from Studentersamfundet and Gløshaugen 85% of my days, four times back and forth 9% of my days and more than four times 1% of my days. This is reasonable, especially once COVID-19 has passed and society opens back up. Other nodes where the same process can be applied include “Quality of inventory”, “Balanced diet”, “Time together” and “Fights”.

| Distance from Studentersamfundet ... | less_than_10_min | between_10_to_20_min | more_than_20_min |
|---|---|---|---|
| less_than_45_mins | 0.9 | 0.05 | 0.05 |
| between_45_and_90_mins | 0.09 | 0.94 | 0.85 |
| more_than_90_mins | 0.01 | 0.01 | 0.1 |

The last uncertain node, “Freedom to decorate home” can be considered solely based on the previous options and some assumptions. When buying an apartment, the owner is usually totally free to make changes as he or she would like. When renting, either alone or as a couple, most owners are reluctant to let the renters do as they please. From this, we can make some assumptions regarding quantification, and get the following probability table. Note that the 0.01-values represent unlikely-but-possible scenarios, such as strict rules in a housing association.

| Move in with Girlfriend | live_separate | rent_together | buy_together |
|---|---|---|---|
| Free_to_decorate | 0.01 | 0.01 | 0.99 |
| Limited_decoration | 0.79 | 0.79 | 0.01 |
| No_decoration | 0.2 | 0.2 | 0 |

### *Quantification of utilities*

Quantifying utilities in scenarios such as the one I have chosen is not an easy task. Obviously, spending time with my girlfriend is a positive thing, but how nice is it compared to, say, not having a fight? How important is rent, compared to not getting COVID-19?

These questions need to be answered. One way to estimate utility is to estimate how much money a person (in this case, I) would be willing to pay to get a certain node-state. To establish a baseline, let the following be the utility table for U5:

| Monthly rent / ... | low_lt_4500 | moderate_4500_to_5500 | high_over_5500 |
|---|---|---|---|
| Value | 1000 | 0 | -1000 |

Using this, we can establish a monetary value for certain attributes, and then convert the monetary value to utility. We see that 100 NOK per month converts to 200 utility. I estimate that I would pay 200 NOK per month for a better view, 300 NOK per month for less noise and 100 NOK per month for nice inventory. Using no view, low noise and poor inventory as baseline and converting this to utility gives us the following utility tables:

| Quality of inventory | Poor | Good |
|---|---|---|
| Value | 0 | 200 |

| View | nice_view | no_view |
|---|---|---|
| Value | 400 | 0 |

| Noise | low_noise | high_noise |
|---|---|---|
| Value | 0 | -600 |

I estimate that I am willing to pay 1000 NOK per month to increase the time spent with my girlfriend from 2 hours per day to more than 3 hours per day. Likewise, I would require 1500 NOK per month to decrease the time spent from 2 hours to less than 1 hour per day. This gives the following utility table:

| Time together | lt_1hr_per_day | between_1_to_3_hrs_pr_day | more_than_3_hrs_pr_day |
|---|---|---|---|
| Value | -3000 | 0 | 2000 |

The same process can be applied to the rest of the utility nodes as well. I estimate that I would pay 1750 NOK per month to be guaranteed not to get COVID-19, 2500 NOK to avoid fights with my girlfriend and 350 NOK to have a balanced diet. This gives the following utility tables:

| Getting COVID-19 | Getting_COVID | Not_getting_COVID |
|---|---|---|
| Value | -3500 | 0 |

| Balanced diet | Balanced_diet | Unbalanced_diet |
|---|---|---|
| Value | 0 | 700 |

| Fights | Fight | No_fight |
|---|---|---|
| Value | -5000 | 0 |

The final utility node, U, is an ALU with a coefficient of 1 for each previous utility node.

### *Verify and refine the model, perform sensitivity analysis*

In order to ensure that the model is fully operational, one should run tests on it, to see how close to reality it comes. If the goal were to help couples in general make this decision, one way of doing this would be to present both random couples and the model with several options (such as renting apartment A together, living separately within 10 minutes from Studentersamfundet, etc.), and tweaking utility values and probabilities until the couples and model prefers the same alternatives. As the goal with this system was to help my girlfriend and I to decide, the test would be to do the same, only for the two of us.

The final step, once the model was implemented, would be to do a sensitivity analysis. This is done by varying probabilities and utilities slightly, to see how if the best decision changes. If the decision changes based on small variations, then the model is not very robust, and likely to be affected by out estimates for probabilities and utilities. One fix to this would be to gather more data, by getting information from the market and by studying my girlfriend and me. If the decision does not change, however, then the model is robust, and we can be safer that it presents the true best option.

As the model is not completely implemented, and to a certain degree based on assumptions for utility values and probabilities, performing these tests and analyses are beyond the scope for this report.