

COMP527  
Data Mining and Visualisation  
Problem Set 0

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**Question 1** Consider two vectors  $\mathbf{x}, \mathbf{y} \in \mathbb{R}^3$  defined as  $\mathbf{x} = (1, 2, -1)^\top$  and  $\mathbf{y} = (-1, 0, 1)^\top$ . Answer the following questions about these two vectors.

- A. Compute the length ( $\ell_2$  norm) of  $\mathbf{x}$  and  $\mathbf{y}$ . **(4 marks)**
- B. Compute the inner product between  $\mathbf{x}$  and  $\mathbf{y}$ . **(2 marks)**
- C. Compute the cosine of the angle between the two vectors  $\mathbf{x}$  and  $\mathbf{y}$ . **(4 marks)**
- D. Compute the Euclidean distance between the end points corresponding to the two vectors  $\mathbf{x}$  and  $\mathbf{y}$ . **(4 marks)**
- E. For any two vectors  $\mathbf{x}, \mathbf{y} \in \mathbb{R}^d$  such that  $\|\mathbf{x}\|_2 = \|\mathbf{y}\|_2 = 1$  show that the following relationship holds between their cosine similarity  $\cos(\mathbf{x}, \mathbf{y})$  and their Euclidean distance  $\text{Euc}(\mathbf{x}, \mathbf{y})$ . **(6 marks)**

$$\text{Euc}(\mathbf{x}, \mathbf{y})^2 = 2(1 - \cos(\mathbf{x}, \mathbf{y}))$$

**Question 2** Consider a matrix  $\mathbf{A} \in \mathbb{R}^{2 \times 2}$  defined as follows:

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$$

Answer the following questions related to  $\mathbf{A}$ .

- A. Compute the transpose  $\mathbf{A}^\top$ . **(2 marks)**
- B. Compute the determinant  $\det(\mathbf{A})$ . **(2 marks)**
- C. Compute the inverse  $\mathbf{A}^{-1}$ . **(4 marks)**
- D. Compute the eigenvalues and eigenvectors of  $\mathbf{A}$ . **(6 marks)**

**Question 3**

- A. Given  $\sigma(x) = \frac{1}{1+\exp(ax+b)}$ , compute  $\sigma'(x)$ , the differential of  $\sigma(x)$  with respect to  $x$ .
- B. Given  $H(p) = -p \log(p) - (1-p) \log(1-p)$ , find the value of  $p$  that maximises  $H(p)$ .
- C. Find the maximum value of  $g(x, y) = x^2 + y^2$  such that  $y \leq -x + 1$ .