13.2 # ØVING 10, side 1 Andreas B Berg

1)

	Våken, 1	Hypnose, 2
$\bar{Y}_{\cdot j}$	18.9	21.7

$\alpha = 0.05$
$k = 2$
$n = 30$
$b = 15$

$\bar{Y}_{\cdot\cdot} = 20.3$

$H_0 : \mu_1 = \mu_2$

$$F = \frac{SSTR/(k-1)}{SSE/((b-1)(k-1))} = \frac{SSTR}{SSE/14}$$

Forkaster H_0 hvis $F \geq F_{0.95,\,1,\,14} = F_{1-\alpha,\,k-1,\,(k-1)(b-1)}$

$$SSTR = \sum_{i=1}^{b}\sum_{j=1}^{k}(\bar{Y}_{\cdot j} - \bar{Y}_{\cdot\cdot})^2 = b\sum_{j=1}^{k}(\bar{Y}_{\cdot j} - \bar{Y}_{\cdot\cdot})^2$$

$$= 15\left((18.9 - 20.3)^2 + (21.7 - 20.3)^2\right) = 58.8$$

$$SSE = \sum_{i=1}^{b}\sum_{j=1}^{k}(Y_{ij} - \bar{Y}_{\cdot j} - \bar{Y}_{i\cdot} + \bar{Y}_{\cdot\cdot})^2$$

			$\bar{Y}_{i\cdot}$	$Y_{ij} - \bar{Y}_{\cdot j} - \bar{Y}_{i\cdot} + \bar{Y}_{\cdot\cdot}$	
	18	25	21.5	-2.1	2.1
	19	20	19.5	0.9	-0.9
	16	26	21	-3.6	3.6
	21	26	23.5	-1.1	1.1
	16	20	18	-0.6	0.6
	20	23	21.5	-0.1	0.1
	20	14	17	4.4	-4.4
	14	16	16	-0.6	0.6
	11	18	14.5	-2.1	2.1
	22	20	21	2.4	-2.4
	19	22	20.5	-0.1	0.1
	29	27	28	2.4	-2.4
	16	19	17.5	-0.1	0.1
	27	27	27	1.4	-1.4
	15	21	18	-1.6	1.6
$\bar{Y}_{\cdot j}$	18.9	21.7	20.3		

$SSE = 159.36$

$$F = \frac{58.8}{159.36/14} = 5.16$$

$F_{0.95,\,1,\,14} = 4.60$

$\Rightarrow F \geq F_{0.05,\,1,\,14} \Rightarrow$ Forkaster H_0

$\Rightarrow$ Signifikant forskjell ved $\alpha = 0.05$

13.2

ØVING 10, side 2

Andreas B. Berg

2)

	ABC (1)	CBS (2)	NBC (3)	$\bar{Y}_{i.}$	$T_{i.}$
A	19.7	16.1	18.2	18.2	54
B	18.6	15.8	17.9	17.43	52.3
C	19.1	14.6	15.3	16.33	49
D	17.9	17.1	18.0	17.67	53
$\bar{Y}_{.j}$	18.825	15,9	17,35	17.36	
$T_{.j}$	75.3	63,6	69,4		208,3

$\alpha = 0,10 \quad k = 3 \quad b = 4 \quad H_0 : \mu_1 = \mu_2 = \mu_3$

La $C = \frac{T_{..}^2}{bk} = \frac{208.3^2}{12} = 3615,74$

$\Rightarrow SSTR = \sum_{j=1}^{k} \frac{T_{.j}^2}{b} - C = \frac{75.3^2}{4} + \frac{63.6^2}{4} + \frac{69.4^2}{4} - 3615.74 = \underline{17.11}$

$SSTOT = \sum_{i=1}^{b}\sum_{j=1}^{k} Y_{ij}^2 - C = 3643.43 - 3615.74 = \underline{27.69}$

$SSB = \sum_{i=1}^{b} \frac{T_{i.}^2}{k} - C = \frac{54^2}{3} + \frac{52.3^2}{3} + \frac{49^2}{3} + \frac{53^2}{3} - 3615.74 = 4,69$

$SSE = SSTOT - SSTR - SSB = 5.89$

$F = \frac{SSTR/(k-1)}{SSE/(k-1)(b-1)} = \frac{17.11/2}{5.89/6} = 8,71$

$F_{0.10, 2, 6} = 3.46$

Ser at $F \geq F_{0.90, 2, 6} \Rightarrow$ Forkaster H_0

$\Rightarrow$ <u>Ikke lik fordeling nyhetstittere hos ABC, CBS & NBC</u>

ØVING 10, side 3

Andreas B. Berg

7) Finn 95% Tukey-intervall for data i 13.2.2) og bruk dem til å sammenlikne parvis ABC, CBS & NBC

$\alpha = 0{,}05$

La $D = Q_{\alpha, k, (b-1)(k-1)}/\sqrt{b} = Q_{0.05, 3, 6}/\sqrt{4} = 4{,}34/2 = \underline{2.17}$

