ØVING 8 side 1 — Andreas B. Berg

5.3

2) Finnes $\lim_{m\to\infty} A^m$? Hvis ja - finn $\lim_{m\to\infty} A^m$

b) $A = \begin{pmatrix} -1.4 & 0.8 \\ -2.4 & 1.8 \end{pmatrix}$. Finner eg. verdier:

$$\det(A - \lambda I) = \det\begin{pmatrix} -1,4-\lambda & 0.8 \\ -2.4 & 1,8-\lambda \end{pmatrix} = (-1,4-\lambda)(1,8-\lambda) + 0.8 \cdot 2.4$$

$$= -2.52 + 1.4\lambda - 1,8\lambda + \lambda^2 + 1,92$$

$$= \lambda^2 - 0,4\lambda - 0,6 = (\lambda - 1)(\lambda + 0,6)$$

$$\Rightarrow \lambda_1 = 1 \qquad \lambda_2 = -0,6$$

$\Rightarrow A$ diagonaliserbar og $\lim_{m\to\infty} A^m$ finnes

$\lambda_1 = 1$

$$A - I = \begin{pmatrix} -2.4 & 0.8 \\ -2.4 & 0.8 \end{pmatrix} \Rightarrow \left(\begin{array}{cc|c} -2.4 & -2.4 & a \\ 0.8 & 0.8 & b \end{array}\right) \sim \left(\begin{array}{cc|c} 0 & 0 & a+3b \\ 0.8 & 0.8 & b \end{array}\right) \Rightarrow v_1 = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

$\lambda_2 = -0,8$

$$A + 0,6I = \begin{pmatrix} -0,8 & 0.8 \\ -2.4 & 2.4 \end{pmatrix} \Rightarrow \left(\begin{array}{cc|c} -0,8 & -2.4 & a \\ 0.8 & 2.4 & b \end{array}\right) \sim \left(\begin{array}{cc|c} -0,8 & -2.4 & a \\ 0 & 0 & b+a \end{array}\right) \Rightarrow v_2 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$Q = \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix} \quad Q^{-1}: \left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 1 & 3 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{cc|cc} 1 & 1 & 1 & 0 \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \end{array}\right) \Rightarrow \left(\begin{array}{cc|cc} 1 & 0 & \frac{3}{2} & -\frac{1}{2} \\ 0 & 1 & -\frac{1}{2} & \frac{1}{2} \end{array}\right)$$

$$\lim_{m\to\infty} A^m = \lim_{m\to\infty} Q \begin{pmatrix} -0,6 & 0 \\ 0 & 1 \end{pmatrix} Q^{-1} = \lim_{m\to\infty} Q \begin{pmatrix} 1^m & 0 \\ 0 & -0.6^m \end{pmatrix} Q^{-1}$$

$$= \begin{pmatrix} 1 & 1 \\ 1 & 3 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} \frac{3}{2} & -\frac{1}{2} \\ -\frac{1}{2} & \frac{1}{2} \end{pmatrix} = \underline{\underline{\begin{pmatrix} -\frac{1}{2} & \frac{1}{2} \\ -\frac{3}{2} & \frac{3}{2} \end{pmatrix}}}$$

c) $A = \begin{pmatrix} 0,4 & 0,7 \\ 0,6 & 0,3 \end{pmatrix}$

$$\det(A - \lambda I) = \det\begin{pmatrix} 0,4-\lambda & 0,7 \\ 0,6 & 0,3-\lambda \end{pmatrix} = (0,4-\lambda)(0,3-\lambda) - 0,42$$

$$= 0,12 + 0,1\lambda + \lambda^2 - 0,42 = \lambda^2 - 0,7\lambda - 0,3 = (\lambda - 1)(\lambda + 0,3)$$

$$\lambda_1 = 1 \qquad \lambda_2 = -0,3$$

$\lambda_1 = 1$:

$$A - I = \begin{pmatrix} -0,6 & 0,7 \\ 0,6 & -0,7 \end{pmatrix} \Rightarrow \left(\begin{array}{cc|c} -0,6 & 0,6 & a \\ 0,7 & -0,7 & b \end{array}\right) \sim \left(\begin{array}{cc|c} -0,6 & 0,7 & a \\ 0 & 0 & 6b+7a \end{array}\right) \Rightarrow v_1 = \begin{pmatrix} 7 \\ 6 \end{pmatrix}$$

