Remi & Whitlock MEC

Background selection

According to Hudson & Kaplan (1995, eq. 8), the effect of background selection (BGS) is apprximately

In[9]:=

```
BHK95[U_, s_, h_, R_] := Exp[-U/(2 s h + R)]
BHK95alt[u_, s_, r_, n_] := Exp[-n (u s)/(2 (s + r)^2)]
BRW18[u_, s_, r_] := Exp[-Sum[(u[[i]] s[[i]])/((s[[i]] + r[[i]] (1 - s[[i]]))^2), {i, 1, Length[u]}]]
BRW18alt[u_, s_, r_, n_] := Exp[-n (u s)/((s + r (1 - s))^2)]
FstFunc[M_, B_] := 1/(1 + 2 M B)
```

$$\mathrm{BHK95}[U\_, s\_, h\_, R\_] := \mathrm{Exp}\left[-\frac{U}{2\,s\,h + R}\right]$$

$$\mathrm{BHK95alt}[u\_, s\_, r\_, n\_] := \mathrm{Exp}\left[-n\,\frac{u\,s}{2\,(s + r)^2}\right]$$

$$\mathrm{BRW18}[u\_, s\_, r\_] := \mathrm{Exp}\left[-\mathrm{Sum}\left[\frac{u[[i]]\,s[[i]]}{(s[[i]] + r[[i]]\,(1 - s[[i]]))^2}, \{i, 1, \mathrm{Length}[u]\}\right]\right]$$

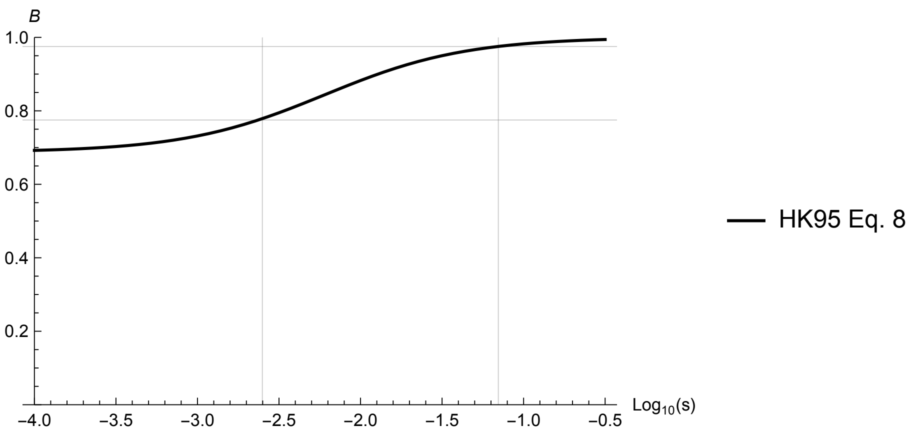
$$\mathrm{BRW18alt}[u\_, s\_, r\_, n\_] := \mathrm{Exp}\left[-n\,\frac{u\,s}{(s + r\,(1 - s))^2}\right]$$

$$\mathrm{FstFunc}[M\_, B\_] := \frac{1}{1 + 2\,M\,B}$$

In[14]:=

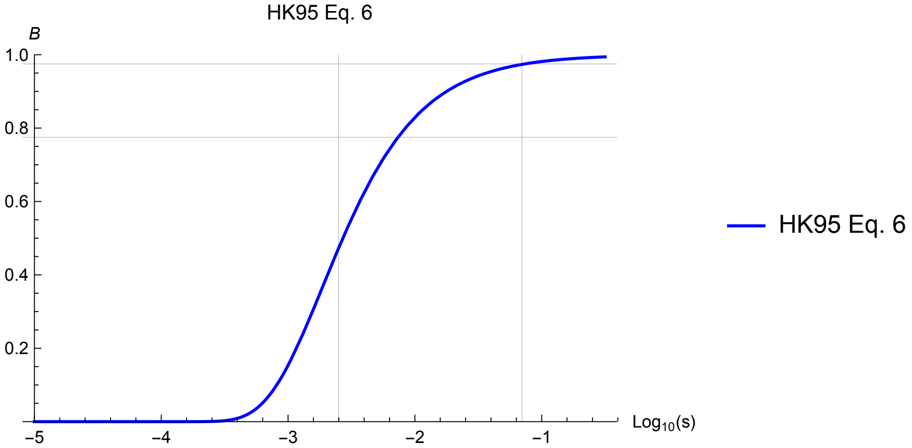
```
l = 10 × 10^6; (* size of region in bp *)
pe = 0.015; (* proportion of region that is exonic *)
μ = 2.5 × 10^-8; (* deleterious mutation rate per bp *)
r = 0.01; (* total map length of region in morgans *)
pl1 = Plot[
  BHK95[pe l μ, 10^sExp, 1, r], {sExp, -4, -0.5},
  PlotRange → {Full, {0, 1}},
  AxesLabel → {"Log10(s)", B},
  GridLines → {{Log[10, 0.0025], Log[10, 0.07]}, {(0.740 + 0.81) / 2, 0.975}},
  PlotStyle → Black
  (*PlotLabel→"HK95 Eq. 8"*),
  PlotLegends → {"HK95 Eq. 8"}
 ]
```

