ME-GY 7943 / ECE-GY9273
Network Robotic Systems Fall 2019
Set of training exercises

Exercise 1

We are given a team of 3 underwater robots with the mission to find the coldest place in a lake. We want to design the complete control system in two steps.

Formation control First, we would like to keep the robots in a triangle formation as shown in Figure **??** while they move in the lake.

a) Define a framework that can describe the desired formation (drawing + formal definition).

b) Is the chosen framework rigid? (Explain why in words). Are there any advantages of a rigid framework over a non-rigid framework for formation control? (explain).

c) Design a control law for the velocity of the robot that regulates the desired formation (i.e. write for each robot a control law of the form $\dot{p}_i = f(p_1, p_2, p_3)$ where $p_i \in \mathbb{R}^2$ is the position of robot i).

d) What are the pros and cons of the control law you chose?

Temperature Estimation When the robots are in formation, we would like to get an approximation of the temperature at the center of the triangle (x_0, y_0) and its gradient given the known robot positions and the temperature measurements coming from each robot.

e) If the robots report their position and temperature measurements to a base station, how can the base station estimate the temperature and gradient at (x_0, y_0)?

f) Assuming that $(x_0, y_0) = (0, 0)$ and that $T_1 = 10$ and $T_2 = T_3 = -5$, find an estimate of the temperature and gradient at (x_0, y_0).

f) If it is not possible to communicate to the base station, what decentralized algorithm would allow each robot to compute these estimates? Describe each part of the algorithm in detail.

Complete system

g) Based on the previous answers, describe a complete control/estimation algorithm that will make sure the robots can find the coldest place in the lake. Write the associated control laws.

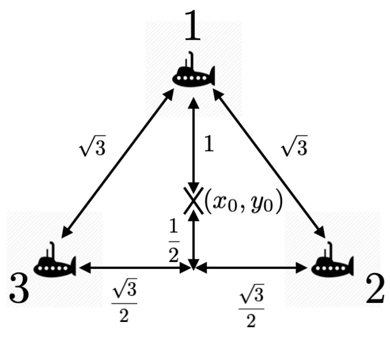

Figure 1: Desired formation of the robots in Exercise 1 and center of the triangle.

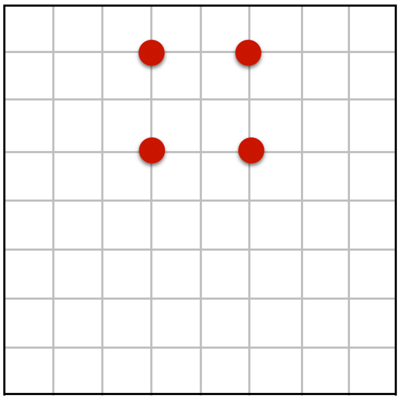

Figure 2: The red dots show the initial configuration of the robots.

Exercise 2

We are interested in the optimal coverage of the 2D space represented in Figure **??** using 4 robots.

a) What is the coverage problem? Describe one concrete application for which coverage can be useful.

b) Draw the first 3 iterations of Lloyd's algorithm using the initial conditions shown in Figure **??**. For each iteration, explain what is being done.

c) Draw the Voronoi diagram that you expect after convergence of the algorithm.

d) If the red dots were mobile robots, describe a decentralized control law for each robot that would ensure appropriate coverage of the space (show the equations and explain how it works).

Exercise 3

Consider the graphs shown in Figure **??**. We would like to implement the consensus algorithm on each of these graphs.

a) What is the consensus algorithm?

b) Write the matrix Laplacian for graph a) and b) and write the equations for the consensus algorithm for each robot for both graphs.

c) What is the rank of the Laplacian of graph b), d) and f)? Justify your answer.

d) We run the consensus protocol with initial conditions $v_1(0) = 0$, $v_2(0) = 10$, $v_3(0) = 5$, $v_4(0) = 20$. For each of the 6 graphs, what will be the value of each v_i when time goes to infinity?

e) In the case of graph a), we would like to ensure that the difference between two connected elements is never higher than 20 (in absolute value) when running the consensus algorithm. Propose a consensus algorithm that will enforce such constraint.

