CS 3630!

Lecture 4:
Inference in
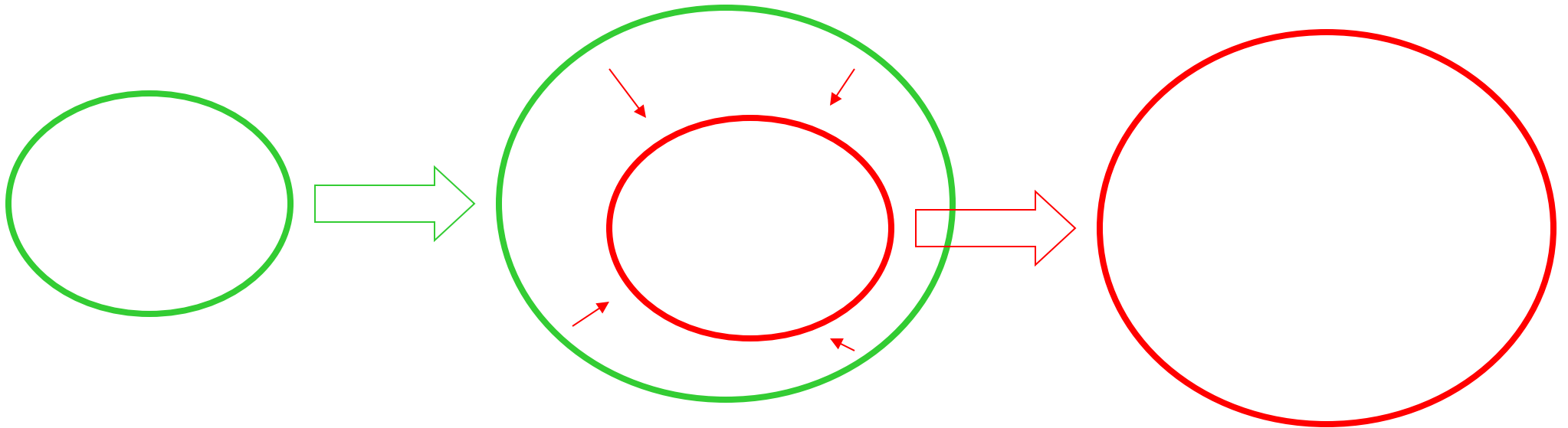Graphical Models

Topics

- **1. Bayes Filter**
- **2. HMMs**
- **3. Factor Graphs**
- **4. Converting Bayes Nets into Factor Graphs**
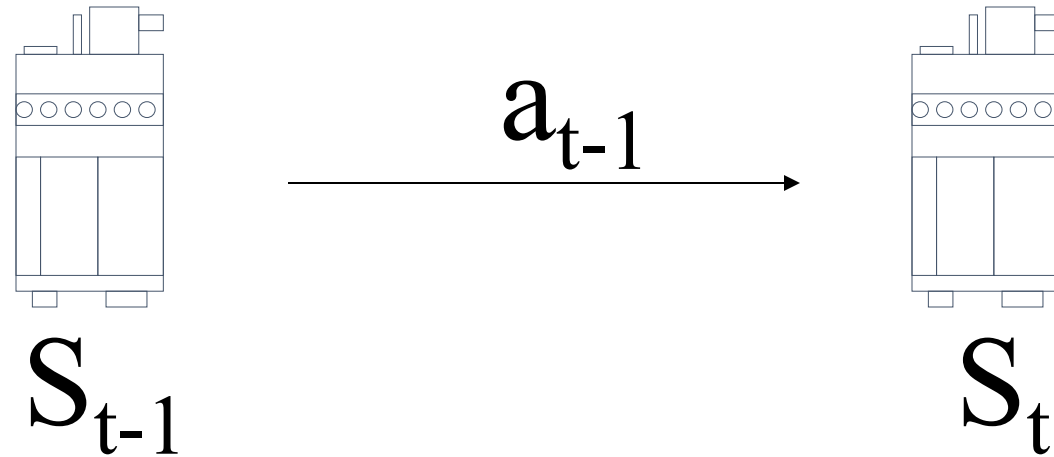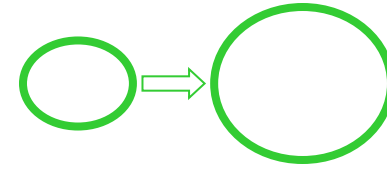- **5. The Max-Product Algorithm for HMMs**