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2)c) $\lambda_2 = -0{,}3$

$$A + 0{,}3I = \begin{pmatrix} 0{,}7 & 0{,}7 \\ 0{,}6 & 0{,}6 \end{pmatrix} \Rightarrow \left(\begin{array}{cc|c} 0{,}7 & 0{,}6 & a \\ 0{,}7 & 0{,}6 & b \end{array}\right) \sim \left(\begin{array}{cc|c} 0{,}7 & 0{,}6 & a \\ 0 & 0 & b-a \end{array}\right) \Rightarrow v_2 = \begin{pmatrix} -1 \\ -1 \end{pmatrix}$$

$$Q = \begin{pmatrix} 7 & -1 \\ 6 & 1 \end{pmatrix} \quad Q^{-1}: \begin{pmatrix} 7 & -1 & 1 & 0 \\ 6 & 1 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 1 & -1 \\ 6 & 1 & 0 & 1 \end{pmatrix} \sim \begin{pmatrix} 1 & -2 & 1 & -1 \\ 0 & 13 & -6 & 7 \end{pmatrix}$$

$$\sim \begin{pmatrix} 1 & -2 & 1 & -1 \\ 0 & 1 & \frac{-6}{13} & \frac{7}{13} \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & \frac{1}{13} & \frac{1}{13} \\ 0 & 1 & \frac{-6}{13} & \frac{7}{13} \end{pmatrix}$$

$$\lim_{m\to\infty} A^m = \lim_{m\to\infty} Q \begin{pmatrix} 1^m & 0 \\ 0 & -3^m \end{pmatrix} Q^{-1} = Q \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} Q^{-1}$$

$$= \begin{pmatrix} 7 & -1 \\ 6 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{13} & \frac{1}{13} \\ \frac{-6}{13} & \frac{7}{13} \end{pmatrix} = \begin{pmatrix} 7 & 0 \\ 6 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{13} & \frac{1}{13} \\ \frac{-6}{13} & \frac{7}{13} \end{pmatrix} = \underline{\underline{\begin{pmatrix} \frac{7}{13} & \frac{7}{13} \\ 6/13 & 6/13 \end{pmatrix}}}$$

d) $A = \begin{pmatrix} -1{,}8 & 4{,}8 \\ -0{,}8 & 2{,}2 \end{pmatrix}$ $\quad \det(A-\lambda I) = \det\begin{pmatrix} -1{,}8-\lambda & 4{,}8 \\ -0{,}8 & 2{,}2-\lambda \end{pmatrix} = (-1{,}8-\lambda)(2{,}2-\lambda) + 3{,}84$

$$= -3{,}96 - 0{,}4\lambda + \lambda^2 + 3{,}84 = \lambda^2 - 0{,}4\lambda - 0{,}12 = (\lambda - 0{,}6)(\lambda + 0{,}2)$$

$$|\lambda| < 1 \ \forall \lambda \Rightarrow \underline{\underline{\lim_{m\to\infty} A^m = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}}}$$

e) $A = \begin{pmatrix} -2 & -1 \\ 4 & 3 \end{pmatrix}$ $\quad \det(A-\lambda I) = \det\begin{pmatrix} -2-\lambda & -1 \\ 4 & 3-\lambda \end{pmatrix} = (-2-\lambda)(3-\lambda) + 4$

$$= -6 - \lambda + \lambda^2 + 4 = \lambda^2 - \lambda - 2 = (\lambda - 2)(\lambda + 1) \Rightarrow \lambda_1 = 2, \lambda_2 = 1$$

$|\lambda_2| > 1 \Rightarrow$ $\underline{\underline{\lim_{m\to\infty} A^m \text{ finnes ikke.}}}$

4) Vis at hvis $A \in M_{n\times n}(\mathbb{C})$ diagonaliserbar, og $L = \lim_{m\to\infty} A^m$, så er $L = I_n$ eller rank $L < n$

A diagonaliserbar $\Rightarrow Q^{-1}AQ = D$, $D = \begin{pmatrix} \lambda_1 & & 0 \\ & \ddots & \\ 0 & & \lambda_n \end{pmatrix}$

$L = \lim_{m\to\infty} Q D^m Q^{-1} \Rightarrow |\lambda_i| < 1$ eller $\lambda_i = 1 \quad \forall i$

$\lambda_i = 1 \ \forall i \Rightarrow L = I_n$

$\exists \lambda_i \neq 1 \Rightarrow |\lambda_i| < 1 \Rightarrow \lim_{m\to\infty} \lambda_i^m = 0 \Rightarrow$ rank $L < 0$

