ASSIGNMENT 2

SECI1013

STRUKTUR DISKRIT (DISCRETE STRUCTURE)

LECTURER: DR. NOORFA HASZLINNA BINTI MUSTAFFA

SECTION: 02

GROUP MEMBERS:

TAN YI YA	**A23CS0187**
TEH RU QIAN	**A23CS0191**
GOE JIE YING	**A23CS0224**

Assignment 2

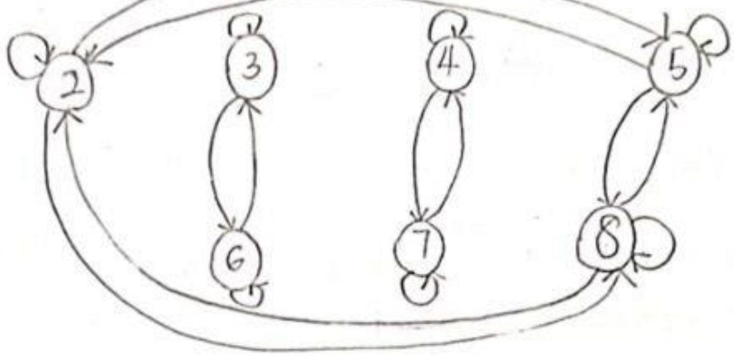
1. $A = \{2,3,4,5,6,7,8\}$, $x - y = 3n$

when $x - y = 3n$ (n = multiple of 3),

$R = \{(8,5),(5,8),(7,4),(4,7),(6,3),(3,6),(5,2),(2,5),(8,2),(2,8)\}$,,

when $x - y = 0$ ($3n = 0$),

$R = \{(2,2),(3,3),(4,4),(5,5),(6,6),(7,7),(8,8)\}$,,



Domain = $\{2,3,4,5,6,7,8\}$

Range = $\{2,3,4,5,6,7,8\}$

Reflexive, Symmetric, Transitive

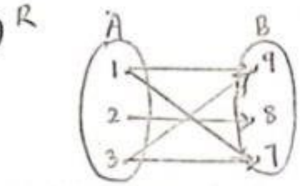
$\therefore$ Equivalance

2. $R = \{(1,9),(1,8),(1,7),(2,9),(2,8),(2,7),(3,9),(3,8),(3,7)\}$

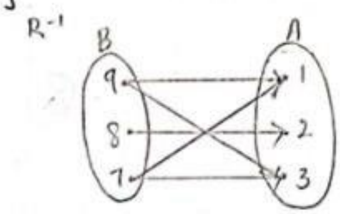
(a) $R = \{(1,9),(1,7),(2,8),(3,9),(3,7)\}$

$R^{-1} = \{(9,1),(7,1),(8,2),(9,3),(7,3)\}$

(b) R



R^{-1}

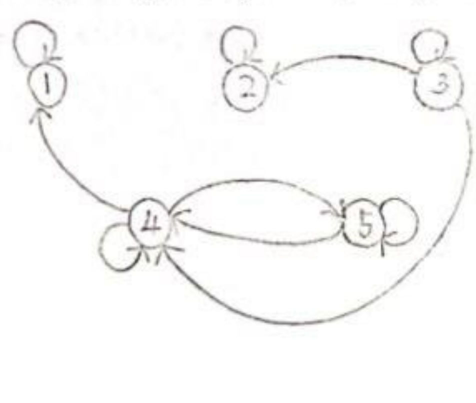


(c) R^{-1} : B to A

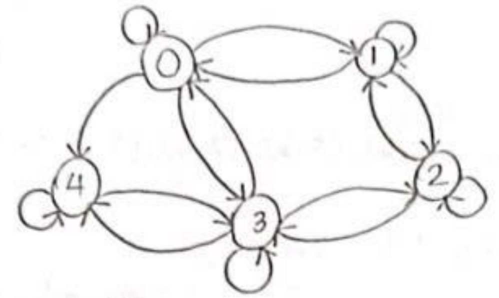
For all $(b,a) \in B \times A$, $b\,R\,a \leftrightarrow b + a$ is an even number.

3. $A = \{1,2,3,4,5\}$

$R = \{(1,1),(2,2),(3,2),(3,3),(3,4)(4,1),(4,4),(4,5),(5,4),(5,5)\}$



	In-degree	Out-degree
1	2	1
2	2	1
3	1	3
4	3	3
5	2	2

4. 

