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THE DIFFRACTION OF LIGHT BY HIGH FREQUENCY SOUND WAVES: PART I.

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AND

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1. *Introduction.*

As is well known, Langevin showed that high frequency sound-waves of great intensity can be generated in fluids by the use of piezo-electric oscillators of quartz. Recently, Debye and Sears[1] in America and Lucas and Biquard[2] in France have described very beautiful experiments illustrating the diffraction of light by such high-frequency sound-waves in a liquid. Amongst the experimenters in this new field, may be specially mentioned R. Bär[3] of Zürich who has carried out a thorough investigation and has published some beautiful photographs of the effect. The arrangement may be described briefly as follows. A plane beam of monochromatic light emerging from a distant slit and a collimating lens is incident normally on a cell of rectangular cross-section and after passing through the medium emerges from the oppsite side. Under these conditions, the incident beam will be undeviated if the medium be homogeneous and isotropic. If, however, the medium be traversed by high-frequency sound-waves generated by introducing a quartz oscillator at the top of the cell, the medium becomes stratified into parallel layers of varying refractive index. Considering the case in which the incident beam is parallel to the plane of the sound-waves, the emerging light from the medium will now consist of various beams travelling in different directions. If the inclination of a beam with the incident light be denoted by θ, it has been found experimentally that the formula

$$\sin\theta = \pm \frac{n\lambda}{\lambda^*}, \quad n \text{ (an integer)} \geqslant 0 \qquad \ldots \qquad \ldots \qquad (1)$$

is in satisfactory agreement with the observed results, where λ and λ^* are the wave-lengths of the incident light and the sound wave in the medium

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[1] P. Debye and F. W. Sears, *Proc. Nat. Acad. Sci.* (Washington), 1932, 18, 409.

[2] R. Lucas and P. Biquard, *Jour. de Phys. et Rad.*, 1932, 3, 464.

[3] R. Bär, *Helv. Phys. Acta*, 1933, 6, 570.

respectively. With sound waves of sufficient intensity, numerous orders of these diffraction spectra have been obtained; a wandering of the intensity amongst these orders has also been noticed by Bär[3] when the experimental conditions are varied.

Various theories of the phenomena have been put forward by Debye and Sears,[1] by Brillouin,[4] and by Lucas and Biquard.[2] The former have not presented quantitative results and it is hard to understand from their theory as to why there should be so many orders and why the intensity should wander between the various orders under varying experimental conditions. In Brillouin's theory, the phenomenon is attributed to the reflection of light from striations of the medium caused by the sound waves. We know, however, from the work of Rayleigh that the reflection of light by a medium of varying refractive index is negligible if the variation is gradual compared with the wave-length of light. Under extreme conditions, we might perhaps obtain the Brillouin phenomenon, but the components of reflection should be very weak in intensity compared to the transmitted ones. As one can see later on in this paper, the whole phenomenon including the positions of the diffracted beams and their intensities can be explained by a simple consideration of the transmission of the light beam in the medium. Lucas and Biquard attribute the phenomenon to an effect of mirage of light waves in the medium. In what way the relation (1) enters in their theory is not clear. The wandering of the intensities of the various components observed by Bär has not found explanation in any of the above theories.

We propose in this paper a theory of the phenomenon on the simple consideration of the regular transmission of light in the medium and the phase changes accompanying it. The treatment is limited to the case of normal incidence. The formula (1) has been established in our theory. Also, a formula for the intensities of the various components has been derived. It is found that the above results are in conformity with the experimental results of Bär.[3]

2. *Diffraction of light from a corrugated wave-front.*

The following theory bears a very close analogy to the theory of the diffraction of a plane wave (optical or acoustical) incident normally on a periodically corrugated surface, developed by the late Lord Rayleigh.[5] He showed therein that a diffraction phenomenon would ensue in which the positions of the various components are given by a formula similar to (1)

[4] L. Brillouin, "La Diffraction de la Lumiere par des Ultra-sons", *Act. Sci. et Ind.*, 1933, 59.

[5] Lord Rayleigh, *Theory of Sound* (Vol. 2), page 89.

and their relative intensities are given by a formula similar to the one we have found.