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8) Hvilke matr. regulær?

a) $A = \begin{pmatrix} 0.2 & 0.3 & 0.5 \\ 0.3 & 0.2 & 0.5 \\ 0.5 & 0.5 & 0 \end{pmatrix}$ $A^2 = \begin{pmatrix} 0.38 & 0.37 & 0.25 \\ 0.37 & 0.38 & 0.25 \\ 0.25 & 0.25 & 0.5 \end{pmatrix}$

$A^2_{ij} > 0 \ \forall \ i,j \Rightarrow$ $\underline{\underline{A \text{ regulær}}}$

b) $A = \begin{pmatrix} 0.5 & 0 & 1 \\ 0.5 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$ $A^2 = \begin{pmatrix} 0.25 & 1 & 0.5 \\ 0.25 & 0 & 0.5 \\ 0.5 & 0 & 0 \end{pmatrix}$

$A^4 = \begin{pmatrix} 0.5625 & 0.25 & 0.625 \\ 0.3125 & 0.25 & 0.125 \\ 0.125 & 0.5 & 0.25 \end{pmatrix}$

$A^4_{i,j} > 0 \ \forall \ i,j \Rightarrow$ $\underline{\underline{A \text{ regulær}}}$

c) $A = \begin{pmatrix} 0.5 & 0 & 0 \\ 0.5 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$ $A^2 = \begin{pmatrix} 0.25 & 0 & 0 \\ 0.25 & 1 & 0 \\ 0.5 & 0 & 1 \end{pmatrix}$ $A^3 = \begin{pmatrix} 0.125 & 0 & 0 \\ 0.625 & 0 & 1 \\ 0.25 & 1 & 0 \end{pmatrix}$

Ser at siste to kolonnene alltid inneholder 0, og kun bytter plass

$\Rightarrow$ $\underline{\underline{A \text{ ikke regulær}}}$

9) Finn $\lim_{m\to\infty} A^m$ der denne finnes

a) $A = \begin{pmatrix} 0.2 & 0.3 & 0.5 \\ 0.3 & 0.2 & 0.5 \\ 0.5 & 0.5 & 0 \end{pmatrix}$ $\det(A-\lambda I) = \det\begin{pmatrix} 0.2-\lambda & 0.3 & 0.5 \\ 0.3 & 0.2-\lambda & 0.5 \\ 0.5 & 0.5 & -\lambda \end{pmatrix}$

$= \Big((0.2-\lambda)(0.2-\lambda)(-\lambda) + 0.3\cdot 0.5\cdot 0.5 + 0.5\cdot 0.3\cdot 0.5\Big) - \Big((0.2-\lambda)(0.25) + 0.3\cdot 0.3(-\lambda) +$

$0.5(0.2-\lambda)0.5\Big)$

$= -0.04\lambda + 0.4\lambda^2 - \lambda^3 + 0.075 + 0.075 - 0.05 + 0.25\lambda + 0.09\lambda - 0.05 + 0.25\lambda$

$= -(\lambda^3 + 0.4\lambda^2 - 0.55\lambda - 0.05)$

$\Rightarrow \lambda_1 = 1 \quad \lambda_2 = -\frac{1}{10} \quad \lambda_3 = -\frac{1}{2}$

$\lambda_1 = 1$:

$A - I = \begin{pmatrix} -0.8 & 0.3 & 0.5 \\ 0.3 & -0.8 & 0.5 \\ 0.5 & 0.5 & -1 \end{pmatrix} \Rightarrow \left(\begin{array}{ccc|c} -0.8 & 0.3 & 0.5 & a \\ 0.3 & -0.8 & 0.5 & b \\ 0.5 & 0.5 & -1 & c \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 0 & 0 & a+b+c \\ & & & b \\ & & & c \end{array}\right) \Rightarrow v_1 = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$

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9) a) $\lambda_2 = -\frac{1}{10}$:

$$(A + 0{,}1I)^T = \left(\begin{array}{ccc|c} 0{,}3 & 0{,}3 & 0{,}5 & a \\ 0{,}3 & 0{,}3 & 0{,}5 & b \\ 0{,}5 & 0{,}5 & 0{,}1 & c \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 0 & 0 & a-b \\ & & & b \\ & & & c \end{array}\right) \Rightarrow v_2 = \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}$$

