standard rules of quantum mechanics that no individual definite state can be attributed to either one of the subsystems. Reduced density matrices of entangled subsystems therefore represent *improper mixtures* [47–49].

The fact that the reduced density matrix may be formally similar to a mixed-state density matrix thus cannot be used to argue that somehow, magically, a definite subsystem state—i.e., $|a_1\rangle$ or $|a_2\rangle$ with equal probabilities, as it would appear from the reduced density matrix (2.51)—has been obtained from the global entangled superposition state (2.42) by means of the (physical) interaction with system $\mathcal{B}$ and the (formal) trace operation. This observation will turn out to be important when discussing the implications of decoherence for the quantum measurement problem (see the next Sect. 2.5). We will also come back to this issue in Chap. 8 (see especially Sect. 8.1).

2.5 The Measurement Problem and the Quantum-to-Classical Transition

In this section, we shall describe the (in)famous *measurement problem* of quantum mechanics that we have already referred to in several places in the text. The choice of the term "measurement problem" has purely historical reasons: Certain foundational issues associated with the measurement problem were first illustrated in the context of a quantum-mechanical description of a measuring apparatus interacting with a system.

However, one may regard the term "measurement problem" as implying too a narrow scope, chiefly for the following two reasons. First, as we shall see below, the measurement problem is composed of three distinct issues, so it would make sense to rather speak of measurement *problems*. Second, quantum measurement and the arising foundational problems are but a special case of the more general problem of the *quantum-to-classical transition*, i.e., the question of how effectively classical systems and properties around us emerge from the underlying quantum domain.

On the one hand, then, the problem of the quantum-to-classical transition has a much broader scope than the issue of quantum measurement in the literal sense. On the other hand, however, many interactions between physical systems can be viewed as measurement-like interactions. For example, light scattering off an object carries away information about the position of the object, and it is in this sense that we thus may view these incident photons as a "measuring device." Such ubiquitous measurement-like interactions lie at the heart of the explanation of the quantum-to-classical transition by means of decoherence. Measurement, in the more general sense, thus retains its paramount importance also in the broader context of the quantum-to-classical transition, which in turn motivates us not to abandon the term "measurement problem" altogether in favor of the more general "problem of the quantum-to-classical transition."

As indicated above, the measurement problem (and the problem of the quantum-to-classical transition) is composed of three parts, all of which we shall describe in more detail in the following:

1. *The problem of the preferred basis* (Sect. 2.5.2). What singles out the preferred physical quantities in nature—e.g., why are physical systems usually observed to be in definite positions rather than in superpositions of positions?
2. *The problem of the nonobservability of interference* (Sect. 2.5.3). Why is it so difficult to observe quantum interference effects, especially on macroscopic scales?
3. *The problem of outcomes* (Sect. 2.5.4). Why do measurements have outcomes at all, and what selects a particular outcome among the different possibilities described by the quantum probability distribution?

Familiarity with these problems will turn out to be important for a proper understanding of the scope, achievements, and implications of decoherence. To anticipate, it is fair to conclude that decoherence has essentially resolved the first two problems. Since these problems and their resolution can be formulated in purely operational terms within the standard formalism of quantum mechanics, the role played by decoherence in addressing these two issues is rather undisputed.

By contrast, the success of decoherence in tackling the third issue—the problem of outcomes—remains a matter of debate, in particular, because this issue is almost inextricably linked to the choice of a specific interpretation of quantum mechanics (which mostly boils down to a matter of personal preference). In fact, most of the overly optimistic or pessimistic statements about the ability of decoherence to solve "the" measurement problem can be traced back to a misunderstanding of the scope that a standard quantum effect such as decoherence may have in resolving the more interpretive problem of outcomes.

2.5.1 The Von Neumann Scheme for Ideal Quantum Measurement

Starting from two separate (nonentangled) systems, how can an entangled composite state come about? How is it possible that the two subsystems lose their individuality to become a quantum-mechanical whole? Quantum entanglement can be viewed as arising from the kinematical concept of the superposition principle combined with the dynamical feature of the linearity of the Schrödinger time evolution. The resulting process is often represented in terms of a *von Neumann measurement*, a scheme devised by von Neumann during the early years of quantum mechanics and discussed in his seminal book of 1932 [60].

Von Neumann's goal was to describe the act of quantum measurement in entirely quantum-mechanical terms as the physical interaction between the measured system and the measuring apparatus, treating not only the system but also the apparatus (and, ultimately, the observer; see Sect. 9.2) as quantum-mechanical objects. It is worth noting that this approach represented a radical departure from the Copenhagen interpretation that had postulated the existence of intrinsically classical measurement apparatuses which were regarded as not subject to the laws of quantum mechanics (see Sect. 8.1).

Despite its name, the scope of the von Neumann measurement scheme goes far beyond the context of quantum measurement in the actual sense. In fact, the von Neumann scheme is the easiest way to understand how quantum entanglement arises. It also illustrates nicely the aforementioned three components of the measurement problem, namely, the problem of the preferred basis, the nonobservability of interference effects, and the problem of outcomes. Finally, it will allow us to introduce the basic formalism of decoherence, by regarding decoherence as a consequence of a von Neumann–type measurement interaction between the system and its environment.

After these introductory remarks, let us now formulate the von Neumann measurement scheme. Its typical ingredients are a (typically microscopic) system $\mathcal{S}$, described by a Hilbert space $\mathcal{H}_{\mathcal{S}}$ with basis vectors $\{|s_i\rangle\}$, and a (usually macroscopic) measuring apparatus $\mathcal{A}$, formally represented by basis vectors $\{|a_i\rangle\}$ in a Hilbert space $\mathcal{H}_{\mathcal{A}}$. Strictly speaking, whether the system and the apparatus are microscopic or macroscopic has no bearing on the following general argument. However, it typically reasonable, from a physical point of view, to associate microscopicity with the system, since we would like to ensure that the system can be easily prepared in a superposition state. On the other hand, the physical realization of a measuring apparatus typically involves a macroscopic system with a large number of degrees of freedom, and from our experience we would expect such an apparatus to behave according to the laws of classical physics (although the von Neumann scheme deliberately treats the apparatus in quantum-mechanical terms).

