PAPER CODE NO. **COMP527**	EXAMINER : Dr. Danushka Bollegala DEPARTMENT : Computer Science Tel. No. 0151 7954283

UNIVERSITY OF LIVERPOOL

Second Semester Examinations 2016/17

Data Mining and Visualisation

TIME ALLOWED : Two and a Half Hours

INSTRUCTIONS TO CANDIDATES

Answer FOUR questions.

If you attempt to answer more questions than the required number of questions (in any section), the marks awarded for the excess questions answered will be discarded (starting with your lowest mark).

Question 1 Consider two sentences S_1 = *I like data mining* and S_2 = *I do not like data science*. Answer the following questions about S_1 and S_2

A. Write all unigrams in S_1. **(2 marks)**

I, like, data, mining

B. Write all bigrams in S_2. **(2 marks)**

I+do, do+not, not+like, like+data, data+science

C. What is meant by *stop words* in text mining? **(2 marks)**

Stop words are non-content features such as prepositions and articles. For example, *the, an, what, etc.*

D. Assuming unigrams to be the feature space, represent S_1 and S_2 respectively by two vectors $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$, where the elements corresponds to the number times the corresponding unigram feature occurs in the sentence. **(4 marks)**

Let us assign indexes to the features as follows: I=0, like=1, data=2, mining=3, not=4, science=5, do=6. Then $\boldsymbol{s}_1 = (1, 1, 1, 1, 0, 0, 0)^\top$ and $\boldsymbol{s}_2 = (1, 1, 1, 0, 1, 1, 1)^\top$.

E. Compute the inner-product between $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$. **(3 marks)**

$\boldsymbol{s}_1{}^\top \boldsymbol{s}_2 = 3$

F. Compute the ℓ_2 norms of $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$. **(2 marks)**

$||\boldsymbol{s}_1||_2 = 2$, $||\boldsymbol{s}_2||_2 = \sqrt{6}$

G. Compute the cosine similarity between the two sentences S_1 and S_2. **(2 marks)**

$\cos(S_1, S_2) = \frac{\boldsymbol{s}_1{}^\top \boldsymbol{s}_2}{||\boldsymbol{s}_1||_2 ||\boldsymbol{s}_2||_2} = 3/2\sqrt{6}$

H. Compute the Manhattan distance between $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$. **(2 marks)**

$|1 - 1| + |1 - 1| + |1 - 1| + |1 - 0| + |0 - 1| + |0 - 1| + |0 - 1| = 4$

I. Compute the Euclidean distance between $\boldsymbol{s}_1$ and $\boldsymbol{s}_2$. **(2 marks)**

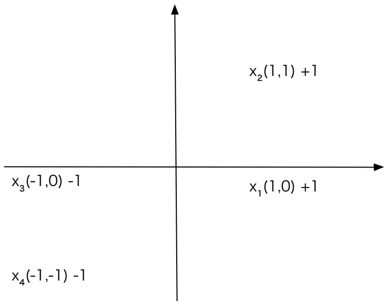
$\sqrt{(1 - 1)^2 + (1 - 1)^2 + (1 - 1)^2 + (1 - 0)^2 + (0 - 1)^2 + (0 - 1)^2 + (0 - 1)^2} = 2$

J. Despite the two sentences are expressing opposite opinions, the cosine similarity measured in **G** gives a value greater than 0.5 indicating a high-degree of similarity. Suggest a solution to overcome this problem. **(4 marks)**

The issue here is that the negation indicator *not* matching (or not matching) between the sentences has equal contribution towards the cosine similarity as with any other feature. We can emphasise negation by assigning higher weights (larger than 1) to negation related features such as *not* to overcome this problem.

Question 2 Consider a training dataset $\{(\boldsymbol{x}_n, t_n)\}_{n=1}^{4}$, where $\boldsymbol{x}_n \in \mathbb{R}^2$ and $t_n \in \{-1, 1\}$. Here, $\boldsymbol{x}_1 = (1,0)^\top$, $\boldsymbol{x}_2 = (1,1)^\top$, $\boldsymbol{x}_3 = (-1,0)^\top$, and $\boldsymbol{x}_4 = (-1,-1)^\top$. Moreover, the labels $t_1 = t_2 = 1$ and $t_3 = t_4 = -1$. Let us consider a support vector machine defined by a weight vector $\boldsymbol{w} = (\alpha, \beta)^\top$ and a bias term b. Here, $\alpha, \beta, b \in \mathbb{R}$.

