# AEEM4063 Airbreathing Propulsion

## Project 1


Team 2

Ben Rupe, Matt Boller, Pierce Elliott, Jon Frueh


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## Hand Calculation Problem Statement

You have been tasked with designing a high-bypass ratio (BPR), two-spool, separate exhaust (unmixed) turbofan engine to meet the mission requirements for a new passenger aircraft. This new aircraft is expected to have a maximum takeoff weight of $W_{TO} = 370{,}000\ lbf$ and wing surface area $S_W = 285\ m^2$ and the primary mission goal is to minimize fuel consumption.

In steady-level flight, the lift is equal to the aircraft weight and the drag on the vehicle is given by $C_D = 0.056C_L^2 - 0.004C_L + 0.0140$. $L = C_L q S_W$ and $D = C_D q S_W$ where $q = \frac{\rho V^2}{2} = \frac{\gamma}{2} P M_0^2$

a) Hand calculate the <u>ideal cycle performance</u> and <u>real cycle performance</u> (accounting for component efficiencies) for an engine with a thrust to offset the drag of the vehicle at the following conditions ($\pi = total\ pressure\ ratio$)

| | | | | |
|---|---|---|---|---|
| $h = 11\ km$ | $\gamma_c = 1.4$ | $\pi_f = 1.5$ | $\eta_{\infty f} = 0.89$ | $\eta_m = 0.99$ |
| $M_0 = 0.85$ | $\gamma_h = 1.333$ | $\pi_c = 36$ | $\eta_{\infty c} = 0.90$ | $\eta_j = 0.99$ |
| $W = 0.8W_{TO}$ | $T_{t4} = 1560\ K$ | $\pi_b = 0.96$ | $\eta_b = 0.99$ | $P_9 = P_{19} = P_0$ |
| $BPR = 10$ | $h_{fuel} = 43{,}100\ kJ/kg$ | $\eta_i = 0.98$ | $\eta_{\infty t} = 0.90$ | |

- Determine the mass flow required to achieve the required thrust and the fan diameter required to capture the required air mass flow.
- Draw a T-s diagram illustrating the thermodynamic stations and paths comparing the ideal and real cycles

## Ideal Cycle performance:

**Lift/Drag**

$$W_{TO} = 370{,}000\ lbf$$

$$L = W = 0.8W_{TO} = 296{,}000\ lbf * \frac{4.44822N}{1\ lbf} = 1{,}316{,}673\ N$$

$$q = \frac{\rho V^2}{2} = \frac{\gamma}{2} P_0 M_0^2$$

$$\frac{P_0}{P_{std}} = 0.2240$$

$$P_{std} = 101{,}325 \frac{N}{m^2}$$

$$P_0 = 0.2240\left(101{,}325 \frac{N}{m^2}\right) = 22{,}696.8 \frac{N}{m^2}$$

$$q = \frac{1.4}{2}\left(22{,}696.8 \frac{N}{m^2}\right)(0.85)^2 = 11{,}478.9 \frac{N}{m^2}$$

$$L = C_L q S_W$$

$$C_L = \frac{L}{qS_W} = \frac{1{,}316{,}673\ N}{\left(11{,}478.9 \frac{N}{m^2}\right)(285\ m^2)} = 0.40247$$

$$C_D = 0.056C_L^2 - 0.004C_L + 0.0140 = 0.02146$$

$$F = D = C_D q S_W = 0.02146\left(11{,}478.9 \frac{N}{m^2}\right)(285\ m^2) = 70{,}206\ N$$

Required thrust is **70,206 N**

**Fan**

$$\frac{T_0}{T_{std}} = 0.7523$$

$$T_{std} = 288.15\ K$$

$$T_0 = 0.7523(288.15\ K) = 216.78\ K$$

$$T_{02} = T_0\left(1 + \frac{\gamma - 1}{2} M^2\right) = 216.78\ K\left(1 + \frac{1.4 - 1}{2} 0.85^2\right) = 248.1\ K$$

$$P_{02} = P_0 \left(\frac{T_{02}}{T_0}\right)^{\frac{\gamma}{\gamma-1}} = 22{,}696.8\,\frac{N}{m^2}\left(\frac{248.1\,K}{216.78\,K}\right)^{\frac{1.4}{1.4-1}} = 36{,}399.1\,\frac{N}{m^2}$$

$$\frac{P_{013}}{P_{02}} = 1.5 = \left(\frac{T_{013}}{T_{02}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$T_{013} = T_{02}\left(\frac{P_{013}}{P_{02}}\right)^{\frac{\gamma-1}{\gamma}} = 248.1\,K(1.5)^{\frac{1.4-1}{1.4}} = 278.6\,K$$

$$P_{013} = 1.5P_{02} = 1.5\left(36{,}399.1\,\frac{N}{m^2}\right) = 54{,}598.7\,\frac{N}{m^2}$$

**Compressor**

$$\frac{P_{03}}{P_{013}} = 36 = \left(\frac{T_{03}}{T_{013}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$T_{03} = 278.6\,K(36)^{\frac{1.4-1}{1.4}} = 775.6\,K$$

$$P_{03} = 36P_{013} = 36\left(54{,}598.7\,\frac{N}{m^2}\right) = 1{,}965{,}553\,\frac{N}{m^2}$$

**Combustor**

$$P_{04} = P_{03} = 1{,}965{,}553\,\frac{N}{m^2}$$

$$f_{ideal} = \frac{c_{p,g}T_{04} - c_{p,a}T_{03}}{h_{fuel} - c_{p,g}T_{04}} = \frac{1.148\,\frac{kJ}{kg-K}(1560\,K) - 1.005\,\frac{kJ}{kg-K}(775.6\,K)}{43{,}100\,\frac{kJ}{kg} - 1.148\,\frac{kJ}{kg-K}(1560\,K)} = 0.02448$$

**Turbine**

$$c_{p,t}(T_{04} - T_{04.5}) = c_{p,c}(T_{03} - T_{013})$$

$$T_{04.5} = T_{04} - \frac{c_{p,c}}{c_{p,t}}(T_{03} - T_{013}) = 1560\,K - \frac{1.005}{1.148}(775.6\,K - 278.6\,K) = 1124.9\,K$$

$$\dot{m}_h c_{p,t}(T_{04.5} - T_{05}) = \dot{m} c_{p,c}(T_{013} - T_{02})$$

$$\dot{m}_h = \frac{\dot{m}}{B+1}$$

$$\frac{\dot{m}}{\dot{m}_h} = \frac{\dot{m}}{\frac{\dot{m}}{B+1}} = B+1$$

$$T_{05} = T_{04.5} - \frac{\dot{m} c_{p,c}}{\dot{m}_h c_{p,t}}(T_{013} - T_{02}) = T_{04.5} - (B+1)\frac{c_{p,c}}{c_{p,t}}(T_{013} - T_{02})$$

$$T_{05} = 1124.9\,K - (10+1)\frac{1.005}{1.148}(278.6\,K - 248.1\,K) = 831.2\,K$$

$$\frac{P_{05}}{P_{04}} = \left(\frac{T_{05}}{T_{04}}\right)^{\frac{\gamma}{\gamma-1}}$$

$$P_{05} = P_{04}\left(\frac{T_{05}}{T_{04}}\right)^{\frac{\gamma}{\gamma-1}} = 1{,}965{,}553\,\frac{N}{m^2}\left(\frac{831.2\,K}{1560\,K}\right)^{\frac{1.333}{1.333-1}} = 158{,}150\,\frac{N}{m^2}$$

**<u>Nozzles</u>**

$$\frac{P_{05}}{P_9} = \left(1 + \frac{\gamma-1}{2}M_9^2\right)^{\frac{\gamma}{\gamma-1}}$$

$$M_9 = \sqrt{\frac{2}{\gamma-1}\left[\left(\frac{P_{05}}{P_0}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} = \sqrt{\frac{2}{1.333-1}\left[\left(\frac{158{,}120}{22{,}696.8}\right)^{\frac{1.333-1}{1.333}} - 1\right]} = 1.936$$

$$M_{19} = \sqrt{\frac{2}{\gamma-1}\left[\left(\frac{P_{013}}{P_{19}}\right)^{\frac{\gamma-1}{\gamma}} - 1\right]} = \sqrt{\frac{2}{1.4-1}\left[\left(\frac{54{,}598.7}{22{,}696.8}\right)^{\frac{1.4-1}{1.4}} - 1\right]} = 1.194$$

$$a = \sqrt{\gamma R T}$$

$$T_9 = \frac{T_{05}}{\left(1 + \frac{\gamma-1}{2}M_9^2\right)} = \frac{831.2\,K}{1 + \frac{1.333-1}{2}(1.936)^2} = 511.8\,K$$

$$C_9 = M_9 a_9 = M_9\sqrt{\gamma R T_9} = 1.936\sqrt{1.333(286.8)(511.8\,K)} = 856.4\,\frac{m}{s}$$

$$T_{19} = \frac{T_{013}}{\left(1 + \frac{\gamma-1}{2}M_{19}^2\right)} = \frac{278.6\,K}{1 + \frac{1.4-1}{2}(1.194)^2} = 216.8\,K$$

$$C_{19} = M_{19} a_{19} = M_{19}\sqrt{\gamma R T_{19}} = 1.194\sqrt{1.4(287)(216.8\,K)} = 352.4\,\frac{m}{s}$$

$$C_0 = 0.85\left(340.3\sqrt{0.7523}\right) = 250.9\frac{m}{s}$$

$$\dot{m} = \frac{F}{\frac{10}{11}C_{19} + \frac{1}{11}C_9 - C_0} = \frac{70{,}206\ N}{\frac{10}{11}\left(352.4\frac{m}{s}\right) + \frac{1}{11}\left(856.4\frac{m}{s}\right) - 250.9\frac{m}{s}} = \boxed{476.6\frac{kg}{s}}\ \boldsymbol{\dot{m}}$$

$$= \boldsymbol{\frac{F}{\frac{10}{11}C_{19} + \frac{1}{11}C_9 - C_0} = \frac{70{,}206\ N}{\frac{10}{11}\left(352.4\frac{m}{s}\right) + \frac{1}{11}\left(856.4\frac{m}{s}\right) - 250.9\frac{m}{s}} = 476.6\frac{kg}{s}}$$

$$\rho = \rho_{std}\left(\frac{\delta}{\theta}\right) = 1.225\frac{kg}{m^3}\left(\frac{0.2240}{0.7523}\right) = 0.3647\frac{kg}{m^3}$$

$$\dot{m} = \rho V A$$

$$A = \frac{\dot{m}}{\rho V} = \frac{476.6\frac{kg}{s}}{\left(0.3647\frac{kg}{m^3}\right)\left(250.9\frac{m}{s}\right)} = 5.21\ m^2$$

$$\boldsymbol{d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(5.21\ m^2)}{\pi}} = 2.58\ m}\ d = \sqrt{\frac{4A}{\pi}} = \sqrt{\frac{4(5.21\ m^2)}{\pi}} = \boxed{2.58\ m}$$