The purpose of the apparatus is now to measure the state of the system $\mathcal{S}$. We can think of the apparatus as having some kind of pointer that moves to the position "i," represented by the state $|a_i\rangle$, if the system is measured to be in the state $|s_i\rangle$ (see Fig. 2.4). Assuming that, before the measurement takes place, the apparatus starts out in some initial "ready" state $|a_\mathrm{r}\rangle$, the dynamical measurement interaction between the system and the apparatus will then be of the form

$$|s_i\rangle\,|a_\mathrm{r}\rangle \longrightarrow |s_i\rangle\,|a_i\rangle \tag{2.52}$$

for all i. Here the initial and final states reside in the tensor-product Hilbert space $\mathcal{H}_{\mathcal{S}} \otimes \mathcal{H}_{\mathcal{A}}$ describing the total $\mathcal{SA}$ system. We see that the measurement has established a one-to-one correspondence between the state of the system and the state of the apparatus: The latter perfectly mirrors the former. Also note that, in writing the right-hand side of (2.52), we have tacitly assumed

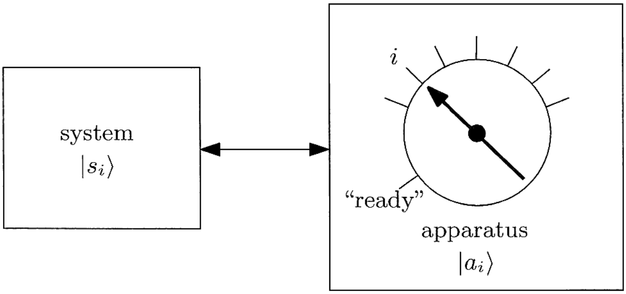

Fig. 2.4. Von Neumann scheme for ideal quantum measurement. Both system and apparatus are treated as quantum systems. The system–apparatus interaction is such that the system's being in state $|s_i\rangle$ causes the apparatus pointer to move from the initial "ready" position on the dial to position "i," represented by a quantum state $|a_i\rangle$ of the apparatus.

that the measurement interaction does not change the state of the system. Because of these assumptions, the measurement scheme (2.52) is often called *ideal*. Such measurements which do not disturb the state of the system are also known as *quantum nondemolition measurements* [69].

Now we come to the key point. Thus far, the interaction (2.52) has not led to any entanglement: The final system–apparatus state is still separable. However, let us consider what happens if the system starts out in a superposition of the basis states $|s_i\rangle$,

$$|\psi\rangle = \sum_i c_i |s_i\rangle . \tag{2.53}$$

In this case, the linearity of the Schrödinger equation implies that the system–apparatus combination $\mathcal{SA}$ will evolve according to

$$|\psi\rangle |a_\mathrm{r}\rangle = \left(\sum_i c_i |s_i\rangle \right) |a_\mathrm{r}\rangle \longrightarrow |\Psi\rangle = \sum_i c_i |s_i\rangle |a_i\rangle . \tag{2.54}$$

This evolution represents the (ideal) von Neumann quantum-measurement scheme.

Inspection of the right-hand side of (2.54) shows that the final state of the system–apparatus combination is in general described by an entangled state, i.e., we can no longer attribute an individual state vector to the system or the apparatus. Entanglement has thus been created *dynamically*. The superposition initially present only in the system has been *amplified* to the level of the (typically macroscopic) apparatus, in the sense that the final superposition involves both the system and the apparatus.

The crucial difficulty is now that it is not at all obvious how one is to regard the dynamical evolution described by (2.54) as representing measurement in the usual sense. This is so because the final state on the right-hand side of (2.54) is, for at least two reasons to be discussed in the following, not sufficient to directly conclude that the measurement has actually been completed. To emphasize this fact, the scheme (2.54) is frequently referred to as *premeasurement.*

2.5.2 The Problem of the Preferred Basis

The first problem is that there exists a basis ambiguity regarding the expansion of the final composite state on the right-hand side of (2.54). We can express this state in many different ways, implying that in the von Neumann scheme stated above the observable that was supposedly measured is not uniquely defined by this final state. In fact, given *any* set of states describing our system $\mathcal{S}$, there exists a corresponding set of apparatus states such that the final composite state takes the form (2.54),

$$|\psi\rangle\,|a_\mathrm{r}\rangle \longrightarrow |\Psi\rangle = \sum_i c_i\,|s_i\rangle\,|a_i\rangle = \sum_i c_i'\,|s_i'\rangle\,|a_i'\rangle = \dots . \tag{2.55}$$

However, this freedom in the choice of the basis is in practice to some degree limited by two constraints. First of all, we will typically require the states $|a_i\rangle$ of the apparatus to be mutually orthogonal. This ensures that these states correspond to classically distinct outcomes of the measurement, such that the possible states $|s_i\rangle$ of the system can be reliably distinguished. Second, for an arbitrary choice of the set of apparatus states $|a_i\rangle$, the relative states $|s_i\rangle$ for the system may fail to be mutually orthogonal. In this case the system observables corresponding to the states $|s_i\rangle$ (i.e., the observables with eigenstates $|s_i\rangle$) will not be Hermitian, which is usually an undesired property.[15]

In view of these arguments, we may therefore require the apparatus states $|a_i\rangle$ to be mutually orthogonal, i.e., $\langle a_i|a_j\rangle = 0$ for $i \neq j$. In this case, it follows from the so-called Schmidt theorem (see Sect. 2.15.1 below) that the decomposition

$$|\Psi\rangle = \sum_i c_i\,|s_i\rangle\,|a_i\rangle\,, \tag{2.56}$$

with c_i real and $\sum c_i^2 = 1$, is unique, *provided* all the coefficients c_i are different from one another.

Let us consider a simple example showing that the decomposition (2.56) is in general *not* unique if this condition on the coefficients does not hold.

[15]However, non-Hermitian observables are not *a priori* forbidden and arise in certain experimental settings. For instance, in quantum optics one often performs measurements that have coherent states as their outcomes. Coherent states, however, form an overcomplete set of states and can therefore not be represented by Hermitian observables. See also the discussion by Zurek in [16].

Suppose both the system $\mathcal{S}$ and the apparatus $\mathcal{A}$ are quantum-mechanical two-state systems represented by spin-$\frac{1}{2}$ particles, described by basis states $|0_z\rangle$ and $|1_z\rangle$. Suppose further that the states $|0_z\rangle_{\mathcal{A}}$ and $|1_z\rangle_{\mathcal{A}}$ of $\mathcal{A}$ act as "pointers" for the spin states $|0_z\rangle_{\mathcal{S}}$ and $|1_z\rangle_{\mathcal{S}}$ of $\mathcal{S}$. That is, the von Neumann interaction (2.52) is here described by the dynamics

$$\begin{aligned} |0_z\rangle_{\mathcal{S}}\, |\text{“ready”}\rangle_{\mathcal{A}} &\longrightarrow |0_z\rangle_{\mathcal{S}}\, |0_z\rangle_{\mathcal{A}}\,, \\ |1_z\rangle_{\mathcal{S}}\, |\text{“ready”}\rangle_{\mathcal{A}} &\longrightarrow |1_z\rangle_{\mathcal{S}}\, |1_z\rangle_{\mathcal{A}}\,. \end{aligned} \tag{2.57}$$