$P = 95\%$ for at $\bar{Y}_{.s} - \bar{Y}_{.t} - D\sqrt{MSE} < \mu_s - \mu_t < \bar{Y}_{.s} - \bar{Y}_{.t} + D\sqrt{MSE}$

$MSE = \frac{SSE}{(b-1)(k-1)} = \frac{5.89}{1 \cdot 2 \cdot 3} = \frac{5.89}{6} = 0{,}98$

$\Rightarrow D\sqrt{MSE} = 2{,}17\sqrt{0.98} = 2{,}14$

par		$\bar{Y}_{.s} - \bar{Y}_{.t}$	Intervall	Konklusjon
ABC - CBS	$\mu_1 - \mu_2$	2,9	(0,76 , 5,04)	Forkast
ABC-NBC	$\mu_1 - \mu_3$	1,48	(-0,66, 3,62)	N/A
CBS-NBC	$\mu_2 - \mu_3$	-1,45	(-3,59, 0,69)	N/A

Ser at vi kan forkaste at like mange nyhetstittere ser ABC som CBS, ellers ingen forkasting ved $\alpha = 0{,}05$

[Kommentar til meg selv/husk:
For blocks: $MSE = \frac{SSE}{(b-1)(k-1)}$]

ØVING 10, side 4

Andreas B. Berg

11) Bevis formlene gitt i likn. 13.2.2, 2.3 og 2.4

13.2.2) $SSTR = \sum_{j=1}^{k}\sum_{i=1}^{n_j}(\bar{Y}_{.j} - \bar{Y}_{..})^2$

$= \sum_{j=1}^{k} n_j (\bar{Y}_{.j} - \bar{Y}_{..})^2 \qquad [n_j = b]$

$= \sum_{j=1}^{k} b(\bar{Y}_{.j} - \bar{Y}_{..})^2$

$= \sum_{j=1}^{k} b(\bar{Y}_{.j}^2 - 2\bar{Y}_{.j}\bar{Y}_{..} + \bar{Y}_{..}^2) \qquad [\bar{Y}_{.j} = \frac{T_{.j}}{b}]$

$= \sum_{j=1}^{k} b\left(\frac{T_{.j}^2}{b^2} - 2\bar{Y}_{.j}\bar{Y}_{..} + \bar{Y}_{..}^2\right)$

$= \sum_{j=1}^{k}\left(\frac{T_{.j}^2}{b} - b2\bar{Y}_{.j}\bar{Y}_{..} + b\bar{Y}_{..}^2\right)$

$= \sum_{j=1}^{k}\frac{T_{.j}^2}{b} - 2b\bar{Y}_{..}\sum_{j=1}^{k}\bar{Y}_{.j} + b\sum_{j=1}^{k}\bar{Y}_{..}^2$

$= \sum_{j=1}^{k}\frac{T_{.j}^2}{b} - 2b\bar{Y}_{..}\sum_{j=1}^{k}\bar{Y}_{.j} + bk\bar{Y}_{..}^2 \qquad [\bar{Y}_{..} = \frac{T_{..}}{bk}]$

$= \sum_{j=1}^{k}\frac{T_{.j}^2}{b} - 2\frac{T_{..}}{k}\sum_{j=1}^{k}\frac{T_{.j}}{b} + \frac{T_{..}^2}{bk}$

$= \sum_{j=1}^{k}\frac{T_{.j}^2}{b} - 2\frac{T_{..}^2}{bk} + \frac{T_{..}^2}{bk}$

$= \sum_{j=1}^{k}\frac{T_{.j}^2}{b} - \frac{T_{..}^2}{bk} \qquad [\frac{T_{..}^2}{bk} = C]$

$= \sum_{j=1}^{k}\frac{T_{.j}^2}{b} - C$

13.2.3) $SSB = \sum_{i=1}^{b}\sum_{j=1}^{k}(\bar{Y}_{i.} - \bar{Y}_{..})^2$

$= \sum_{i=1}^{b}\sum_{j=1}^{k}(\bar{Y}_{i.}^2 - 2\bar{Y}_{i.}\bar{Y}_{..} + \bar{Y}_{..}^2)$

$= k\sum_{i=1}^{b}(\bar{Y}_{i.}^2 - 2\bar{Y}_{i.}\bar{Y}_{..} + \bar{Y}_{..}^2)$

$= k\left(\sum_{i=1}^{b}\bar{Y}_{i.}^2 - 2\bar{Y}_{..}\sum_{i=1}^{b}\bar{Y}_{i.} + b\bar{Y}_{..}^2\right) \qquad [\bar{Y}_{i.} = \frac{T_{i.}}{k},\ \bar{Y}_{..} = \frac{T_{..}}{bk}$

$= k\left(\sum_{i=1}^{b}\frac{T_{i.}^2}{k^2} - 2\frac{T_{..}}{bk}\sum_{i=1}^{b}\frac{T_{i.}}{k} + \frac{T_{..}^2}{bk^2}\right)$