Motivation

- Probability -> simulate robots !
- Our example: grid world
- Probabilistic statements about state: Bayesian inference

1. The Bayes filter

- Two phases: a. Prediction Phase
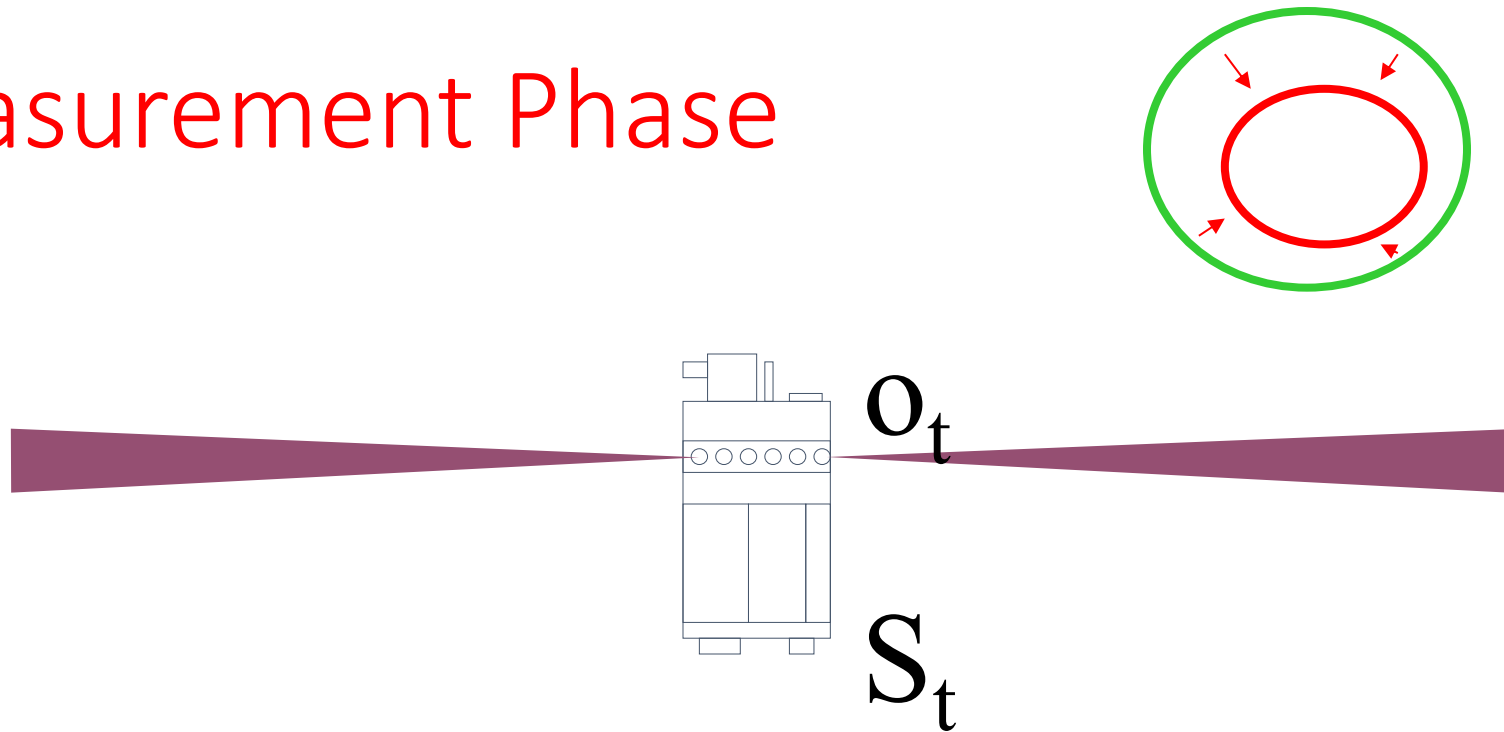   b. Measurement Phase