Assuming the system $\mathcal{S}$ starts out in the superposition $\left(|0_z\rangle_{\mathcal{S}} + |1_z\rangle_{\mathcal{S}}\right)/\sqrt{2}$, it follows from (2.57) and the linearity of the Schrödinger equation [see also (2.54)] that the final composite entangled spin–apparatus state will be

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|0_z\rangle_{\mathcal{S}}\, |0_z\rangle_{\mathcal{A}} + |1_z\rangle_{\mathcal{S}}\, |1_z\rangle_{\mathcal{A}}\right). \tag{2.58}$$

Let us now use (2.35) to rewrite the z-spin states $|0_z\rangle_i$ and $|1_z\rangle_i$, $i = \mathcal{S}, \mathcal{A}$, in terms of the eigenstates $|0_x\rangle_i$ and $|1_x\rangle_i$ of the Pauli spin operator $\hat{\sigma}_x$. Expressed in the new basis $\{|0_x\rangle_i\,, |1_x\rangle_i\}$, the state (2.58) reads

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\left(|0_x\rangle_{\mathcal{S}}\, |0_x\rangle_{\mathcal{A}} + |1_x\rangle_{\mathcal{S}}\, |1_x\rangle_{\mathcal{A}}\right). \tag{2.59}$$

Recall that we regarded $\mathcal{A}$ as a measuring device for the spin of the system $\mathcal{S}$. Equations (2.58) and (2.59) then imply that the apparatus $\mathcal{A}$ has formed one-to-one correlations with *both* the z-spin and x-spin states of $\mathcal{S}$. If we interpret, in the spirit of the von Neumann scheme (i.e., without the assumption of any subsequent wave-function collapse), this formation of system–apparatus correlations as a complete measurement, this state of affairs seems to imply the following. Once $\mathcal{A}$ has measured the spin of $\mathcal{S}$ along the z axis [see the final state (2.58)], $\mathcal{A}$ may be considered as having measured also the spin of $\mathcal{S}$ along the x axis, where the latter "measurement" is represented by the equivalent form (2.59) of (2.58).

Thus our device $\mathcal{A}$ would appear to have simultaneously measured two *noncommuting* observables of the system, namely, $\hat{\sigma}_x$ and $\hat{\sigma}_z$, in apparent contradiction with the laws of quantum mechanics. What is more, it is easy to show that the state (2.58) can in fact be rewritten in infinitely many equivalent ways using the spin-$\frac{1}{2}$ basis along *any* arbitrary axis. Thus it would appear that, once our apparatus $\mathcal{A}$ has "measured" the spin of $\mathcal{S}$ along the z axis—in the von Neumann sense of the formation of correlations of the type (2.58)—it has also "measured" spin along any other spatial direction.

Such a situation of simultaneous measurement of a set of noncommuting observables is not only forbidden by quantum mechanics, but also contradicts our experience that measuring devices seem to be *designed* to measure only very particular quantities. In our example, the apparatus $\mathcal{A}$ may be realized in form of a Stern–Gerlach device (see Sect. 2.2.2), with the states $|0_z\rangle_{\mathcal{A}}$

and $|1_z\rangle_{\mathcal{A}}$ corresponding to the two separated paths through the apparatus (with the magnetic field aligned along the z axis) that distinguish the spin states $|0_z\rangle_{\mathcal{S}}$ and $|1_z\rangle_{\mathcal{S}}$ of the system $\mathcal{S}$. This Stern–Gerlach apparatus is therefore set up to measure spin only along the z axis, but not along any other axis. Performing a measurement along an axis different from z would require a physical rotation of the magnetic field and would thus correspond to a *physically different* setup.

The existence of such a "preferred observable" (or of a "preferred basis") is thus not explained by the final system–apparatus state arrived at through a von Neumann measurement. As we have seen, the form of this state will in general not uniquely fix the observable of the system that is recorded by the apparatus via the formation of quantum correlations. This *problem of the preferred basis* was first cleanly separated out from the problem of wave-function collapse and the intimately related problem of outcomes (see Sect. 2.5.4 below) by Zurek [8], who emphasized the distinct and important role played by the preferred-basis problem in any account of quantum measurement. Before the problem of outcomes may play any role, we ought to solve this preferred-basis problem, since it does not make sense to even inquire about specific outcomes if the set of possible outcomes is not clearly defined.

Zurek also recognized [8, 9] that the preferred-basis problem plays a key role in the problem of the quantum-to-classical transition, well beyond the narrow context of quantum measurement. Here, we encounter the core question of why we perceive systems, especially macroscopic ones, in only a tiny subset of the physical quantities in principle allowed by the superposition principle. Most notably, for example, macroscopic systems are always found in definite spatial positions but not in superpositions thereof. What singles out position as the preferred quantity? As first suggested by Zeh [4] and spelled out in detail by Zurek [8, 9], to answer these questions and to overcome the preferred-basis problem one must consider the system and the apparatus as open quantum systems, i.e., as interacting with their environment. We will discuss this approach in detail in Sect. 2.8.

2.5.3 The Problem of the Nonobservability of Interference

In many experiments, we can observe interference patterns indicative of the presence of quantum superpositions of component states in a particular basis (see Sect. 2.2.2). However, as we go to larger scales, such interference patterns are typically observed to vanish. For example, if we carry out the double-slit experiment with microscopic particles such as electrons, interference fringes appear on the distant detecting screen (see Fig. 6.6 in Chap. 6). However, as we perform the experiment with atoms or molecules, the interference pattern usually disappears rapidly.

In traditional textbook accounts, this inability to observe interference patterns for mesoscopic and macroscopic objects in the double-slit experiment has typically been explained by an analogy with classical light-wave interference. In the latter setting, it is well known from basic optics that the separation between the diffracting slits must be on the order of the wavelength of the light in order for an interference pattern to be observed. The analogous argument is then applied to the quantum case of the double-slit experiment with matter. Since the (de Broglie) wavelength of particles such as atoms is extremely short, it is simply impossible in practice to manufacture slits whose width and spacing would be of similar magnitude as the wavelength of the particles. In other words, in this picture our inability to observe interference patterns for massive particles would be rooted in the insufficient "resolution" of the experimental device, preventing us from unlocking the quantum nature of the particles.