A. Plot the training dataset in two dimensional space. **(2 marks)**



B. Explain what is meant by the inequality $t_n(\boldsymbol{w}^\top \boldsymbol{x}_n + b) \geq 1$ with respect to each training data point. **(2 marks)**

This inequality means that each train data point must be classified correctly by the decision hyperplane $\boldsymbol{w}$ with a margin of at least 1.

C. Write down the four inequalities that must be satisfied by the four data points in the training dataset if they are to be correctly classified. **(4 marks)**

$$\begin{aligned} \alpha + b &\geq 1 \\ \alpha + \beta + b &\geq 1 \\ \alpha - b &\geq 1 \\ \alpha + \beta - b &\geq 1 \end{aligned}$$

One mark is awarded for each inequality.

D. Show that the maximisation of the margin corresponds to the minimisation of $\alpha^2 + \beta^2$. **(4 marks)**

Maximisation of the margin corresponds to the minimisation of the ℓ_2 norm of the vector w, which is $\alpha^2 + \beta^2$.

E. Using Lagrange multipliers for the four inequalities, write the objective function $L(\alpha, \beta, b, \boldsymbol{\lambda})$ where $\boldsymbol{\lambda}$ is a vector containing the four Lagrange multipliers given by $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \lambda_3, \lambda_4)^\top$ each respectively for the inequalities corresponding to the four data points x_1, x_2, x_3, x_4. **(2 marks)**

$L(\alpha, \beta, b, \boldsymbol{\lambda}) = \frac{1}{2}(\alpha^2 + \beta^2) - \lambda_1(\alpha + b - 1) - \lambda_2(\alpha + \beta + b - 1) - \lambda_3(\alpha - b - 1) - \lambda_4(\alpha + \beta - b - 1)$

F. Find the values of α, β, b by minimising the objective function defined in **E**. **(7 marks)**

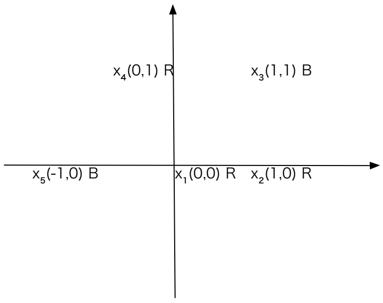
By setting the gradients of L w.r.t. α, β, b to zero, we obtain three equations and w.r.t. each Lagrangian multiplier another four equations. By solving these equations we obtain $\alpha = 1, \beta = 0, b = 0, \lambda_1 = \lambda_3 = 1/2, \lambda_2 = \lambda_4 = 0$. One mark is awarded for each variable.

G. Show that the decision hyperplane you obtained in **F** correctly classifies the four data points in the training dataset. **(4 marks)**

The weight vector is $(1, 0)$. Therefore, we have $y_1 = y_2 = 1 > 0$ and $y_3 = y_4 = -1 < 0$. One mark is awarded for each data point.

Question 3 Consider five data points in $\mathbb{R}^2$ given by $\boldsymbol{x}_1 = (0,0)^\top$, $\boldsymbol{x}_2 = (1,0)^\top$, $\boldsymbol{x}_3 = (1,1)^\top$, $\boldsymbol{x}_4 = (0,1)^\top$, and $\boldsymbol{x}_5 = (-1,0)^\top$. Here, data points $\boldsymbol{x}_1, \boldsymbol{x}_2$, and $\boldsymbol{x}_4$ are labelled with the colour red, whereas data points $\boldsymbol{x}_3$ and $\boldsymbol{x}_5$ are labelled with the colour blue. Answer the following questions about this dataset.