$\therefore$ R is reflexive because $(0,0),(1,1),(2,2),(3,3),(4,4) \in R$

R is symmetric because $(0,1)$ and $(1,0) \in R$, $(1,2)$ and $(2,1) \in R$, $(2,3)$ and $(3,2) \in R$, $(3,4)$ and $(4,3) \in R$.

R is not transitive because $(0,1)$ & $(1,2) \in R$, but $(0,2) \notin R$

5. $R = \{(1,3),(2,6),(3,9),(4,12)\}$

(a) R is irreflexive because $(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(7,7),(8,8),(9,9),(10,10),(11,11),(12,12),(13,13),(14,14) \notin R$. $(x,x) \notin R$

(b) R is antisymmetric because $(1,3) \in R$ but $(3,1) \notin R$.

(c) R is not transitive because $(1,3)$ and $(3,9) \in R$ but $(1,9) \notin R$.

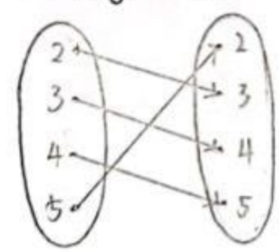
6. (a) $RS = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \otimes \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ //

(b) $SR = \begin{bmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix} \otimes \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \end{bmatrix}$ //

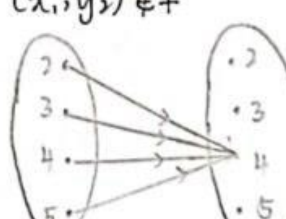
7. Relation is a set of ordered pairs, function is a specific type of relation where each input value (element of the domain) is associated with exactly one output value (element of the codomain).

8. $A = \{2,3,4,5\}$

(i) $\{(2,3),(3,4),(4,5),(5,2)\}$ is a function because $(x_1, y_1) \in f$ and $(x_1, y_2) \notin f$.

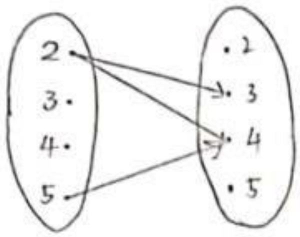


(ii) $\{(2,4),(3,4),(5,4),(4,4)\}$ is a function because $(x_1, y_1) \in f$ and $(x_1, y_2) \notin f$



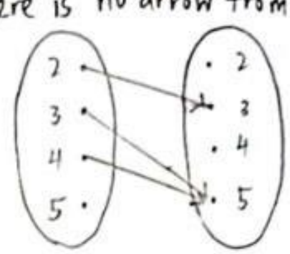
(iii) $\{(2,3),(2,4),(5,4)\}$

is not a function because $(2,3)$ and $(2,4) \in f$, $3 \neq 4$



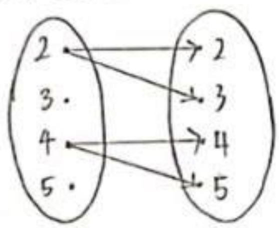
(iv) $\{(2,3),(3,5),(4,5)\}$

is not a function because $(5,y) \notin f$, there is no arrow from 5



(v) $\{(2,2),(2,3),(4,4),(4,5)\}$

is not a function because $(2,2)$ and $(2,3) \in f$, $2 \neq 3$



9. $R = \{(1,6),(2,7),(3,8),(4,9),(5,10)\}$

Domain $= \{1,2,3,4,5\}$

Range $= \{6,7,8,9,10\}$

10. (v) $f = R \to R$, $f(x) = 1-2x$

$f(x_1) = f(x_2)$

$1-2x_1 = 1-2x_2$

$-2x_1 = -2x_2$

$x_1 = x_2$ $\therefore$ one to one

$f(x) = 1-2x$

let $f(x) = y$

$2x = 1-y$

$x = \frac{1-y}{2}$

$\therefore$ there exist an x for every y, hence it is onto

$\therefore$ Since the function is both one to one and onto, it is bijective

(vi) $f = R \to R$, $f(x) = 5x^2-1$

$f(x_1) = f(x_2)$

$5x_1^2-1 = 5x_2^2-1$

$5x_1^2 = 5x_2^2$

$x_1^2 = x_2^2$

$x_1 \neq x_2$ $\therefore$ not one to one

$f(x) = 5x^2-1$

$5x^2 = y+1$

$x^2 = \frac{y+1}{5}$

$\therefore$ there do not exist x for every y, hence the function is not onto

$\therefore$ Since the function is not one to one and not onto, it is not bijective.