Out[18]=



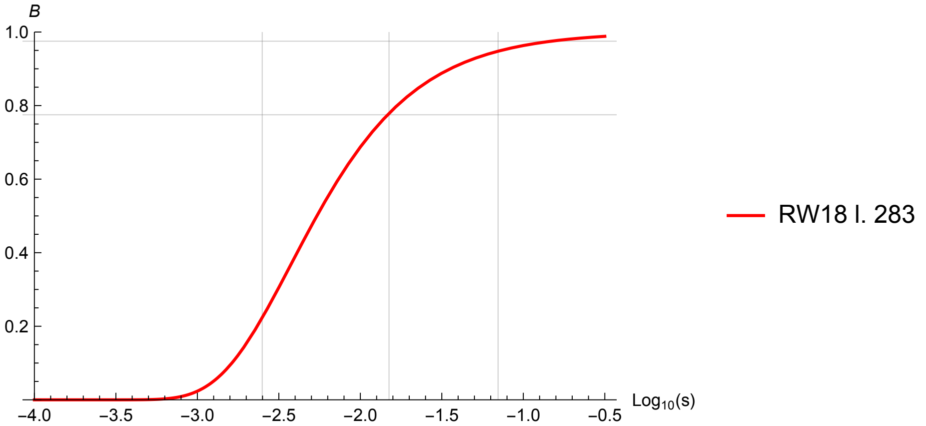
In[19]:=
```
myu = 2.5 × 10^-8;
myr = r / l;
myn = pe l;
pl2 = Plot[
  BHK95alt[myu, 10^sExp, myr, myn], {sExp, -5, -0.5},
  PlotRange → {Full, {0, 1}},
  AxesLabel → {"Log10(s)", B},
  GridLines → {{Log[10, 0.0025], Log[10, 0.07]}, {(0.740 + 0.81) / 2, 0.975}},
  PlotLabel → "HK95 Eq. 6",
  PlotStyle → Blue,
  PlotLegends → {"HK95 Eq. 6"}
 ]
```

Out[22]=



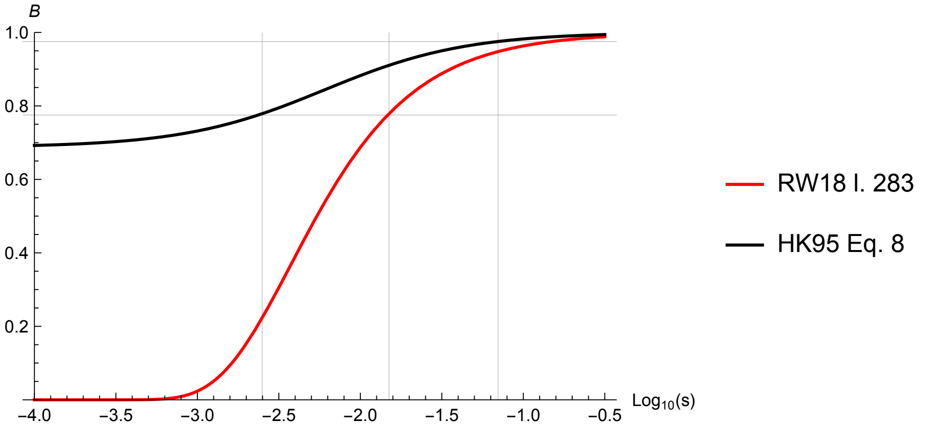
In[23]:=
```
myu = 2.5 × 10^-8;
myr = r / l;
myn = pe l;
pl3 = Plot[
  BRW18alt[myu, 10^sExp, myr, myn], {sExp, -4, -0.5},
  PlotRange → {Full, {0, 1}},
  AxesLabel → {"Log10(s)", B},
  GridLines →
   {{Log[10, 0.0025], Log[10, 0.07], Log[10, 0.015]}, {(0.740 + 0.81) / 2, 0.975}},
  (*PlotLabel→"RW18 l. 283",*)
  PlotStyle → Red,
  PlotLegends → {"RW18 l. 283"}
 ]
```

