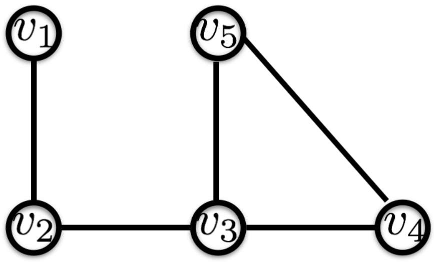
ME-GY 7943 - Network Robotic Systems, Cooperative Control and Swarming

Set of Training Exercises

Exercise 1

Consider the following graph



a) Compute the adjacency, degree and incidence matrices of the graph.

b) Compute the graph Laplacian

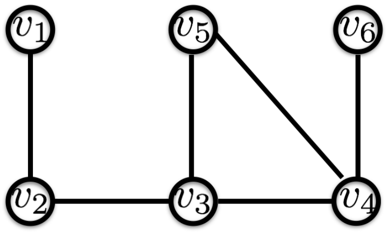
c) What is the relationship between the adjacency, degree, incidence matrices and the Laplacian matrix?

d) Without computing the eigenvalues of the Laplacian, what signs do you expect for all the eigenvalues? What value do you expect for the smallest one? And for the second smallest one?

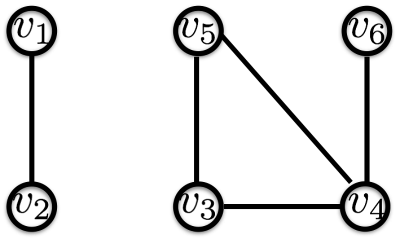
e) Suppose we remove the edge between vertex 1 and vertex 2, how do you expect the eigenvalues of the Laplacian to change?

Exercise 2

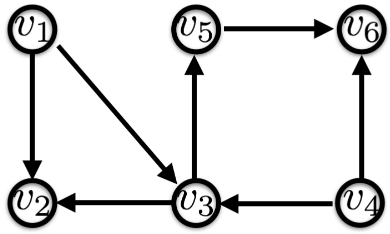
We want to implement the consensus protocol to measure the average temperature of a room with a set of 6 robots. We are given several possible communication schemes, defined by the following graphs



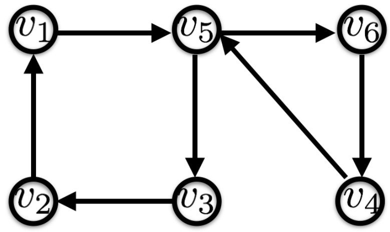
Graph 1



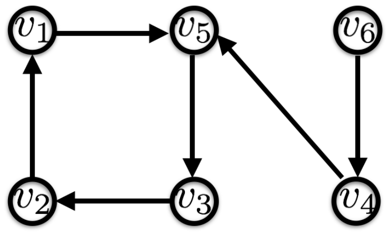
Graph 2



Graph 3



Graph 4



Graph 5

a) What is the consensus protocol? (Write down the associated equations).

b) For which of the above graphs will the protocol converge to consensus? (justify your answer)

c) For all the graphs for which the protocol would converge to consensus, which ones would ensure convergence to the average of the initial temperature measurements? Can you guess the values to which the other graphs will converge? What will be the rate of convergence? (justify you answer)

Exercise 3

We would like to design a distributed controller to keep the formation of 3 robots in a 2D space. In order to design the controller, we consider a framework composed of 3 vertexes and edges such that they form a complete graph with a function $p : \mathcal{V} \to \mathbb{R}^2 = \{(v_1, (0, 0)),\ (v_2, (1, d)),\ (v_3, (2, 0))\}$.

a) Draw the framework (include the coordinate axes)

b) Write down the distance constraints defined by the framework and compute the rigidity matrix of the framework

c) What are the types of robot motions that preserve the formation?

d) In the general case of N agents in a 2D space, what should be the rank of the rigidity matrix to ensure infinitesimal rigidity?

e) For which values of d is the framework infinitesimally rigid? (justify)

f) For which values of d is the framework rigid? (justify)

g) Assume that each robot has the following integrator dynamics

$$\begin{aligned}\dot{x}_i &= u_{x,i} \\ \dot{y}_i &= u_{y,i}\end{aligned}$$

Propose a control law for $u_{x,i}, u_{y,i}$ that will ensure proper formation control.

h) Give an example of a framework that is rigid but not globally rigid (including a drawing of the framework)

j) Give an example of a framework that is minimally rigid (include a drawing of the framework)

Exercise 4

Consider the following linear differential equation

$$\dot{x} = -x + y \tag{1}$$
$$\dot{y} = -y \tag{2}$$

a) What are the fixed points of the system?

b) What is the solution of this equation? (hint: write it using a matrix exponential)

c) Knowing that the eigenvalues of the matrix $\begin{bmatrix} -1 & 1 \\ 0 & -1 \end{bmatrix}$ are -1 and -1, can you deduce the stability of the system? What does it mean?

d) What does Lyapunov stable means? Is the system Lyapunov stable?

e) Can you find a Lyapunov function for the system? What conclusions can you draw?

f) Is the origin globally asymptotically stable?

Exercise 5

Answer the following questions

a) Are the three vectors $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ linearly independent? Justify.

b) Give an example of a symmetric positive definite matrix. What can you expect of its eigenvalues?

c) What is the rank of a matrix? What is the rank of matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \tag{3}$$

d) What is the nullspace of a matrix? Compute the nullspace of matrix

$$\begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \end{bmatrix} \tag{4}$$

e) If the rank of a matrix A of size 10x10 is 6, what is the dimension of its nullspace?