**Performance**

Calculating other mass flows:

$$\dot{m}_f = \frac{\dot{m} * f}{(BPR+1)} = \frac{476.6\frac{kg}{s} * 0.02448}{11} = 1.061\frac{kg}{s}$$

$$\dot{m}_c = \frac{B}{(BPR+1)} * \dot{m} = 433.272\frac{kg}{s}$$

$$\dot{m}_h = \frac{1}{(BPR+1)} * \dot{m} = 43.327\frac{kg}{s}$$

Using TSFC equation:

$$TSFC = \frac{\dot{m}_f}{F_{total}} = \frac{1.061\frac{kg}{s}}{70{,}206\ N}$$

$$TSFC = 15.11\frac{g}{s * kN}$$

Using Thermal Efficiency equation:

$$\eta_T = \frac{0.5 * \dot{m}_c {C_{19}}^2 + 0.5 * \dot{m}_h {C_9}^2 - 0.5 * \dot{m} {C_0}^2}{\dot{m}_f h_{fuel}}$$

$$= \frac{0.5 * 433.272\frac{kg}{s}\left(352.4\frac{m}{s}\right)^2 + 0.5 * 543.327\frac{kg}{s}\left(856.4\frac{m}{s}\right)^2 - 0.5 * 476.6\frac{kg}{s}(250.9\frac{m}{s})^2}{1.061\frac{kg}{s} * 43100 * 1000\frac{J}{kg}}$$

$$\eta_T = 0.608$$

Using Propulsive Efficiency equation:

$$\eta_P = \frac{F_{total} * C_0}{0.5 * \dot{m}_c {C_{19}}^2 + 0.5 * \dot{m}_g {C_9}^2 - 0.5 * \dot{m} {C_0}^2}$$

$$= \frac{70{,}206\ N * 250.9 \frac{m}{s}}{0.5 * 433.272 \frac{kg}{s}\left(351.611 \frac{m}{s}\right)^2 + 0.5 * 543.327 \frac{kg}{s}\left(580.921 \frac{m}{s}\right)^2 - 0.5 * 476.6 \frac{kg}{s}(250.916 \frac{m}{s})^2}$$

$$\eta_P = 0.634$$

Using Overall Efficiency equation:

$$\eta_O = \eta_P * \eta_T = 0.634 * 0.608$$

$$\eta_O = 0.385$$

## Real Cycle performance:

**Lift/Drag**

From ISA Tables (Anderson): at $h = 11\ km$: $P_0 = 22.7\ kPa, \rho_0 = 0.3647\ \frac{kg}{m^3}, T_0 = 216.8K$

From given equations:

$$q = \frac{\gamma}{2} P M_0^2 = \frac{1.4}{2} * 22.7kPa * 0.85^2 = 11.4805\ kPa$$

$$C_L = \frac{L}{q * S_w} = \frac{0.8 * W_{TO} * 4.44822162 \frac{N}{lbf}}{q * S_w} = \frac{0.8 * 370000 * 4.448\ N}{11.4805\ kPa * 285m^2} = 0.40241$$

$$C_D = 0.056 * {C_L}^2 - 0.004 * C_L + 0.014 = 0.02146$$

$$D = C_D q S_W = 0.02146 * 11.4805kPa * 285m^2 = \boxed{70211.975\ N}$$

**Fan**

Using Isentropic Relations:

$$T_{t2} = T_0 \left[1 + \frac{\gamma_c - 1}{2} {M_0}^2\right] = 216.8K[1 + 0.2(0.85^2)] = 248.128K$$

$$P_{t2} = P_0 \left[1 + \eta_i \frac{\gamma_c - 1}{2} {M_0}^2\right]^{\frac{\gamma_c}{\gamma_c - 1}} = 22.7kPa[1 + 0.98 * 0.2(0.85^2)]^{3.5} = 36.086kPa$$

Definition of pressure ratio:

$$P_{t13} = P_{t2} * \pi_f = 36.086kPa * 1.5 = 54.129kPa$$

Polytropic Equation:

$$\frac{T_{t13}}{T_{t2}} = (\pi_f)^{\frac{\gamma_c - 1}{\gamma_c * \eta_{\infty f}}} = (1.5)^{\frac{0.4}{1.4*0.89}} = 1.1390$$

$$T_{t13} = \frac{T_{t13}}{T_{t2}} * T_{t2} = 1.1390 * 248.128K = 282.621K$$

**Fan Nozzle**

$$\dot{m}_c = \frac{\dot{m} * BPR}{BPR + 1} = 0.909\dot{m}, \;\; T_{t13} = T_{t19}, \;\; P_{t13} = P_{t19}, \;\; P_{19} = P_0$$

(Real) Isentropic Relations:

$$T_{t19} - T_{19} = \eta_j * T_{t19}\left[1 - \left(\frac{P_{19}}{P_{t19}}\right)^{\frac{\gamma_c - 1}{\gamma_c}}\right] = 0.99 * 282.621K * \left[1 - \left(\frac{22.7kPa}{54.129kPa}\right)^{\frac{0.4}{1.4}}\right] = 61.517K$$

$$T_{19} = T_{t19} - (T_{t19} - T_{19}) = 282.621K - 61.517K = 221.105K$$

Using Mach Number Equation:

$$M_{19} = \sqrt{\left[\left(\frac{T_{t19}}{T_{19}}\right) - 1\right] * \frac{2}{\gamma_c - 1}} = \sqrt{\left[\left(\frac{282.621K}{221.105K}\right) - 1\right] * \frac{2}{0.4}} = 1.179$$

$$C_{19} = M_{19} * \sqrt{\gamma_c R T_{19}} = 1.179 * \sqrt{1.4 * 287.1\frac{J}{kg * K} * 221.105K} = 351.611\frac{m}{s}$$

$$C_0 = M_0 * \sqrt{\gamma_c R T_0} = 0.85 * \sqrt{1.4 * 287.1\frac{J}{kg * K} * 216.8K} = 250.916\frac{m}{s}$$

Power Equation:

$$\dot{W}_f = \dot{m} * Cp_c(T_{t13} - T_{t2}) = \dot{m} * 1005\frac{J}{kg * K}(282.621K - 248.128K) = 34666.354\dot{m}\,W$$

**Compressor**

$$\dot{m}_h = \frac{\dot{m}}{BPR + 1} = 0.0909\dot{m}$$

Call compressor inlet stage 2.5:

$$T_{t13} = T_{t2.5}, \;\; P_{t13} = P_{t2.5}$$

Definition of pressure ratio:

$$P_{t3} = P_{t2.5} * \pi_c = 54.129kPa * 36 = 1948.641kPa$$

Polytopic Equation:

$$T_{t3} = T_{t2.5} * (\pi_c)^{\frac{\gamma_c - 1}{\gamma_c * \eta_{\infty c}}} = 282.595K * (36)^{\frac{0.4}{1.4*0.90}} = 881.596K$$

Total cold-side power (fan + compressor):

$$\begin{aligned}\dot{W}_c &= \dot{W}_f + \dot{m}_h Cp_c (T_{t3} - T_{t2.5}) \\ &= 34666.354\dot{m}\, W + \dot{m} * 0.0909 * 1005 \frac{J}{kg * K}(881.596K - 282.621K) \\ &= 89390.816\dot{m}\, W\end{aligned}$$

**Burner**

Definition of pressure ratio:

$$P_{t4} = P_{t3} * \pi_b = 1948.641kPa * 0.96 = 1870.695kPa$$

Given exit temperature:

$$T_{t4} = 1560K$$

Fuel/Air ratio equation:

$$f = \frac{Cp_h T_{t4} - Cp_c T_{t3}}{\eta_b (h_{fuel} - Cp_h T_{t4})} = \frac{1.148 \frac{kJ}{kg * K} * 1560K - 1.005 \frac{kJ}{kg * K} * 881.596K}{0.99(43100 \frac{kJ}{kg} - 1.148 \frac{kJ}{kg * K} * 1560K)} = 0.02213$$

$$\dot{m}_g = \frac{\dot{m}(1 + f)}{BPR + 1} = 0.0929\dot{m}$$

**Turbine**

Power balance:

$$\eta_m \dot{m}_g Cp_h (T_{t4} - T_{t5}) = \dot{W}_c$$

$$T_{t4} - T_{t5} = \frac{\dot{W}_c}{\eta_m \dot{m}_g Cp_h} = \frac{89390.816\dot{m}\, W}{0.99(0.0929\dot{m}) * 1148 \frac{J}{kg * K}} = 846.455K$$

$$T_{t5} = T_{t4} - (T_{t4} - T_{t5}) = 1560K - 846.455K = 713.545K$$

Polytropic equation:

$$\frac{P_{t5}}{P_{t4}} = (\frac{T_{t5}}{T_{t4}})^{\frac{\gamma_h}{\eta_{\infty t}(\gamma_h - 1)}} = (\frac{713.545K}{1560K})^{\frac{1.333}{0.90(0.333)}} = 0.03084$$

Definition of pressure ratio:

$$P_{t5} = P_{t4} * \frac{P_{t5}}{P_{t4}} = 1870.695kPa * 0.03084 = 57.687kPa$$

**Core Nozzle**

$$T_{t5} = T_{t9}, \ \ P_{t5} = P_{t9}$$

Nozzle is fully expanded:

$$P_9 = P_0$$

(Real) Isentropic Equations

$$T_{t9} - T_9 = \eta_j * T_{t9}\left[1 - \left(\frac{P_9}{P_{t9}}\right)^{\frac{\gamma_h - 1}{\gamma_h}}\right] = 0.98 * 713.545K * \left[1 - \left(\frac{22.7kPa}{57.687kPa}\right)^{\frac{0.333}{1.333}}\right] = 146.820K$$

$$T_9 = T_{t9} - (T_{t9} - T_9) = 713.545K - 146.820K = 566.725K$$

Using Mach Number Equation:

$$M_9 = \sqrt{\left[\left(\frac{T_{t9}}{T_9}\right) - 1\right] * \frac{2}{\gamma_h - 1}} = \sqrt{\left[\left(\frac{713.545K}{566.725K}\right) - 1\right] * \frac{2}{0.333}} = 1.247$$

Rearranging Mach number equation:

$$C_9 = M_9\sqrt{\gamma_h R T_9} = 1.247\sqrt{1.333 * 287.1\frac{J}{kg * K} * 566.725K} = 580.921\frac{m}{s}$$

Thrust equation (no pressure thrust):

Total thrust:

$$F_{total} = \frac{B}{B+1}C_{19} + \frac{1+f}{B+1}C_9 - C_o = 122.710\dot{m}\,N$$

$$\boxed{\frac{F}{\dot{m}} = 122.710\ \frac{N * s}{kg}}$$

Setting equal to Drag and solving for mass flow:

$$120.34\dot{m}\,N = 70211.975\ N \ \rightarrow \boxed{\boldsymbol{\dot{m} = 572.180\frac{kg}{s}}}$$

Using mass flow equation to solve for area of the inlet/fan:

$$A = \frac{\dot{m}}{\rho_0 C_0} = \frac{572.180\frac{kg}{s}}{0.3647\frac{kg}{m^3} * 250.916\frac{m}{s}} = 6.2527m^2$$

Using area of a circle equation to find diameter:

$$D_f = \sqrt{4\frac{A}{\pi}} = \sqrt{4\frac{6.2527m^2}{\pi}} \rightarrow \boxed{\boldsymbol{D = 2.8216\ m}}$$

**Performance**

Calculating other mass flows:

$$\dot{m}_f = \frac{\dot{m} * f}{(BPR+1)} = \frac{572.180\frac{kg}{s} * 0.02213}{11} = 1.151\frac{kg}{s}$$

$$\dot{m}_c = 0.909 * \dot{m} = 520.164\frac{kg}{s}$$

$$\dot{m}_g = 0.0929 * \dot{m} = 53.167\frac{kg}{s}$$

Using TSFC equation:

$$TSFC = \frac{\dot{m}_f}{F_{total}} = \frac{\dot{m} * f}{(BPR+1)F_{total}} = \frac{572.180\frac{kg}{s} * 0.02213}{11 * 70211.975\ N} \rightarrow \boxed{TSFC = 16.39\frac{g}{s * kN}}$$

