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Report of the Experiment with  
**Long Half Lives**

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# 1 Introduction

## 1.1 Motivation

This experiment is about finding the half-life of two elements, Samarium  $^{147}\text{Sm}$  as an  $\alpha$ -ray source and potassium  $^{40}\text{K}$  as  $\beta$ -ray source. Because the half-life of both samples is very high, in the range of  $10^9$  to  $10^{11}$  years, we can not observe the change of the rate. With

$$A = -\dot{N} = \lambda \cdot N = \frac{\ln(2)}{T_{\frac{1}{2}}} \cdot N \quad (1)$$

we can find out  $T_{\frac{1}{2}}$  if we know the number of active and participating atoms  $N$  which we can find out by measuring the mass or surface.

## 1.2 Set-Up

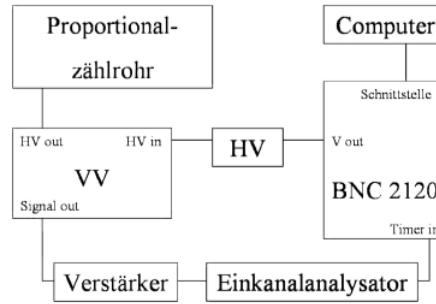


Figure 1: Sheme of the set-up

In figure 1 we see the set-up. We use a Geiger-Counter filled with methane. The energy is provided by a high voltage source (HV). The signal gets amplified in a preamplifier (VV). Afterwards it passes the main amplifier. In the following Single-Chanal-Analyser the signal gets modified to a digital signal and counted in the BNC. That one provides the data for the computer that runs LabView.

## 1.3 Theoretical Basics

### 1.3.1 Samarium

Samarium decays under emission of  $\alpha$ -rays. These are blocked very easily and only have a limited reach  $R_{\text{Sm}_2\text{O}_3}$  so the rate  $n$  is

$$n = A_v \frac{F}{4} R_{\text{Sm}_2\text{O}_3} \quad (2)$$

With  $F$  the surface and  $A_V$  the activity per volume. Further we use the relation

$$\sqrt{m_A} = \sum_i p_i \sqrt{m_{A,i}} \quad (3)$$

and get  $\sqrt{m_A}$  for air.

$$Air = Nitrogen + Oxygen + Argon \quad (4)$$

$$\sqrt{m_A} = 0,75518\sqrt{m_N} + 0,23135\sqrt{m_O} + 0,01288\sqrt{m_{Ar}} \quad (5)$$

Straight forward we can figure out  $C$  with

$$R_O \rho = C \sqrt{m_A} \quad (6)$$

$$R_O = 1,13cm \quad (7)$$

$$\rho = 0,001226 \frac{g}{cm^3}.$$

This we can use to calculate

$$R_{Sm_2O_3} \rho_{Sm_2O_3} = C \sqrt{m_{Sm_2O_3}} = 0,004380787 \frac{g}{cm^2}. \quad (8)$$

The values we used and were not given in the manual<sup>1</sup>:

$$m_N = 2,32587 \cdot 10^{-23}g$$

$$m_O = 2,65676 \cdot 10^{-23}g$$

$$m_{Ar} = 6,63355 \cdot 10^{-23}g$$

$$m_{^{147}Sm} = 2,439579 \cdot 10^{-22}g$$

The half-life  $T_{\frac{1}{2}}$  is given by

$$T_{\frac{1}{2}} = \frac{\ln(2) \cdot N}{A} = \frac{\ln(2) \cdot N}{A_v \cdot F \cdot d} = \frac{\ln(2) \cdot N \cdot R_{Sm_2O_3}}{4 \cdot n \cdot d} \quad (9)$$

with  $d$  the thickness of the sample and  $A_V = A/V$ .

