Swarm Robotics Assignment 2

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1 Exercise 1

After testing the `get_laplacian()` function the result is the same as in the given example.

Then I found the two smallest and two largest eigenvalues for C_5, C_{15} and C_{199} are,

Graph	Smallest	Second Smallest	Second Largest	Largest
C_5	0.00	1.38	3.62	3.62
C_{15}	0.00	0.17	3.96	3.96
C_{199}	0.00	9.97e-4	4.00	4.00

2 Exercise 2

For the given x_0 we have the following relationship between the state and time shown in figure 1a, as we can it takes 12.831s to reach consensus which is where all states becomes 16.11, this aligns with the theoretical prediction.

If we cut the connection between 2 and 6 and 2 and 4 it becomes a disconnected graph, thus it will not converge at all, which is verified by figure 1b.

Using a complete graph the system will converge a lot faster, this is verified by figure 1c, all the states converge to 16.11 in 0.866s, this aligns with the theoretical prediction.

Using a cycle graph the system will converge, however the rate will be much slower than the complete graph, which is shown in figure 1d. The states will also converge to 16.11, this aligns with the theoretical prediction.

3 Exercise 3

For a directed graph to satisfy $\lambda_2 > 0$ we should have the Laplacian to be symmetric, for a directed graph we have the property $Re(\lambda_2) \geq 0$ when it contains a rooted-out branching.

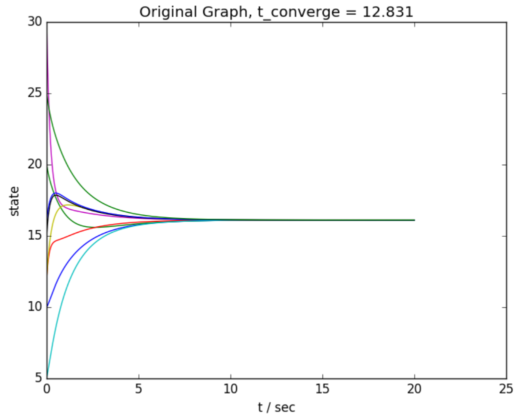
To prove this we have the theorem that a digraph D contains a rooted-out branching as a subgraph if and only if. rank(L(D)) = N − 1. Thus we have the null space spanned by $\mathbf{1}$ which is also the eigenvector corresponding to the eigenvalue 0.

Since we have $Re(\lambda_2) \geq 0$ and only one eigenvalue is 0, we can have $Re(\lambda_2) > 0$.

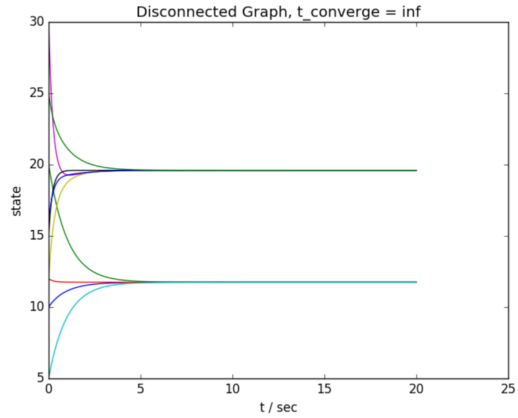
Now we prove that $\mathbf{q}_1^T\mathbf{x}$ is a constant of motion.

$$\frac{d}{dt}(\mathbf{q}_1^T\mathbf{x}) = \mathbf{q}_1^T L(\mathcal{D})\mathbf{x}(t) = 0 \cdot \mathbf{x}(t) = 0$$

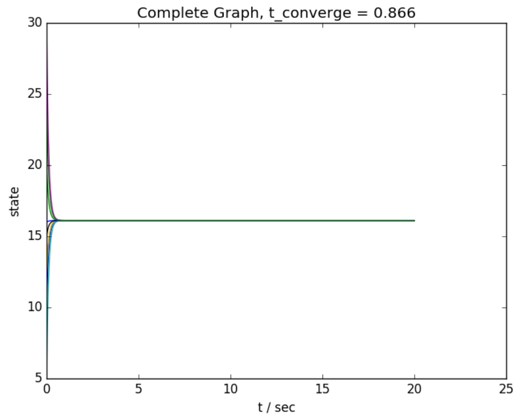
thus we can see that it is a constant of motion.



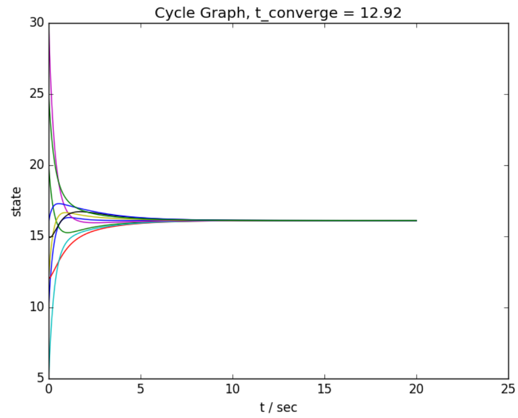
(a) State evolution for the original graph