$\lambda_3 = -\frac{1}{2}$

$$(A + 0{,}5I)^T = \left(\begin{array}{ccc|c} 0{,}7 & 0{,}3 & 0{,}5 & a \\ 0{,}3 & 0{,}7 & 0{,}5 & b \\ 0{,}5 & 0{,}5 & 0{,}5 & c \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 0 & 0 & a+b-2c \\ & & & b \\ & & & c \end{array}\right) \Rightarrow v_3 = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

$$Q = \begin{pmatrix} 1 & -1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & -2 \end{pmatrix} \qquad Q^{-1} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{2} & \frac{1}{2} & 0 \\ \frac{1}{6} & \frac{1}{6} & -\frac{1}{3} \end{pmatrix}$$

$$\lim_{m\to\infty} A^m = \lim_{m\to\infty} Q \begin{pmatrix} 1^m & 0 & 0 \\ 0 & -0{,}1^m & 0 \\ 0 & 0 & -0{,}5^m \end{pmatrix} Q^{-1} = Q \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} Q^{-1}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ & & \\ & & \end{pmatrix} = \underline{\underline{\begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}}}$$

9) b) $$A = \begin{pmatrix} 0{,}5 & 0 & 1 \\ 0{,}5 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \quad \det(A - \lambda I) = \det \begin{pmatrix} 0{,}5-\lambda & 0 & 1 \\ 0{,}5 & -\lambda & 0 \\ 0 & 1 & -\lambda \end{pmatrix}$$

$$= (0{,}5-\lambda)(-\lambda)(-\lambda) + 0{,}5 - 0 = -\lambda^3 + 0{,}5\lambda^2 + 0{,}5$$

$$\lambda_1 = 1 \qquad \lambda_2 = -\frac{1}{4} + \frac{\sqrt{7}}{4}i \qquad \lambda_3 = -\frac{1}{4} - \frac{\sqrt{7}}{4}i$$

$\lambda_1 = 1$:

$$(A - I)^T = \left(\begin{array}{ccc|c} -0{,}5 & 0{,}5 & 0 & a \\ 0 & -1 & 1 & b \\ 1 & 0 & -1 & c \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 0 & 0 & 2a+b+c \\ & & & b \\ & & & c \end{array}\right) \Rightarrow v_1 = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$$

$\lambda_2 = -\frac{1}{4} + \frac{\sqrt{7}}{4}i$

$$(A - \lambda_2 I) = \begin{pmatrix} 0{,}75 - \frac{\sqrt{7}}{4}i & 0 & 1 \\ 0{,}5 & \frac{1}{4} - \frac{\sqrt{7}}{4}i & 0 \\ 0 & 1 & \frac{1}{4} - \frac{\sqrt{7}}{4}i \end{pmatrix} \Rightarrow \left(\begin{array}{ccc|c} 0{,}75 - \frac{\sqrt{7}}{4}i & 0{,}5 & 0 & a \\ 0 & 0{,}25 - \frac{\sqrt{7}}{4}i & 1 & b \\ 1 & 0 & 0{,}75 - \frac{\sqrt{7}}{4}i & c \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 0{,}5(0{,}75 + \frac{\sqrt{7}}{4}i) & 0 & (0{,}75 + \frac{\sqrt{7}}{4}i)\,a \\ 0 & 0{,}5 & 0{,}25 + \frac{\sqrt{7}}{4}i & (0{,}25 + \frac{\sqrt{7}}{4}i)\,b \\ 1 & 0 & 0{,}25 - \frac{\sqrt{7}}{4}i & c \end{array}\right)$$

Jeg aner ikke hvor jeg skal eller om jeg er langt på viddene. Gir opp denne oppg. !!

9) c) $A = \begin{pmatrix} 0.5 & 0 & 0 \\ 0.5 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$

Har sett at siste to kolonnene konstant bytter plass, uten annen endring. Der for finnes ikke $\lim_{m\to\infty} A^m$.