The number  $N$  of the  $^{147}Sm$ -Cores one gets by

$$N = 2 \cdot N_{Sm_2O_3} \cdot p \quad (10)$$

$p$  is the empirical probability of  $^{147}Sm$  in Samarium. Its given in the manual as  $p = 0,1487$ . If one put this in (9) and use

$$N_{Sm_2O_3} = \frac{m \cdot N_A}{m_{rel}} \quad (11)$$

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<sup>1</sup>source: Wolfram Mathematica

you get the final equation for  $T_{\frac{1}{2}}$

$$T_{\frac{1}{2}} = \frac{\ln(2) \cdot R_{Sm_2O_3} \cdot \rho_{Sm_2O_3} \cdot N_A \cdot p \cdot F}{2 \cdot n \cdot m_{rel}} = 3.89811 \cdot 10^{17} \cdot \frac{F}{n}. \quad (12)$$

Here is  $N_A = 6,022141 \cdot 10^{23}$  the Avogadro's number<sup>2</sup>,  $m_{rel} = 348,72 \text{g/mol}$ <sup>3</sup> the molar mass and  $F$  the surface.

### 1.3.2 Potassium

Potassium decays under emission of a  $\beta$ -ray. Therefore we can not assume a discrete range but an exponential. So we expect a curve suitable with

$$n(m) = a(1 - \exp(-bm)) \quad (13)$$

with  $a$  and  $b$  which look like

$$n(m) = f_B \frac{\Omega}{4\pi} \frac{A}{m} \frac{f\rho}{\mu} \left(1 - \exp\left(-\frac{\mu}{f\rho}m\right)\right) \quad (14)$$

Where  $\Omega = 2\pi$  due to the limited range of the detector. From this it follows that

$$a = \frac{1}{2} f_B \frac{A}{m} b^{-1}. \quad (15)$$

The half-life  $T_{\frac{1}{2}}$  is:

$$T_{\frac{1}{2}} = \frac{\ln(2)}{1.12} \frac{N}{A} = \frac{\ln 2}{1.12} \frac{N_A h_{rel}}{m_{mol, KCl}} \frac{m}{A} \quad (16)$$

$$\stackrel{(15)}{=} \frac{\ln 2}{2 \cdot 1.12} \frac{N_A h_{rel}}{m_{mol, KCl}} \frac{f_b}{ab} \quad (17)$$

where  $f_B = 1.29$ ,  $h_{rel} = 1.18 \cdot 10^{-4}$  and the molar mass<sup>4</sup>  $m_{mol, KCl} = 74.551 \text{g/mol}$ . The other values are taken from the manual. The errors of these quantities are neglectable.

### 1.3.3 Calculation of measure time for an error of 2%

The radioactive decay is within the laws of Poisson. The error on  $N$  is therefore:

$$\sigma(N) = \sqrt{N} \quad (18)$$

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<sup>2</sup>source: CODATA

<sup>3</sup>source: Wolfram Mathematica

<sup>4</sup>source: Wolfram Mathematica

We want an error of 2% so we need

$$\sqrt{N} = 0,02 \cdot N \quad (19)$$

$$\Rightarrow N = 2500 \quad (20)$$

$$\Rightarrow t_N = \frac{2500}{n} \quad (21)$$

The error for the underground should be much smaller so that we can ignore it at the end. We decided that the error for the underground needs to be a 5th.

$$\frac{\sigma_U}{t_U} < \frac{\sigma_N}{5 \cdot t_N} \quad (22)$$

$$\Rightarrow \frac{\sqrt{U}}{t_U} < \frac{\sqrt{N}}{5 \cdot t_N} \quad (23)$$

$$\Rightarrow \frac{U}{t_U^2} < \frac{N}{25 \cdot t_N^2} \quad (24)$$

$$\Rightarrow \frac{u}{t_U} < \frac{n}{25 \cdot t_N} \quad (25)$$

$$\Rightarrow t_U > 25 \cdot \frac{u}{n} \cdot t_N. \quad (26)$$

Out of this we get our time for the measurement of the underground by

$$t_U > 62500 \cdot \frac{u}{n^2}. \quad (27)$$

## 2 Measurement

The measurements were done in a different order, but to increase the clearness, we sorted them by sample. For all measurements a threshold setting of  $3.01 \pm 0.01$  was used.