Using Thermal Efficiency equation:

$$\eta_T = \frac{0.5 * \dot{m}_c {C_{19}}^2 + 0.5 * \dot{m}_g {C_9}^2 - 0.5 * \dot{m} {C_0}^2}{\dot{m}_f h_{fuel}}$$

$$= \frac{0.5 * 520.164\frac{kg}{s}\left(351.611\frac{m}{s}\right)^2 + 0.5 * 53.167\frac{kg}{s}\left(580.921\frac{m}{s}\right)^2 - 0.5 * 572.180\frac{kg}{s}(250.916\frac{m}{s})^2}{1.151\frac{kg}{s} * 43100 * 1000\frac{J}{kg}}$$

$$\boxed{\eta_T = 0.466}$$

Using Propulsive Efficiency equation:

$$\eta_P = \frac{F_{total} * C_0}{0.5 * \dot{m}_c {C_{19}}^2 + 0.5 * \dot{m}_g {C_9}^2 - 0.5 * \dot{m} {C_0}^2}$$

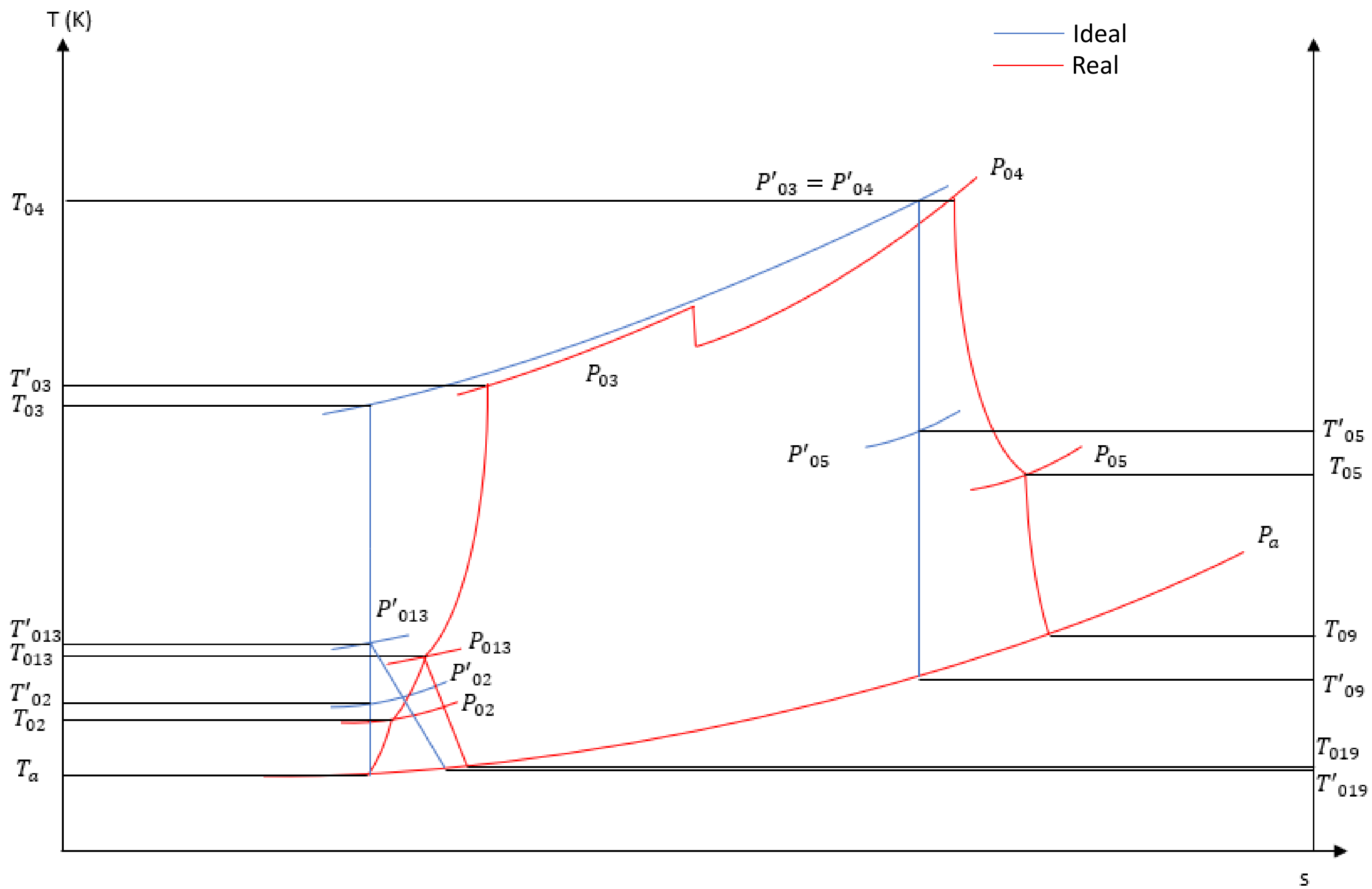
$$= \frac{70211.975\ N * 250.905\frac{m}{s}}{0.5 * 520.164\frac{kg}{s}\left(351.611\frac{m}{s}\right)^2 + 0.5 * 53.167\frac{kg}{s}\left(580.921\frac{m}{s}\right)^2 - 0.5 * 572.180\frac{kg}{s}(250.905\frac{m}{s})^2}$$

$$\boxed{\eta_P = 0.762}$$

Using Overall Efficiency equation:

$$\eta_O = \eta_P * \eta_T = 0.762 * 0.462 \rightarrow \boxed{\eta_O = 0.355}$$

## T-s Diagram


T (K)
Ideal
Real
$T_{04}$
$T'_{03}$
$T_{03}$
$T'_{013}$
$T_{013}$
$T'_{02}$
$T_{02}$
$T_a$
$P'_{03} = P'_{04}$
$P_{04}$
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$P'_{013}$
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# Parametric Studies

## Bypass Ratio Study

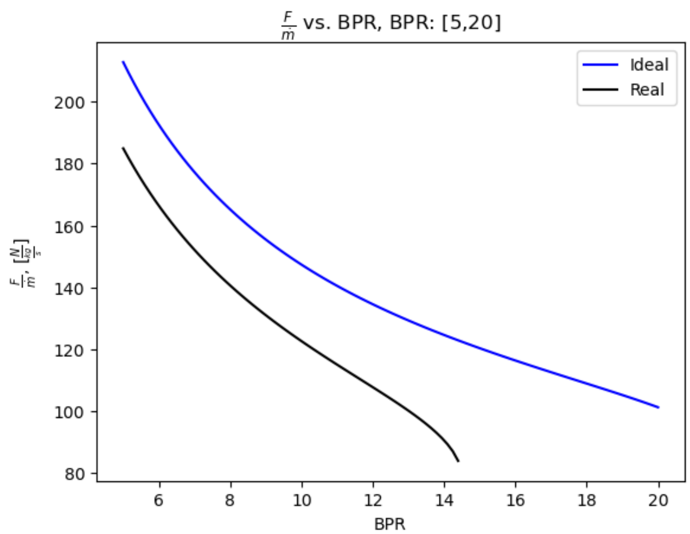


*Figure 1: Specific Thrust vs. BPR for BPR Ranging from 5 to 20*

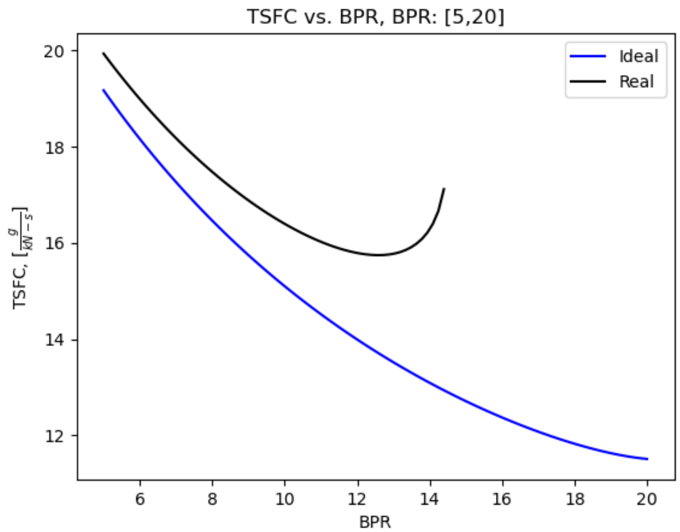


*Figure 2: Thrust Specific Fuel Consumption (TSFC) vs. BPR for BPR Ranging from 5 to 20*