a. Prediction Phase

$$P(S_t) = \sum_{s_{t-1}} P(S_t | s_{t-1}, a_{t-1}) P(s_{t-1})$$

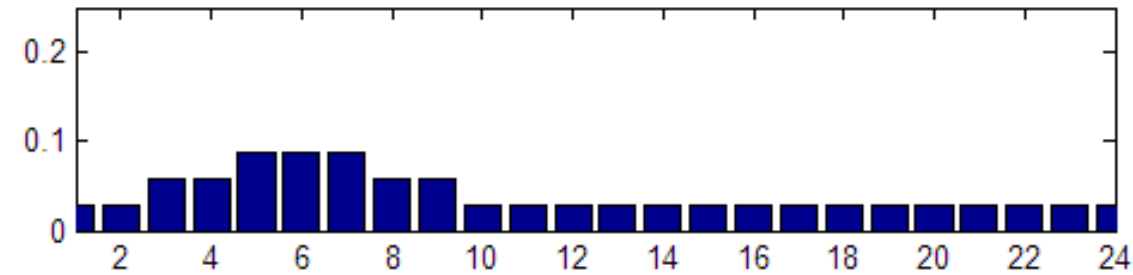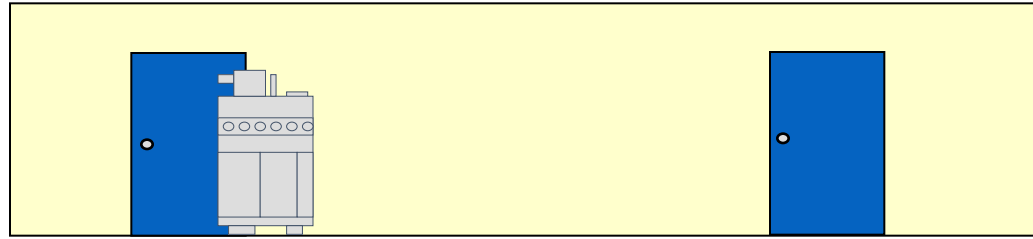State transition model

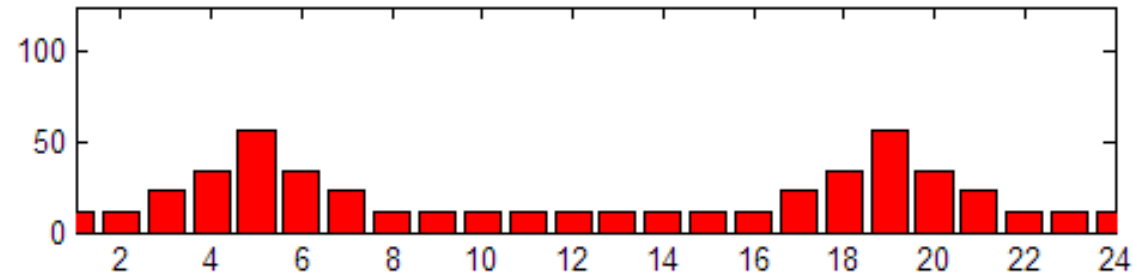Previous state prior

b. Measurement Phase

o_t

S_t

$$P(S_t|O_t = o_t) \propto P(o_t|S_t)P(S_t)$$
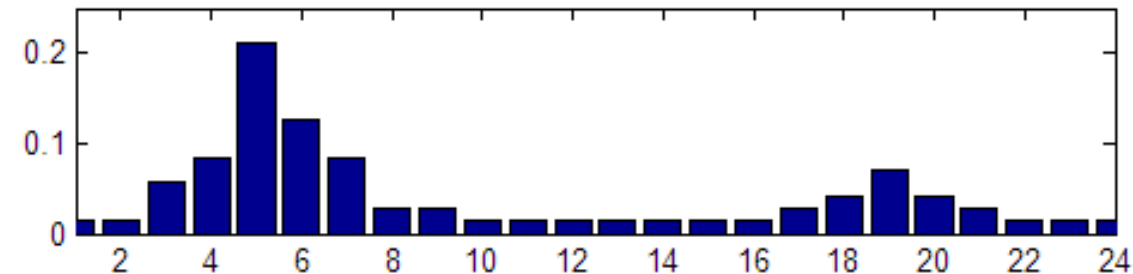$$\propto L(S_t|o_t)P(S_t)$$