A. Plot this dataset in two-dimensional space. **(3 marks)**



B. Assuming that we used k-means clustering to cluster this dataset into two clusters, and we set the initial cluster centres at $\boldsymbol{x}_2$ and $\boldsymbol{x}_3$. Compute the clusters after the first assignment. **(4 marks)**

$c_1 = \{3, 4\}, c_2 = \{1, 2, 5\}$

C. How many further iterations does it take for the k-means algorithms to cluster for this dataset. Justify your answer. **(4 marks)**

0 further iterations. The k-means clustering has converged because the points 4 and 3 are the closest to c_1 and points 5, 1, and 2 are the closest to c_2. Therefore, the cluster centroids do not get updated any further. Answers without the justification will receive only 2 marks.

D. Using the B-cubed method compute the precision for the final two clusters obtained at convergence. **(4 marks)**

$P(x_4) = P(x_3) = 1/2, P(x_1) = P(x_2) = 2/3, P(x_5) = 1/3$. Therefore, the overall precision will be the summation of individual precisions divided by the total number of instances (5) in the dataset, which is 8/15.

E. Using the B-cubed method compute the recall for the final two clusters obtained at convergence. **(4 marks)**

$R(x_4) = 1/3, R(x_3) = 1/2, R(x_1) = R(x_2) = 2/3, R(x_5) = 1/2$. Therefore, the overall recall will be the summation of individual precisions divided by the total number of instances (5) in the dataset, which is 8/15.

F. Using the precision and recall values, compute the F-score for the final two clusters obtained at convergence. **(3 marks)**

$F = \frac{2PR}{P+R} = 8/15$

G. Instead of creating two clusters, we would like to obtain three clusters via k-means where we use the data points 5, 1 and 2 as the centroids of the three initial clusters. Write down the final three clusters at convergence. **(3 marks)**

$\{5\}, \{1, 4\}, \{2, 3\}$

Question 4

A. Consider a biased coin that returns head with probability p. Let us assume that when this coin was flipped n times, we got $k(< n)$ heads. Answer the following questions about this event.

(a) Compute the likelihood of observing $k(< n)$ heads when this coin was flipped for n times. **(2 marks)**

$L = p^k(1-p)^{n-k}$

(b) Using the maximum likelihood estimation, compute the most likely value of p for this probabilistic event. **(4 marks)**

$\frac{\partial \log(L)}{\partial p} = \frac{\partial}{\partial p} k \log(p) + (n-k)\log(1-p) = 0$. This gives, $k/p - (n-k)/(1-p) = 0$, from which we obtain $p = k/n$.

B. Consider the dataset shown in Table 1 consisting of five reviews x_1, x_2, x_3, x_4, x_5 defined over three integer-valued attributes a_1, a_2, a_3 corresponding to the frequency of occurrences of a particular word in the document. The positive or negative sentiments label (t) of each review are denoted respectively by +1 and -1 in Table 1. Assuming that the three attributes to be mutually independent, and the probability of a review to be given by the product of the probabilities of individual attributes raised to their occurrences in the review, answer the following questions.

Review	a_1	a_2	a_3	label (t)
x_1	1	1	0	1
x_2	2	1	0	1
x_3	1	2	3	-1
x_4	0	1	1	-1
x_5	1	0	1	-1

Table 1: A set of five reviews represented using three attributes.

(a) Compute the marginal probabilities $p(a_1), p(a_2)$ and $p(a_3)$. **(3 marks)**

$p(a_1) = p(a_2) = p(a_3) = 1/3$

(b) Compute the conditional probabilities $p(a_1|t = 1), p(a_2|t = 1)$ and $p(a_3|t = 1)$. **(3 marks)**

$p(a_1|t=1) = 3/5$, $p(a_2|t=1) = 2/5$, and $p(a_3|t=1) = 0$

(c) Assuming that the prior probabilities of the sentiment labels to be equal (i.e. $p(t = 1) = p(t = -1) = 0.5$), compute $p(t = -1|x_3)$, the probability of x_3 being negative. **(4 marks)**

$p(t=1|x_3) = \frac{p(x_3|t=1)}{p(x_3}$.
However, note that

$$p(x_3|t=1) = p(a_1|t=1)^1 \times p(a_2|t=1)^2 \times p(a_3|t=1)^3 = 0$$

because $p(a_3|t=1) = 0$. Therefore, $p(t=-1|x_3) = 1 - p(t=1|x_3) = 1$.

