BIGEY
Raphaël

Ex 1.6.1 :

1) * $\varepsilon_{1pq}\, u_p \wedge u_q = \varepsilon_{123}\, u_2 \wedge u_3 + \varepsilon_{132}\, u_3 \wedge u_2$

$= u_2 \wedge u_3 - u_3 \wedge u_2$

$= 2\, u_2 \wedge u_3$

Donc $\underline{\frac{1}{2} \varepsilon_{1pq}\, u_p \wedge u_q = u_2 \wedge u_3}$

* $\varepsilon_{2pq}\, u_p \wedge u_q = \varepsilon_{213}\, u_1 \wedge u_3 + \varepsilon_{231}\, u_3 \wedge u_1$

$= -u_1 \wedge u_3 + u_3 \wedge u_1$

$= 2\, u_3 \wedge u_1.$

Donc $\underline{\frac{1}{2} \varepsilon_{2pq}\, u_p \wedge u_q = u_3 \wedge u_1.}$

* $\varepsilon_{3pq}\, u_p \wedge u_q = \varepsilon_{312}\, u_1 \wedge u_2 + \varepsilon_{321}\, u_2 \wedge u_1$

$= u_1 \wedge u_2 - u_2 \wedge u_1$

$= 2\, u_1 \wedge u_2.$

Donc $\underline{\frac{1}{2} \varepsilon_{3pq}\, u_p \wedge u_q = u_1 \wedge u_2.}$

2) si $\underline{j=1}$: $(Cof(A))_{i1} = (u_2 \wedge u_3)_i = \left(\frac{1}{2} \varepsilon_{1pq}\, u_p \wedge u_q\right)_i$

si $\underline{j=2}$: $(Cof(A))_{i2} = (u_3 \wedge u_1)_i = \left(\frac{1}{2} \varepsilon_{2pq}\, u_p \wedge u_q\right)_i$

si $\underline{j=3}$: $(Cof(A))_{i3} = (u_1 \wedge u_2)_i = \left(\frac{1}{2} \varepsilon_{3pq}\, u_p \wedge u_q\right)_i$

Ainsi $(Cof(A))_{ij} = \boxed{\left(\frac{1}{2} \varepsilon_{jpq}\, u_p \wedge u_q\right)_i}$

$= \left(\frac{1}{2} \varepsilon_{jpq}\, \varepsilon_{\ell mn}\, (u_p)_m\, (u_q)_n\, e_\ell\right)_i$

$= \boxed{\frac{1}{2} \varepsilon_{jpq}\, \varepsilon_{imn}\, A_{mp}\, A_{nq}}$

3) $(\mathrm{Cof}(A^t))_{ij} = \frac{1}{2} \varepsilon_{jpq} \varepsilon_{imn} (A^t)_{mp} (A^t)_{nq}$
$= \frac{1}{2} \varepsilon_{jpq} \varepsilon_{imn} A_{pm} A_{qn}$
$= \frac{1}{2} \varepsilon_{imn} \varepsilon_{jpq} A_{pm} A_{qn}$.
$= \frac{1}{2} \varepsilon_{ipq} \varepsilon_{jmn} A_{mp} A_{nq}$, en échangeant: $p \longleftrightarrow m$, $q \longleftrightarrow n$
$= (\mathrm{Cof}(A))_{ji} = (\mathrm{Cof}(A))^t_{ij}$.

Donc $\mathrm{Cof}(A^t) = \mathrm{Cof}(A)^t$

4) $(A(\mathrm{Cof}(A^t)))_{ij} = A_{ik} (\mathrm{Cof}(A^t))_{kj}$
$= A_{ik} \frac{1}{2} \varepsilon_{jpq} \varepsilon_{kmn} A_{pm} A_{qn}$
$= \frac{1}{2} \varepsilon_{jpq} \varepsilon_{kmn} A_{ik} A_{pm} A_{qn}$
$= \frac{1}{2} \varepsilon_{jpq} \varepsilon_{ipq} \det(A)$, par (1.2.12)
$= \frac{1}{2} \times 2 \delta_{ij} \det(A)$
$= \delta_{ij} \det(A)$
$= [(\det A) I]_{ij}$.

Donc $A\,\mathrm{Cof}(A^t) = \det(A) I$.

↳ Si A inversible: $\mathrm{Cof}(A^t) \times \frac{1}{\det(A)} = A^{-1}$.
en multipliant par A^{-1}.

5) admis.
6) A inversible $\Rightarrow \mathrm{Cof}(A)\mathrm{Cof}(A^{-1}) = \mathrm{Cof}(I) = I$.
↳ et ainsi $\mathrm{Cof}(A)^{-1} = \mathrm{Cof}(A^{-1})$.

7) $Au \wedge Av = \varepsilon_{pqr} (Au)_q (Av)_r e_p$
$= \varepsilon_{pqr} (A_{qj} u_j)(A_{rk} v_k) e_p$
...

8) /