Partial summary of work done in summer of 2016

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1 Heating a disc

Laplace equation. Solution to Laplace equation with the appropriate boundary conditions leads naturally to Fourier series. Questions of convergence raised. Must be answered in pieces.

2 The fourier series

Rather than dealing with regular convergence, dealt with Abel summability of the fourier series. Poisson kernel.

3 Digression: All about kernels

Convolution operation. Kernels. Dirac sequences of kernels. Proved that $f * D_n$ converges uniformly to f.

4 Weaker notions of convergence

Master theorem showed that the fourier series is Abel summable. Now create similar kernels for finite fourier series (Dirichlet kernel) and averages of first n terms (Fejér kernel). Fejér kernels form a Dirac sequence, hence the fourier series is Cesàro summable.

5 Orthonormal basis for $C(T)$

Proved exponential polynomials dense in $C(T)$: two different proofs using Poisson and Fejér kernels (Fejér kernel gives explicit approximation). Two line proof using Stone-Weirstrass, and a much nastier proof using Weirstrass approximation.

6 Strengthening the conditions on f

Used density result to show $\lim_{n\to\infty} \hat{f}(n) = 0$ for a continuous f (Riemann-Lebesgue lemma). Proved the principle of localisation. Then showed that if $\hat{f}(n)$ is $O\left(\frac{1}{n}\right)$, then the fourier series converges. Subsequently showed that if $f \in C^1(T)$, then $\hat{f}(n)$ is $o\left(\frac{1}{n}\right)$