### 2.1 Detector Measurements

#### 2.1.1 Characteristic of Detector

file:UraniumCharacteristic.txt

Sample: Uranium

$V_{\min}$  = 1kV

$V_{\max}$  = 4kV

$V_{\text{step}}$  = 100V

$\Delta t$  = 50s

$t_{\text{dead}}$  = 15s

#### 2.1.2 Underground for all $V$

file:Untergrund1500-4000V.txt

Sample: empty, clean Aluminium dish

$V_{\min}$	= 1.5kV
$V_{\max}$	= 4kV
$V_{\text{step}}$	= 100V
$\Delta t$	= 100s
$t_{\text{dead}}$	= 15s

## 2.2 Potassium

### 2.2.1 $\beta$ Plateau

file:Plateau\_Kalium\_3000-4000V.txt

Sample:	Potassium
$V_{\min}$	= 3kV
$V_{\max}$	= 4kV
$V_{\text{step}}$	= 100V
$\Delta t$	= 100s
$t_{\text{dead}}$	= 15s
$m_{\text{both}}$	= 3.0869g
$m_{\text{dish}}$	= 1.3259g

From this measurement we selected the voltage of the plateau for  $\beta$ -decay measurements. The selected Voltage is  $V_{\beta} = 3.5\text{kV}$

### 2.2.2 Estimation of $n$ at $V_{\beta}$

Sample:	Potassium
$V_{\text{fixed}}$	= 3.5kV
$\Delta t$	= 200s
$m_{\text{both}}$	= 3.0869g
$m_{\text{dish}}$	= 1.3259g
Rate $n$	= 7.42Hz

We estimated the required time for an 2% error  $t_N = 336,9\text{s}$  with the equation (21).

### 2.2.3 Underground at $V_{\beta}$

Sample:	empty, clean Aluminium dish
$V_{\text{fixed}}$	= 3.5kV
$\Delta t$	= 55800s = 15.5h (over night)
Rate $u$	= 5.806Hz
Counts $U$	= 323952

The flow of gas might be disturbed during the night.



## 2.3 Potassium main measurement

From equation (27) we know the rate  $n_{\min}$  must not be below 0,876Hz whether the underground error should be neglectable.

$$\begin{aligned} V_{\text{fixed}} &= 3,5\text{kV} \\ \Delta t &= 340\text{s} \end{aligned}$$

$n \cdot s$	$m/g$
4,709	2,1486
3,962	1,8504
1,594	1,4553
5,944	3,1387
5,156	3,1387

## 2.4 Samarium

### 2.4.1 $\alpha$ -Plateau

file:Plateau.Samarium.txt

$$\begin{aligned} \text{Sample:} & \quad \text{Samarium} \\ V_{\min} &= 1.3\text{kV} \\ V_{\max} &= 3.7\text{kV} \\ V_{\text{step}} &= 100\text{V} \\ \Delta t &= 200\text{s} \\ t_{\text{dead}} &= 15\text{s} \end{aligned}$$

From this measurement we selected the voltage of the plateau for  $\alpha$  decay measurements. The selected voltage is  $V_{\alpha} = 2\text{kV}$ .

We estimated the required time for an 2% error  $t_N = 4629\text{s}$  with the equation (21).

## 2.5 Samarium main measurement

$$\begin{aligned} V_{\text{fixed}} &= 2\text{kV} \\ \Delta t &= 4630\text{s} \\ s_d &= 0,05\text{mm} \end{aligned}$$

Diameter $d/\text{mm}$	Rate $n \cdot s$	Counts
28,90; 28,90; 28,85; 28,80; 28,90	0,539	2496
17,00; 17,00; 16,95; 16,95; 16,00	0,203	938
9,95; 9,95; 9,90; 10,00; 9,95	0,080	370

Because of a lack of time we were not able to increase the time for each measurement to reach our required 2500 counts.

### 2.5.1 Underground at $V_\alpha$

Sample: empty, clean Aluminium dish  
 $V_{\text{fixed}}$  = 2kV  
 $\Delta t$  = 55800s = 15.5h (over night)  
Rate  $u$  = 0.07Hz  
Counts  $U$  = 3881

In the morning the gas was continuously flowing with 3-5 bubbles per second.