However, this is only one part of the story. For example, it *is* possible to observe spatial interference patterns for mesoscopic molecules in experimental setups that are similar in spirit to the double-slit experiment but circumvent the obstacle of having to manufacture microscopic slits (see Sect. 6.2 for details on these experiments). Yet, when certain experimental parameters *unrelated to the diffraction process* are changed (for instance, the density of air surrounding the diffracted molecules), the interference pattern is observed to decay. Thus, although the diffraction stage of the experiment clearly allows for the creation of the spatial superposition (which could then be observed in form of an interference pattern), there are other factors that prevent us from observing the pattern. This clearly indicates that our difficulties in "seeing interferences" cannot solely be due to the problem of generating the superposition in the first place.

Furthermore, there exist many interference experiments involving various physical systems that do not at all fall into the category of diffraction experiments with matter. For instance, as already briefly mentioned earlier, there are experiments that allow us to generate superpositions of electrical currents flowing in opposite directions, which would lead to a temporal interference pattern in form of a current oscillating back and forth between the two directions (such experiments will be described in more detail in Sect. 6.3). Yet, despite the fact that the experimental conditions allow for the generation of the superposition state, observation of interference often fails or at least requires extremely refined experimental conditions. Clearly, this problem can no longer be described by the simple analogy between the diffraction of classical light waves and quantum "probability waves."

A forteriori, from the final state (2.54) of the von Neumann measurement scheme it follows that superpositions involving macroscopic measurement devices should be ubiquitous in nature. Why, then, do we not seem to observe interferences between different pointer positions of the apparatus in the everyday world around us? Why is it so difficult to observe *any* interference effects in the mesoscopic and macroscopic regime, although the von Neumann scheme clearly suggests that superpositions should be easily amplified from the microlevel to the macrolevel?

This, in essence, constitutes the problem of the nonobservability of interference. Shortly, we will see how decoherence provides a very elegant and general answer to this problem, by explaining the observed decay of interference patterns (or the complete inability to experimentally observe such patterns in the first place) as a result of interactions with environmental degrees of freedom.

2.5.4 The Problem of Outcomes

Our experience tells us that every measurement results in a definitive value of the measured quantity. In fact, the very definition of terms such as "outcome," "value," "quantity," etc., inherently relies on this definiteness. On the other hand, the final state (2.54) obtained via the von Neumann scheme represents a superposition of system–apparatus states. From our discussion in Sect. 2.2.1 we know that such a superposition is fundamentally different from a classical ensemble of states, i.e., from a situation in which the system–apparatus combination actually is in only one of the component states $|s_i\rangle\,|a_i\rangle$ but we simply do not know in which (see also the analysis in Sect. 2.4.2 above). Therefore, unless we supply some additional physical process (say, some collapse mechanism) or provide a suitable interpretation of such a superposition, it is not clear how to account, given the final composite state, for the definite pointer positions that are observed as the result of an actual measurement.

This problem can be further broken down into two distinct aspects. First, we are faced with the question of why we do not perceive the pointer of the apparatus in a superposition of different pointer positions $|a_i\rangle$ at the conclusion of the measurement (whatever it would actually *mean* to observe such a superposition), i.e., why measurements seem to have outcomes at all. And second, we may ask what "selects" a specific outcome. That is, why do we observe, in each run of the experiment that realizes the measurement, a *particular* pointer position i (and thus a particular pointer state $|a_i\rangle$), as opposed to one of the other possible states $|a_{j\neq i}\rangle$? We shall refer to both issues jointly as the *problem of outcomes.*

The problem of outcomes directly underlies the Schrödinger-cat paradox described in Chap. 1. Recall that the first part of the paradox is concerned with the fact that quantum mechanics seems to predict that the final composite atom–cat state is described by a superposition of classically mutually exclusive states (for simplicity, we shall here refrain from explicitly including the hammer and the poison in our discussion). This part can be understood as simply arising from a von Neumann–type measurement-like interaction between the atom and the cat. Here the atom corresponds to the microscopic system $\mathcal{S}$, while the cat represents the macroscopic apparatus (in the sense that its vitality is an indicator—a "pointer"—of the state of the atom). Following (2.52), the "measurement" scheme therefore reads

$$|\text{“atom not decayed”}\rangle\,|c_\mathrm{r}\rangle \longrightarrow |\text{“atom not decayed”}\rangle\,|\text{“cat alive”}\rangle\,, \tag{2.60a}$$

$$|\text{“atom decayed”}\rangle\,|c_\mathrm{r}\rangle \longrightarrow |\text{“atom decayed”}\rangle\,|\text{“cat dead”}\rangle\,. \tag{2.60b}$$

Here $|c_\mathrm{r}\rangle$ denotes the initial state of the cat. Now, according to quantum mechanics, an unstable atom is at all times described by a superposition of the decayed state and the undecayed state of the atom,

$$|\psi\rangle = \alpha\,|\text{“atom not decayed”}\rangle + \beta\,|\text{“atom decayed”}\rangle\,, \tag{2.61}$$

where α and β are time-dependent coefficients with $|\alpha|^2 + |\beta|^2 = 1$. Just as in the general von Neumann scheme (2.54), the fact that the Schrödinger equation is linear implies that the final composite atom–cat system is then described by an entangled state of the form

$$\begin{aligned}|\psi\rangle\,|c_\mathrm{r}\rangle \longrightarrow\; &\alpha\,|\text{“atom not decayed”}\rangle\,|\text{“cat alive”}\rangle \\ &+\beta\,|\text{“atom decayed”}\rangle\,|\text{“cat dead”}\rangle\,.\end{aligned} \tag{2.62}$$

No individual quantum state can now be attributed to the cat, and it would thus appear that “the cat is neither alive nor dead,” as the situation described by the final state on the right-hand side of (2.62) is often interpreted.[16] Note, though, that it is not the cat itself that is described by a superposition of the states “alive” and “dead.” Rather, it is the *composite* atom–cat state that is represented by a superposition of atom–cat states (again, we have omitted the hammer and the poison from the picture).

As explained in Chap. 1, the second part of the Schrödinger-cat paradox refers to the fact that the superposition (2.62) persists until, at least according to the standard interpretation of quantum mechanics (see Sect. 8.1), the box is opened and its contents are directly observed (leading to the postulated “collapse of the wave function”). This poses the question of how and why the fate of the cat could possibly be left to the intervention (von Neumann's *“erster Eingriff”*) of an external observer. Of course, this is in essence nothing else than the problem of outcomes: How can we account for the observer's experience of a definite state of the cat (i.e., of a cat that is either alive or dead) given the superposition state (2.62)?