Consider a beam of light with a plane wave-front emerging from a rectangular slit and falling normally on a plane face of a medium with a rectangular cross-section and emerging from the opposite face parallel to the former. If the medium has the same refractive index at all its points, the incident beam will emerge from the opposite face with its direction unchanged. Suppose we now create layers of varying refractive index in the medium, say by suitably placing a quartz oscillator in the fluid. If the distance between the two faces be small, the incident light could be regarded as arriving at the opposite face with variations in the phase at its different parts corresponding to the refractive index at different parts of the medium. The change in the phase of the emerging light at any of its parts could be simply calculated from the optical lengths found by multiplying the distance between the faces and the refractive index of the medium in that region. This step is justified for $\int \mu(x, y, z)ds$ taken over the actual path is minimum, *i.e.*, it differs from the one taken over a slightly varied hypothetical path by a differential of the second order. So, the incident wave-front becomes a periodic corrugated wave-front when it traverses a medium which has a periodic variation in its refractive index. The origin of the axes of reference is chosen at the centre of the incident beam projected on the emerging face, the boundaries of the incident beam being assumed to be parallel to the boundaries of the face. The X-axis is perpendicular to the sound-waves and the Z-axis is along the direction of the incident beam of light. If the incident wave is given by

$$Ae^{2\pi i\nu t}$$

it will be

$$Ae^{2\pi i\nu\{t - L\mu(x)/c\}}$$

when it arrives at the other face where L is the distance between the two faces and $\mu(x)$ the refractive index of the medium at a height x from the origin. It is assumed that the radii of curvature of the corrugated wave-front are large compared with the distance between the two faces of the cell. If μ_0 be the refractive index of the whole medium in its undisturbed state, we can write $\mu(x)$ as given by the equation

$$\mu(x) = \mu_0 - \mu \sin \frac{2\pi x}{\lambda^*}$$

ignoring its time variation, μ being the *maximum variation* of the refractive index from μ_0.

The amplitude due to the corrugated wave at a point on a distant screen parallel to the face of the medium from which light is emerging whose join with the origin has its x-direction-cosine l, depends on the evaluation of the diffraction integral

$$\int_{-p/2}^{p/2} e^{2\pi i\{lx+\mu L \sin(2\pi x/\lambda^*)\}/\lambda}\, dx$$

?

where p is the length of the beam along the X-axis. The real and the imaginary parts of the integral are

$$\int_{-p/2}^{p/2} \{\cos ulx \cos(v \sin bx) - \sin ulx \sin(v \sin bx)\} dx$$

and

$$\int_{-p/2}^{p/2} \{\sin ulx \cos(v \sin bx) + \cos ulx \sin(v \sin bx)\} dx$$

where $u = 2\pi/\lambda$, $b = 2\pi/\lambda^*$ and $v = u\mu L = 2\pi\mu L/\lambda$.

We need the well-known expansions

$$\cos(v \sin bx) = 2\sum_{r=0}^{\infty}{}' J_{2r} \cos 2rbx$$

$$\sin(v \sin bx) = 2\sum_{0}^{\infty} J_{2r+1} \sin \overline{2r+1}\, bx$$

to evaluate the integrals, where $J_n[=J_n(v)]$ is the Bessel function of the nth order and a dash over the summation sign indicates that the co-efficient of J_0 is half that of the others. The real part of the integral is then

$$2\sum_{0}^{\infty}{}' J_{2r} \int_{-p/2}^{p/2} \cos ulx \cos 2rbx\, dx - 2\sum_{0}^{\infty} J_{2r+1} \int_{-p/2}^{p/2} \sin ulx \sin \overline{2r+1}\, bx\, dx$$

or

$$\sum_{0}^{\infty}{}' J_{2r} \int_{-p/2}^{p/2} \{\cos(ul+2rb)x + \cos(ul-2rb)x\} dx$$

$$+\sum_{0}^{\infty} J_{2r+1} \int_{-p/2}^{p/2} \{\cos(ul+\overline{2r+1}\, b)x - \cos(ul-\overline{2r+1}\, b)x\} dx$$