## 3 Evaluation

### 3.1 Characteristics of the Detector

In 2.1.1 we have measured the characteristics of the detector with uranium. This means we have measured the rates  $n$  depending on the applied voltage  $V$ . Afterwards in 2.1.2 we measured the rate of the underground  $u$  also depending on the voltage  $V$ . The error on a rate is give by

$$\sigma(u) = \frac{\sqrt{U}}{\Delta t} = \sqrt{\frac{u}{\Delta t}}. \quad (28)$$

The same applies for  $n$ .

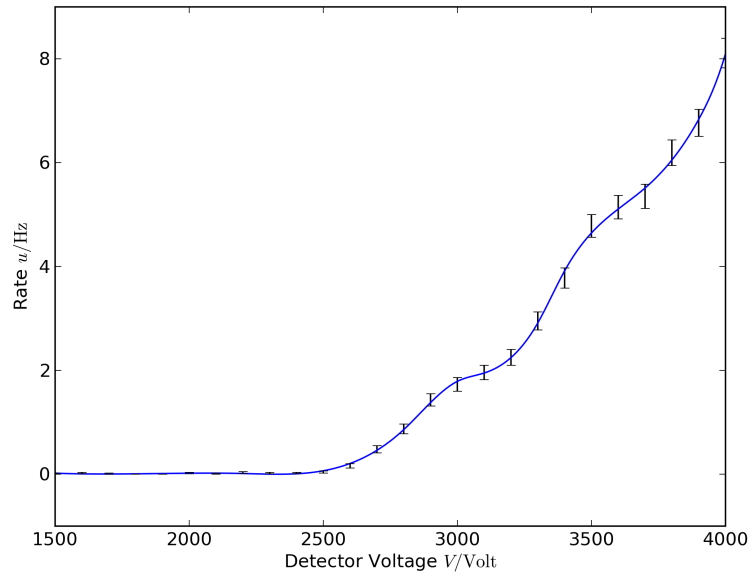


Figure 2: Underground measurement for all voltages. The points are fitted with a spline.

The measured rates are not the real rates of our sample, because we also measured some radiation from the environment. The additional radiation is called

underground. To subtract the underground from our signal – to gain the real rate  $n_0$  – we have to combine both error  $\sigma(n)$  and  $\sigma(u)$  as follows

$$\begin{aligned} n_0 &= n - u \\ \sigma(n_0) &= \sqrt{\sigma^2(n) + \sigma^2(u)} \end{aligned} \tag{29}$$

The real rate  $n_0$  is shown in figure 3 as a function of the voltage. At roughly 2000V one can see the  $\alpha$ -plateau and above 3000V one can see the  $\beta$ -plateau.

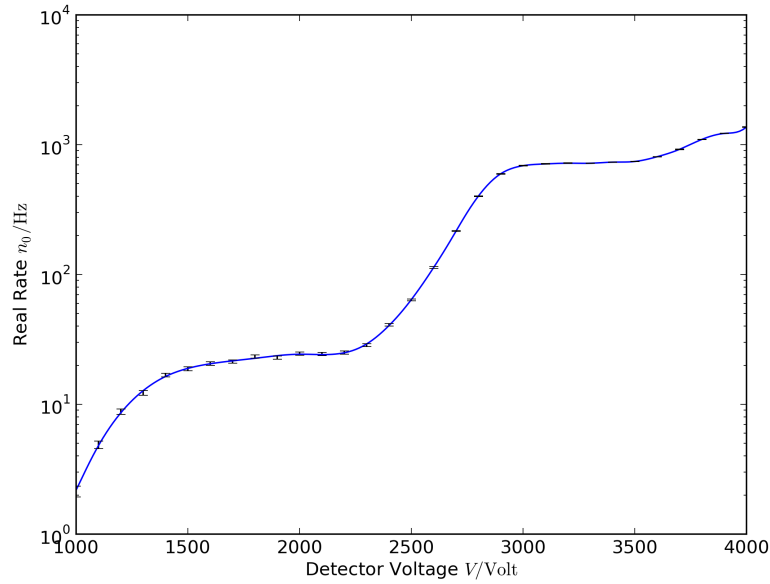


Figure 3: Detector characteristics without underground using a uranium sample. The points are fitted with a spline.