Out[26]=



In[27]:=
```
Show[pl3, pl1]
```

Out[27]=

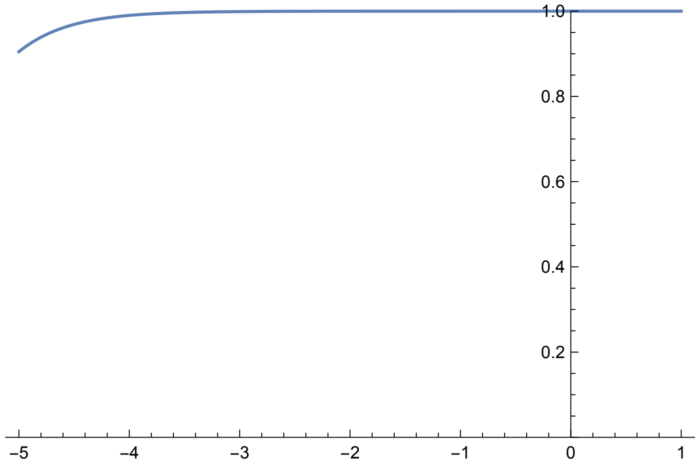


In[28]:= $10^{-1.8}$

Out[28]= 0.0158489

In[29]:=
```
myu = 1 × 10^-8;
myr = 0.0001 × 10^-8;
myL = 10^2;
Plot[BRW18[Table[myu, {myL}], Table[10^sExp, {myL}], Table[myr, {myL}]],
 {sExp, -5, 1}, PlotRange → {Full, {0, 1}}]
```

Out[32]=



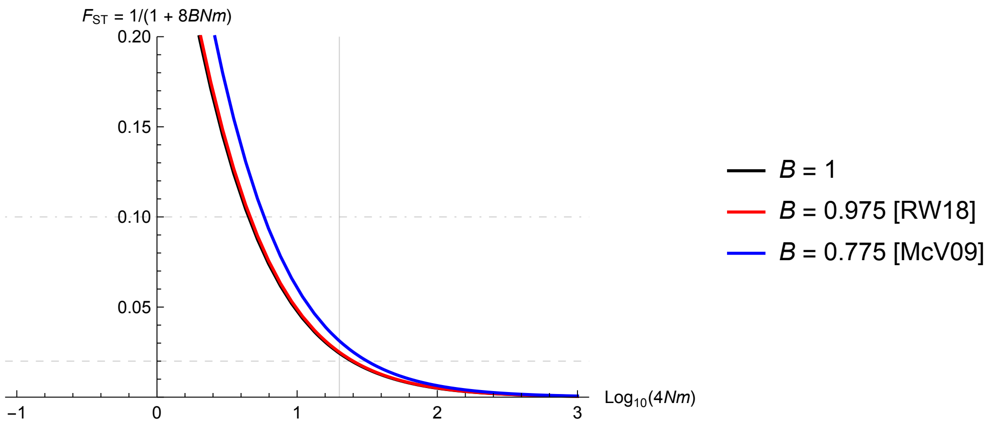
In[33]:=
```
Table[1, {10}]
```

Out[33]=
```
{1, 1, 1, 1, 1, 1, 1, 1, 1, 1}
```

In[34]:=

```
In[84]:= B1 = 0.975;
B2 = (0.74 + 0.81) / 2;
Plot[
 {
  FstFunc[10^MExp, 1],
  FstFunc[10^MExp, B1],
  FstFunc[10^MExp, B2]
 }, {MExp, -1, 3},
 PlotStyle → {Black, Red, Blue},
 PlotRange → {Full, {0, 0.2}},
 PlotLegends → {"B = 1", "B = 0.975 [RW18]", "B = 0.775 [McV09]"},
 AxesLabel → {"Log10(4Nm)", "FST = 1/(1 + 8BNm)"},
 GridLines → {{(*Log[10,4 0.05 1000],*)Log[10, 4 × 0.005 × 1000]
   (*,Log[10,4 0.05 10000]*)}, {{0.02, Dashed}, {0.1, DotDashed}}}
]
```

