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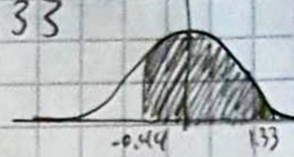
1) a) $\int_{-0.44}^{1.33} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \, dz$

$=$ Omr. under normalfordelt kurve fra $-0.44 \to 1.33$

$= 0{,}9082 - 0{,}3300 = \underline{\underline{0{,}5782}}$

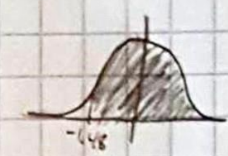


b) $\int_{-\infty}^{0{,}94} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \, dz$

$=$ Omr. opp til $0.94 = \underline{\underline{0{,}8264}}$

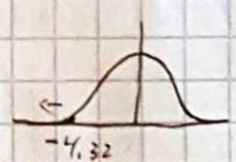
c) $\int_{-1.48}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \, dz$

$= 1 -$ omr. til $-1.48 = 1 - 0{,}0694 = \underline{\underline{0{,}9306}}$



evt $=$ omr. til 1.48 pga. symmetri

d) $\int_{-\infty}^{-4{,}32} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \, dz = \underline{\underline{0{,}0000}}$ [utenfor tabell]



4) a) $\int_{0}^{1{,}24} e^{-z^2/2} \, dz = \sqrt{2\pi} \int_{0}^{1{,}24} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \, dz$

$\sqrt{2\pi}\left(0{,}8925 - 0{,}5000\right) = \underline{\underline{\sqrt{2\pi}\ 0.3925}} \approx \underline{\underline{0{,}9839}}$

b) $\int_{-\infty}^{\infty} 6e^{-z^2/2} \, dz = 6\sqrt{2\pi} \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \, dz$

$= \underline{\underline{6\sqrt{2\pi}}} \approx \underline{\underline{15{,}0398}}$

5) Anta $Z \sim N(0,1)$. For hvilke z stemmer:

a) $P(Z \leq z) = 0{,}33$. Tabell: $\underline{\underline{z = -0{,}44}}$

b) $P(Z \geq z) = 0{,}2236 \Rightarrow P(Z \leq z) = 0{,}7764$. Tabell: $\underline{\underline{z = 0{,}76}}$

c) $P(-1{,}00 \leq Z \leq z) = 0{,}5004$

$\underline{\underline{z = 0{,}41}}$

$P(Z \leq z) - P(Z \leq -1{,}00) = P(Z \leq z) - 0.1587 \Rightarrow P(Z \leq z) = 0{,}6591$

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5) d) $P(-z \leq Z \leq z) = P(Z \leq z) - P(Z \leq -z) = 0{,}80$

$$\xrightarrow{\text{symmetri}} P(Z \leq z) - P(Z \geq z) = 0{,}80$$

$$P(Z \leq z) - (1 - P(Z \leq z)) = 0{,}80$$

$$2P(Z \leq z) = 1{,}80$$

$$P(Z \leq z) = 0{,}9 \Rightarrow \underline{\underline{z = 1{,}28}}$$

e) $P(z \leq Z \leq 2{,}03) = P(Z \leq 2{,}03) - P(Z \leq z)$

$$= 0{,}9788 - P(Z \leq z) = 0{,}15$$

$$P(Z \leq z) = 0{,}8288 \Rightarrow \underline{\underline{z = 0{,}95}}$$

13) Hvis $p_X(k) = \binom{10}{k}(0{,}7)^k(0{,}3)^{10-k}$, $k = 0, 1, \ldots, 10$, kan du tilnærme

at $P(4 \leq X \leq 8) = P\left(\frac{3{,}5 - 10(0{,}7)}{\sqrt{10(0{,}7)(0{,}3)}} \leq Z \leq \frac{8{,}5 - 10(0{,}7)}{\sqrt{10(0{,}7)(0{,}3)}}\right)$?

Bør ha $n > 9\frac{p}{1-p}$ $\quad n = 10, \quad p = 0{,}7$

$$9\frac{p}{1-p} = 9\frac{0{,}7}{0{,}3} = 21$$

$$10 < 21$$

<u>Ser at $n < 9\frac{p}{1-p}$, så tilnærmelsen fungerer dårlig</u>

23) La $X \sim N(\mu, \sigma^2)$, X = dollar til TV-evangelist. Finn

$P(X > 30000)$ $\quad \mu = 20000, \quad \sigma = 5000$

La $Z = \frac{X - \mu}{\sigma} = \frac{X - 20000}{5000}$ $\quad Z \sim N(0,1)$

$$P(X > 30000) = P(\sigma Z + \mu > 30000) = P\left(Z > \frac{30000 - 20000}{5000}\right)$$

$$= P\left(Z > \frac{10000}{5000}\right) = 1 - P(Z \leq 2) \overset{\text{tabell}}{=} 1 - 0{,}9772$$

$$= \underline{\underline{0{,}0228}}$$

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34) La $Y_1, \dots, Y_n$ tilf. resultater fra $Y \sim N(\overset{\mu}{2}, \overset{\sigma^2}{4})$. Hvor stor må n være for at $P(1{,}9 \le \bar{Y} \le 2{,}1) \ge 0{,}99$