(vii) $f = R \to R$, $f(x) = x^4$

$f(x_1) = f(x_2)$

$x_1^4 = x_2^4$

$x_1 \neq x_2$ $\therefore$ not one to one

$y = x^4$

$\therefore$ there do not exist x for every y, hence the function is not onto

$\therefore$ Since the function is not one to one and not onto, it is not bijective.

(viii) $f = R \to R$, $f(x) = \left(\frac{x-2}{x-3}\right)$

$f(x_1) = f(x_2)$

$\left(\frac{x_1 - 2}{x_1 - 3}\right) = \left(\frac{x_2 - 2}{x_2 - 3}\right)$

$(x_1 - 2)(x_2 - 3) = (x_2 - 2)(x_1 - 3)$

$x_1 \neq x_2$ $\therefore$ not one to one

$y = \frac{x-2}{x-3}$

$y(x-3) = x - 2$

$xy - 3y = x - 2$

$x(y-1) = 3y - 2$

$x = \frac{3y-2}{y-1}$ $\therefore$ onto

$\therefore$ Since the function is not both one to one and onto, it is not bijective.

11. (ix) $f(g(x)) = f(x^2 - 1)$

$= 3(x^2 - 1) - 1$

$= 3x^2 - 4$ //

$\underline{x = 0}$

$fg(0) = 3(0)^2 - 4$

$= -4$ //

$\underline{x = 1}$

$fg(1) = 3(1)^2 - 4$

$= -1$ //

$\underline{x = 2}$

$fg(2) = 3(2)^2 - 4$

$= 8$ //

$\underline{x = 3}$

$fg(3) = 3(3)^2 - 4$

$= 23$ //

(x) $f(g(x)) = f(5x - 6)$

$= (5x - 6)^2$

$= 25x^2 - 30x - 30x + 36$

$= 25x^2 - 60x + 36$ //

$\underline{x = 0}$

$fg(0) = 25(0)^2 - 60(0) + 36$

$= 36$ //

$\underline{x = 1}$

$fg(1) = 25(1)^2 - 60(1) + 36$

$= 1$ //

$\underline{x = 2}$

$fg(2) = 25(2)^2 - 60(2) + 36$

$= 16$ //

$\underline{x = 3}$

$fg(3) = 25(3)^2 - 60(3) + 36$

$= 81$ //

(xi) $f(g(x)) = f(x^3 + 1)$

$= x^3 + 1 - 1$

$= x^3$ //

$\underline{x = 0}$

$fg(0) = 0^3$

$= 0$ //

$\underline{x = 1}$

$fg(1) = 1^3$

$= 1$ //

$\underline{x = 2}$

$fg(2) = 2^3$

$= 8$ //

$x = 3$

$fg(3) = 3^3$

$= 27$ //

12. (xii) $a_n = 6a_{n-1} - 9a_{n-2}$; $a_0 = 1$ and $a_1 = 6$

$a_2 = 6(6) - 9(1) = 27$

$a_3 = 6(27) - 9(6) = 108$ $\quad\therefore 1, 6, 27, 108, 405, \ldots$

$a_4 = 6(108) - 9(27) = 405$

(xiii) $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$; $a_0 = 2, a_1 = 5, a_2 = 15$

$a_3 = 6(15) - 11(5) + 6(2) = 47$

$a_4 = 6(47) - 11(15) + 6(5) = 147$ $\quad\therefore 2, 5, 15, 47, 147, 455, \ldots$

$a_5 = 6(147) - 11(47) + 6(15) = 455$

(xiv) $a_n = -3a_{n-1} - 3a_{n-2} + a_{n-3}$; $a_0 = 1, a_1 = -2, a_2 = -1$

$a_3 = -3(-1) - 3(-2) + 1 = 10$

$a_4 = -3(10) - 3(-1) + (-2) = -29$ $\quad\therefore 1, -2, -1, 10, -29, 56, \ldots$

$a_5 = -3(-29) - 3(10) + (-1) = 56$

13. (i) $a_{n+1} = 5a_n - 3$; $a_1 = k$

$a_2 = 5k - 3$

$a_3 = 5(5k - 3) - 3$

$= 25k - 18$

$a_4 = 5(25k - 18) - 3$

$= 125k - 90 - 3$

$= 125k - 93$ //

(ii) $a_4 = 7$

$7 = 125k - 93$

$k = 0.8$ //