Swarm Robotics Assignment 3

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1 Exercise 2

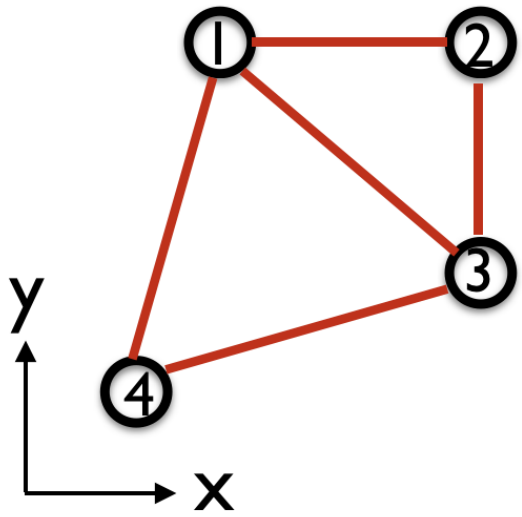

Figure 1: Framework with distance constraints $d_{12} = 1$, $d_{23} = 1$, $d_{13} = \sqrt{2}$, $d_{14} = 1.5$, $d_{34} = 1.5$

From the figure 1 we can see that the constrains for the system are,

$$
\begin{aligned}
d_{12} &= g_{\mathcal{G}}^1 = \|p_1 - p_2\|^2 = 1 \\
d_{23} &= g_{\mathcal{G}}^2 = \|p_2 - p_3\|^2 = 1 \\
d_{13} &= g_{\mathcal{G}}^3 = \|p_1 - p_3\|^2 = \sqrt{2} \\
d_{14} &= g_{\mathcal{G}}^4 = \|p_1 - p_4\|^2 = 1.5 \\
d_{34} &= g_{\mathcal{G}}^5 = \|p_3 - p_4\|^2 = 1.5
\end{aligned}
$$

thus we have the rigidity matrix $R_{\mathcal{G}}$ as,

$$
R_{\mathcal{G}} = \begin{bmatrix}
\frac{\partial g_{\mathcal{G}}^1}{\partial p_{1,x}} & \frac{\partial g_{\mathcal{G}}^1}{\partial p_{1,y}} & \cdots & \frac{\partial g_{\mathcal{G}}^1}{\partial p_{4,y}} \\
\vdots & \vdots & \ddots & \vdots \\
\frac{\partial g_{\mathcal{G}}^5}{\partial p_{1,x}} & \frac{\partial g_{\mathcal{G}}^5}{\partial p_{1,y}} & \cdots & \frac{\partial g_{\mathcal{G}}^5}{\partial p_{4,y}}
\end{bmatrix}
$$

we can construct the coordinate system as follow,

$$
\begin{aligned}
p_1 &= (0, 0) \\
p_2 &= (1, 0) \\
p_3 &= (1, -1) \\
p_4 &= \left(\frac{2 - \sqrt{14}}{4}, -\frac{\sqrt{14} + 2}{4}\right)
\end{aligned}
$$

to calculate the rigidity matrix we can write $g_{\mathcal{G}}^1$ as $\|p_1 - p_2\|^2 = (p_{1,x} - p_{2,x})^2 + (p_{1,y} - p_{2,y})^2$, and we have

$$\frac{\partial g_{\mathcal{G}}^1}{\partial p_{1,x}} = 2p_{1,x} - 2p_{2,x}$$
$$\frac{\partial g_{\mathcal{G}}^1}{\partial p_{1,y}} = 2p_{1,y} - 2p_{2,y}$$
$$\frac{\partial g_{\mathcal{G}}^1}{\partial p_{2,x}} = 2p_{2,x} - 2p_{1,x}$$
$$\frac{\partial g_{\mathcal{G}}^1}{\partial p_{2,y}} = 2p_{2,y} - 2p_{1,y}$$

and we can rewrite the rigidity matrix $R_{\mathcal{G}}$ as using the notation $p_{ij}^k = p_{i,k} - p_{j,k}$,

$$R_{\mathcal{G}} = \begin{bmatrix} 2p_{1,2}^x & 2p_{1,2}^y & 2p_{2,1}^x & 2p_{2,1}^y & 0 & 0 & 0 & 0 \\ 0 & 0 & 2p_{2,3}^x & 2p_{2,3}^y & 2p_{3,2}^x & 2p_{3,2}^y & 0 & 0 \\ 2p_{1,3}^x & 2p_{1,3}^y & 0 & 0 & 2p_{3,1}^x & 2p_{3,1}^y & 0 & 0 \\ 2p_{1,4}^x & 2p_{1,4}^y & 0 & 0 & 0 & 0 & 2p_{4,1}^x & 2p_{4,1}^y \\ 0 & 0 & 0 & 0 & 2p_{3,4}^x & 2p_{3,4}^y & 2p_{4,3}^x & 2p_{4,3}^y \end{bmatrix}$$
$$= \begin{bmatrix} -2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & -2 & 0 & 0 \\ -2 & 2 & 0 & 0 & 2 & -2 & 0 & 0 \\ (\sqrt{14}-2)/2 & (\sqrt{14}+2)/2 & 0 & 0 & 0 & 0 & (2-\sqrt{14})/2 & -(\sqrt{14}+2)/2 \\ 0 & 0 & 0 & 0 & (2+\sqrt{14})/2 & (\sqrt{14}-2)/2 & -(2+\sqrt{14})/2 & (2-\sqrt{14})/2 \end{bmatrix}$$

we can see that the rank of $R_{\mathcal{G}}$ is 5, thus the dimension of the kernel is 3 and the dimension of the range space is 5.

We can see that three kind of movements: translation along x and y plus rotation about z preserves the distance constraints of the framework.

For the infinitesimal motions of all the agents in the x direction we use $\dot{p}_x = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}^T$, for infinitesimal motions of all the agents in the y direction we use $\dot{p}_y = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}^T$. For infinitesimal motions of all the agents rotating about a fixed point p^* we have,

$$\dot{p}^* = \begin{bmatrix} p_y^* - p_{1,y} \\ p_{1,x} - p_x^* \\ p_y^* - p_{2,y} \\ p_{2,x} - p_x^* \\ p_y^* - p_{3,y} \\ p_{3,x} - p_x^* \\ p_y^* - p_{4,y} \\ p_{4,x} - p_x^* \end{bmatrix} = \begin{bmatrix} p_y^* \\ -p_x^* \\ p_y^* \\ 1 - p_x^* \\ p_y^* + 1 \\ 1 - p_x^* \\ p_y^* + \frac{\sqrt{14}+2}{4} \\ \frac{2-\sqrt{14}}{4} - p_x^* \end{bmatrix}$$

we can see that $R_{\mathcal{G}}\dot{p}_x = R_{\mathcal{G}}\dot{p}_y = R_{\mathcal{G}}\dot{p}^* = \mathbf{0}$. Also we have the rank of the matrix $\begin{bmatrix} \dot{p}_x & \dot{p}_y & \dot{p}^* \end{bmatrix}$ as 3 thus it forms a basis for the kernel.