ØVING 2 side 1

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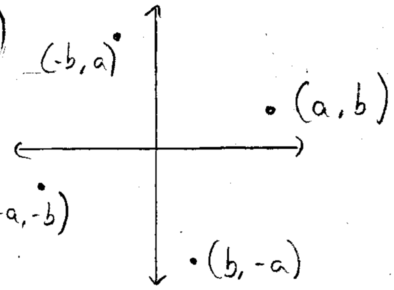
T 1.2

2)



Hvis $(a,b) = (3, 1)$

$(-a,-b) = (-3,-1)$

$(b,-a) = (1,-3)$

$(-b, a) \quad (-1, 3)$

4) Avstand $\overset{a}{(-7,8)} \to \overset{b}{(6,-3)}$

Vektor $\vec{v}$ fra $a \to b = (6+7, -3-8) = (13, -11)$

Avstand $a \to b = |\vec{v}| = \sqrt{169+121} = \underline{\underline{\sqrt{290}}} \approx 17{,}03$

6) $(-2,3) \cdot (4,1) = -8 + 3 = \underline{\underline{-5}}$

$\Theta = \cos^{-1}\left(\frac{(-2,3)\cdot(4,1)}{|(-2,3)|\cdot|(4,1)|}\right) = \cos^{-1}\left(\frac{-5}{\sqrt{13}\,\sqrt{17}}\right) = \underline{\underline{\cos^{-1}\left(\frac{-5}{\sqrt{221}}\right)}} \approx 1{,}9 \text{ rad}$

11) $a = (4,3) = b + c$, $b \parallel d$, $c \perp d$, $d = (1,2)$

La $c = (c_1, c_2)$,

$c \cdot d = c_1 + 2c_2 = 0$

$c_2 = -\frac{1}{2} c_1 \Rightarrow c = (-2c_2, c_2)$

$b = zd = (z, 2z) = (b_1, 2b_1)$

$(4,3) = (b_1, 2b_1) + (-2c_2, c_2)$

$4 = b_1 - 2c_2$ $\qquad$ $3 = 2b + c_2$

$\qquad\qquad\qquad\qquad c_2 = 3 - 2b_1$

$4 = b_1 - 6 + 4b_1$

$10 = 5b_1$ $\qquad$ $c_2 = 3 - 4$

$b_1 = 2$ $\qquad$ $c_2 = -1$

$b = (2, 4) \qquad c = (2, -1)$

$\underline{\underline{a = (4,3) = b + c = (2,4) + (2,-1)}}$

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15) Hvorfor kan ikke $|\vec{a}| = 7$, $|\vec{b}| = 2$ og $|\vec{a} - \vec{b}| = 4$?

Se på $|\vec{a}|$

$$|\vec{a}| = |\vec{a} + \vec{b} - \vec{b}| = |\vec{a} - \vec{b} + \vec{b}| \overset{\Delta\text{-ulikhet}}{\leq} |\vec{a} - \vec{b}| + |\vec{b}|$$

Snu om, og få:

$$|\vec{a} - \vec{b}| \geq |\vec{a}| - |\vec{b}|$$

Settes inn tall:

$$4 \geq 7 - 2 = 5$$

<u>Dette stemmer ikke, så tallene kan ikke stemme.</u>

17) $d(\vec{a}, \vec{b}) = |\vec{b} - \vec{a}|$ Vis at $d(\vec{a}, \vec{b}) \leq d(\vec{a}, \vec{c}) + d(\vec{c}, \vec{b}) \quad \forall \vec{a}, \vec{b}, \vec{c}$

Skal vise: $|\vec{b} - \vec{a}| \leq |\vec{c} - \vec{a}| + |\vec{b} - \vec{c}|$

[Regner kun med vektorer, så skriver uten $\rightarrow$ pil over videre.]

$$|b - a| = |b - a + c - c| = |c - a + b - c|$$

Δ-ulikhet: $\leq |c - a| + |b - c| \quad \square$

Geometrisk: lengden av en vektor fra a til b er mindre eller lik lengden av en vektor fra a til c pluss en vektor fra c til b.

22) Finn param. fremst. for linje gjennom $(2, -1)$ og $(3, 8)$

$\vec{v}$ = vektor parallell med linjen $= (3 - 2, 8 + 1) = (1, 9)$

Har at linjen går gjennom $(2, -1)$, så

$$\underline{\underline{\vec{r}(t) = (2 + t, -1 + 9t)}}$$

ØVING 2 side 3 . Andreas B. Berg

A) 1) $\vec{r}(t) = (2\sin t, 3\cos t) \quad t \in \mathbb{R}$

$\underline{\vec{v}(t)} = \vec{r}\,'(t) = \underline{\underline{(2\cos t, -3\sin t)}}$

$\underline{v(t)} = |\vec{v}(t)| = \sqrt{4\cos^2 t + 9\sin^2 t} = \sqrt{4(\cos^2 t + \sin^2 t) + 5\sin^2 t}$

$= \underline{\underline{\sqrt{4 + 5\sin^2 t}}}$

$\underline{\vec{a}(t)} = \vec{v}\,'(t) = \underline{\underline{(-2\sin t, -3\cos t)}}$

$\underline{a(t)} = v'(t) = \frac{1}{2\sqrt{4+5\sin^2 t}} \cdot 5 \cdot 2\sin t \cdot \cos t$

$= \underline{\underline{\frac{5\sin(2t)}{2\sqrt{4+5\sin^2 t}}}}$

2) $x = 2\sin t \qquad x^2 = 4\sin^2 t \qquad \sin^2 t = \frac{x^2}{4}$

$y = 3\cos t \qquad y^2 = 9\cos^2 t \qquad \cos^2 t = \frac{y^2}{9}$

$\frac{x^2}{4} + \frac{y^2}{9} = \sin^2 t + \cos^2 t = 1$

$\underline{\underline{\frac{x^2}{4} + \frac{y^2}{9} = 1}}$ - $\underline{\underline{\text{ellipse}}}$

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B) 1) $\vec{r}(t) = (4\cosh t \ , \ 5\sinh t) \quad t \in \mathbb{R}$

$\underline{\vec{v}(t)} = \vec{r}\,'(t) = \underline{\underline{(4\sinh t \ , \ 5\cosh t)}}$

$\underline{v(t)} = |\vec{v}(t)| = \underline{\underline{\sqrt{16\sinh^2 t + 25\cosh^2 t}}}$

$\underline{\vec{a}(t)} = \vec{v}\,'(t) = \underline{\underline{(4\cosh t \ , \ 5\sinh t)}}$

$\underline{a(t)} = v'(t) = \frac{1}{2\sqrt{16\sinh^2(t) + 25\cosh^2(t)}} \cdot (32\sinh(t)\cosh(t) + 50\cosh(t)\sinh(t))$

$= \underline{\underline{\frac{41\sinh(t)\cosh(t)}{\sqrt{16\sinh^2(t) + 25\cosh^2(t)}}}}$

2). $x = 4\cosh t \qquad x^2 = 16\cosh^2 t \qquad \cosh^2 t = \frac{x^2}{16}$

$y = 5\sinh t \qquad y^2 = 25\sinh^2 t \qquad \sinh^2 t = \frac{y^2}{25}$

$\frac{x^2}{16} - \frac{y^2}{25} = \cosh^2 t - \sinh^2 t = 1$

$\underline{\underline{\frac{x^2}{16} - \frac{y^2}{25} = 1}}$ - $\underline{\underline{\text{hyperbel}}}$