Integrating the above, we obtain

$$p\sum_{0}^{\infty}{}' J_{2r} \left\{ \frac{\sin(ul+2rb)p/2}{(ul+2rb)p/2} + \frac{\sin(ul-2rb)p/2}{(ul-2rb)p/2} \right\}$$

$$+p\sum_{0}^{\infty} J_{2r+1} \left\{ \frac{\sin(ul+\overline{2r+1}\, b)p/2}{(ul+\overline{2r+1}\, b)p/2} - \frac{\sin(ul-\overline{2r+1}\, b)p/2}{(ul-\overline{2r+1}\, b)p/2} \right\} \quad .. \quad (2)$$

The integral corresponding to the imaginary part of the diffraction integral is zero. One can see that the magnitude of each individual term of (2) attains its highest maximum (the other maxima being negligibly small compared to the highest) when its denominator vanishes. Also, it can be seen that when any one of the terms is maximum, all the others have negligible values as the numerator of each cannot exceed unity and the denominator is some integral non-vanishing multiple of b which is sufficiently large. So the maxima of the magnitude of (2) correspond to the maxima of the magnitudes of the individual terms. Hence the maxima occur when

$$ul \pm nb = 0 \quad n(\text{an integer}) \geqslant 0 \quad \ldots \quad (3)$$

where n is any even or odd positive integer. The equation (3) gives the directions in which the magnitude of the amplitude is maximum which correspond also to the maximum of the intensity. If θ denotes the angle between such a direction in the XZ-plane along which the intensity is maximum and the direction of the incident light, (3) can be written as

$$\sin\theta = \pm \frac{n\lambda}{\lambda^*} \quad \ldots \quad (4)$$

remembering that $u = 2\pi/\lambda$ and $b = 2\pi/\lambda^*$. This formula is identical with the formula (1) given in the first section. The magnitudes of the various components in the directions given by (4) can be calculated if we know,

$$J_n \text{ or } J_n(v) \text{ or } J_n(2\pi\mu L/\lambda).$$

Thus the relative intensity of the mth component to the nth component is given by

$$\frac{J_m^2(v)}{J_n^2(v)} \quad \text{where } v = 2\pi\mu L/\lambda.$$

In the undisturbed state of the medium there is no variation of the refractive index, *i.e.*, $\mu = 0$. In this case all the components vanish except the zero component for

$$J_m(0) = 0 \text{ for all } m \neq 0 \text{ and } J_0(0) = 1.$$

In the disturbed state, the relative intensities depend on the quantity v or $2\pi\mu L/\lambda$ where λ is the wave-length of the incident light, μ is the maximum variation of the refractive index and L is the path traversed by light in the medium. We have calculated the relative intensities of the various components which are observable for values of v lying between 0 and 8 at different steps (Fig. 1).

Fig. 1 shows that the number of observable components increases as the value of v increases. When $v = 0$, we have only the central component. As v increases from 0, the first orders begin to appear. As v increases still more, the intensity of the central component decreases steadily and the first orders increase steadily in their intensity till they attain maximum intensity when the zero order will nearly vanish and the second orders will have just appeared. As v increases still more, the zero order is reborn and increases