Sensor   State

Model   prior

Markov Localization: a 1D Example
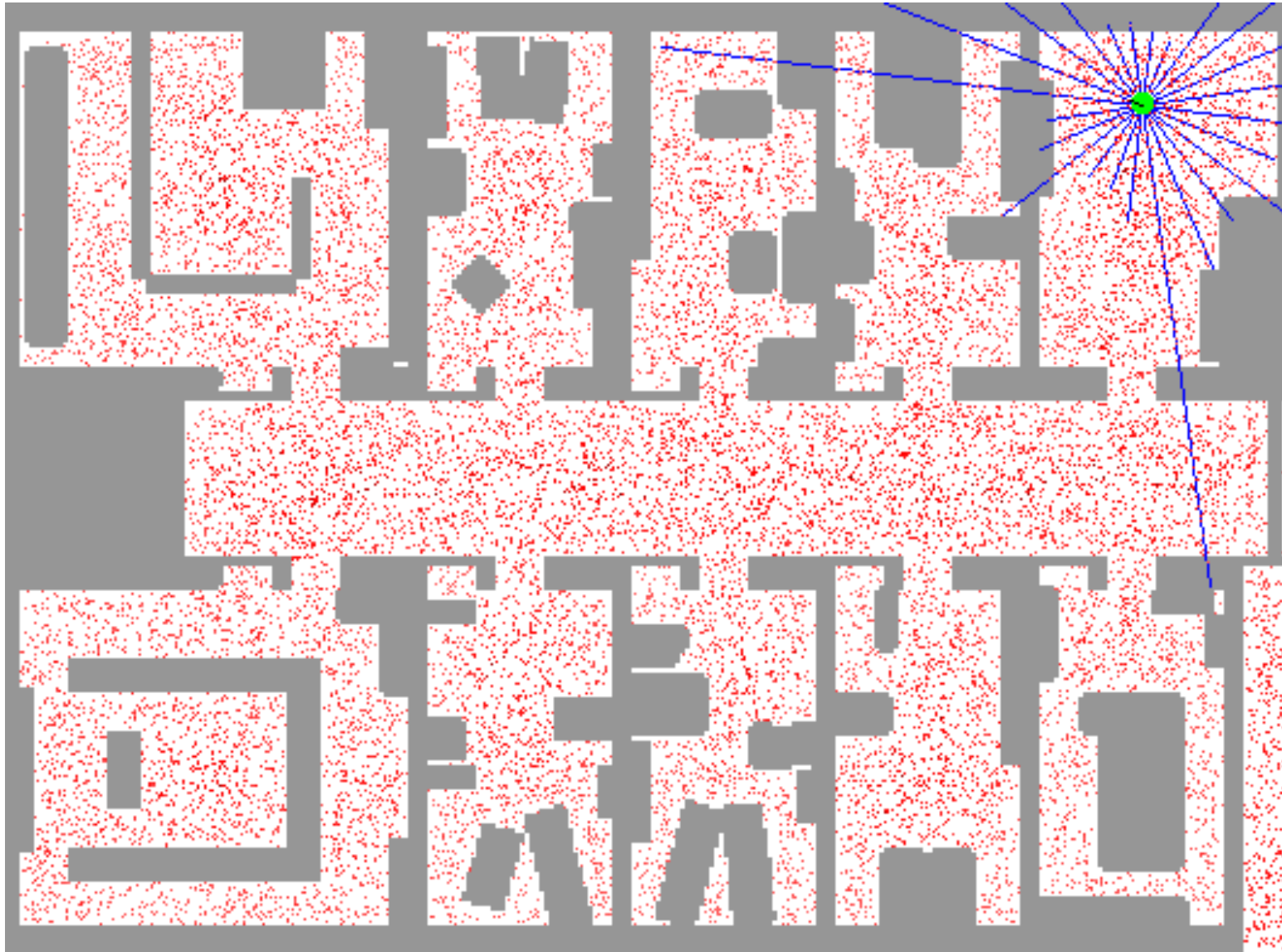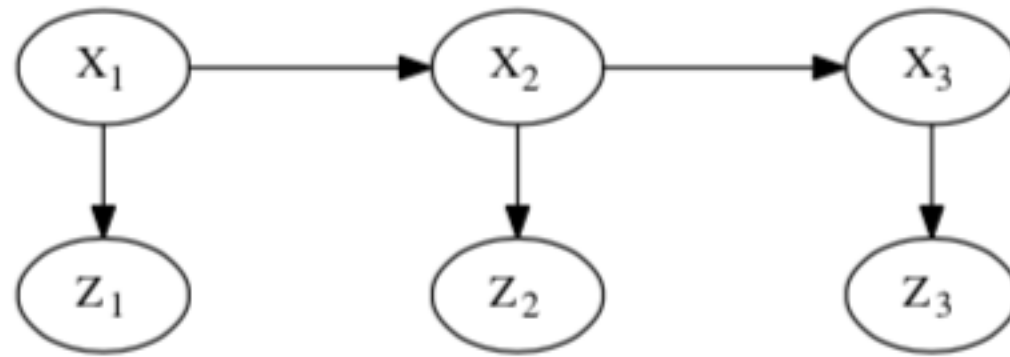