$\bar{Y} = \frac{1}{n}\sum_{i=1}^{n} Y_i \qquad \bar{Y} \sim N(\mu, \sigma^2/n)$

$$\text{La } Z = \frac{\bar{Y} - \mu}{\sigma/\sqrt{n}} = \frac{\bar{Y} - 2}{\frac{2}{\sqrt{n}}} \qquad Z \sim N(0,1)$$

$$P(1{,}9 \le \bar{Y} \le 2{,}1) = P\left(\frac{1{,}9-2}{2/\sqrt{n}} \le Z \le \frac{2{,}1-2}{2/\sqrt{n}}\right)$$

$$= P\left(\frac{-0{,}1\sqrt{n}}{2} \le Z \le \frac{0{,}1\sqrt{n}}{2}\right) = P(Z \le 0{,}05\sqrt{n}) - P(Z \le 0{,}05\sqrt{n})$$

$$\overset{\text{symmetri}}{\underset{\text{(se 5.d)}}{=}} \quad 2P(Z \le 0{,}05\sqrt{n}) - 1 \ge 0{,}99$$

$$P(Z \le 0{,}05\sqrt{n}) \ge 0{,}995$$

$$\text{tabell} \Rightarrow 0{,}05\sqrt{n} \ge 2{,}58$$

$$(\sqrt{n})^2 \ge (51{,}6)^2$$

$$\underline{\underline{n \ge 2663}}$$

37) Bevis korr. 4.3.1 og 4.3.2 med momentgen. funk

1) $M_{\bar{Y}}(t) = M_{Y_1}(\tfrac{1}{n}t) \cdot \dots \cdot M_{Y_n}(\tfrac{1}{n}t)$

$$= \left(e^{\frac{t}{n}\mu + \frac{1}{2}\sigma^2 \frac{t^2}{n^2}}\right)^n$$

$$= e^{t\mu + \frac{1}{2}\sigma^2 \frac{t^2}{n}} = e^{\mu t + \frac{1}{2}\frac{\sigma^2}{n}t^2}$$

$$\Rightarrow \underline{\underline{\bar{Y} \sim N(\mu, \tfrac{\sigma^2}{n})}}$$

2) $Y_1 \sim N(\mu_1, \sigma_1^2), \dots, Y_n \sim N(\mu_n, \sigma_n^2)$

$Y = a_1 Y_1 + \dots + a_n Y_n$

$$M_Y(t) = M_{Y_1}(a_1 t) \cdot \dots \cdot M_{Y_n}(a_n t)$$

$$= e^{a_1 t\mu_1 + \frac{1}{2}\sigma_1^2 a_1^2 t^2} \cdot \dots \cdot e^{\mu_n a_n t + \frac{1}{2}\sigma_i^2 a_n^2 t^2}$$

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$$= e^{(a_1\mu_1 + \dots + a_n\mu_n)t + \frac{1}{2}(a_1^2\sigma_i^2 + \dots + a_n^2\sigma_n^2)t^2}$$

$$\Rightarrow Y \sim N(\mu, \sigma^2) \text{ der}$$

$$\underline{\underline{\mu = \sum_{i=1}^{n} a_i\mu_i}} \qquad \underline{\underline{\sigma^2 = \sum_{i=1}^{n} a_i^2\sigma_i^2}}$$

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2) $X \sim N(\mu, \sigma^2)$ $\mu = 0{,}0165$, $\sigma = 0{,}073$

a) Finn $P(X>0)$, $P(|X|<0{,}1)$

La $Z = \frac{X - 0.0165}{0.073}$, $Z \sim N(0, 1)$

$$P(X>0) = P\left(Z > \frac{0 - 0.0165}{0{,}073}\right) = 1 - P(Z \leq -0{,}23)$$

$$= 1 - 0{,}4090 = \underline{\underline{0{,}5910}}$$

$$P(|X| < 0{,}1) = P(-0{,}1 < X < 0{,}1) = P\left(\frac{-0{,}1 - 0.0165}{0{,}073} < Z < \frac{0{,}1 - 0{,}0165}{0{,}073}\right)$$

$$= P(-1{,}60 < Z < 1{,}14) = P(Z < 1{,}14) - P(Z \leq -1{,}60)$$

$$= 0{,}8729 - 0{,}0548 = \underline{\underline{0{,}8181}}$$

b) $Y_i = \frac{S(i)}{S(i-1)}$ uavh. $\ln(Y_i) \sim N(\mu, \sigma^2)$ som i (a)

Finn $P(Y_i > 1) = P(\ln(Y_i) > 0) = P(X>0) \overset{(a)}{=} \underline{\underline{0{,}5910}}$

Finn $P\left(\frac{S(2)}{S(0)} > 1\right) = P\left(\frac{S(2)}{S(1)} \cdot \frac{S(1)}{S(0)} > 1\right) = P(Y_1 \cdot Y_2 > 1)$

$$= P(\ln(Y_1) + \ln(Y_2) > 0) = P(2X > 0) = P(X>0)$$

$$= \underline{\underline{0{,}5910}}$$

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2) c) Finn $E(Y_i)$ og $Var(Y_i)$

$$Y_i = e^{\ln(Y_i)} = e^{X}$$

$$M_X(t) = e^{\mu t + \frac{1}{2}\sigma^2 t^2}$$

$$M_{Y_i}(t) = M_{e^X}(t) = E\left(e^{te^X}\right)$$

Jeg er tom for tid, og vet ikke hvordan jeg skal løse dette. Kommer LF?

Merk: er usikker på overgangen $M_X(t) \to M_{e^X}(t)$, og om det finnes "lovlige" direkte overganger