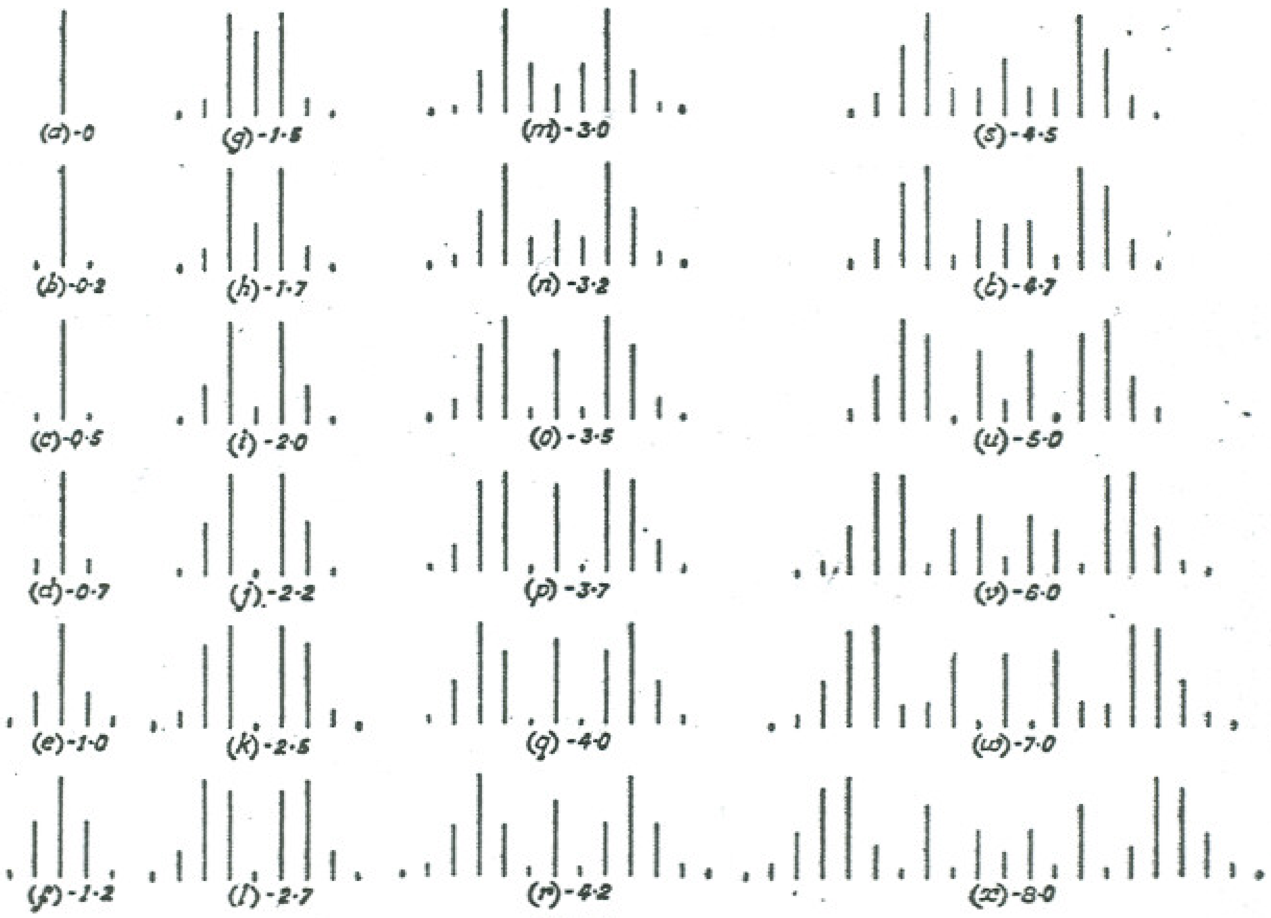

FIG. 1.

Relative intensities of the various components in the diffraction spectra.
(For tables, see Watson's *Bessel Functions and Report of the British Association*, 1915.)

in its intensity, the first orders fall in their intensity giving up their former exalted places to the second orders, while the third orders will have just appeared and so on.

Our theory shows that the intensity relations of the various components depend on the quantity v or $2\pi\mu L/\lambda$. Thus an *increase* of μ (*i.e.*, an increase of the supersonic intensity which creates a greater variation in the refractive index of the medium) or an *increase* of L or a *decrease* of λ should give similar effects *except* in the last case where the directions of the various beams will be altered in accordance with (4).

3. *Interpretation of Bär's Experimental Results.*

(*a*) *Dependence of the effect on the supersonic intensity.*—Bär has observed that only the zero order (strong) and the first orders (faint) are present when the supersonic intensity is not too great. He found that more orders appear as the supersonic intensity is increased but that the intensity of the zero order decreases while the first orders gain in their intensity. Increasing the supersonic intensity more, he found that the first order would become very faint while the second and third orders will have about the same intensity. The figures 1*a* of his paper may very well be compared with our figures 1(*c*), 1(*h*) and 1(*k*). Thus, we are able to explain the appearance of more and more components and the wandering of the intensity amongst them as the supersonic intensity is increased, in a satisfactory manner.