Remedying the problem of outcomes, i.e., solving the apparent conflict between the paramount and experimentally confirmed role of the superposition principle and the observation of single definite outcomes in measurements, has been one of the core motivations behind any interpretation of quantum mechanics. The standard (or “orthodox”) interpretation of quantum mechanics (see Sect. 8.1 for details) prescribes that an observable corresponding to a physical quantity has a definite value if and only if the system is in an eigenstate of this observable. On the other hand, if the system is described by a superposition of such eigenstates, it is considered meaningless to speak

[16] Indeed, it is difficult to properly put such a state of affairs into words, as it is simply not part of our experience.

of the (physical) state of the system as having any definite value of the observable at all. This rule of orthodox quantum mechanics is often referred to as the *eigenvalue–eigenstate link* (sometimes also called, more to the point, the *value–eigenstate link*).

However, the eigenvalue–eigenstate link is not necessitated by the structure of quantum mechanics or by any empirical constraints [68]. Furthermore, the concept of an "exact" eigenvalue–eigenstate link leads to difficulties of its own. For instance, outcomes of measurements are typically registered by pointers localized in position space. But these pointers are never perfectly localized, i.e., they cannot be described by exact eigenstates of the position operator, since such eigenstates are unphysical (they correspond to an infinite spread in momentum and therefore to an infinite amount of energy to be contained in the system). Therefore the states corresponding to different pointer positions cannot be exactly mutually orthogonal.

The concept of classical "values" that can be ascribed through the eigenvalue–eigenstate link based on observables and the existence of exact eigenstates of these observables has therefore frequently been either weakened or altogether abandoned. Either the quantum formalism and the concept of measurement have then been reinterpreted in certain ways, or actual modifications of quantum mechanics itself have been introduced. In the former category, relative-state and modal interpretations aim to interpret the final composite system–apparatus state arising in the von Neumann scheme (2.54) in such a way as to explain the existence, or at least the subjective perception, of outcomes in spite of the fact that the quantum state has the form of a superposition. In the latter category, physical collapse models (already briefly mentioned in Sect. 2.2.3) postulate the existence of some fundamental mechanism in nature that breaks the unitarity of the Schrödinger evolution and leads to an "objective" reduction of the wave function onto one of its components. These various interpretations and their relation to decoherence will be discussed in more detail in Chap. 8.

Generally, the problem of outcomes is rooted in the question of what actualizes a particular result in a probabilistic theory. In classical probabilistic theories, answering this question does not, at least in principle, pose any difficulties. Here, the notion of probability is simply a consequence of a convenient coarse-graining procedure that may simplify the treatment of certain problems. At the fundamental level of the physical system, however, the particular outcome is completely specified by the underlying, deterministic laws of physics (however complicated they may look in practice for, say, a system composed of billions of atoms). Thus, in principle, we could always rid classical physics of any probabilistic aspect.

By contrast, as we have discussed in detail in Sect. 2.1, quantum mechanics appears to possess an intrinsically probabilistic character. Pure quantum states already represent a complete description of the (physical) state of the system, and their evolution is given by the deterministic Schrödinger equation. Yet, there exists no fundamental mechanism that would determine which particular outcome is realized in each measurement instance. Therefore, the problem of outcomes is fundamental to quantum mechanics itself. Accordingly, the best hope we can have for decoherence to help us solve this problem is by explaining why only one of the possible outcomes is actually observed in a measurement (rather than a superposition of outcomes). However, the question of why a *particular* outcome appears to the observer rather than another one of the possible outcomes, none of which is formally singled out in any way in the final von Neumann state (2.54), pertains to fundamental issues in the interpretation of quantum mechanics outside of the scope of decoherence. We will come back to this topic in Chap. 8.

2.6 Which-Path Information and Environmental Monitoring

Having laid out the key elements of the formalism and interpretation of quantum mechanics relevant to decoherence—namely, quantum states, the superposition principle, entanglement, density matrices, and the measurement problem—we are now in an excellent position to finally approach our actual subject of interest, namely, decoherence. We shall go about this task by first revisiting the well-known double-slit experiment, which will provide us with a very intuitive and accessible illustration of the basic mechanism of decoherence.

2.6.1 The Double-Slit Experiment, Which-Path Information, and Complementarity

Let us have a look back at Fig. 2.2, where we sketched the usual double-slit setup. Particles (such as electrons) approaching from the left are incident on a screen with two slits. After passage through the slits, they hit a distant detector screen, leaving a permanent spot. As predicted by quantum mechanics and confirmed by experiment, there exist two different limiting regimes (see also our discussion in Sect. 2.2.2):

1. *The "wave" scenario* (illustrated on the right of Fig. 2.2). If we refrain from measuring through which slit each particle has passed, the particle density observed at the level of the detecting screen corresponds to an interference pattern given by $\varrho(x) = \frac{1}{2}\left|\psi_1(x) + \psi_2(x)\right|^2$, i.e., by the probability density corresponding to a *quantum-mechanical superposition* of the partial waves $\psi_1(x)$ and $\psi_2(x)$ representing passage through slit 1 and 2, respectively.
2. *The "particle" scenario* (shown in the center of Fig. 2.2). If we place a detector at one of the slits to find out whether the particle has passed

through this slit, the interference pattern disappears. Now the density pattern on the distant screen is simply equal to a *classical addition* of the pattern created by all particles that have traversed slit 1 (that is, the pattern that would be obtained if slit 2 was covered) and the pattern created by all particles that have passed through slit 2: $\varrho(x) = \frac{1}{2}|\psi_1(x)|^2 + \frac{1}{2}|\psi_2(x)|^2$.

The standard explanation of the second case (the "particle" scenario) goes as follows. According to quantum mechanics, the state of the particle at the level of the slits is given by the superposition $\Psi(x) = (\psi_1(x) + \psi_2(x))/\sqrt{2}$ of the partial waves $\psi_1(x)$ and $\psi_2(x)$. This superposition is spatially spread out over the region encompassing the two slits. If we now measure the position of the particle at one of the slits and indeed find the particle to be present at this slit, we localize $\Psi(x)$ to the corresponding spatial region. That is, since $\psi_1(x)$ and $\psi_2(x)$ have negligible overlap at the level of the slits, we may say that, using the standard collapse postulate of quantum mechanics, the measurement has collapsed the superposition $\Psi(x)$ onto either one of the component states $\psi_1(x)$ or $\psi_2(x)$. Thus we can no longer obtain interference between these partial waves, and hence the interference pattern on the distant screen disappears.

In other words, whenever we try to obtain *which-path ("Welcher-Weg") information* about the particle in order to see which of the two slits the particle has traversed—i.e., whenever we attempt to make sense of the quantum-mechanical superposition of the two paths that would seem to describe a counterintuitive simultaneous passage through *both* slits—it seems that we cannot help but destroy the ability of the particle to exhibit the quantum property of (spatial) interference. Thus there exists, to use Niels Bohr's famous term [70], a *complementarity* between which-path information (the "particle" aspect") and interference (the "wave" aspect). Depending on the experimental setup (namely, depending on whether we measure the path of the particle or not), we seem to observe either "particle-like" or "wave-like" behavior. This so-called "wave–particle duality" has been considered a cornerstone of quantum theory and has been the subject of countless discussions among physicists and philosophers of physics alike.