## 3.2 Potassium

In 2.2.1 we did a more detailed measurement of the  $\beta$ -plateau with KCl as source. To gain the real rate  $n_0$  we subtracted the underground measured in 2.1.2 again using (29). The result is shown in figure 4. From this data we have selected the voltage  $V_\beta = 3.5\text{kV}$  for all other measurements with the KCl sample

In order to account to the requirement that the error of the underground should be neglectable, we use (27). Using  $\tilde{u} = 4.78\text{Hz}$  from 2.1.2 and  $\tilde{n} = 7.42\text{Hz}$  from 2.2.2 we get  $t_U = 5426\text{s} \approx 1.5\text{h}$ . This sets the parameter for our underground measurement 2.2.3 which we ran over night, so the time  $t_U$  is much higher, which results in even lower relative errors.

The underground measurement counted  $U = 323952$  hits in  $t_U = 55800\text{s}$  which

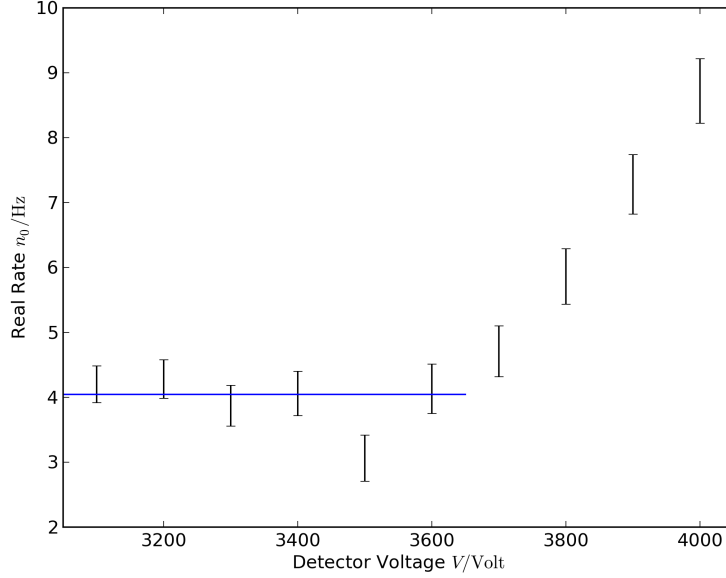


Figure 4:  $\beta$ -plateau without underground using a KCl sample. The plateau is fitted with the mean value.

results in a rate of

$$u = (5.806 \pm 0.010)\text{Hz} \quad (30)$$

where (28) was used again to calculate the error. This rate is even higher than the rate we have measured in the overall underground measurement 2.1.2. Because of this we suppose that the potential disturbed flow of methane did not have a negative effect on our measurement. A lack of methane would have resulted in a lower rate.

As stated in 2.2.2 we have measured an estimation of the rate  $\tilde{n} = 7.42\text{Hz}$ . From this we calculated with (21) that we have to measure the  $\beta$  decays for at least  $t_N = 336\text{s}$  to achieve the requirement of a relative error below 2%. This sets the parameters for the measurement 2.3.

To calculate the real decay rates of the potassium sample, we have to subtract the underground from our main measurement 2.3 using (29). Here we have a problem, because there is only one decay rate of the sample which is higher than the underground  $u = (5.806 \pm 0.010)\text{Hz}$ . We think our long measurement 2.2.3 is correct. When we compare this with the over all underground measurement 2.1.2 which gives  $u = 4.8 \pm 0.2$  we suppose that the underground might not be constant in time. There are be may sources of radiation, which could inflict our measurement, such as solar flares, other experiments in the building or radio active residuals from other groups. This could explain the low rates of the potassium sample which we have measured.