13) Endring 1975-1985 (stor-med-liten):

$$A = \begin{pmatrix} 0.7 & 0.1 & 0 \\ 0.3 & 0.7 & 0.1 \\ 0 & 0.2 & 0.9 \end{pmatrix}$$

Startvektor (1975): $P = \begin{pmatrix} 0.4 \\ 0.2 \\ 0.4 \end{pmatrix}$

1995:

$$A^2P = \begin{pmatrix} 0.52 & 0.14 & 0.01 \\ 0.42 & 0.54 & 0.16 \\ 0.06 & 0.32 & 0.83 \end{pmatrix}\begin{pmatrix} 0.4 \\ 0.2 \\ 0.4 \end{pmatrix} = \begin{pmatrix} 0.24 \\ 0.34 \\ 0.42 \end{pmatrix}$$

I 1995: 24% stor, 34% middels, 42% liten bil

Finn $\lim_{m\to\infty} A^m P$

$$\det(A-\lambda I) = \det\begin{pmatrix} 0.7-\lambda & 0.1 & 0 \\ 0.3 & 0.7-\lambda & 0.1 \\ 0 & 0.2 & 0.9-\lambda \end{pmatrix}$$

$$= (0.7-\lambda)^2(0.9-\lambda) - (0.7-\lambda)0.02 - (0.9-\lambda)0.03$$

$$= -\lambda^3 + 2.3\lambda^2 - 1.7\lambda + 0.4$$

$$\Rightarrow \lambda_1 = 1, \quad \lambda_2 = \frac{1}{2} \quad \lambda_3 = \frac{4}{5}$$

$\lambda_1 = 1$:

$$(A-I)^T = \left(\begin{array}{ccc|c} -0.3 & 0.3 & 0 & a \\ 0.1 & -0.3 & 0.2 & b \\ 0 & 0.1 & -0.1 & c \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 0 & 0 & a+3b+6c \\ & & & b \\ & & & c \end{array}\right) \Rightarrow v_1 = \begin{pmatrix} 1 \\ 3 \\ 6 \end{pmatrix}$$

$\lambda_2 = \frac{1}{2}$:

$$(A-\tfrac{1}{2}I)^T = \left(\begin{array}{ccc|c} 0.2 & 0.3 & 0 & a \\ 0.1 & 0.2 & 0.2 & b \\ 0 & 0.1 & 0.4 & c \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 0 & 0 & a-2b+c \\ & & & b \\ & & & c \end{array}\right) \Rightarrow v_2 = \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}$$

$\lambda_3 = \frac{4}{5}$:

$$(A-\tfrac{4}{5}I)^T = \left(\begin{array}{ccc|c} -0.1 & 0.3 & 0 & a \\ 0.1 & -0.1 & 0.2 & b \\ 0 & 0.1 & 0.1 & c \end{array}\right) \sim \left(\begin{array}{ccc|c} 0 & 0 & 0 & a+b-2c \\ & & & b \\ & & & c \end{array}\right) \Rightarrow v_3 = \begin{pmatrix} 1 \\ 1 \\ -2 \end{pmatrix}$$

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13) $Q = \begin{pmatrix} 1 & 1 & 1 \\ 3 & -2 & 1 \\ 6 & 1 & -2 \end{pmatrix}$ $\qquad Q^{-1} = \begin{pmatrix} \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\ \frac{2}{5} & -\frac{4}{15} & \frac{1}{15} \\ \frac{1}{2} & \frac{1}{6} & -\frac{1}{6} \end{pmatrix}$

$$\lim_{m\to\infty} A^m = \lim_{m\to\infty} Q \begin{pmatrix} 1^m & 0 & 0 \\ 0 & (\frac{1}{2})^m & 0 \\ 0 & 0 & (\frac{4}{5})^m \end{pmatrix} Q^{-1} = Q \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} Q^{-1}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 3 & 0 & 0 \\ 6 & 0 & 0 \end{pmatrix} \begin{pmatrix} \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\ \text{---} & \text{---} & \text{---} \\ \text{---} & \text{---} & \text{---} \end{pmatrix} = \begin{pmatrix} \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\ 3/10 & 3/10 & 3/10 \\ 6/10 & 6/10 & 6/10 \end{pmatrix}$$

$$\lim_{m\to\infty} A^m P = \begin{pmatrix} \frac{1}{10} & \frac{1}{10} & \frac{1}{10} \\ \frac{3}{10} & \frac{3}{10} & \frac{3}{10} \\ \frac{3}{5} & \frac{3}{5} & \frac{3}{5} \end{pmatrix} \begin{pmatrix} 0,4 \\ 0,2 \\ 0,4 \end{pmatrix} = \underline{\underline{\begin{pmatrix} 0,1 \\ 0,3 \\ 0,6 \end{pmatrix}}}$$

<u>Det går mot 10% stor, 30% middels, 60% liten bil</u>