The complementarity principle and its application was the subject of a famous debate between Einstein and Bohr at the Fifth Solvay Conference in Brussels in 1927 [71]. At the conference, Einstein had challenged Bohr with the following thought experiment involving the standard double-slit setup. He argued that, based on the law of momentum conservation, the passage of the particle through the top (bottom) slit should result in a recoil of the screen containing the slits in the upward (downward) direction. Suppose now we could measure the direction of the recoil of the screen (see Fig. 2.5). This measurement would allows us to infer, at least in principle, the path of the particle, thus providing us with which-path information. Einstein argued that, since the interference pattern is solely due to the interference between

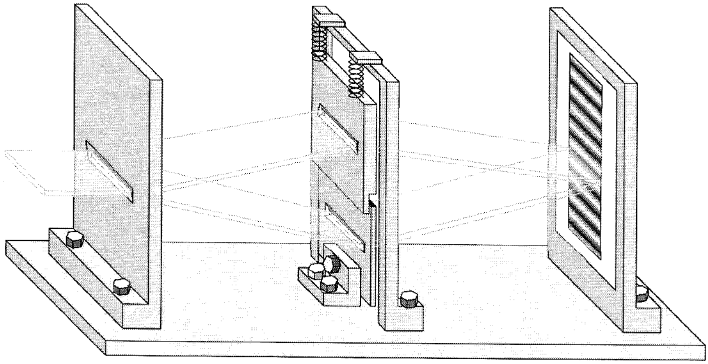

Fig. 2.5. Einstein's thought experiment for obtaining which-path information in the double-slit experiment. Particles leaving the collimating slit on the left pass through a double slit (center) and are registered on the detection screen on the right. The passage of the particle through the slit transfers momentum to the slit, which should in principle be measurable as a recoil of the screen containing the slits. While the bottom slit is kept fixed, the upper slit is suspended by springs. A particle passing through the upper slit would therefore induce a tiny oscillatory motion of the suspended slit that could (at least in principle) be detected, which would allow us to infer which slit the particle has traversed. The illustration is based on original drawings of Bohr [72] and is reproduced from [73] by permission from Macmillan Publishers Ltd: Nature, copyright 2001.

the two wave packets emerging from the slits, the determination of the direction of the recoil at the stage of the screen cannot have any effect on the subsequent evolution, and thus an interference pattern should be observable, in apparent contradiction with Bohr's principle of complementarity.

Bohr countered the challenge in the following way. The recoil imparted on the slits by an individual particle will typically be extremely small. To resolve this tiny change in momentum, we must know the initial momentum of the screen containing the slits within a range at least as small as the to-be-detected recoil. According to the uncertainty principle, measuring the momentum of the screen with such high accuracy implies a large uncertainty in the position of the screen. Bohr then showed that already due to the measurement of the initial momentum of the screen (before the incident particle even reaches the slits), the resulting indeterminacy of the position of the screen translates into a range of possible positions of the interference fringes on the detecting screen that would be on the order of the characteristic spacing between these fringes. Thus, if we average the interference pattern over this range of positions, the pattern is "washed out": The position of a maximum in the pattern for one of the possible positions coincides with the position of a minimum in the pattern for another position of the screen, and hence the net interference pattern disappears.

Bohr's central claim is therefore that obtaining which-path information implies an inevitable disturbance of the system, which is indeed true in the Bohr–Einstein example of gaining which-path information from a measurement of the recoil of the screen. However, as first shown by Wootters and Zurek [74] and further investigated by Scully and Drühl [75], in certain situations it is also possible to gather which-path information in such a way that there is no significant change in the spatial wave function of the particles, moderating the effect of the position–momentum uncertainty principle pointed out by Bohr. This will be discussed in the next section.

2.6.2 The Description of the Double-Slit Experiment in Terms of Entanglement

Thus far, the complementarity between obtaining which-path information and observing an interference pattern has been introduced as a discontinuous either–or distinction. However, we may now ask whether we could retain parts of the interference pattern by gathering only *some* which-path information, e.g., by performing an imprecise measurement of which slit the particle has traversed [74].

It turns out that the answer to this question is in the affirmative. However, to discuss and explain this feature of quantum mechanics, the description in terms of a wave-function collapse (as used above) will no longer be suitable. By postulate, the collapse is a discontinuous, irreversible process and therefore cannot account for smooth, reversible changes in the amount of which-path information and the degree of interference. Instead, we shall pursue a purely quantum-mechanical account in terms of the von Neumann measurement scheme (Sect. 2.5.1) and entanglement. This description will then also become the basis for our description of the process of decoherence, where the environment will assume the role of the which-path detector. The connection between the observability of interference in the double-slit experiment and entanglement was first discussed by Wootters and Zurek [74].

Let us denote the quantum states of the particle corresponding to passage through slit 1 and 2 by $|\psi_1\rangle$ and $|\psi_2\rangle$, respectively. As before, we place a detector at each of the two slits, with both detectors initially in the "ready" state. We can prepare our particle in, say, the state $|\psi_1\rangle$ by covering slit 2, and by placing the particle source directly behind slit 1 such that the particle will be guaranteed to pass through this slit. Consequently, the detector associated with slit 1 will trigger, while the detector at slit 2 will remain in the untriggered "ready" state. In the following we shall refer to the two individual detectors jointly as "the detector." We denote the joint "ready" state of the (composite) detector by $|\text{“ready”}\rangle$, and the quantum state of this detector system after preparation of the state $|\psi_1\rangle$ (as described above) by $|1\rangle$, indicating the passage of the particle through slit 1. Thus, in this case, the evolution of the state of the composite particle–detector system will be of the form

$$|\psi_1\rangle\,|\text{“ready”}\rangle \longrightarrow |\psi_1\rangle\,|1\rangle\,. \tag{2.63}$$

Repeating the above argument with the role of the slits reversed yields the evolution

$$|\psi_2\rangle\,|\text{“ready”}\rangle \longrightarrow |\psi_2\rangle\,|2\rangle\,. \tag{2.64}$$

Equations (2.63) and (2.64) correspond to the general description (2.52) of a measurement interaction.

