COMP527
Data Mining and Visualisation
Problem Set 0

Danushka Bollegala

Question 1   Consider two vectors $x, y \in \mathbb{R}^3$ defined as $x = (1, 2, -1)^\top$ and $y = (-1, 0, 1)^\top$. Answer the following questions about these two vectors.

   A. Compute the length (ℓ_2 norm) of x and y.   **(4 marks)**

   B. Compute the inner product between x and y.   **(2 marks)**

   C. Compute the cosine of the angle between the two vectors x and y. **(4 marks)**

   D. Compute the Euclidean distance between the end points corresponding to the two vectors x and y.   **(4 marks)**

   E. For any two vectors $x, y \in \mathbb{R}^d$ such that $||x||_2 = ||y||_2 = 1$ show that the following relationship holds between their cosine similarity $\cos(x, y)$ and their Euclidean distance $\mathrm{Euc}(x, y)$.   **(6 marks)**

$$\mathrm{Euc}(x, y)^2 = 2(1 - \cos(x, y))$$

Question 2   Consider a matrix $\mathbf{A} \in \mathbb{R}^{2 \times 2}$ defined as follows:

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

Answer the following questions related to $\mathbf{A}$.

A. Compute the transpose $\mathbf{A}^{\top}$. **(2 marks)**

B. Compute the determinant $\det(\mathbf{A})$. **(2 marks)**

C. Compute the inverse $\mathbf{A}^{-1}$. **(4 marks)**

D. Compute the eigenvalues and eigenvectors of $\mathbf{A}$. **(6 marks)**

Question 3

A. Given $\sigma(x) = \frac{1}{1+\exp(ax+b)}$, compute $\sigma'(x)$, the differential of $\sigma(x)$ with respect to x.

B. Given $H(p) = -p\log(p) - (1-p)\log(1-p)$, find the value of p that maximises $H(p)$.

C. Find the maximum value of $g(x,y) = x^2 + y^2$ such that $y \leq -x + 1$.