(*b*) *Dependence of the effect on the wave-length of the incident light.*—We have already pointed out that the effects due to an increase of μ caused by an increase of supersonic intensity are similar due to those with a decrease of λ except for the fact that the positions of the components of the emerging light alter in accordance with (4). Bär has obtained two patterns of the phenomenon by using light with wave-lengths 4750Å and 3650Å. He obtained, using the former seven components and using the latter eleven components in all. He also observed great variations in the intensities of the components. Not only is the increase in the number of components an immediate consequence of our theory, but we can also find the pattern with 3650Å if we assume the pattern with 4750Å. The pattern with the latter in Bär's paper shows a strong resemblance to our figure 1(*p*) for which $2\pi\mu L/\lambda$ is 3·7. Thus we can calculate $2\pi\mu L/\lambda$ when λ is 3650Å. It comes to about 4·8. Actually our figure for which $2\pi\mu L/\lambda$ is 4·8 closely corresponds to Bär's pattern with 3650Å.

(*c*) *Dependence of the effect on the length of the medium which the light traverses.*—It is clear from our theory that an increase of L corresponds to an increase of v and that the effects due to this variation would be similar to those with an increase of the supersonic intensity. But the basis of our theory does not actually cover any large change in L. However, we should find more components and the wandering of the intensity amongst the various components.

4. *Summary.*

(*a*) A theory of the phenomenon of the diffraction of light by sound-waves of high frequency in a medium, discovered by Debye and Sears and Lucas and Biquard, is developed.

(*b*) The formula

$$\sin\theta = \pm\frac{n\lambda}{\lambda^*} \quad n \text{ (an integer)} \geqslant 0$$

which gives the directions of the diffracted beams from the direction of the incident beam and where λ and λ^* are the wave-lengths of the incident light and the sound wave in the medium, is established. It has been found that the relative intensity of the mth component to the nth component is given by

$$J_m^2(2\pi\mu L/\lambda) \;/\; J_n^2(2\pi\mu L/\lambda)$$

where the functions are the Bessel functions of the mth order and the nth order, μ is the maximum variation of the refractive index and L is the path traversed by light. These theoretical results interpret the experimental results of Bär in a very gratifying manner.

THE DIFFRACTION OF LIGHT BY SOUND WAVES OF HIGH FREQUENCY : PART II.

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1. *Introduction.*

IN the first[1] of this series of papers, we were concerned with the explanation of the diffraction effects observed when a beam of light traverses a medium filled by sound waves of high frequency. For simplicity, we confined our attention to the case in which a plane beam of light is normally incident on a cell of the medium with rectangular cross-section and travels in a direction strictly perpendicular to the direction along which the sound waves are propagated in the medium. By taking into account the corrugated form of the wave-front on emergence from the cell, the resulting diffraction-effects were evaluated. This treatment will be extended in the present paper to the case in which the light waves travel in a direction inclined at a definite angle to the direction of the propagation of the sound waves. The extension is simple, but it succeeds in a remarkable way in explaining the very striking observations of Debye and Sears[2] who found a characteristic variation of the intensity of the higher orders of the diffraction spectrum when the angle between the incident beam of light and the plane of the sound waves was gradually altered.

We shall first set out a simple geometrical argument by which the changes in the diffraction phenomenon which occur with increasing obliquity can be inferred from the results already given for the case of the normal incidence. An analytical treatment then follows which confirms the results obtained geometrically.

2. *Elementary Geometrical Treatment.*

The following diagrams illustrate the manner in which the amplitude of the corrugation in the emerging wave-front alters as the incidence of light on the planes of the sound waves is gradually changed. In the diagrams,

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[1] C. V. Raman and N. S. Nagendra Nath, *Proc. Ind. Acad. Sci.*, 1935, **2**, 406—412.