Exercise 4

We would like to use several robots to transport objects.

a) What are the advantages and disadvantages of using several robots to transport objects?

b) Describe three different strategies to move objects with a swarm of robots. What are their pros and cons?

c) We are using 100 robots to transport a piano of mass M, subject to friction force $\mu\dot{\mathbf{x}}$, where $\mathbf{x}$ is the position of the center of mass of the piano. Assume that each robot can apply a force F_i on the piano. What are the equations of motion of the piano's center of mass?

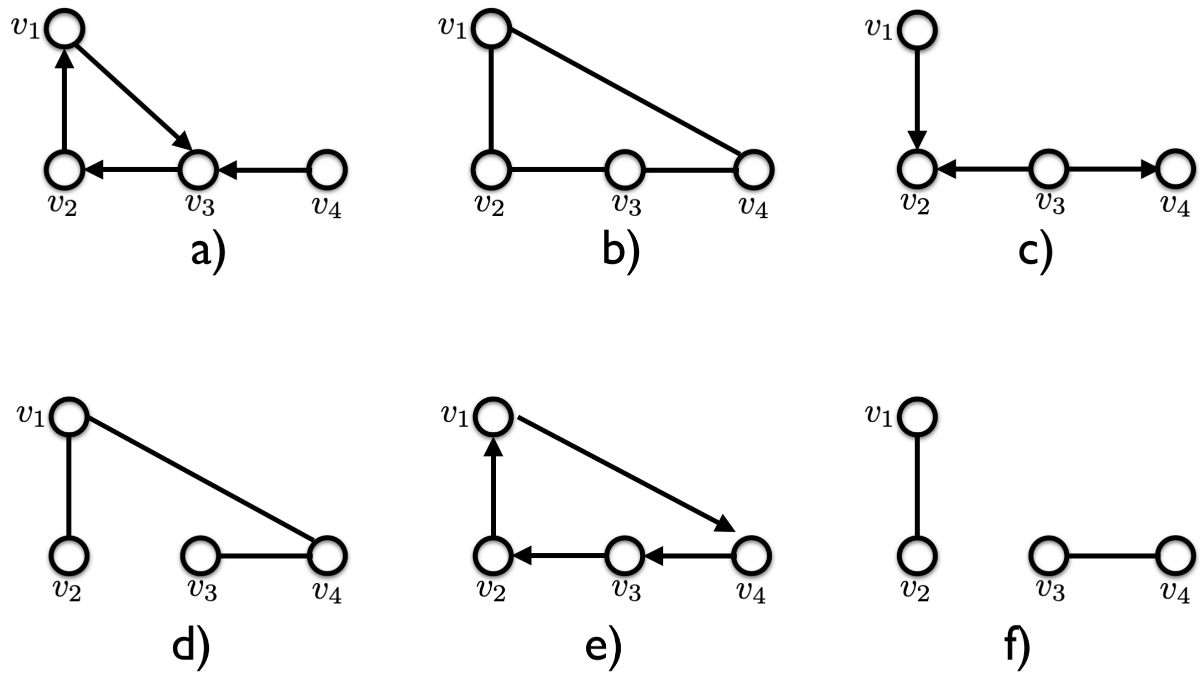

Figure 3: Graphs considered in Exercise 3

d) Assuming that the robots are not allowed to communicate with each other but can measure the piano's center of mass position, velocity and acceleration, propose a control law for each robot that will allow to move the piano towards a goal $x_{desired}$. What will be the force applied by each robot?

Exercise 5

a) What is a Lyapunov function? Why is it useful?

b) If a framework is rigid does it mean that it is infinitesimally rigid? Is the converse true? Justify your answers.

c) Show an example of a framework with 5 elements that is minimally rigid? Explain why it is minimally rigid.

d) Give two examples of individual behaviors in animals that lead to coordinated motions.