(b) State evolution when edges connecting 2 and 6 and 2 and 4 are cut



(c) State evolution for a complete graph



(d) State evolution for the cycle graph

Given that $\mathbf{q}_1^T\mathbf{x}$ is a constant of motion we can have

$$\begin{aligned}
\mathbf{q}_1^T\mathbf{x}_0 &= \mathbf{q}_1^T\mathbf{x}_{\text{final}} \\
\mathbf{q}_1^T\mathbf{x}_0 &= x_{\text{final}}\mathbf{q}_1^T\mathbf{1} \\
\frac{\mathbf{q}_1^T\mathbf{x}_0}{\sum_i q_i} &= x_{\text{final}}
\end{aligned}$$

We have the states converge to $\dfrac{\mathbf{q}_1^T\mathbf{x}_0}{\sum_i q_i}$ which in this case we have

$$\mathbf{q}_1 = \begin{bmatrix} 0 \\ 0.378 \\ 0.378 \\ 0.378 \\ 0.756 \end{bmatrix}$$

and we have

$$\frac{\mathbf{q}_1^T\mathbf{x}_0}{\sum_i q_i} = 0.38$$

this can be verified by figure 2a.

After removing the edges to the graph to make it balanced, shown in figure 2d, we have,

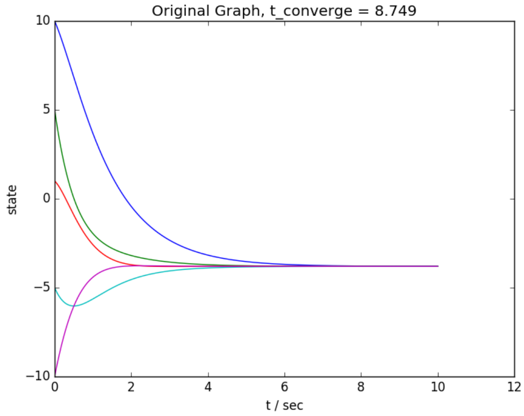
$$\mathbf{q}_1 = \begin{bmatrix} 0.447 \\ 0.447 \\ 0.447 \\ 0.447 \\ 0.447 \end{bmatrix}$$

and we have it to converge to $x_{\text{final}} = 0.2$, which is confirmed by figure 2c.

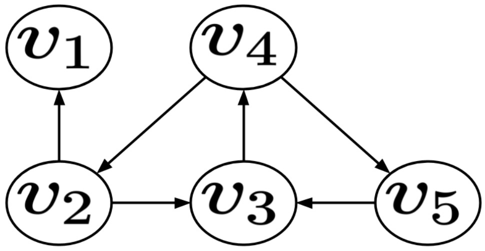
Then when making the leader v_4, shown in figure 2f, we have,

$$\mathbf{q}_1 = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1.0 \\ 0 \end{bmatrix}$$

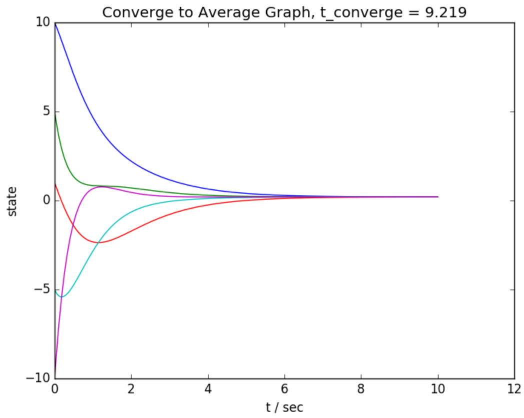
and we have it to converge to $x_{\text{final}} = 0.5$, which is confirmed by figure 2e.



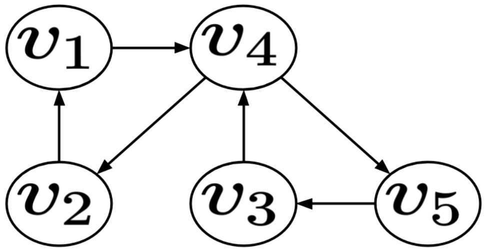
(a) State evolution for the original graph



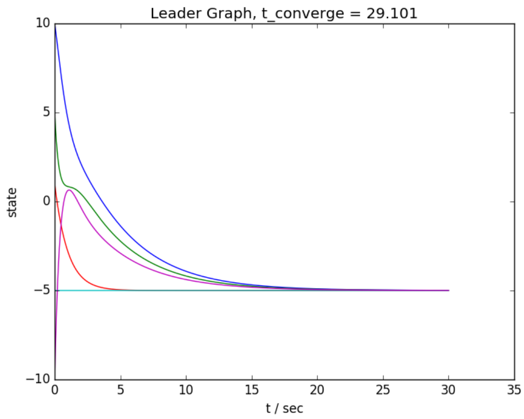
(b) Structure of the original graph



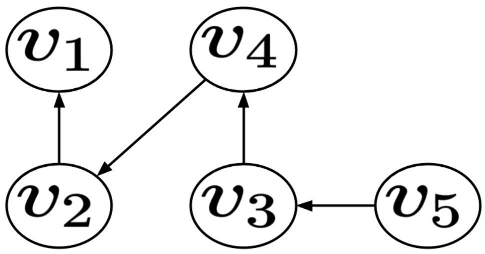
(c) State evolution for the balanced graph



(d) Structure of the balanced graph



(e) State evolution for a rooted-out branching with leader graph



(f) Structure of the rooted-out branching with leader graph