Out[86]=

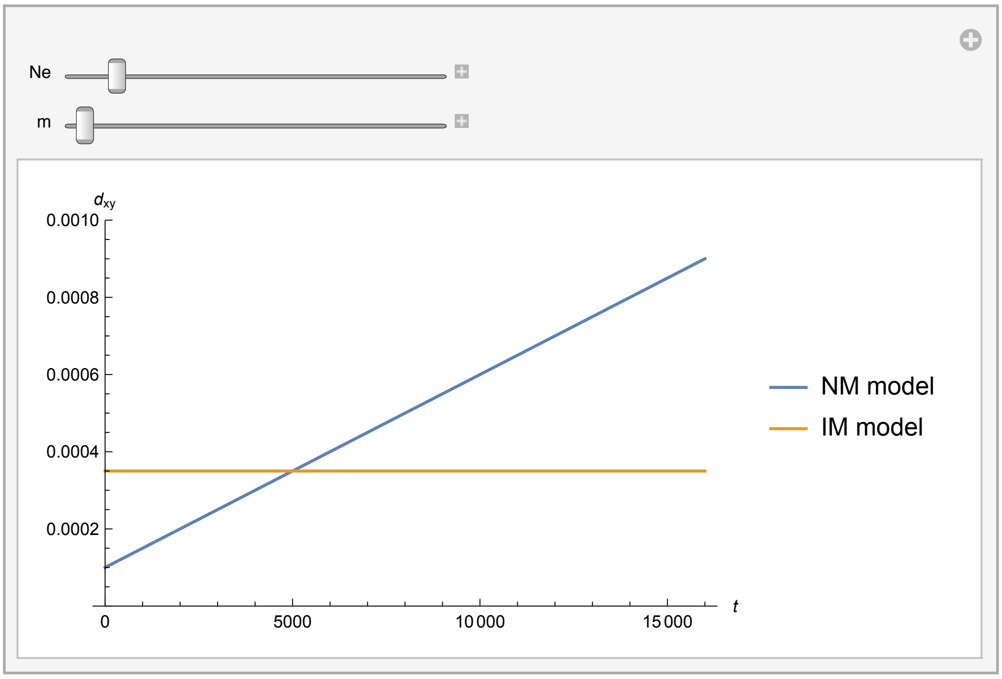



```
In[71]:= 10^.75 / (4 * 1000)
Out[71]= 0.00140585
```

Plotting d_{xy} as a function of time for the complete-isolation (NM) model and the isolation-with-migration (IM) model at equilibrium.

```
In[123]:= Manipulate[
  Plot[
   {2 μ (2 Ne + t),
    2 μ (2 Ne + 1/(2 m))
   }, {t, 0, 16 Ne},
   PlotRange → {Full, {0, 0.001}},
   AxesLabel → {"t", "dxy"},
   PlotLegends → {"NM model", "IM model"}
  ],
  {{Ne, 10^3}, 10^1, 10^4}, {{m, 10^-4}, 10^-5, 10^-2}
 ]
```