[2] P. Debye and F. W. Sears, *Proc. Nat. Acad. Sci.* (Washington), 1932, **18**, 409.

the planes of maximum and minimum density caused by the sound waves at any instant of time are indicated by thick and thin lines (*e.g.*, AB and CD) respectively. The paths of the light rays are represented by dotted lines in Figs. 1 (*b*), (*c*) and (*d*). As we are mainly interested in the calculation of the phase-changes which the incident wave undergoes before it emerges from the cell, the bending of the light rays within the medium may, in virtue of Fermat's well-known principle, be ignored without a sensible error, *provided* the total depth of the cell is not excessive.

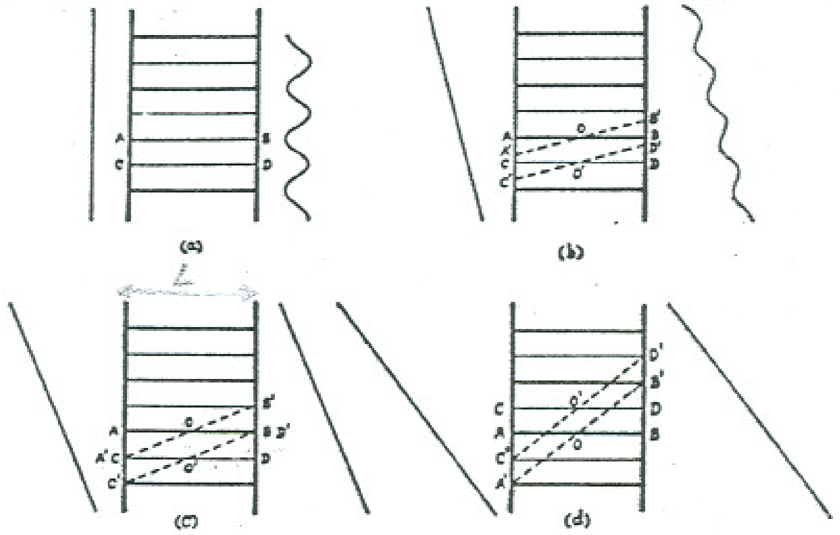

FIG. 1.

Considering the variation in the refractive index to be simply periodic, the neighbouring light-paths with maximum and minimum optical lengths AB and CD respectively, in the case of normal incidence, are shown in Fig. 1(*a*). The lines AB and CD are separated by $\lambda^*/2$ where λ^* is the wave-length of the sound waves. The difference between the maximum and the minimum optical lengths gives a measure of the corrugation of the wave-front on emergence. Considering now a case in which the light rays make an angle ϕ with the planes of the sound waves, we may denote the maximum and the minimum optical lengths by A′B′ and C′D′ respectively. These would be symmetrically situated with respect to AB and CD, and would tend to coincide with them as ϕ is decreased. The optical length of A′B′ is *less* than that of AB, for the refractive index at any point except at O is less than the constant maximum refractive index along AB, ϕ being small. On the other hand, the optical length of C′D′ is *greater* than that of CD, for the refractive index is minimum along CD. A simple consideration of the above shows that the difference between the optical lengths of A′B′ and C′D′ is less than that between those of AB and CD. As this difference gives twice the amplitude of the corrugation of the emerging wave-front, it follows, in the case shown in Fig. 1 (*b*), that the amplitude of the corrugation of the emerging wave-front is less than that in the case of Fig. 1 (*a*).

Fig. 1 (*c*) illustrates a case when the maximum optical length is just equal to the minimum optical length. This occurs when the direction of the incident beam is inclined to the planes of the sound-wave-fronts at an angle α_1 given by $\tan^{-1}\dfrac{\mathrm{B'B}}{\mathrm{OB}} = \tan^{-1}\dfrac{\lambda^*/2}{\mathrm{L}/2} = \tan^{-1}(\lambda^*/\mathrm{L})$. That the optical lengths of A'B' and C'D' in Fig. 1 (*c*) are equal follows by a very simple geometrical consideration. Thus, when light rays are incident on the sound waves at an angle $\tan^{-1}(\lambda^*/\mathrm{L})$, the amplitude of the corrugation of the emerging wave-front vanishes, *i.e.*, a plane incident beam of light remains so when it emerges from the medium. This result would also be true whenever $\alpha_n = \tan^{-1}(n\lambda^*/\mathrm{L})$, $n \neq 0$. The case when $n = 2$ is illustrated in Fig. 1 (*d*). In all these cases the diffraction effects disappear. As the corrugation vanishes when ϕ is α_{n+1} or α_n, there is an intermediate direction which makes an angle β_n with the sound waves giving the maximum corrugation if light travels along that direction. We can take $\beta_0(=0)$ to represent the case when the incident beam of light is parallel to the sound waves.