Review	a_1	a_2	a_3	label (t)
x_1	2	2	1	1
x_2	3	2	1	1
x_3	2	3	4	-1
x_4	1	2	2	-1
x_5	2	1	2	-1

Table 2: Smoothed counts.

(d) Apply Laplace smoothing for the occurrences of attributes in reviews shown in Table 1. Compute $p(t = 1|x_3)$ using the smoothed counts. **(4 marks)**

Smoothed counts are shown in Table 2. $p(a_1) = p(a_2) = p(a_3) = 10/30 = 1/3$, $p(a_1|t = 1) = 5/10, p(a_2|t = 1) = 4/10, p(a_3|t = 1) = 2/10$. Therefore, we have,

$$\begin{aligned} p(t = 1|x_3) &= \frac{p(x_3|t = 1)p(t = 1)}{p(x_3)} \\ &= \frac{p(a_1|t = 1)^2 p(a_2|t = 1)^3 p(a_3|t = 1)^4 p(t = 1)}{p(a_1)^2 p(a_2)^3 p(a_3)^4} \\ &= \frac{(1/2)^2(2/5)^3(1/5)^4(1/2)}{(1/3)^2(1/3)^3(1/3)^4}. \end{aligned}$$

2 marks are awarded for computing the smoothed counts and another 2 marks for correctly computing $p(t = 1|x_3)$. No penalties for not simplifying the answers.

(e) Assuming the reviews to be independent, compute the likelihood of this dataset using the smoothed counts. **(5 marks)**

Likelihood of the dataset is given by $p(x_1)p(x_2)p(x_3)p(x_4)p(x_5)$. By substituting $p(a_1), p(a_2), p(a_3)$ and aggregating the powers for each attributes we have, $p(a_1)^{10}p(a_2)^{10}p(a_3)^{10} = \frac{1}{3^{30}}$.

Question 5 We would like to project the four data points $x_1 = (0, 2), x_2 = (-1, 0), x_3 = (0, -2), x_4 = (1, 0)$ shown in Figure 1 onto the $y = \tan(\theta)x$ line that passes through the origin $O = (0, 0)$ and has an angle $0 < \theta < \pi/2$ with the positive direction of the x-axis. The four corresponding projected points along the line segment are shown by A_1, A_2, A_3 and A_4. Answer the following questions.