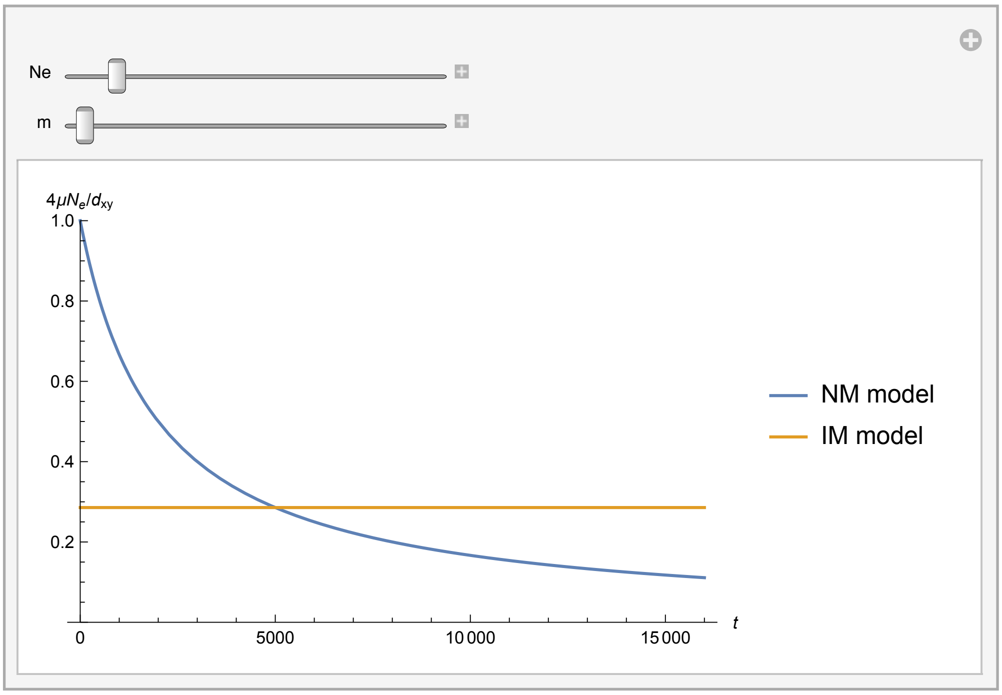
Out[123]=



Plotting the proportion of d_{xy} explained by $4\,N\,\mu$.

In[122]:=
```
Manipulate[
 Plot[
  {4 μ Ne / (2 μ (2 Ne + t)),
   4 μ Ne / (2 μ (2 Ne + 1/(2 m)))
  }, {t, 0, 16 Ne},
  PlotRange → {Full, {0, 1}},
  AxesLabel → {"t", "4μNe/dxy"},
  PlotLegends → {"NM model", "IM model"}
 ],
 {{Ne, 10^3}, 10^1, 10^4}, {{m, 10^-4}, 10^-5, 10^-2}
]
```

Out[122]=



Under the NM and the IM models, F_{ST} converges to 1 and $\frac{1}{1+8N_e m}$, respectively. Therefore, without migration, there will be a point in time where BGS will no longer affect F_{ST}, but with sufficient migration, BGS can affect F_{ST} independently of time.

In[104]:= $\text{TtIM} = \frac{1}{2}\, 2\,\text{Ne} + \frac{1}{2}\left(2\,\text{Ne} + \frac{1}{2\,\text{m}}\right)$

Out[104]= $\text{Ne} + \frac{1}{2}\left(\frac{1}{2\,\text{m}} + 2\,\text{Ne}\right)$

In[105]:= `TtIM // Simplify`

Out[105]= $\frac{1}{4\,\text{m}} + 2\,\text{Ne}$

In[106]:= `TsIM = 2 Ne`

Out[106]= 2 Ne

In[107]:=
```
TtNM = 1/2 2 Ne + 1/2 (2 Ne + t)
```
$$\text{Out[107]=}\quad Ne + \frac{1}{2}\,(2\,Ne + t)$$

In[110]:=
```
TsNM = 2 Ne
```
Out[110]= 2 Ne

In[109]:=
```
FstIM = (TtIM - TsIM) / TtIM // Simplify
```
$$\text{Out[109]=}\quad \frac{1}{1 + 8\,m\,Ne}$$