Prior P(S)

Likelihood L(S;o)

Posterior P(S|o)

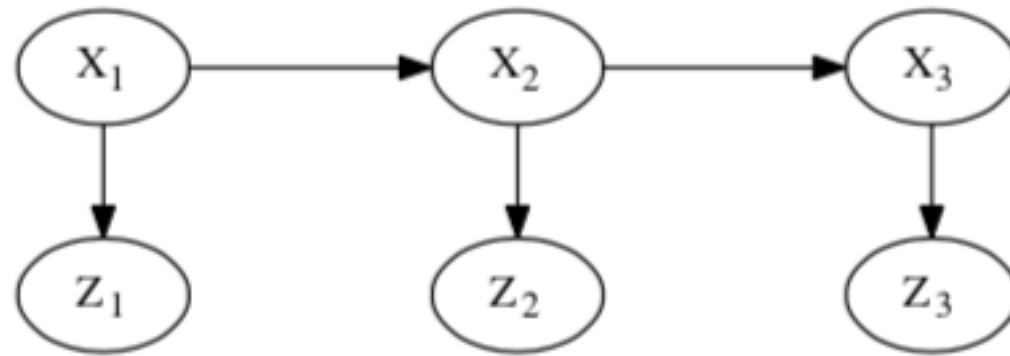# Later: Monte Carlo Localization

2. Hidden Markov Models

- Hidden states and given measurements.
- Joint distribution:

$$P(\mathcal{X}, \mathcal{Z}) = P(X_1)P(Z_1|X_1)P(X_2|X_1)P(Z_2|X_2)P(X_3|X_2)P(Z_3|X_3)$$

Bayes' rule for inference
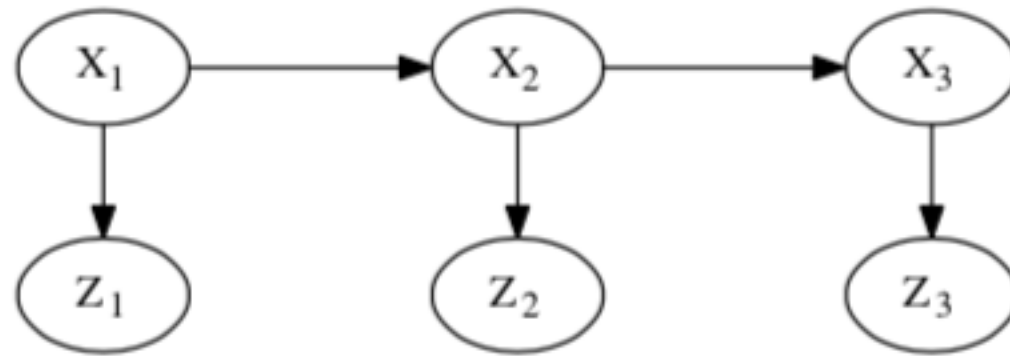

- Consider states and measurements as sets:

$$P(\mathcal{X}, \mathcal{Z}) = P(\mathcal{Z}|\mathcal{X})P(\mathcal{X})$$

- Bayes' rule:

$$P(\mathcal{X}|\mathcal{Z}) \propto P(\mathcal{Z} = \mathfrak{z}|\mathcal{X})P(\mathcal{X})$$
$$= L(\mathcal{X}; \mathcal{Z} = \mathfrak{z})P(\mathcal{X})$$