There is an other way, to calculate the real decay rates (without underground)

Sample Mass $m/\text{g}$	Rate $u/\text{Hz}$	Expected $\sigma_e(u)/\text{Hz}$	Chosen $\sigma_e(u)/\text{Hz}$
0.8227	4.709	0.12	0.3
0.5245	3.962	0.11	0.3
0.1294	1.594	0.07	0.3
1.8128	5.944	0.13	0.4
1.8128	5.156	0.12	0.3

Table 1: Results of the measurement

from the data we have measured. The real rate  $n_0(m)$  as a function of the sample mass  $m$  is expressed by

$$n_0(m) = n(m) - u = a(1 - e^{-bm}) \quad (31)$$

where  $a$  and  $b$  are some fit parameters and  $u$  the unknown underground rate. We can solve this for  $n(m)$  – which we have actually measured – and obtain

$$n(m) = a(1 - e^{-bm}) + u \quad (32)$$

where  $a$ ,  $b$  and  $u$  are now fit parameters. That's what we are going to do, but first we have to deal with the masses.

The rates were measured as a function of the sample mass plus the mass of the dish  $m_{\text{both}} = m_{\text{dish}} + m$ . So we have to subtract the mass of the dish  $m_{\text{dish}}$ . The errors on these masses are neglectable compared to the errors from the Poisson distributed counts.

The masses of our sample  $m$  and the corresponding rates  $n$  are shown in table 1. The table also shows the standard deviations which we expect due to the Poisson distribution, see (28). The last two measurements were done with the exact same sample, but the rates differ much more than the expected standard deviation. The error seems to be greater, because there are many influences. Therefore we increase the standard deviation by a factor 3, to account the greater errors. These errors are shown in table 1.

Figure 5 shows the measured values and the fitted function. The fitted parameters are  $a = (5.3 \pm 0.6)\text{Hz}$ ,  $b = (2.3 \pm 0.6)1/\text{g}$ ,  $u = (0.2 \pm 0.7)\text{Hz}$ . The covariance matrix of the fitted parameters is

$$C_p = \begin{pmatrix} \sigma^2(a) & \text{cov}(a, b) & \text{cov}(a, u) \\ \text{cov}(b, a) & \sigma^2(b) & \text{cov}(b, u) \\ \text{cov}(u, a) & \text{cov}(u, b) & \sigma^2(u) \end{pmatrix} = \begin{pmatrix} 0.398 & 0.167 & -0.373 \\ 0.167 & 0.418 & -0.324 \\ -0.373 & -0.324 & 0.455 \end{pmatrix} \quad (33)$$

Using these values in (17 we get

$$T_{\frac{1}{2}} = \frac{\ln 2}{2 \cdot 1.12} \frac{N_A h_{\text{rel}}}{m_{\text{mol, KCl}}} \frac{f_b}{ab} = \frac{c}{ab} = (1.0 \pm 0.3) \cdot 10^9 \text{a} \quad (34)$$

where  $c = 3.80489 \cdot 10^{17} \cdot 1/\text{g}$ . The error was calculated using the Gaussian error