Now, if both slits are open, the particle must be described by a superposition $|\psi\rangle = (|\psi_1\rangle + |\psi_2\rangle)/\sqrt{2}$ of the two components $|\psi_1\rangle$ and $|\psi_2\rangle$. Using (2.63) and (2.64), we therefore obtain a dynamical evolution of the von Neumann type (2.54), leading to an entangled composite particle–detector state,

$$\frac{1}{\sqrt{2}}\left(|\psi_1\rangle + |\psi_2\rangle\right)|\text{“ready”}\rangle \longrightarrow \frac{1}{\sqrt{2}}\left(|\psi_1\rangle\,|1\rangle + |\psi_2\rangle\,|2\rangle\right). \tag{2.65}$$

Once again, the detector states $|1\rangle$ and $|2\rangle$ act as “pointers” for the relative states $|\psi_1\rangle$ and $|\psi_2\rangle$ of the system.

What can we now learn about the particle by performing a measurement on it, for instance by letting it impinge on the detection screen, which corresponds to a measurement of position? As we know from Sect. 2.4.6, the quantity of interest is now the reduced density matrix for the particle, which for the pure state on the right-hand side of (2.65) is given by [see (2.49)]

$$\hat{\rho}_{\text{particle}} = \frac{1}{2}\left\{|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2| + |\psi_1\rangle\langle\psi_2|\langle 2|1\rangle + |\psi_2\rangle\langle\psi_1|\langle 1|2\rangle\right\}. \tag{2.66}$$

This density matrix corresponds to a particle density $\varrho(x)$ at the detecting screen given by

$$\begin{aligned}\varrho(x) &\equiv \rho_{\text{particle}}(x,x)\\ &\equiv \langle x|\,\hat{\rho}_{\text{particle}}\,|x\rangle\\ &= \frac{1}{2}\,|\psi_1(x)|^2 + \frac{1}{2}\,|\psi_2(x)|^2 + \mathrm{Re}\left\{\psi_1(x)\psi_2^*(x)\langle 2|1\rangle\right\},\end{aligned} \tag{2.67}$$

where $\psi_i(x) = \langle x|\psi_i\rangle$, $i = 1, 2$. The last term describes the well-known interference pattern, and we see that the visibility of this interference pattern is quantified by the overlap $\langle 2|1\rangle$. This observation directly connects with our previous discussion in Sect. 2.4.6.

In particular, the limiting case of perfect distinguishability of the detector states $|1\rangle$ and $|2\rangle$, $\langle 2|1\rangle = 0$, corresponds to the “particle” regime,

$$\varrho(x) = \frac{1}{2}\,|\psi_1(x)|^2 + \frac{1}{2}\,|\psi_2(x)|^2\,. \tag{2.68}$$

Conversely, if $|1\rangle$ and $|2\rangle$ are completely unable to resolve the path of the particle, $\langle 2|1\rangle = 1$ (disregarding phase factors), the “wave” scenario of full interference pattern applies,

$$\varrho(x) = \frac{1}{2}\left|\psi_1(x)\right|^2 + \frac{1}{2}\left|\psi_2(x)\right|^2 + \mathrm{Re}\left\{\psi_1(x)\psi_2^*(x)\right\}. \tag{2.69}$$

The situation in which the detector obtains some but not full which-path information is formally represented by an overlap of the two detector states $|1\rangle$ and $|2\rangle$ that is nonzero but less than one. From (2.66) we see that we will then be able to observe an interference pattern, but the pattern will decay progressively as the overlap $\langle 2|1\rangle$ *decreases* (i.e., as the detector states $|1\rangle$ and $|2\rangle$ become more distinguishable) and thus the amount of which-path information obtained by the detector *increases.*

Thus we have shown that it is indeed possible to simultaneously observe an interference pattern and obtain some information about the path of the particle through the slits, provided this information remains incomplete. As soon as the information acquired by the detector is sufficient to enable us to infer with certainty which path the particle has taken, the interference pattern disappears. In their analysis of this fundamental trade-off between which-path information and interference and thus of the intimate connection between complementarity and entanglement, Wootters and Zurek [74] pointed out that one can obtain a fairly large amount of which-path information while retaining a visible interference pattern. Specifically, they showed that 90% certainty about the which-path question still allows for roughly 50% contrast of the interference pattern.

In summary, the degree to which an interference pattern can be observed is simply determined by the available which-path information encoded in some system entangled with the object of interest, and this amount can be changed without necessarily influencing the spatial wave function of the object itself. Thus complementarity can be regarded as a *consequence* of quantum entanglement.

2.6.3 The Environment as a Which-Path Monitor

Let us consider this book. It is immersed into a large environment of air molecules, light and thermal photons, even cosmic neutrinos and radioactive background radiation. In every second, a huge number of these particles will collide with and scatter off the book. Each of these collision will deflect the particle to some extent, depending on the position and orientation of the book. Let us look at a particular air molecule. It starts from some initial position with a certain velocity, scatters off the book, and then flies away along a trajectory deflected by a certain angle with respect to the incoming path.

Suppose now the book was oriented at a different angle. Now the air molecule would in general fly off along a different direction than in the first case. Thus, these two distinct paths of the scattered molecule would allow us to *distinguish* the existence of two different spatial orientations of the book. In other words, the scattered particles—the air molecules—carry away information about the position and orientation of the scattering object, which is here represented by the book. Using our now-familiar terminology, they therefore encode which-path information about the system. We remark that the term "which-path information" should here (as in the remainder of the text) be understood in the more general sense of "which-state information." The former term is historically motivated by the example of the double-slit experiment in which the information of interest concerns the path of the particle through the slits. Nonetheless, we shall often use the well-established term "which-path information" even in cases where the relevant information does not actually pertain to the trajectory of a particle.

A single air molecule colliding with the book may not carry away much information about the orientation of the book. After all, the scattering takes place only in a tiny region compared to the size of the book. However, as mentioned above, there are millions of such molecules scattering off the surface of the book in any given moment. The which-path (or, maybe more appropriately, "which-orientation") information encoded in all these molecules taken together will certainly be completely sufficient to distinguish two different positions and orientations of the book, even if they are very similar.

Let us express our argument schematically in terms of the quantum formalism. Suppose we focus on all N particles that will scatter off the book in the span of one second. Let us represent the quantum state of all N environmental particles before the collision by $|E_0\rangle$. After the collision, each particle will fly off along a certain trajectory with some velocity. For another orientation of the book, the particle may scatter into a slightly different direction. Now suppose all N particles have scattered off the book. Depending on the orientation of the book, we denote the post-scattering state of the environment by $|E_1\rangle$ and $|E_2\rangle$, respectively.