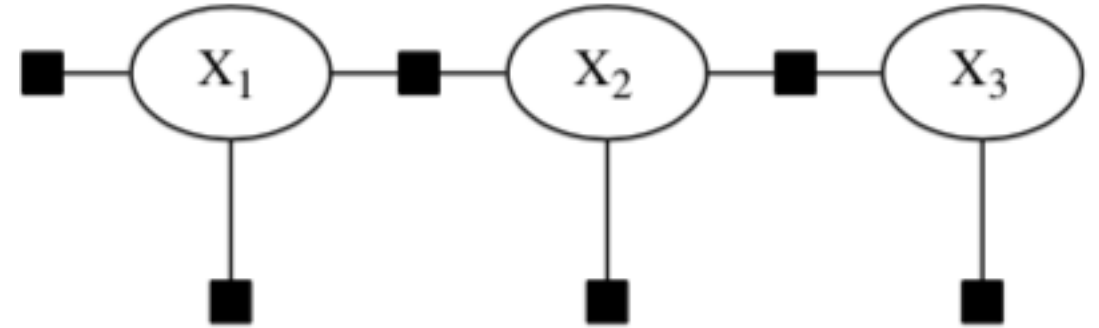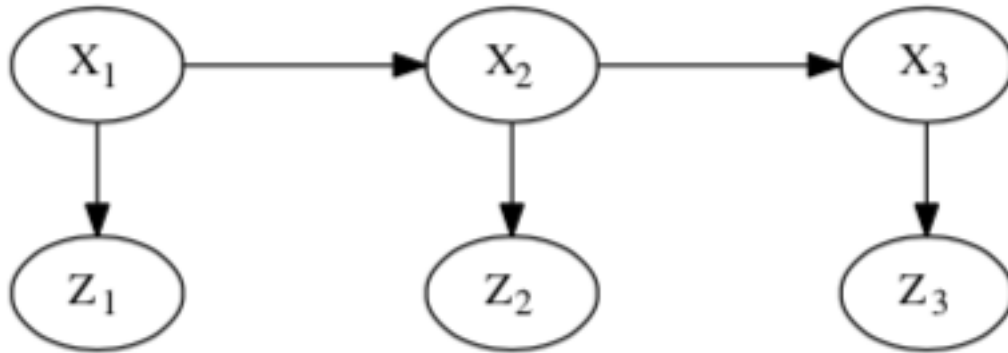$$L(\mathcal{X}; \mathcal{Z} = \mathfrak{z}) = P(z_1|X_1)P(z_2|X_2)P(z_3|X_3)$$
$$= L(X_1; Z_1)L(X_2; Z_2)L(X_3; Z_3)$$

Three efficient inference methods

1. Branch & bound
2. Dynamic programming
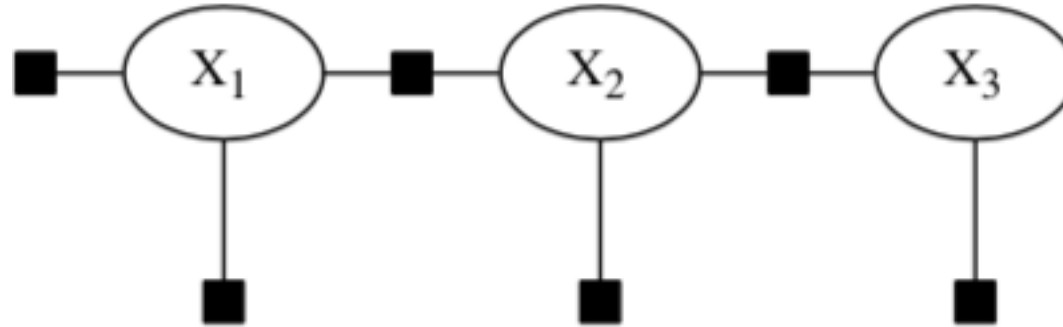3. Inference using factor graphs

3. Factor graphs

- Measurements are given: get rid of them!

$$P(\mathcal{X}|\mathcal{Z}) \propto P(X_1)L(X_1; z_1)P(X_2|X_1)L(X_2; z_2)P(X_3|X_2)L(X_3; z_3)$$

- This becomes:

$$\phi(\mathcal{X}) = \phi_1(X_1)\phi_2(X_1)\phi_3(X_1, X_2)\phi_4(X_2)\phi_5(X_2, X_3)\phi_6(X_3)$$