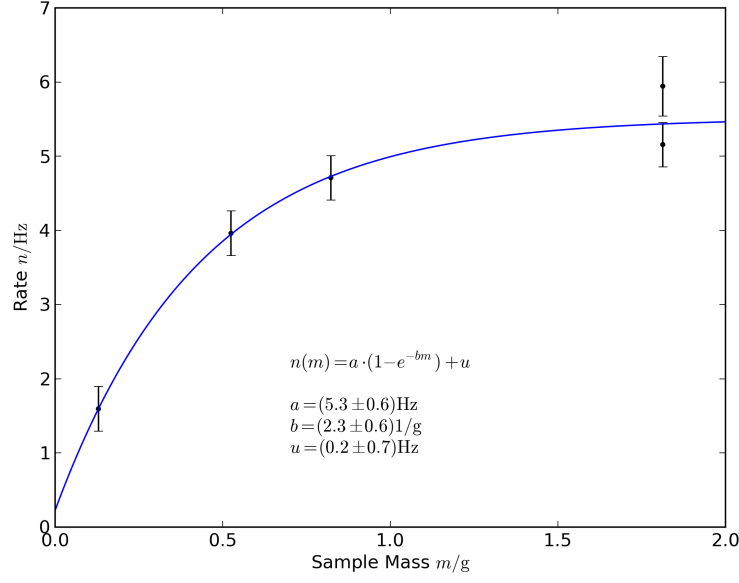


Figure 5: Measured  $\beta$ -decay rate (with underground) as a function of the sample mass. The data points are fitted with (32).

propagation  $C_{T_{1/2}} = GC_p G^t = \sigma^2(T_{1/2})$  where the  $G$  is the derivative vector

$$G = \left( \frac{\partial T_{1/2}}{\partial a}, \frac{\partial T_{1/2}}{\partial b}, \frac{\partial T_{1/2}}{\partial u} \right) = \left( \frac{c}{ba^2}, \frac{c}{ab^2}, 0 \right) \quad (35)$$

$$= (5.88929 \cdot 10^{15}, 1.35710 \cdot 10^{16}, 0)$$

Our result is in agreement within the standard deviations with the value from the literature  $n_0 = 1.28 \cdot 10^9 \text{ a}$ . We did not reach the required relative standard deviations, but this is due to the inconsistent measurements (underground greater than signal).

### 3.3 Samarium

In 2.4.1 we did a more precise measure for the  $\alpha$ -plateau. On this base, after subtracting the underground, we decided to use the Voltage  $U_\alpha = 2000 \text{ kV}$  for the following measurements. Shown in Figure 6. Above 2500V the underground is greater than our signal, as shown in the graphics. This is a sign again that the underground changed in time. With (21) we chose the  $t_N = 4630 \text{ s} \approx 80 \text{ min}$  as the optimal time. The underground should have a neglectable error so we found with (27) and our estimation of the underground in 2.1.2  $u(2 \text{ kV}) = 0,02 \text{ Hz}$  and the rate in 2.4.1  $n = 0,540 \text{ Hz}$

$$t_U > 62500 \cdot \frac{u}{n^2} = 62500 \cdot \frac{0,02}{0,54^2} = 4286,7 \text{ s} \approx 72 \text{ min} \quad (36)$$

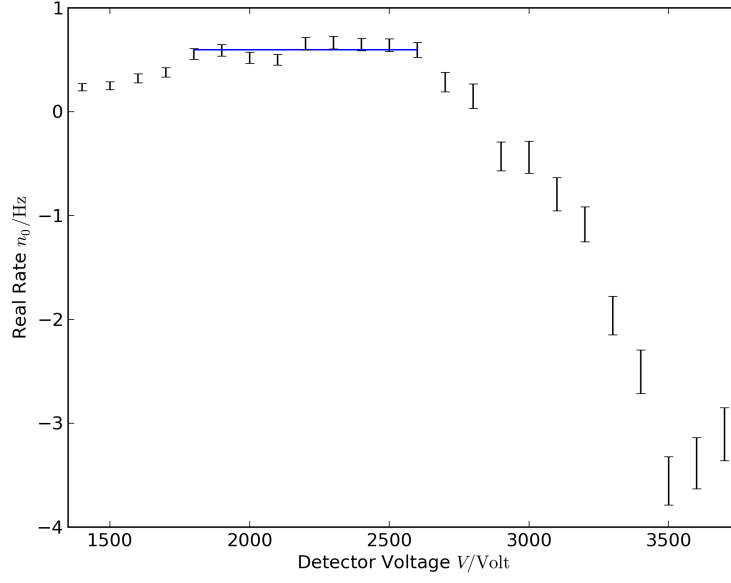


Figure 6:  $\alpha$ -plateau without underground. The plateau is fitted with its mean value.

We did the measurement for the underground during the night for 55800s = 15.5h so the error is small enough.