Thus, we have deduced that the corrugation of the emerging wave-front is maximum when the direction of light is parallel to the sound waves $[\beta_0(=0)]$, decreases steadily to zero as the inclination ϕ between the incident light and the sound waves is increased to α_1, increases to a smaller maximum as ϕ increases from α_1 to β_1, decreases to zero as ϕ increases from β_1 to α_2, increases to a still smaller maximum as ϕ increases from α_2 to β_2, and so on. (I)

As the variation of the refractive index is simply periodic along the direction normal to the sound-wave-fronts, it follows that the optical length of the light path is also simply periodic along the same direction when the incident light rays are parallel to the sound waves. This means that the corrugation of the emerging wave-front is also simply periodic. When the incident light rays are incident at an angle ϕ to the sound waves, the optical length of the light path would be simply periodic in a direction perpendicular to the light rays. This means that the emerging wave-front would be tilted by the angle ϕ about the line of the propagation of the sound waves and that its corrugation would be simply periodic along the same line.

We have shown in our previous paper that a simply periodic corrugated wave is equivalent to a number of waves travelling in directions which make angles, denoted by θ, with the direction of the incident beam given by

$$\sin\theta = \pm \frac{n\lambda}{\lambda^*} \qquad n \text{ (an integer)} \geqslant 0 \qquad \ldots \qquad \ldots \quad (1)$$

where λ is the wave-length of the incident light. In view of the results obtained in the previous paragraph, the formula (1) would also hold good when the incident light is a small angle with the sound waves.

The relative intensities of the various diffraction spectra which depend on the amplitude of the corrugation should obey a law similar to the one in the case of the normal incidence.

Thus, we find that the results in the case of an oblique incidence would be similar to those of the normal incidence with the amplitude of the corrugation modified. Hence, we deduce, in virtue of the statement I, the following results, assuming the results, in the case of normal incidence, obtained in our earlier paper.

The diffraction spectrum will be most prominent when $\phi = 0$. The intensity of the various components wander when ϕ is increased. When ϕ increases from zero to α_1, the number of the observable orders in practice decreases and when $\phi = \alpha_1$ all the components disappear except the central one which will attain maximum intensity. This does not mean that the intensities of all the orders except the central one decrease to zero monotonically as ϕ varies from zero to α_1, but some of them may attain maxima and minima in their intensities before they attain the zero intensity when $\phi = \alpha_1$. This is obvious in virtue of the property that the intensity of the nth component depends on the square of the Bessel function J_n. As ϕ increases from α_1 to β_1 the intensity of the central component falls and the other orders are reborn one by one. As ϕ increases from β_1 to α_2, the number of observable orders decreases and when $\phi = \alpha_2$ all the orders vanish except the central one which will attain the maximum intensity and so on.

3. *Analytical Treatment.*

In the following, we employ the same notation as in our earlier paper. The optical length of a path in the medium parallel to the direction of the incident light making an angle ϕ with the sound waves may be easily calculated. It is

$$\int_0^{L\sec\phi} \mu(s)\,ds$$

or

$$\mu_0 L \sec\phi - \mu \int_0^{L\sec\phi} \sin b(x - s\sin\phi)\,ds.$$

Integrating we obtain the integral as

$$\mu_0 L \sec\phi - \frac{\mu}{b\sin\phi}\{\sin(bL\tan\phi)\sin bx + [\cos(bL\tan\phi)-1]\cos bx\}.$$