General definition of Factor graphs

- Bipartite graph of variables and factors

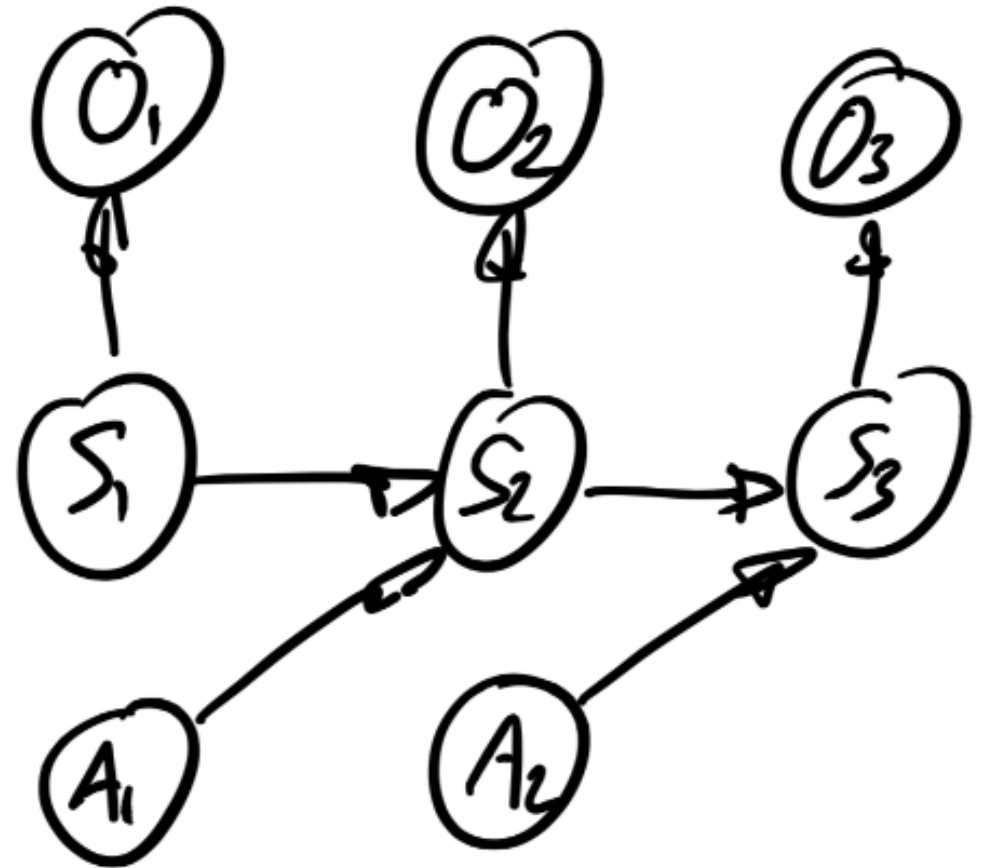$$\phi(\mathcal{X}) = \prod_i \phi_i(\mathcal{X}_i)$$

- Each $\mathcal{X}_i$ is the subset of variables connected to factor ϕ_i
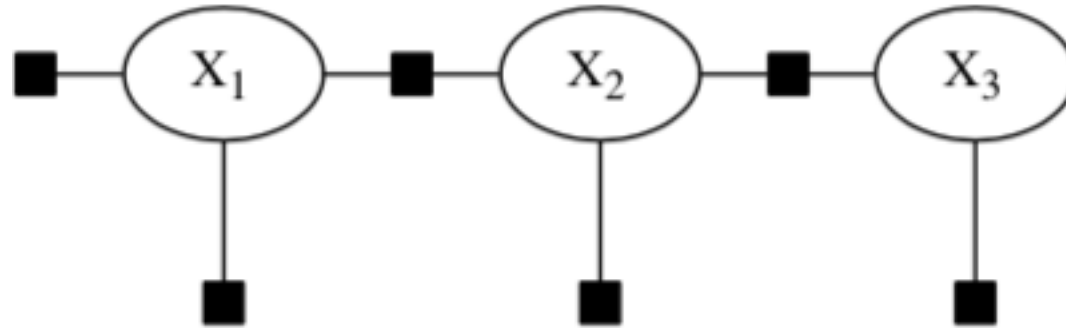
4. Converting Bayes nets to factor graphs

Every node in Bayes net becomes a factor, incl. evidence nodes.

Try converting when:

1. No known quantities
2. Observations and actions known
3. States known ??

5. The Max-product Algorithm for HMMs

Done in class:

- Eliminate X_1
- Eliminate X_2
- Eliminate X_3
- *Back-substitute*

Summary

- 1. Bayes Filter is great for localization

- 2. HMMs model entire trajectories

- 3. Factor Graphs make inference tractable

- 4. You can convert any Bayes Nets into a Factor Graph

- 5. The Max-Product Algorithm does MPE in linear time