METHODOLOGICAL NOTE
Estimation of Potential GDP Using a Particle Filter

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1.1 Objective

The objective is to estimate potential GDP from an annual series of real GDP, using a stochastic trend model estimated through a particle filter. The aim is to obtain a path for potential GDP and an output gap consistent with the standard macroeconomic notion, This allows isolating the long-term trend and distinguishing it from cyclical fluctuations in observed GDP.

1.2 Data and transformation

The variable used is real Gross Domestic Product (level), at annual frequency, taken from the file Serie.xlsx. For the estimation, GDP is transformed into logarithms:

$y_t = \ln(GDP_t)$

The use of the natural logarithm allows:

(i) Facilitating the interpretation of variations as approximate growth rates.

(ii) Partially linearizing the dynamics.

(iii) Reducing heteroskedasticity associated with increasing levels of the series.

1.3 Structural model: stochastic trend (local level model)

The model chosen for the trend–cycle decomposition is a local level model:

$y_t = \mu_t + \varepsilon_t$

$\mu_t = \mu_{t-1} + \eta_t$

where:

– μ_t is the stochastic trend component (log of potential GDP).

– ε_t is a transitory component or measurement noise.

– η_t is the innovation to the trend, governing long-term evolution.

The terms ε_t and η_t are independent and normally distributed:

$\varepsilon_t \sim N(0, \sigma_\varepsilon^2)$

$\eta_t \sim N(0, \sigma_\eta^2)$

This model implies that the trend follows a random walk and that the cycle is found in the difference between observed GDP and the trend.

1.4 Calibration of σ_ε^2 and σ_η^2 based on the variance of growth

Instead of estimating σ_ε^2 and σ_η^2 by maximum likelihood, a calibration is used that guarantees a clear separation between trend and cycle.

Let:

$\Delta y_t = y_t - y_{t-1}$

and its empirical variance:

$s^2_{\Delta y} = \mathrm{Var}(\Delta y_t)$

For the local level model, the following approximate identity holds:

$\mathrm{Var}(\Delta y_t) = 2\sigma_\varepsilon^2 + \sigma_\eta^2$

The ratio:

$r = \sigma_\varepsilon^2 / \sigma_\eta^2$

is introduced, which controls the relative importance of the transitory noise versus the permanent component.

Solving the system:

$\sigma_\eta^2 = s^2_{\Delta y} / (2r + 1)$

$\sigma_\varepsilon^2 = r \cdot \sigma_\eta^2$

A search over different values of r allows matching a plausible cycle amplitude. The value r = 1.0 generates a smooth trend and output gaps with a standard deviation of approximately 2–3%, which is consistent with standard macroeconomic analysis.

1.5 Implementation of the particle filter (bootstrap particle filter)

The particle filter approximates the posterior distribution of the state $p(\mu_t \mid y_{1:t})$ using a set of particles $\{\mu_t^{(i)}, w_t^{(i)}\}$.

Steps:

(1) Initialization:

– Particles are generated around $\mu_1 \approx y_1$.

– Equal weights are assigned: $w_1^{(i)} = 1/N$.

(2) Prediction:

$$\mu_t^{(i)} = \mu_{t-1}^{(i)} + \eta_t^{(i)}, \quad \text{with } \eta_t^{(i)} \sim N(0, \sigma_\eta^2)$$

(3) Update:

– The likelihood is computed:

$$\ell_t^{(i)} = N(y_t \,;\, \mu_t^{(i)}, \sigma_\varepsilon^2)$$

– The weights are updated:

$$\tilde{w}_t^{(i)} = w_{t-1}^{(i)} \cdot \ell_t^{(i)}$$

– They are normalized:

$$w_t^{(i)} = \tilde{w}_t^{(i)} / \sum_j \tilde{w}_t^{(j)}$$

(4) Computation of the filtered state:

$$\hat{\mu}_t = \sum_i w_t^{(i)} \mu_t^{(i)}$$

(5) Systematic resampling:

– Equally spaced positions u_i are generated.

– New particles are selected according to the cumulative distribution of weights.

– Weights are reset to $w_t^{(i)} = 1/N$.

This procedure is repeated for all periods $t = 1,\dots,T$, using $N \approx 4000$ particles to ensure numerical stability and smoothness in the estimation.

1.6 Reconstruction of potential GDP and calculation of the output gap

Once the filtered trajectory $\hat{\mu}_t$ has been estimated, potential GDP in levels is obtained as:

$$GDP\_pot_t = \exp(\hat{\mu}_t)$$

The output gap is defined as:

$$OutputGap_t = (GDP\_obs_t - GDP\_pot_t)/GDP\_pot_t \times 100$$

The final result provides:

– a smoothed trajectory of potential GDP,

– visible cycles in output,

– and an interpretation consistent with macroeconomic trend–cycle analysis.