When we substitute the underground  $u_\alpha = 0.07\text{Hz}$  from measurement 2.5.1 and average the diameter we get the new tabular and figure 7.

radius $r = d/2\text{mm}$	Rate $n \cdot s$	error $s_n = \frac{\sqrt{N}}{t_N}$
14,435	0,469	0,010
8,392	0,133	0,007
4,975	0,010	0,004

Error  $s_r = 0,04\text{mm}^5$

With equation (12) we find for  $T_{\frac{1}{2}}$  the three values

$$T_{\frac{1}{2}} = 1,72529 \cdot 10^{11}; \quad 2,05627 \cdot 10^{11}; \quad 9,61139 \cdot 10^{11}$$

Gauss gives us the equation for the error which is

$$s_{T_{\frac{1}{2}}} = 3,89811 \cdot 10^{17} \cdot \pi \sqrt{\left(2 \frac{r}{n} \sigma\right)^2 + \left(-\frac{r^2}{n^2} \sigma_n\right)} \quad (37)$$

so we have for  $T_{\frac{1}{2}}$

$$(1,72529 \pm 0.04) \cdot 10^{11} \text{a}$$

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$$^5 s_r = \frac{r}{2 \cdot \sqrt{5}} \cdot s_d$$

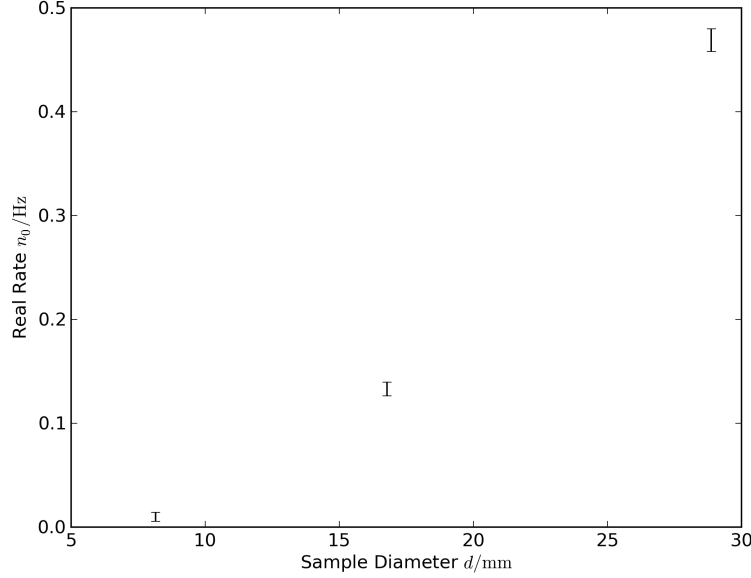


Figure 7:  $\alpha$ -Samarium without underground.

$$(2,05627 \pm 0.11) \cdot 10^{11} \text{a}$$

$$(9,61139 \pm 4) \cdot 10^{11} \text{a}$$

We use the weighted average and got the final value  $T_{\frac{1}{2}} = (1,765 \pm 0,04) \cdot 10^{11} \text{a}$ . The given value from the manual is  $T_{\frac{1}{2}} = 1,06 \cdot 10^{11} \text{a}$ . Because all our values are too small it is presumable to look for systematic errors. Probably our measurement of the underground is too high. This is likely because the underground for high voltages had been higher than the rates.

The underground is not stable all the time and it is thinkable that during the night of our measurement the underground was different. To verify this we would need to measure the underground again. Same phenomena we observed in the potassium experiment.

### 3.4 Conclusion

Although we had serious problems with our underground we came to a solution for potassium within the range of the errors. The underground we found in the fit is much smaller than the measured underground. The solution for potassium is:

$$T_{\frac{1}{2},Pot} = (1 \pm 0,3) \cdot 10^9 \text{a}$$

The result for samarium is not within the errors to the value given in the manual. Also here we had problems with the underground which affected the values for the



small diameter most. Our value for  $T_{\frac{1}{2}}$  of samarium is:

$$T_{\frac{1}{2},Sam} = (1,765 \pm 0,04) \cdot 10^{11} \text{a}$$