11) 13.2.3)

$$= \sum_{i=1}^{b} \frac{T_{i.}^2}{k} - 2\frac{T_{..}^2}{bk} + \frac{T_{..}^2}{bk} \qquad \left[C = \frac{T_{..}^2}{bk}\right]$$

$$= \underline{\underline{\sum_{i=1}^{b} \frac{T_{i.}^2}{k} - C}}$$

13.2.4) $SSTOT = \sum_{i=1}^{b}\sum_{j=1}^{k} (Y_{ij} - \bar{Y}_{..})^2$

$$= \sum_{i=1}^{b}\sum_{j=1}^{k} \left(\bar{Y}_{ij}^2 - 2Y_{ij}\bar{Y}_{..} + \bar{Y}_{..}^2\right)$$

$$= \sum_{i=1}^{b}\left(\sum_{j=1}^{k} \bar{Y}_{ij}^2 - 2\bar{Y}_{..}\sum_{j=1}^{k}\bar{Y}_{ij} + k\bar{Y}_{..}^2\right)$$

$$= \sum_{i=1}^{b}\sum_{j=1}^{k} \bar{Y}_{ij}^2 - 2\bar{Y}_{..}\sum_{i=1}^{b}\sum_{j=1}^{k}\bar{Y}_{ij} + bk\bar{Y}_{..}^2 \qquad \left[\bar{Y}_{..} = \frac{T_{..}}{bk}\right]$$

$$= \sum_{i=1}^{b}\sum_{j=1}^{k} \bar{Y}_{ij}^2 - 2\frac{T_{..}^2}{bk} + \frac{T_{..}^2}{bk} \qquad \left[\frac{T_{..}^2}{bk} = C\right]$$

$$= \underline{\underline{\sum_{i=1}^{b}\sum_{j=1}^{k} \bar{Y}_{ij}^2 - C}}$$

14) For et sett med tilf. blokkdata som sammenlikner k behandlinger i b blokker, finn a) $E(SSB)$, b) $E(SSE)$

a) Anta blokkeffekten er lik for alle blokker, altså for β = blokkeffekt er $\beta_1 = \beta_2 = .. = \beta_b$. Da gir teorem 13.2.2 b) at

$\frac{SSB}{\sigma^2} \sim \chi^2$ med $b-1$ d.f.

Da er $E\left(\frac{SSB}{\sigma^2}\right) = \frac{1}{\sigma^2}E(SSB) = E(\chi^2_{b-1}) = b-1$

$$\Rightarrow E(SSB) = \underline{\underline{\sigma^2(b-1)}}$$

13.2

ØVING 10, side 6

Andreas B. Berg

14) b) Har fra teorem 13.22 c) at $\frac{SSE}{\sigma^2} \sim \chi^2$ med $(b-1)(k-1)$ d.f.

$$\Rightarrow E(SSE) = \sigma^2 E\left(\frac{SSE}{\sigma^2}\right) = \sigma^2 E\left(\chi^2_{(b-1)(k-1)}\right)$$

$$\underline{\underline{= \sigma^2 (b-1)(k-1)}}$$

a igjen)

Anta $\beta_1 = \beta_2 = \dots = \beta_b$ ikke er sann. La $SSB = \sum_{i=1}^{b} \frac{T_{i.}^2}{k} - \frac{T_{..}^2}{bk}$

$$E(SSB) = E\left(\sum_{i=1}^{b} \frac{T_{i.}^2}{k} - \frac{T_{..}^2}{bk}\right)$$

$$= \frac{1}{k} \sum_{i=1}^{b} E(T_{i.}^2) - \frac{1}{bk} E(T_{..}^2)$$

[Vet at $E(X^2) = Var(X) + \mu^2$. La μ_i gjelde for $T_{i.}$ og $\mu_{..}$ for $T_{..}$. La $Var(T_{i.}) = k\sigma^2$, $Var(T_{..}) = bk\sigma^2$]

$$= \frac{1}{k} \sum_{i=1}^{b} (k\sigma^2 + \mu_i^2) - \frac{1}{bk}(bk\sigma^2 - \mu_{..}^2)$$

$$= b\sigma^2 + \frac{1}{k} \sum_{i=1}^{b} \mu_i^2 - \sigma^2 + \frac{1}{bk} \mu_{..}^2$$

$$\underline{\underline{= \sigma^2 (b-1) + \frac{1}{k} \sum_{i=1}^{b} \mu_i^2 + \frac{1}{bk} \mu_{..}^2}}$$

Der μ_i = Forv. verdi av summen av rad i

$$= \sum_{j=1}^{k} E(Y_{ij}) = E(T_{i.})$$

$\mu_{..}$ = Forv. verdi av summen av alle elementer

$$= \sum_{i=1}^{b} \sum_{j=1}^{k} E(Y_{ij}) = E(T_{..}) = \sum_{i=1}^{b} E(T_{i.})$$