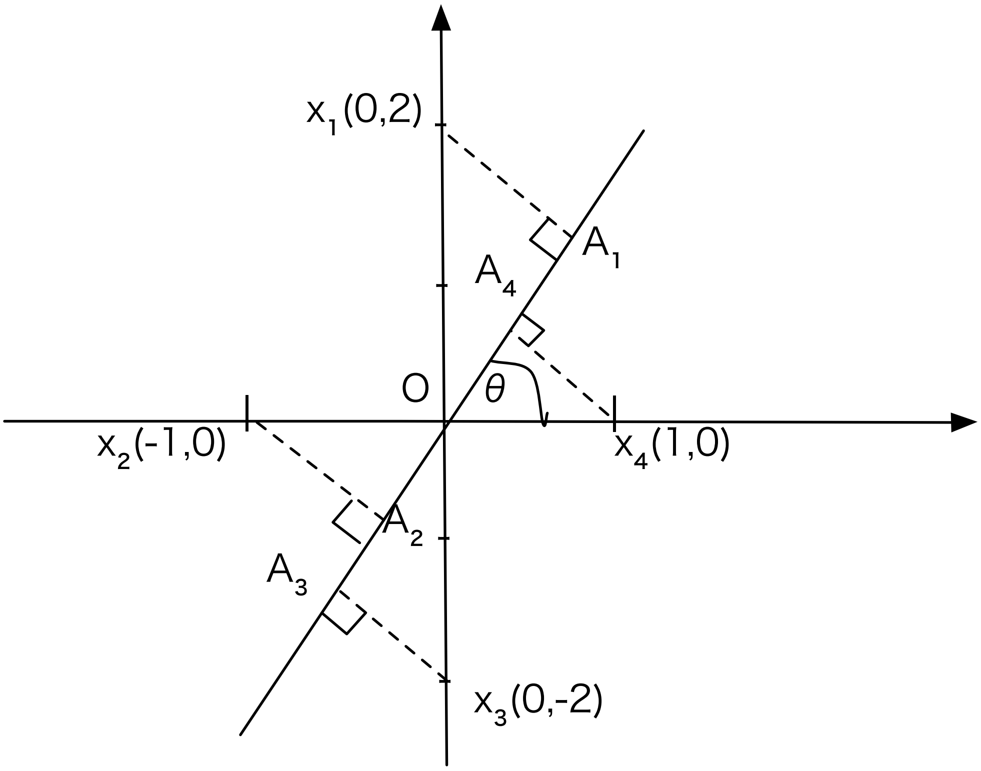


Figure 1: Four data points projected onto a straight line.

A. State two methods that can be used to project high dimensional data onto a lower dimensional space. **(2 marks)**

Principle Component Analysis, Singular Value Decomposition

B. Compute the perpendicular distances x_1A_1, x_2A_2, x_3A_3, and x_4A_4 to the projection line $y = \tan(\theta)x$ respectively from the four points x_1, x_2, x_3, and x_4. **(4 marks)**

Let $d_1 = x_1A_1, d_2 = x_2A_2, d_3 = x_3A_3, d_4 = x_4A_4$. Then we have, $d_1 = 2\cos(\theta), d_2 = \sin(\theta), d_3 = 2\cos(\theta), d_4 = \sin(\theta)$

C. Find the value of $\tan(\theta)$ for which the projection error is minimised. **(4 marks)**

The total projection error, $e = d_1 + d_2 + d_3 + d_4 = 2\sin(\theta) + 4\cos(\theta)$. To find the minimum value of this error, we differentiate w.r.t. θ and set the result to zero.
$\frac{\partial e}{\partial \theta} = 2\cos(\theta) - 4\sin(\theta) = 0$, which gives $\tan(\theta) = 1/2$.

D. Compute the distances $e_1 = OA_1$, $e_2 = OA_2$, $e_3 = OA_3$ and $e_4 = OA_4$ to each of the projected points A_1, A_2, A_3, A_4 from the origin O. **(4 marks)**

$e_1 = 2\sin(\theta), e_2 = \cos(\theta), e_3 = 2\sin(\theta), e_4 = \cos(\theta)$

E. Compute the mean $\mathbb{E}[e]$ of the distances e_1, e_2, e_3, e_4. **(2 marks)**

Adding all four values and dividing by four we get, $\sin(\theta) + 0.5\cos(\theta)$.

F. Compute the variance $\mathbb{V}[e]$ of the distances e_1, e_2, e_3, e_4. (You may use the formula, $\mathbb{V}[e] = \mathbb{E}[e^2] - (\mathbb{E}[e])^2$ if required.) **(4 marks)**

$\mathbb{E}[e^2] = 0.25(4\sin^2(\theta) + \cos^2(\theta) + 4\sin^2(\theta) + \cos^2(\theta)) = 0.5(3\sin^2(\theta) + 1)$. Therefore, $\mathbb{V}[e] = \frac{3\sin^2(\theta)+1}{2} - \left(\sin(\theta) + \frac{1}{2}\cos(\theta)\right)^2 = 3/4\sin^2(\theta)\cos(\theta) - \frac{1}{2} - 12\sin(2\theta)$.

G. Compute the value of θ that maximises the variance of the projected points. (You may use the identities $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$, $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$, and $\tan(2\theta) = \frac{2\tan(\theta)}{1-\tan^2(\theta)}$, if required.) **(5 marks)**

To find the optimal value of θ we set $\frac{\partial \mathbb{V}[e]}{\partial \theta}$ to zero. This gives, $3/2\sin(\theta)\cos(\theta) - \cos(2\theta) = 3/4\sin(2\theta) - \cos(2\theta) = 0$. Solving this for θ we get $(\tan(\theta) + 2)(2\tan(\theta) - 1) = 0$. Because $0 < \theta < \pi/2$, we have $\tan(\theta) = 1/2$. Therefore, maximising the projection error and minimising the variance of the projected points lead to the same optimal solution.