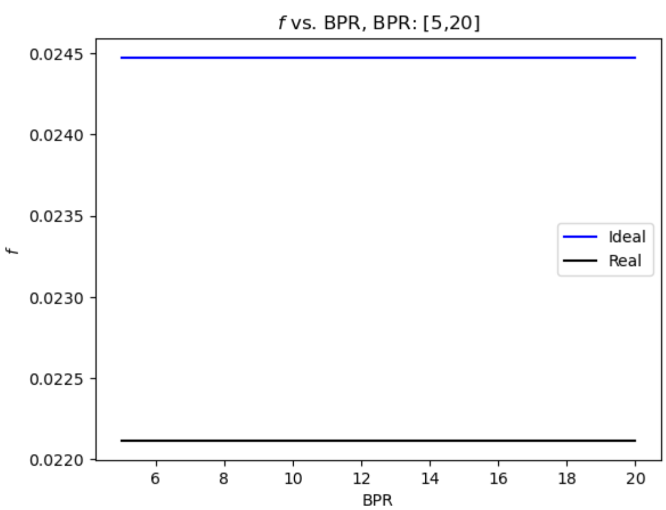


*Figure 3: Fuel-Air Ratio vs. BPR for BPR Ranging from 5 to 20*

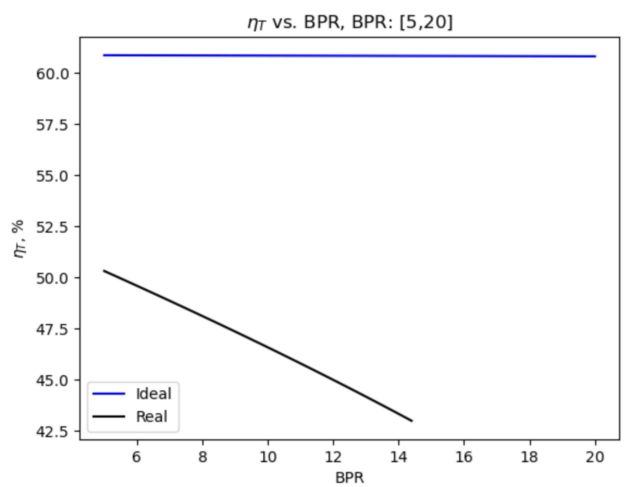


*Figure 4: Thermal Efficiency vs. BPR for BPR Ranging from 5 to 20*

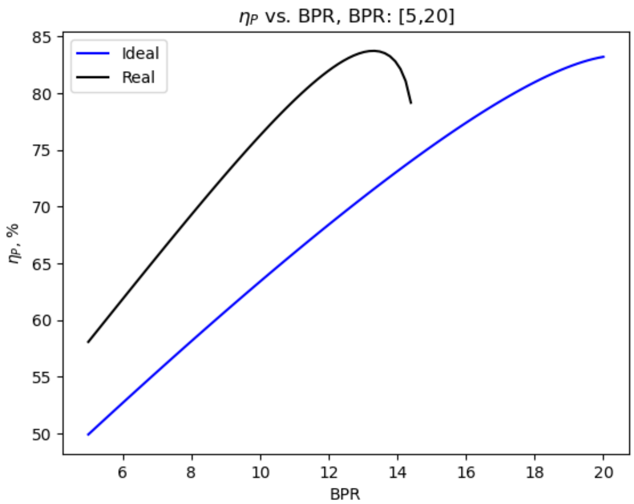


*Figure 5: Propulsive Efficiency vs. BPR for BPR Ranging from 5 to 20*

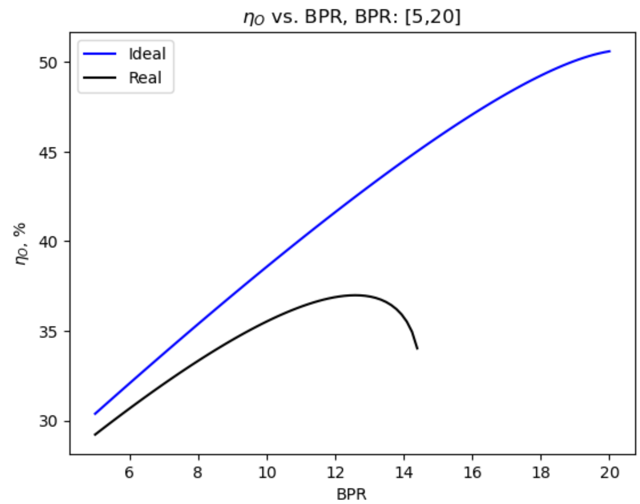


*Figure 6: Overall Efficiency vs. BPR for BPR Ranging from 5 to 20*

The bypass ratio study displays some interesting results. For this study, BPR was varied from 5 to 20, while compressor pressure ratio was held at 36 and fan pressure ratio was held at 1.5. As can be seen in every Figure 1-6 except for Figure 3, there is a maximum bypass ratio for the real cycle of about 14.5. After this value, the required exit pressure becomes impossible to reach, and an asymptote forms. Shown in Figure 1, specific thrust decreases in both the real and ideal cycle for increasing bypass ratio. Thrust specific fuel consumption decreases for both cycles as bypass ratio increases, until the asymptote is reached for the real cycle (Figure 2). Fuel air ratio remains constant for both cycles across the range of bypass ratios, as seen in Figure 3. Figure 4 shows that the thermal efficiency is constant for the ideal cycle but decreases for the real cycle with increasing bypass ratio. Both the propulsive efficiency and overall efficiency increase for both cycles with increasing bypass ratio, as shown in Figures 5-6. From these results, since the goal is to minimize fuel consumption, the bypass ratio should be increased to the maximum value before the asymptote occurs for the real cycle, which corresponds to $BPR \approx 13$.

## Fan Pressure Ratio Study

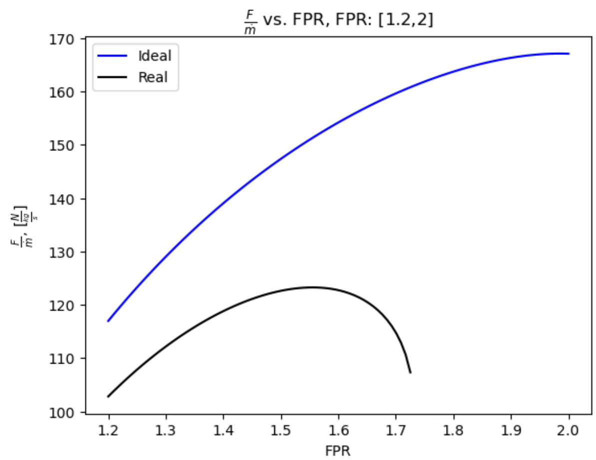


*Figure 7: Specific Thrust vs. FPR for FPR Ranging from 1.2 to 2*

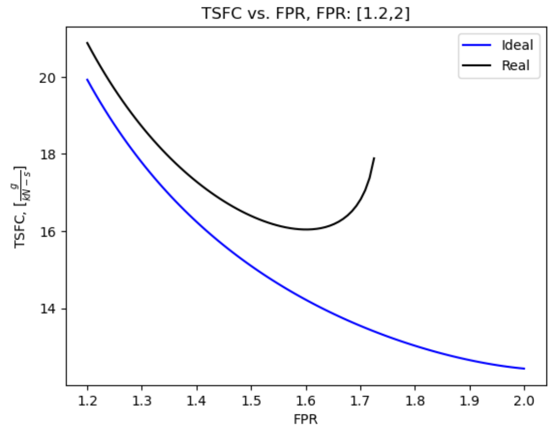


*Figure 8: Thrust Specific Fuel Consumption vs. FPR for FPR Ranging from 1.2 to 2*

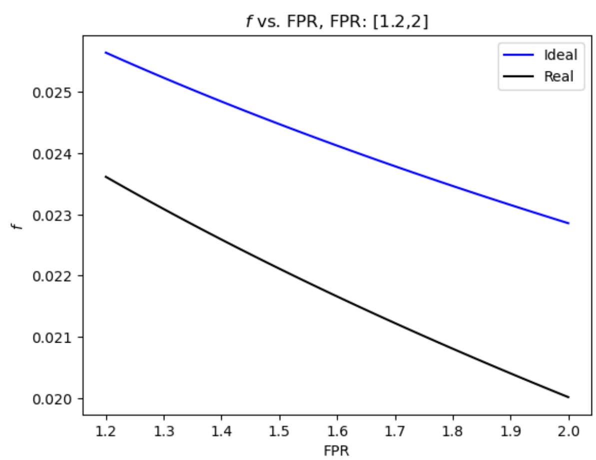


*Figure 9: Fuel-Air Ratio vs. FPR for FPR Ranging from 1.2 to 2*

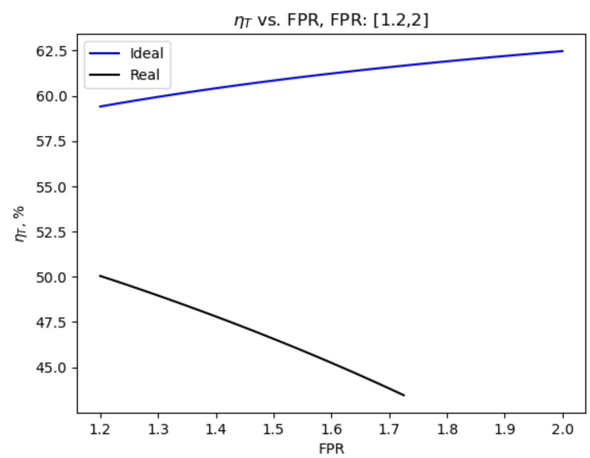


*Figure 10: Thermal Efficiency vs. FPR for FPR Ranging from 1.2 to 2*

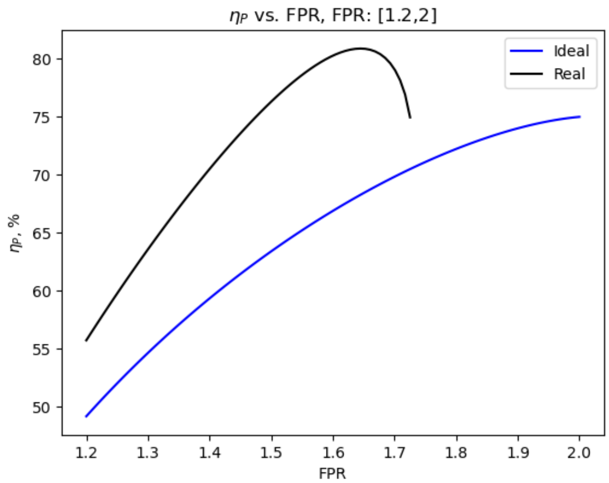


*Figure 11: Propulsive Efficiency vs. FPR for FPR Ranging from 1.2 to 2*

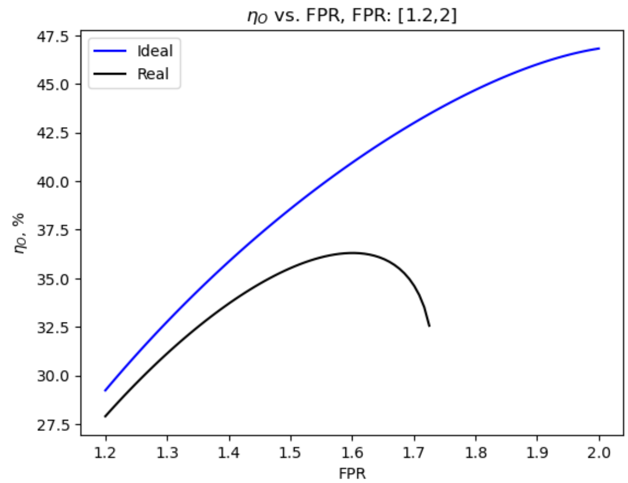


*Figure 12: Overall Efficiency vs. FPR for FPR Ranging from 1.2 to 2*

For the fan pressure ratio study, BPR was held at 10, CPR was held at 36, and FPR was varied from 1.2 to 2. Figures 7, 8, 11, and 12 display the same surge asymptote as was seen in the bypass ratio study at a fan pressure ratio of 1.72. Many of the charts from this study look like their counterparts in the bypass ratio study. The three charts that differ significantly are Figures 7, 9, and 10. In Figure 7, the real cycle line experiences a maximum at a FPR of 1.55, then decreases until FPR reaches the surge point. Meanwhile, the ideal cycle line shows increasing specific thrust throughout the entire range of FPR values, flattening off at the end. Figure 9 shows that for both the real and ideal cycles, fuel-to-air ratio decreases as FPR increases. Shown in Figure 10, the real and ideal cycles experienced opposite reactions for thermal efficiency as FPR increases. The real line decreases to a minimum value before reaching surge, while the ideal line increases over the entire range of FPR. Figures 11, and 12 show that for both real and ideal cycles, propulsive and overall efficiency increase as FPR increases, respectively. Figure 8 shows that both the real and ideal cycles experience decreasing thrust specific fuel consumption as FPR is increased. Like the bypass ratio study, if the goal is to minimize fuel consumption, the maximum values (minimum in the case of TSFC) before the surge asymptote in the real cycle should be selected. This value is $FPR \approx 1.6$.

## Compressor Pressure Ratio Study

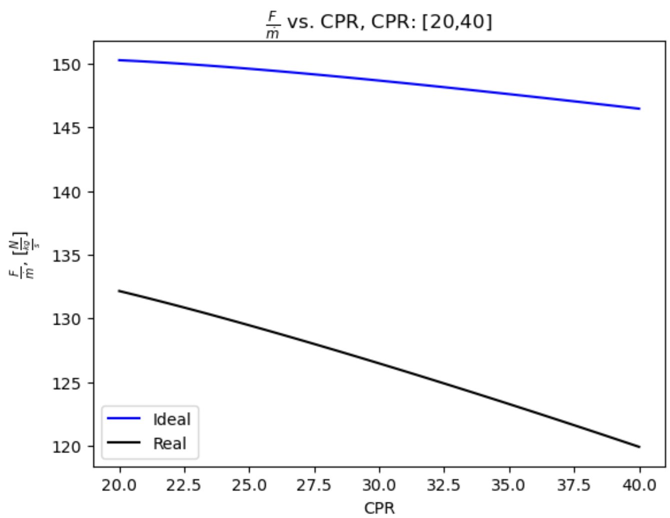


*Figure 13: Specific Thrust vs. CPR for CPR Ranging from 20 to 40*

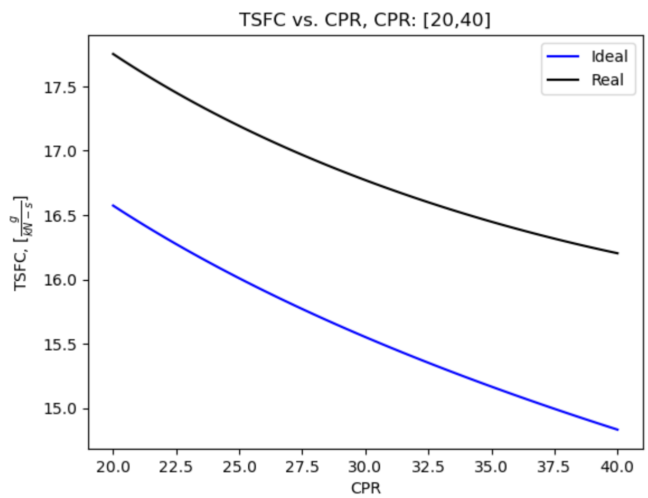


*Figure 14: Thrust Specific Fuel Consumption vs. CPR for CPR Ranging from 20 to 40*