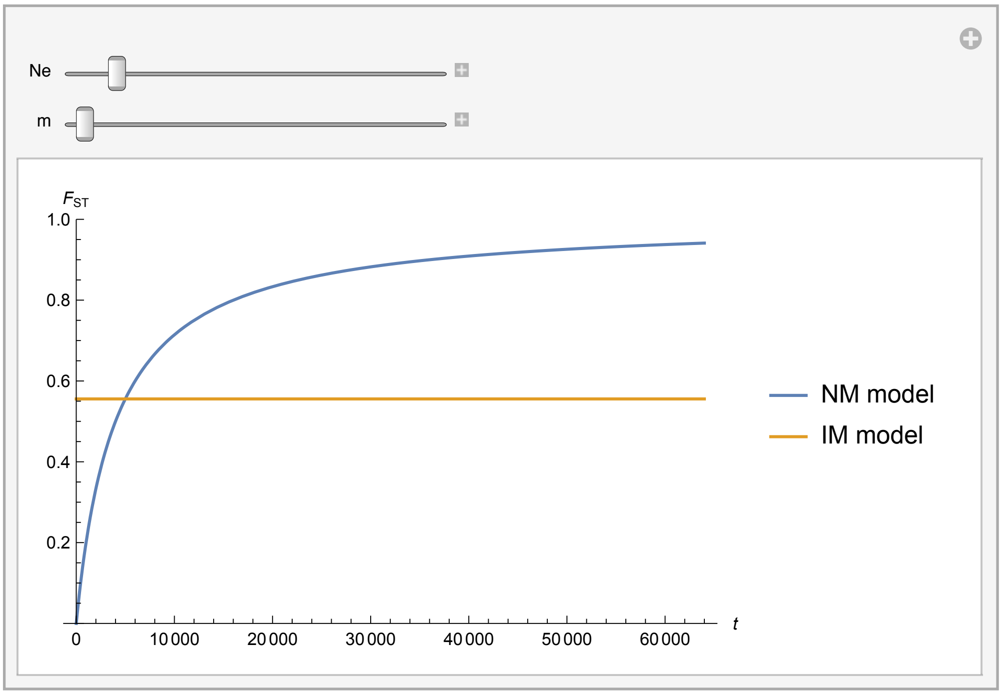
In[111]:=
```
FstNM = (TtNM - TsNM) / TtNM // Simplify
```
$$\text{Out[111]=}\quad \frac{t}{4\,Ne + t}$$

In[112]:=
```
Limit[FstNM, t → ∞]
```
Out[112]= 1

Plotting F_{ST} as a function of time (IM model: equilibrium only).

In[128]:=
```
Manipulate[
 Plot[
  {t/(4 Ne + t),
   1/(1 + 8 Ne m)
  }, {t, 0, 64 Ne},
  PlotRange → {Full, {0, 1}},
  AxesLabel → {"t", "F_ST"},
  PlotLegends → {"NM model", "IM model"}
 ],
 {{Ne, 10^3}, 10^1, 10^4}, {{m, 10^-4}, 10^-5, 10^-2}
]
```

Out[128]=



Derivative of d_{xy} and F_{ST} w.r.t. to N_e:

In[138]:=
```
DerDxyNM = D[2 u (2 Ne + t), Ne] // Simplify
```

Out[138]= 4 u

In[143]:=
```
DerDxyIM = D[2 u (2 Ne + 1/(2 m)), Ne] // Simplify
```

Out[143]= 4 u

In[140]:=
```
DerFstNM = D[t/(4 Ne + t), Ne] // Simplify
```

Out[140]= $-\frac{4\,t}{(4\,Ne + t)^2}$

In[147]:=
```
Limit[DerFstNM, t → ∞]
```

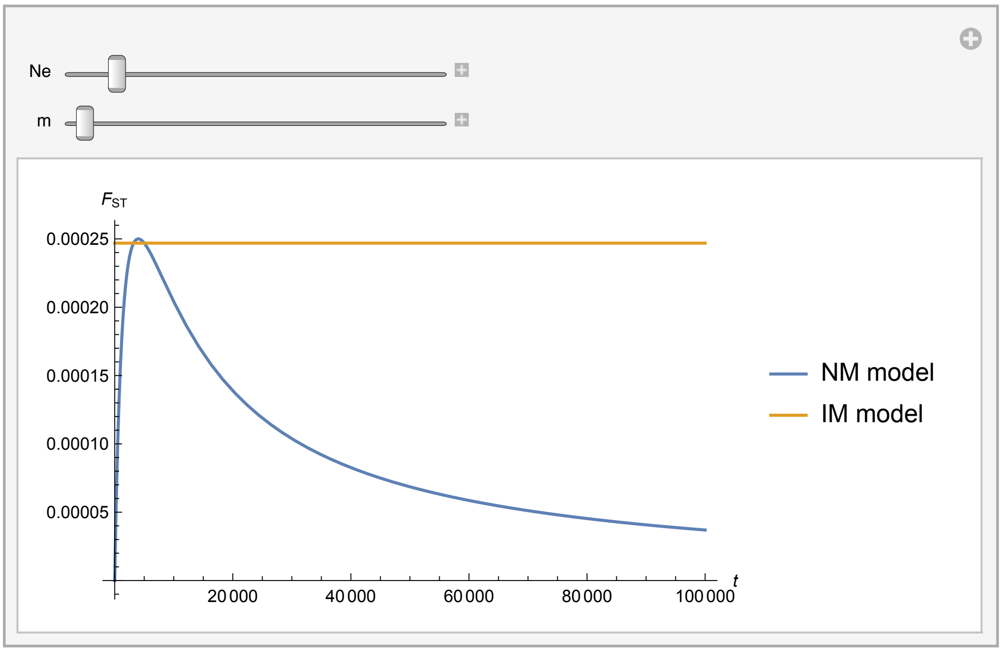
Out[147]= 0

In[142]:=
```
DerFstIM = D[1/(1 + 8 Ne m), Ne] // Simplify
```

Out[142]= $-\frac{8\,m}{(1+8\,m\,Ne)^2}$

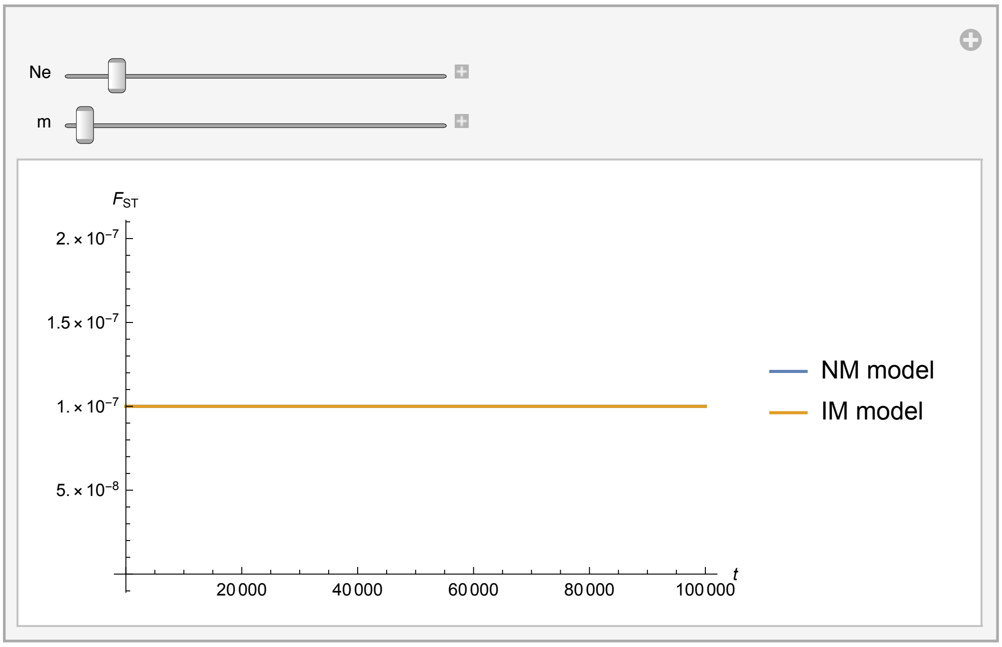
In[148]:=
```
Manipulate[
 Plot[
  {Abs[-(4 t)/(4 Ne + t)^2],
   Abs[-(8 m)/(1 + 8 m Ne)^2]
  }, {t, 0, 100 Ne},
  PlotRange → {Full, Automatic},
  AxesLabel → {"\!\(\*StyleBox[\"t\",FontSlant->\"Italic\"]\)",
    "\!\(\*SubscriptBox[StyleBox[\"F\",FontSlant->\"Italic\"], \"ST\"]\)"},
  PlotLegends → {"NM model", "IM model"}
 ],
 {{Ne, 10^3}, 10^1, 10^4}, {{m, 10^-4}, 10^-5, 10^-2}
]
```

Out[148]=



In[149]:=
```
Manipulate[
 Plot[
  {4 μ,
   4 μ
  }, {t, 0, 100 Ne},
  PlotRange → {Full, Automatic},
  AxesLabel → {"\!\(\*StyleBox[\"t\",FontSlant->\"Italic\"]\)",
    "\!\(\*SubscriptBox[StyleBox[\"F\",FontSlant->\"Italic\"], \"ST\"]\)"},
  PlotLegends → {"NM model", "IM model"}
 ],
 {{Ne, 10^3}, 10^1, 10^4}, {{m, 10^-4}, 10^-5, 10^-2}
]
```

Out[149]=



In[152]:=
```
ListPlot[{1, 1 - s, (1 - s)^2} /. {s → 0.07},
 Joined → True]
```

Out[152]=