Following our above argument, these two states are clearly distinguishable, since the information contained in these two states is sufficient to discriminate between the two orientations of the book. Therefore the overlap between $|E_1\rangle$ and $|E_2\rangle$ will be negligibly small.[17] We clearly see the connection with the which-path formalism introduced in the previous Sect. 2.6.2: The environmental states $|E_1\rangle$ and $|E_2\rangle$ simply correspond to the states of a which-path (or, in this case, which-orientation) detector.

The crucial point is that these environmental which-path detectors are present everywhere in nature. Every object interacts with its environment, which in turn will obtain information about certain physical properties of the system. We may be able to shield our book from air molecules to some extent by placing it in a good vacuum, we may block out visible photons (light), we

[17] We can think of $|E_1\rangle$ and $|E_2\rangle$ as products of states pertaining to each individual molecule, $|E_1\rangle = \prod_{i=1}^{N} |e_1\rangle_i$ and $|E_2\rangle = \prod_{i=1}^{N} |e_2\rangle_i$. While the overlap of each individual pair of molecular states will be close to one, $(\langle e_1|e_2\rangle)_i \approx 1$, the product states $|E_1\rangle$ and $|E_2\rangle$ of a large number N of these individual states will be approximately orthogonal.

may even try to reduce the influence of thermal photons by cooling the book. Still, as we shall see in the next Chap. 3, particles from other sources remain that will continue to "measure" the position of the system. This crucial role of the environment as a ubiquitous "measuring device" which continuously performs effective measurements (in the von Neumann sense) on the system was first clearly recognized and discussed by Zurek in the early 1980s [8,76].

We emphasize that this monitoring process does not require a human observer of any sort. The fact that the which-path information encoded in the environment could *in principle* be read out is sufficient for the interference pattern to disappear, i.e., for the particles to "lose their wave nature." This lends a more precise, observer-independent meaning to Heisenberg's statement (already quoted in Sect. 2.1.3) that "the particle trajectory is created by our act of observing it" [42, p. 185]. This positivist attitude had resulted in much criticism directed at the quantum theory, as famously represented by Einstein's rhetorical question (mentioned in Chap. 1) of whether "the moon exists only when I look at it" [3].

We can now go through a formal argument analogous to that of Sect. 2.6.2. Suppose the system is described by a coherent superposition of two quantum states $|\psi_1\rangle$ and $|\psi_2\rangle$ representing localization around two different positions x_1 and x_2 (in the case of the double-slit experiment, the corresponding position-space wave functions $\psi_1(x)$ and $\psi_2(x)$ would represent the partial waves at the slits). Before the scattering of environmental particles takes place, the combined system–environment state has the product form

$$|\psi\rangle\,|E_0\rangle = \frac{1}{\sqrt{2}}\left(|\psi_1\rangle + |\psi_2\rangle\right)|E_0\rangle\,. \tag{2.70}$$

Following our above discussion, the system–environment interaction dynamics are given by

$$|\psi_1\rangle\,|E_0\rangle \longrightarrow |\psi_1\rangle\,|E_1\rangle\,, \tag{2.71a}$$
$$|\psi_2\rangle\,|E_0\rangle \longrightarrow |\psi_2\rangle\,|E_2\rangle\,. \tag{2.71b}$$

That is, the state of the environment evolves into $|E_1\rangle$ or $|E_2\rangle$ depending on the state of the system. Then the linearity of the Schrödinger equation implies the usual von Neumann evolution

$$\frac{1}{\sqrt{2}}\left(|\psi_1\rangle + |\psi_2\rangle\right)|E_0\rangle \longrightarrow \frac{1}{\sqrt{2}}\left(|\psi_1\rangle\,|E_1\rangle + |\psi_2\rangle\,|E_2\rangle\right). \tag{2.72}$$

We see that the relative states $|\psi_1\rangle$ and $|\psi_2\rangle$ of the system have become entangled with the environmental states $|E_1\rangle$ and $|E_2\rangle$ that encode which-path information. The superposition initially confined to the system has now spread to the larger, composite system–environment state. Correspondingly, coherence between the components $|\psi_1\rangle$ and $|\psi_2\rangle$ is no longer a property of the system alone: It has become a shared property of the global system–environment state. One therefore often says that coherence has been "delocalized into the larger system" [77, p. 5], which now includes the environment.

The dynamical system–environment evolution described by (2.72) is the basic formal representation of the decoherence process, and we shall now discuss its consequences.

Heißt also: auch wenn diese hochenerget
(= hochfrequenten) Zustände keine Prod
zustände sind, sollen sie ≈ 0 Überlapp h

2.7 Decoherence and the Local Damping of Interference

The reduced density matrix $\hat{\rho}_{\text{particle}}$ of the system for the state (2.72) is given by [see (2.49) and (2.66)]

$$\hat{\rho}_{\text{particle}} = \frac{1}{2}\left\{|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2| + |\psi_1\rangle\langle\psi_2|\langle E_2|E_1\rangle + |\psi_2\rangle\langle\psi_1|\langle E_1|E_2\rangle\right\}. \tag{2.73}$$

As usual, the last two terms correspond to interference between the component states $|\psi_1\rangle$ and $|\psi_2\rangle$. Provided the environment has indeed recorded sufficient which-path information (which will certainly be the case for our above example of air molecules scattering off a macroscopic object over a period of one second), the final environmental states $|E_1\rangle$ and $|E_2\rangle$ will be approximately orthogonal, $\langle E_2|E_1\rangle \approx 0$. Then interferences in the reduced density matrix (2.73) will become suppressed,

$$\hat{\rho}_{\text{particle}} \approx \frac{1}{2}\left\{|\psi_1\rangle\langle\psi_1| + |\psi_2\rangle\langle\psi_2|\right\}. \tag{2.74}$$

Only measurements that include both the system and the environment could possibly reveal the persistent coherence between the components in the superposition state (2.72). However, in practice, it is impossible to include in our observation all the many environmental degrees of freedom that have interacted with the system. Joos and Zeh [7, p. 224] poignantly summarized this state of affairs as "the interference terms still exist, but they are not *there*." That is, the interference terms remain present at the *global* level of the system–environment superposition (2.72) but have become unobservable at the *local* level of the system as described by the reduced density matrix (2.74). Of course, typically some of the environmental degrees of freedom will be part of our observation. For example, we will directly intercept a certain fraction of the light scattered off the system.[18] We can then simply regard these environmental degrees of freedom as part of the system. However, the important point is that there still remains a comparably large number of other environmental degrees of freedom that will not be observed directly.

Because of this very large number of environmental degrees of freedom interacting with the system and our inability to directly manipulate them, the creation of system–environment entanglement described by (2.72) is virtually impossible to undo in practice. Thus the environment-induced loss of

[18] In fact, this is how in practice observers will usually gather information about the system (see Sect. 2.9).