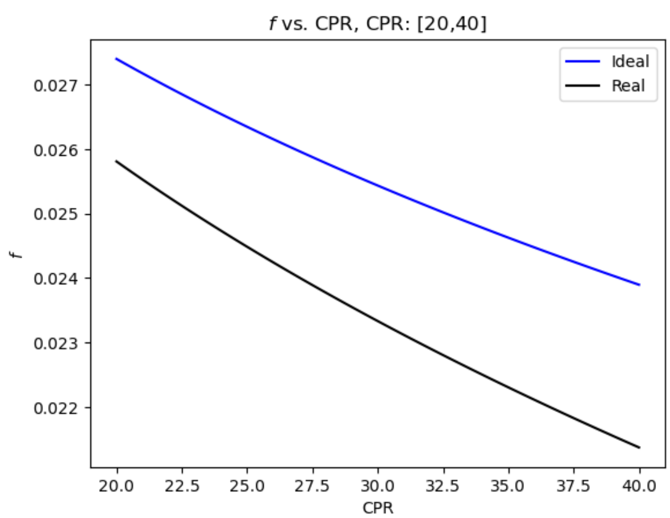


*Figure 15: Fuel-Air Ratio vs. CPR for CPR Ranging from 20 to 40*

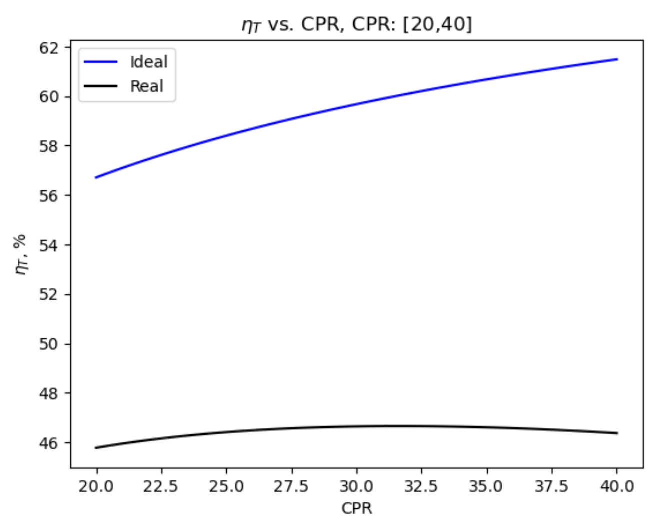


*Figure 16: Thermal Efficiency vs. CPR for CPR Ranging from 20 to 40*

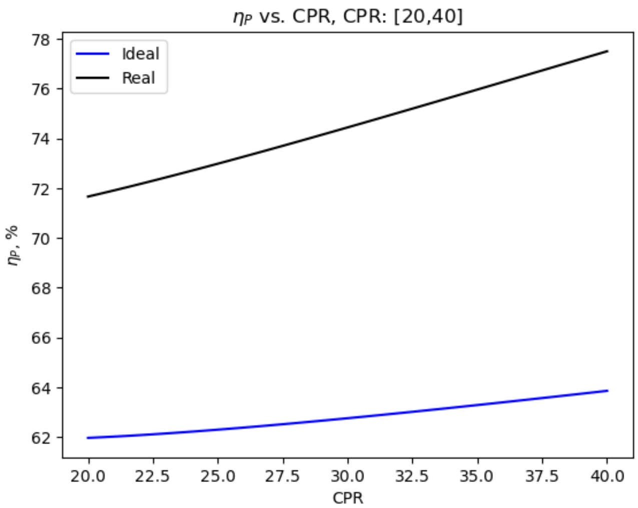


*Figure 17: Propulsive Efficiency vs. CPR for CPR Ranging from 20 to 40*

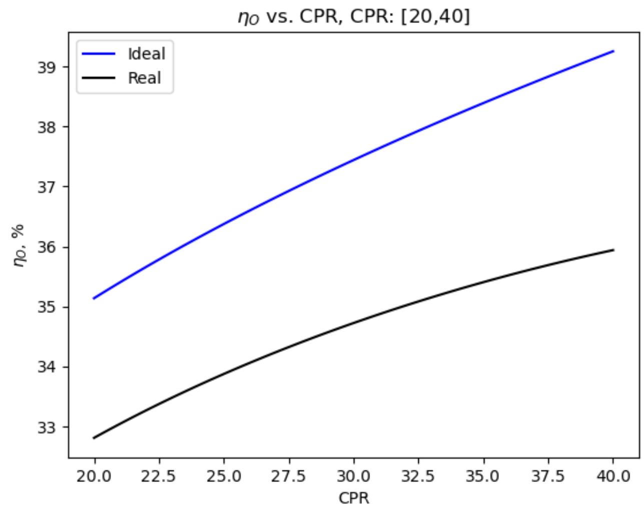


*Figure 18: Overall Efficiency vs. CPR for CPR Ranging from 20 to 40*

For the compressor ratio study, there are no asymptotes found for the pressure ratios used in this study. The CPR was varied from 20 to 40, and BPR was held at 10 and FPR was held at 1.5. Figure 13 shows that the specific thrust decreases with increasing CPR for both cycles. Thrust specific fuel consumption and fuel to air ratio both decrease for increasing CPR for both cycles (Figures 14-15). All of the efficiencies increase with increasing CPR for both cycles (Figures 16-18). Since the goal of this study is to decrease fuel consumption, the best CPR to use is the maximum allowable value, which in this study is $CPR = 40$.

## Overall 2D Study Takeaways

Regarding the bypass ratio, for real cycle optimization, the goal would be to maximize the bypass ratio of the engine at the desired flight condition and constant CPR and FPR values. As was seen in the graphs, a bypass ratio around 14 results in engine surge due to the ambient pressure exceeding the pressure at the nozzle outlet. In terms of the fan pressure ratio, maximizing the FPR is also desired up until the surge FPR, which was around 1.72 for the given flight condition. Finally, maximizing the CPR is also desired for cycle optimization, and there was no apparent surge point, leading to a CPR value of 40 to be optimal.

## Constant Bypass Ratio Study

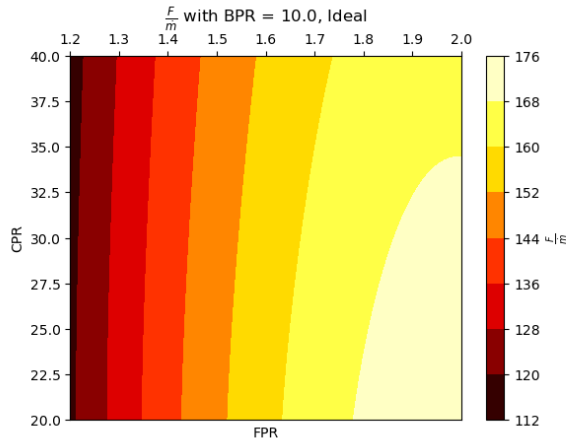


*Figure 19: Ideal Specific Thrust with BPR Constant at 10 for FPR: 1.2 to 2.0 and CPR: 20 to 40*

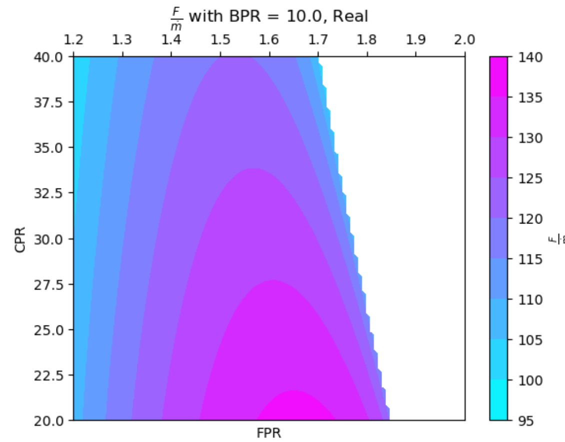


*Figure 20: Real Specific Thrust with BPR Constant at 10 for FPR: 1.2 to 2.0 and CPR: 20 to 40*

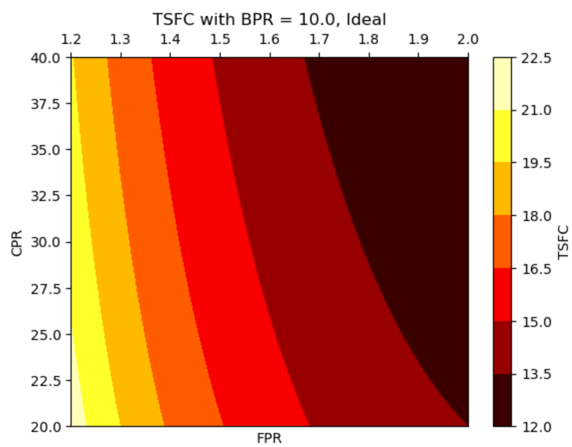


*Figure 21: Ideal TSFC with BPR Constant at 10 for FPR: 1.2 to 2.0 and CPR: 20 to 40*

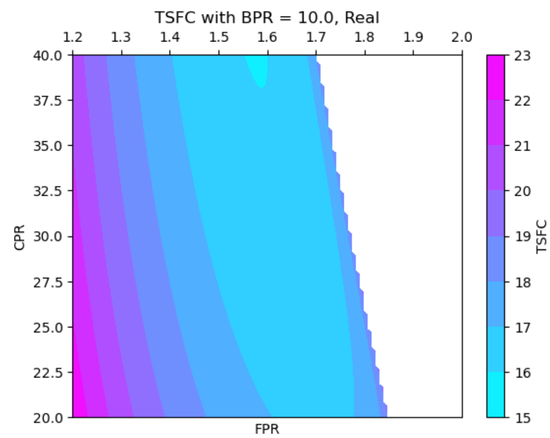


*Figure 22: Real TSFC with BPR Constant at 10 for FPR: 1.2 to 2.0 and CPR: 20 to 40*

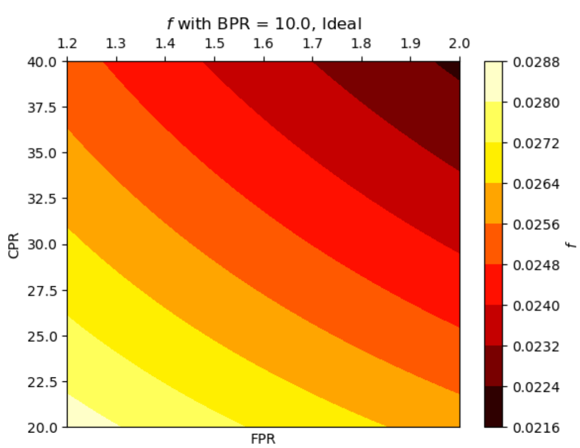


*Figure 23: Ideal Fuel-Air Ratio with BPR Constant at 10 for FPR: 1.2 to 2.0 and CPR: 20 to 40*

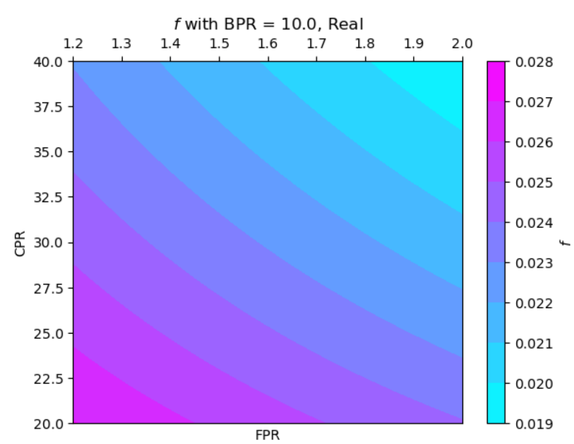


*Figure 24: Real Fuel-Air Ratio with BPR Constant at 10 for FPR: 1.2 to 2.0 and CPR: 20 to 40*

