ME-GY 7943/ECE-GY9273
Network Robotic Systems, Cooperative Control and Swarming

Exercise series 1

Please typeset your answers (e.g. using LaTeX). For all questions, justify clearly your answers.

Exercise 1

For each of the following matrices

$$A_1 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 0 & 1 & 2 & 1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}, \quad A_4 = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & -1 \end{bmatrix}$$

answer the following questions and justify your answers[1]

a) What is the rank of the matrix?

b) Compute the nullspace of the matrix (give an orthonormal basis for the space). What is its dimension?

c) Compute the range space of the matrix (give an orthonormal basis for the space). What is its dimension?

Exercise 2

In python, use the functions *eig* or *eigval* from the Linear algebra package of numpy, to compute the eigenvalues and eigenvectors of the following matrices[2]. For each matrix, how many real and complex eigenvalues did you find? How many 0 eigenvalues? Can you infer the rank and size of the nullspace of each matrix?

$$A_1 = \begin{bmatrix} 0 & 1 & 0 & 2 & 3 \\ 1 & 0 & 0 & 3 & 2 \\ 3 & -1 & 0 & 2 & 3 \\ 1 & -1 & 3 & 0 & 4 \\ -2 & 1 & -2 & -2 & 3 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 0 & 2 & 3 & 3 & 1 \\ 2 & 0 & -1 & 2 & 3 \\ 3 & -1 & 0 & 5 & 1 \\ 3 & 2 & 5 & 0 & 2 \\ 1 & 3 & 1 & 2 & 6 \end{bmatrix},$$

[1] Remember that for $A : n \times m$:

- $rank(A)$ is the number of linearly independent columns of A and that it is the same as the number of linearly independent rows of A
- $null(A) = \{x | Ax = 0\}$
- $range(A) = \{y | y = Ax, \forall x \in \mathbb{R}^m\}$
- $rank(A) + dim(null(A)) = m$

[2] `https://docs.scipy.org/doc/numpy/reference/routines.linalg.html`

Exercise 3

We want to deploy a team of robots for a recognition and rescue mission. We have two drones and four wheeled robots and create two teams of robot. Each team is composed of two wheeled robots and one drone. Due to terrain conditions, wheeled robots can only communicate to each other inside the same team and cannot communicate with wheeled robots or the drone of a different team. The drones can communicate with each other and with the wheeled robots of their own team.

- Draw the communication graph associated to this scenario (label the vertexes with the robot type)
- Write down the graph in terms of a set of edges and vertexes. Is the graph directed or undirected?
- Is the graph connected? Why?
- Compute the adjacency, degree and incidence matrices of the graph.
- Compute the graph Laplacian. What property of the Laplacian is a necessary and sufficient condition for having a connected graph? Use this property to test if the graph is connected (use a computer if necessary)

Exercise 4

Consider the following matrices

$$L_1 = \begin{bmatrix} 2 & -1 & -1 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 2 & -1 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 2 & -1 & 1 & 0 \\ -1 & 1 & 0 & 0 \\ -1 & 0 & 2 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}, \quad L_3 = \begin{bmatrix} 3 & 0 & -1 & -1 & -1 \\ 0 & 2 & 0 & -1 & -1 \\ -1 & 0 & 2 & -1 & 0 \\ -1 & -1 & -1 & 3 & 0 \\ -1 & -1 & 0 & 0 & 2 \end{bmatrix}$$

$$L_4 = \begin{bmatrix} 2 & 0 & -1 & -1 & -1 \\ 0 & 1 & 0 & -1 & -1 \\ -1 & 0 & 2 & -1 & 0 \\ -1 & -1 & -1 & 3 & 0 \\ -1 & -1 & 0 & 0 & 2 \end{bmatrix}, \quad L_5 = \begin{bmatrix} 2 & 0 & -1 & -1 & 0 \\ 0 & 1 & 0 & 0 & -1 \\ -1 & 0 & 2 & -1 & 0 \\ -1 & 0 & -1 & 2 & 0 \\ 0 & -1 & 0 & 0 & 1 \end{bmatrix}$$

Which of these matrices could be the Laplacian of a graph? Why? For the matrices that could be a Laplacian of a graph

a) Draw the associated graph

b) Is the graph connected?

c) How many positive, negative and zero eigenvalues do you expect the Laplacian to have? Why? Compute them to verify your answer.