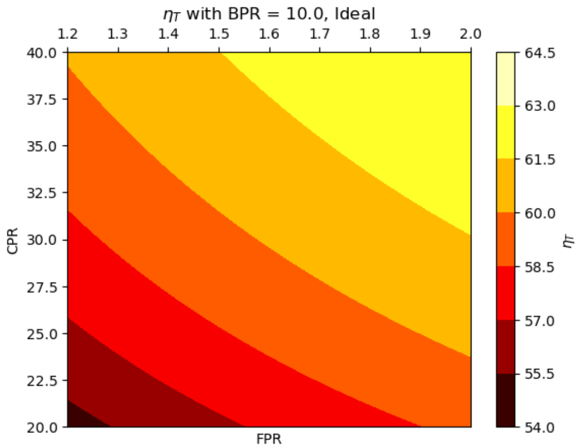


*Figure 25: Ideal Thermal Efficiency with BPR Constant at 10 for FPR: 1.2 to 2.0 and CPR: 20 to 40*

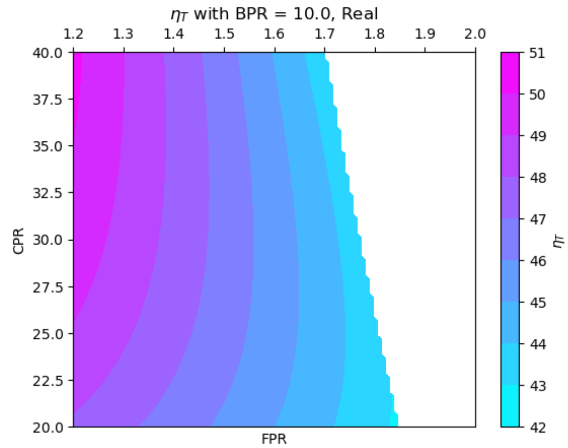


*Figure 26: Real Thermal Efficiency with BPR Constant at 10 for FPR: 1.2 to 2.0 and CPR: 20 to 40*

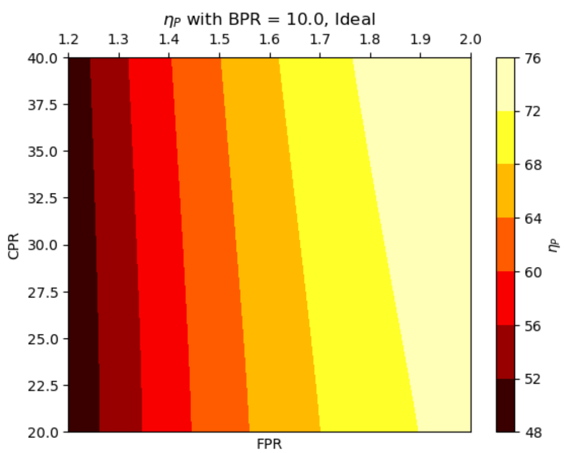


*Figure 27: Ideal Propulsive Efficiency with BPR Constant at 10 for FPR: 1.2 to 2.0 and CPR: 20 to 40*

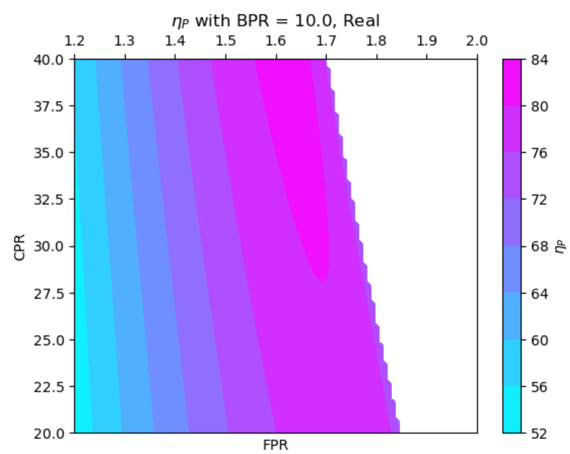


*Figure 28: Real Propulsive Efficiency with BPR Constant at 10 for FPR: 1.2 to 2.0 and CPR: 20 to 40*

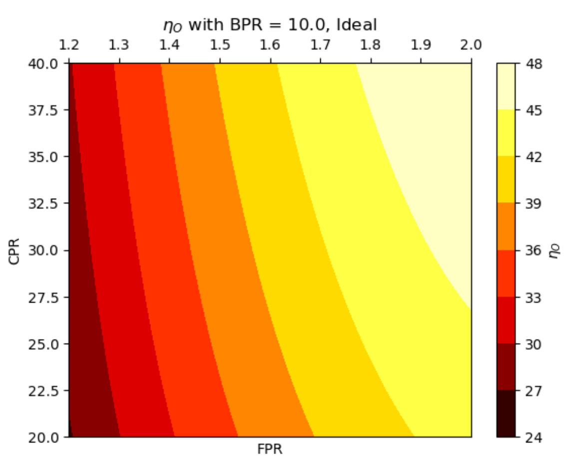


*Figure 29: Ideal Overall Efficiency with BPR Constant at 10 for FPR: 1.2 to 2.0 and CPR: 20 to 40*

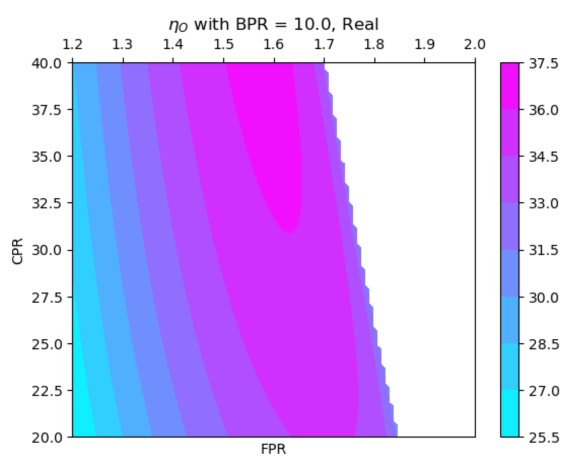


*Figure 30: Real Overall Efficiency with BPR Constant at 10 for FPR: 1.2 to 2.0 and CPR: 20 to 40*

Figures 19-30 above provide some more detail as to the effect of varying the different parameters for this engine. This was a constant bypass ratio study, where the BPR was held at 10, and the CPR was varied from 20-40 and the FPR was varied from 1.2-2.0. By varying two variables simultaneously, it is possible to understand more about the effects of these variables. From Figure 30, it can be seen that the maximum overall efficiency is achieved at $CPR = 40$ and $FPR = 1.55$ for the real cycle. Minimum thrust specific fuel consumption is at this same point for the real cycle. Also, note that the asymptotes that showed up in the previous study show up as white space in these contour plots, so now it is possible to see ranges of invalid design parameters.

## Constant Fan Pressure Ratio Study

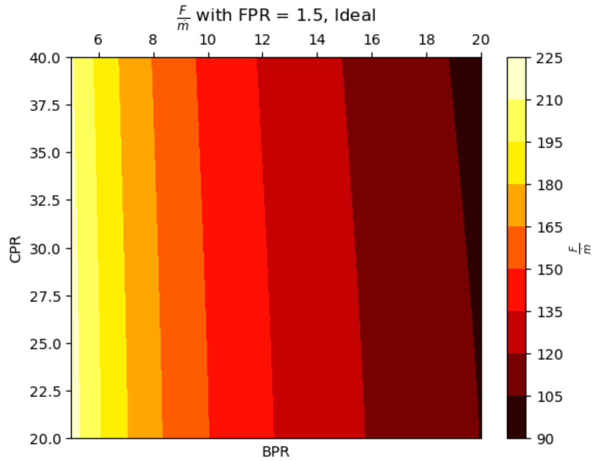


*Figure 31: Ideal Specific Thrust with FPR Constant at 1.5 for BPR: 5 to 20 and CPR: 20 to 40*

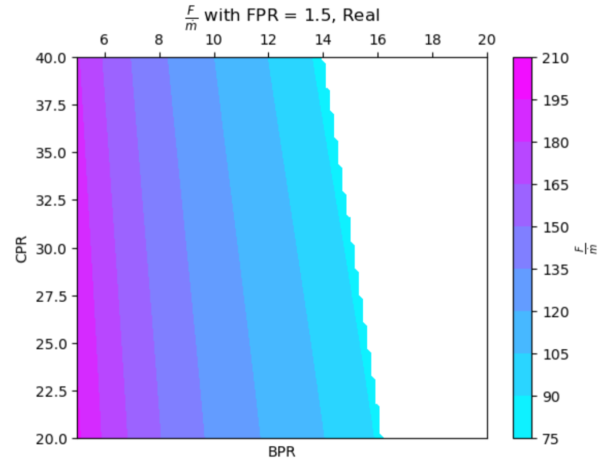


*Figure 32: Real Specific Thrust with FPR Constant at 1.5 for BPR: 5 to 20 and CPR: 20 to 40*

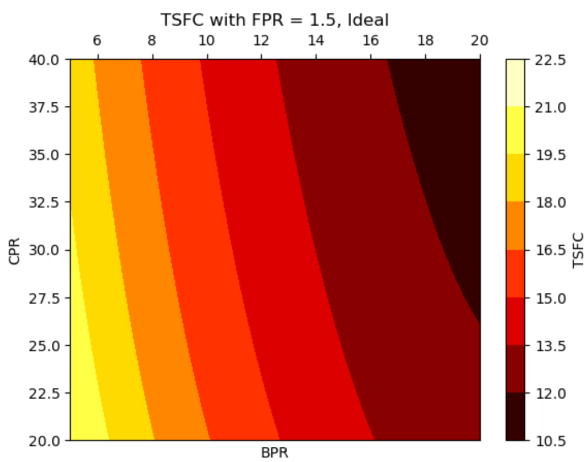


*Figure 33: Ideal TSFC with FPR Constant at 1.5 for BPR: 5 to 20 and CPR: 20 to 40*

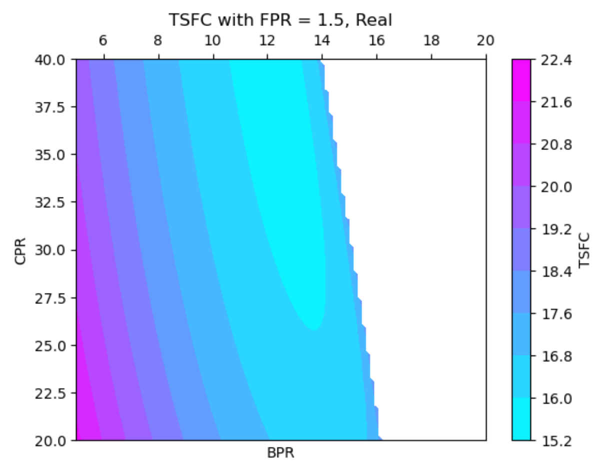


*Figure 34: Real TSFC with FPR Constant at 1.5 for BPR: 5 to 20 and CPR: 20 to 40*

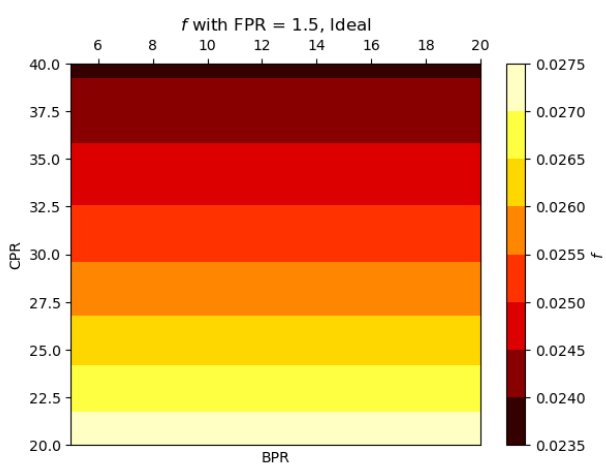


*Figure 35: Ideal Fuel-Air Ratio with FPR Constant at 1.5 for BPR: 5 to 20 and CPR: 20 to 40*

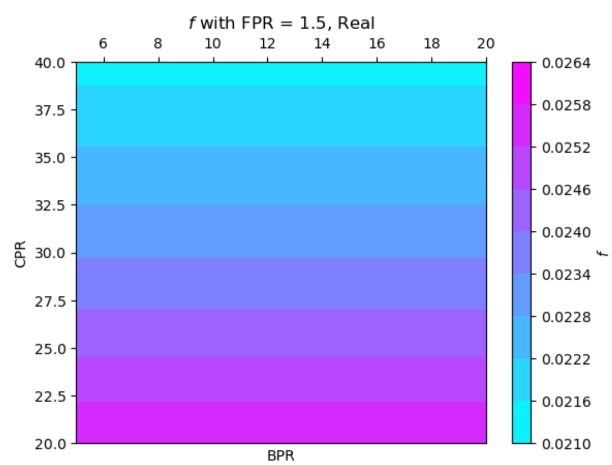


*Figure 36: Real Fuel-Air Ratio with FPR Constant at 1.5 for BPR: 5 to 20 and CPR: 20 to 40*

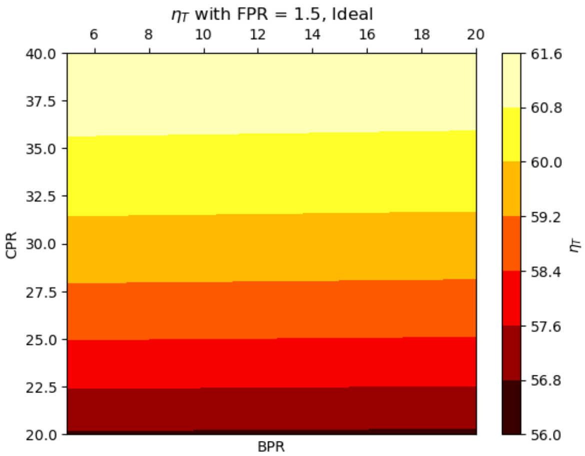


*Figure 37: Ideal Thermal Efficiency with FPR Constant at 1.5 for BPR: 5 to 20 and CPR: 20 to 40*

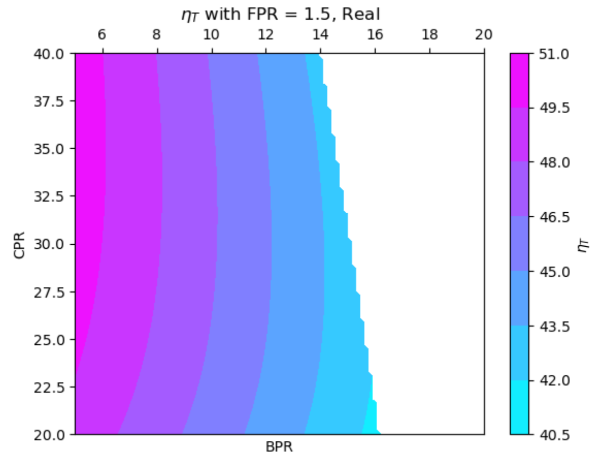


*Figure 38: Real Thermal Efficiency with FPR Constant at 1.5 for BPR: 5 to 20 and CPR: 20 to 40*

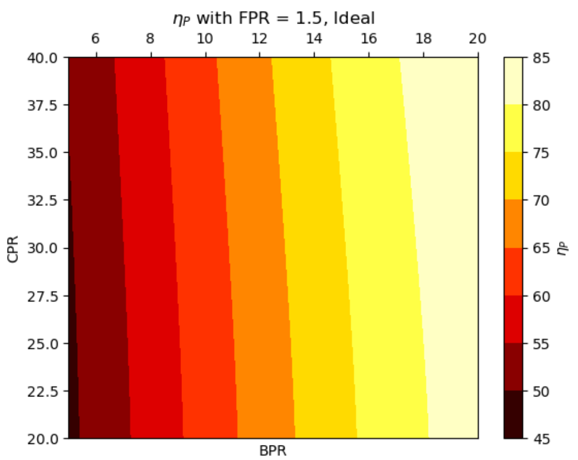


*Figure 39: Ideal Propulsive Efficiency with FPR Constant at 1.5 for BPR: 5 to 20 and CPR: 20 to 40*

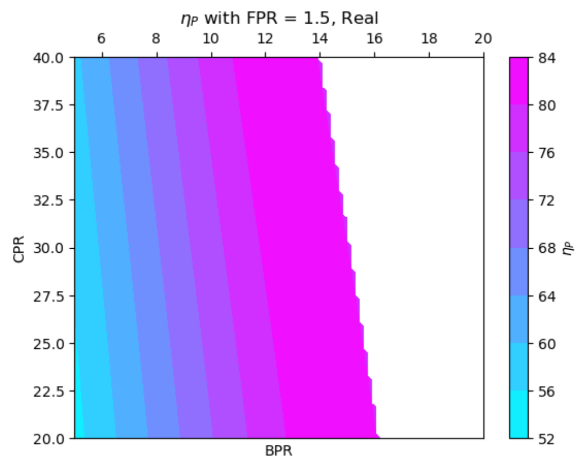


*Figure 40: Real Propulsive Efficiency with FPR Constant at 1.5 for BPR: 5 to 20 and CPR: 20 to 40Figure*

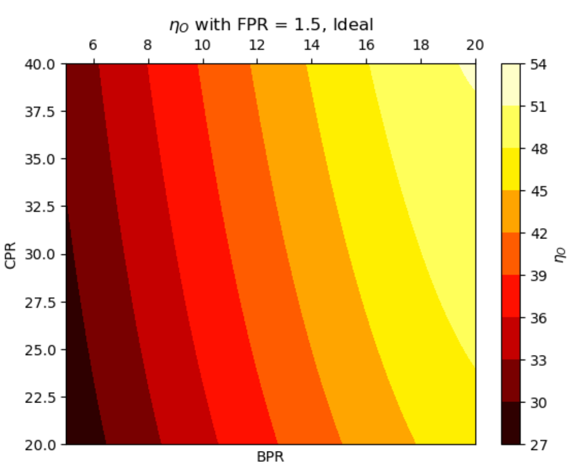


*Figure 41: Ideal Overall Efficiency with FPR Constant at 1.5 for BPR: 5 to 20 and CPR: 20 to 40*

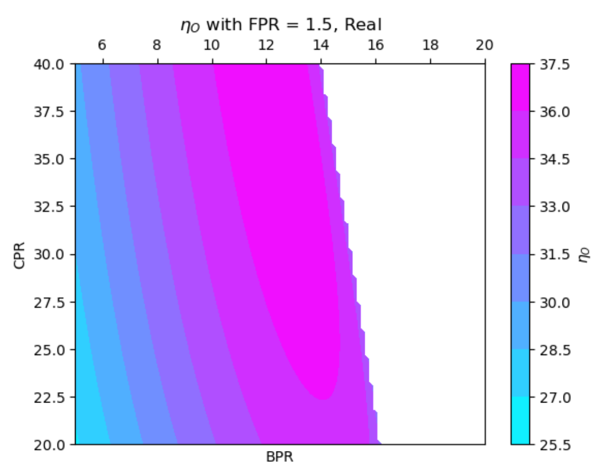


*Figure 42 : Real Overall Efficiency with FPR Constant at 1.5 for BPR: 5 to 20 and CPR: 20 to 40*

Figures 31-42 detail the simultaneous parameter choices that occur when the FPR is held constant at 1.5 and both the BPR and CPR are varied simultaneously, with the BPR ranging from values of 5 to 20 and the CPR ranging from 20 to 40. For the real cycle, regarding minimizing TSFC, Figure 34 demonstrates that the minimum TSFC value occurs at a maximum CPR of 40 and a bypass ratio near 12, which is on the high end of the BPR range that was described to be maximized above. This corroborates the 2D parametric study in that both the BPR and CPR should be maximized for best TSFC. This is also seen in the overall efficiency contour of Figure 42, where the same BPR and CPR yielded the highest efficiency value of ~38%. Therefore, both the 2D and 3D studies verify for a constant FPR, a high BPR of ~12 and CPR = 40 are desirable for maximum efficiency at the required design point.

## Constant Compressor Ratio Study

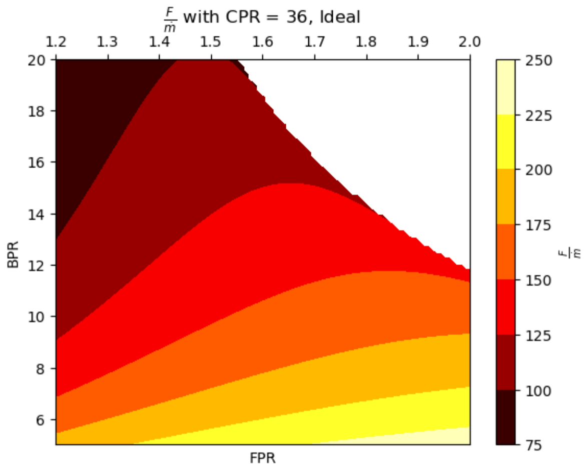


*Figure 43: Ideal Specific Thrust with CPR Constant at 36 for BPR: 5 to 20 and FPR: 1.2 to 2*

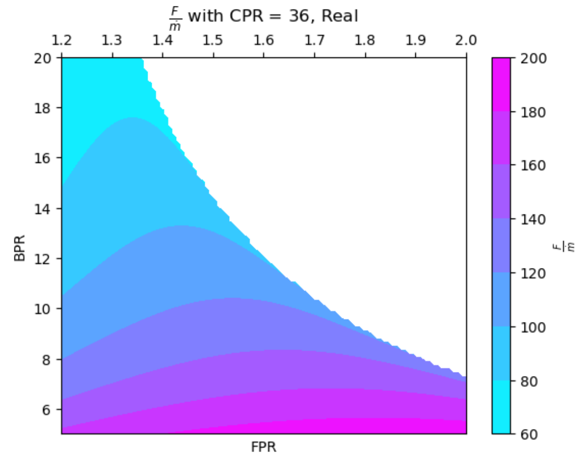


*Figure 44: Real Specific Thrust with CPR Constant at 36 for BPR: 5 to 20 and FPR: 1.2 to 2*

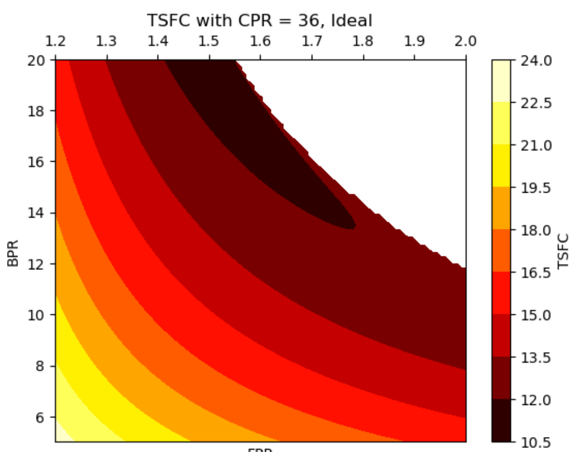


*Figure 45: Ideal TSFC with CPR Constant at 36 for BPR: 5 to 20 and FPR: 1.2 to 2*

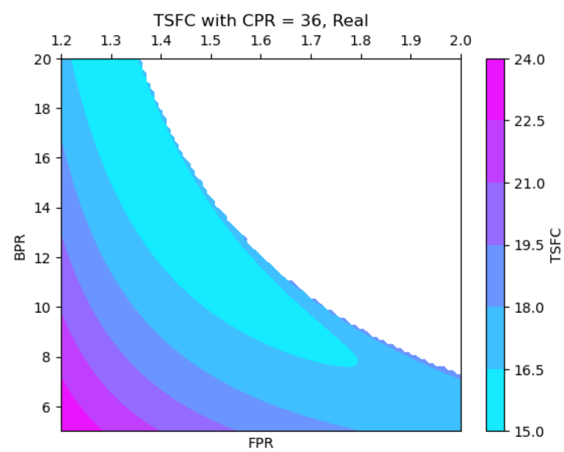


*Figure 46: Real TSFC with CPR Constant at 36 for BPR: 5 to 20 and FPR: 1.2 to 2*

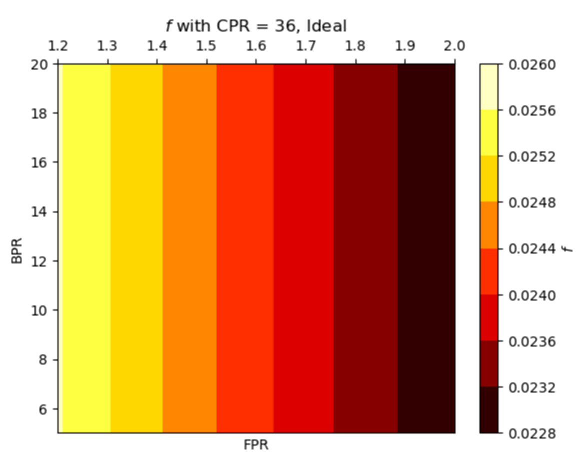


*Figure 47: Ideal Fuel-Air Ratio with CPR Constant at 36 for BPR: 5 to 20 and FPR: 1.2 to 2*

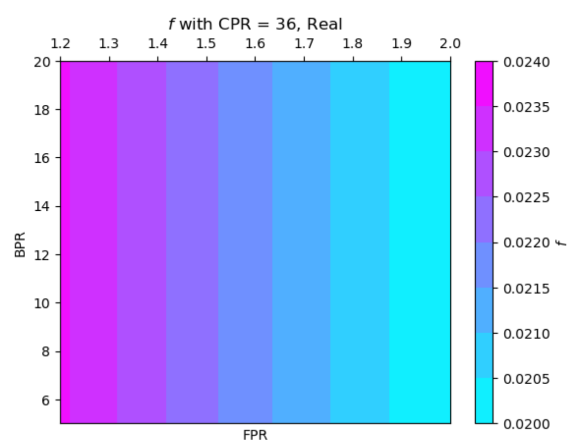


*Figure 48: Real Fuel-Air Ratio with CPR Constant at 36 for BPR: 5 to 20 and FPR: 1.2 to 2*

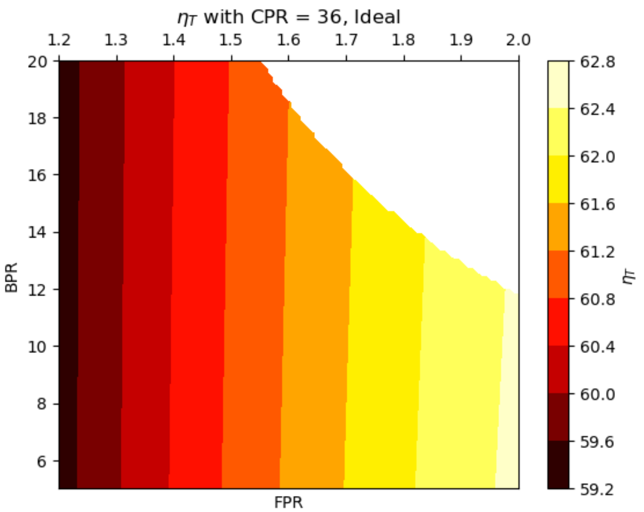


Figure 49: Ideal Thermal Efficiency with CPR Constant at 36 for BPR: 5 to 20 and FPR: 1.2 to 2

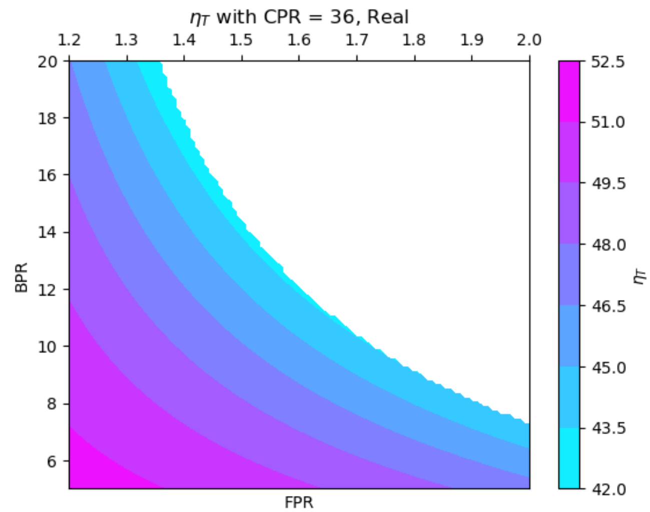


Figure 50: Real Thermal Efficiency with CPR Constant at 36 for BPR: 5 to 20 and FPR: 1.2 to 2

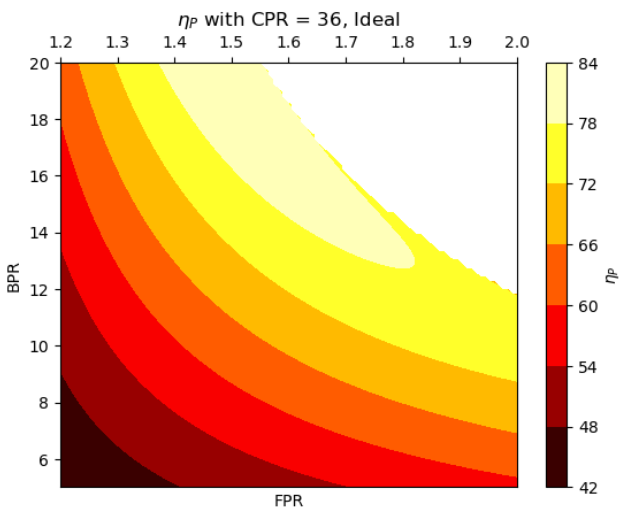


Figure 51: Ideal Propulsive Efficiency with CPR Constant at 36 for BPR: 5 to 20 and FPR: 1.2 to 2

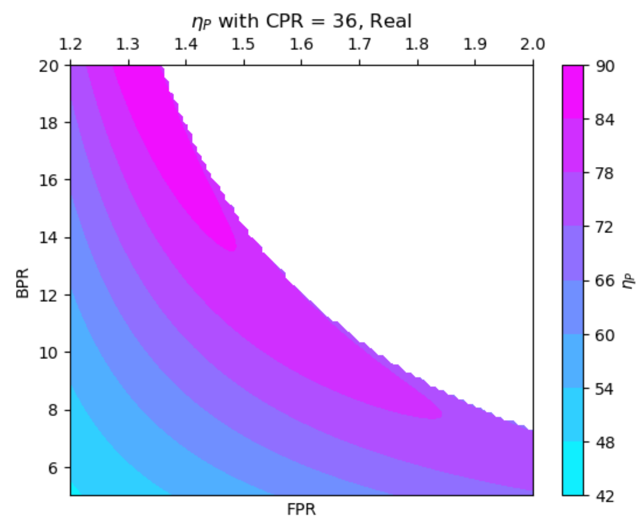


Figure 52: Real Propulsive Efficiency with CPR Constant at 36 for BPR: 5 to 20 and FPR: 1.2 to 2

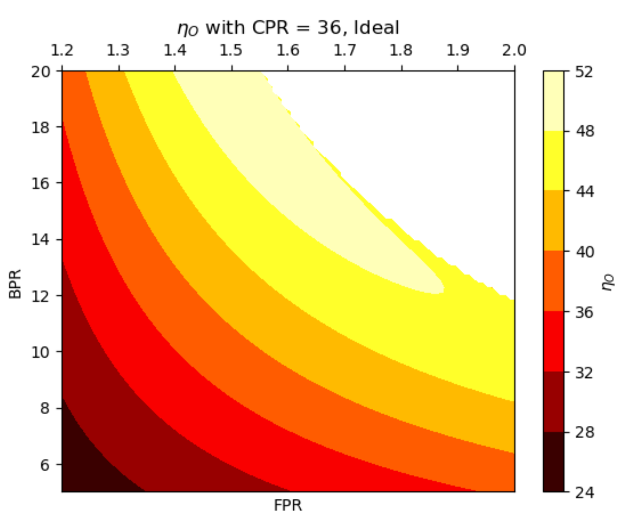


Figure 53: Ideal Overall Efficiency with CPR Constant at 36 for BPR: 5 to 20 and FPR: 1.2 to 2

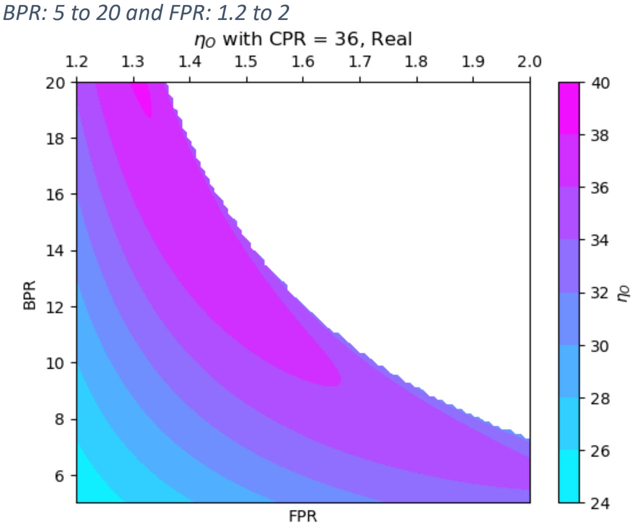


Figure 54: Real Overall Efficiency with CPR Constant at 36 for BPR: 5 to 20 and FPR: 1.2 to 2

Figures 43-54 above provide some more detail as to the effect of varying the different parameters for this engine. This was a constant CPR study, where the CPR was held at 36, and the BPR was varied from 5-20 and the FPR was varied from 1.2-2.0. From Figure 54, it can be seen that the maximum overall efficiency is achieved at $BPR = 20$ and $FPR = 1.3$ for the real cycle. Minimum thrust specific fuel consumption is at this same point for the real cycle. Once again, the white area constitutes invalid design parameters.

## Overall 3D Study Takeaways

When comparing multiple variables at a time, it starts to become clear that optimizing for one design parameter could limit another. For example, minimum TSFC can be achieved at $BPR = 20$ and $FPR = 1.3$ if the CPR is held constant at $CPR = 36$, but minimum TSFC can be achieved at $CPR = 40$ and $BPR = 12$ if the FPR is held constant at $FPR = 1.5$. Therefore, to truly understand the absolute optimized values for the system, a three variable minimization calculation would have to be performed.

## Team Member Contributions

Matt Boller and Pierce Elliott created the code.

Jon Frueh performed hand calculations for the ideal cycle.

Ben Rupe performed hand calculations for the real cycle and created the T-s diagram.

Matt, Pierce, and Ben wrote about the takeaways from the design studies.