The last term can be written as

$$-A\sin bx + B\cos bx$$

where

$$A = \frac{\mu}{b\sin\phi}\sin(bL\tan\phi)$$

$$B = -\frac{\mu}{b\sin\phi}[\cos(bL\tan\phi)-1].$$

Thus the optical length of the path can be written as

$$\mu_0 L\sec\phi - \sqrt{(A^2+B^2)}\sin b\left(x - \tan^{-1}\frac{B}{A}\right).$$

Ignoring the constant phase factor, the optical length is

$$\mu_0 L\sec\phi - \frac{2\mu}{b\sin\phi}\sin\left(\frac{bL\tan\phi}{2}\right)\sin bx.$$

If the incident light is

$$\exp\left[2\pi i\nu\left(t - \frac{x\sin\phi}{c}\right)\right]$$

when it arrives at the face of the cell, it will be

$$\exp\left[\frac{2\pi i}{\lambda}\left(ct - x\sin\phi - \int_0^{L\sec\phi}\mu(s)ds\right)\right]$$

when it arrives at the face from which it emerges.

The amplitude of the corrugated wave at a point on the screen whose join with the origin has its x-direction-cosine l, depends on the evaluation of the diffraction integral

$$\int_{-p/2}^{p/2}\exp\left[\frac{2\pi i}{\lambda}\left\{(l-\sin\phi)x + \frac{2\mu}{b\sin\phi}\sin\left(\frac{bL\tan\phi}{2}\right)\sin bx\right\}\right]dx.$$

The evaluation of the integral and the discussion of its behaviour with respect to l may be effected in the same way as in our earlier paper. Maxima of the intensity due to the corrugated wave occur in directions making angles, denoted by θ, with the direction of the incident beam when

$$\sin(\theta+\phi) - \sin\phi = \pm\frac{n\lambda}{\lambda^*} \qquad n \text{ (an integer)} \geqslant 0 \qquad \ldots \quad (1)$$

The relative intensity of the mth order to the nth order is given by

$$\frac{J_m^2(v)}{J_n^2(v)} \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \qquad \ldots \quad (2)$$

where

$$v = \frac{2\pi}{\lambda} \cdot \frac{2\mu}{b \sin\phi} \sin\left(\frac{bL\tan\phi}{2}\right)$$

$$= \frac{2\pi\mu L}{\lambda} \sec\phi \frac{\sin t}{t} \text{ where } t = \frac{bL\tan\phi}{2} = \frac{\pi L\tan\phi}{\lambda^*}.$$

The expression for the relative intensities in our earlier paper can be obtained from (2) by making $\phi \to 0$ when $v \to \frac{2\pi\mu L}{\lambda} = v_0$. So the expression for the relative intensities

$$J_m^2(v_0)/J_n^2(v_0) \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (3)$$

in the case of normal incidence will change to

$$J_m^2(v)/J_n^2(v)$$

where

$$v = v_0 \sec\phi \frac{\sin t}{t} \quad \ldots \quad \ldots \quad \ldots \quad \ldots \quad (4)$$

and

$$t = \frac{\pi L\tan\phi}{\lambda^*}.$$

Even if ϕ be small so that $\sin\phi \backsimeq \tan\phi \backsimeq \phi$, it is *not* justifiable to write $\sin t \backsimeq t$ *unless* $\pi L\phi/\lambda^*$ is also small to admit the approximation. As $\pi L/\lambda^*$ is sufficiently large we should expect great changes in the diffraction phenomenon even if ϕ be a fraction of a degree. v vanishes when

$$t = n\pi \qquad n \text{ (an integer)} > 0,$$

that is, when $L\tan\phi = n\lambda^*$,

or

$$\phi = \tan^{-1}\frac{n\lambda^*}{L}, \; n \text{ (an integer)} > 0,$$

confirming the same result obtained geometrically. Whenever v vanishes, it can be seen that the amplitude of the corrugation of the wave-front also vanishes. The statement I in Section 2 and the consequences with regard to the behaviour of the intensity among the various orders can all be confirmed by the expression (3).

In the numerical case when $L = 1$ cm., and $\lambda^* = 0{\cdot}01$ cm., the amplitude of the corrugation vanishes $\tan\alpha_1 = 0{\cdot}01$ or $\alpha_1 = 0° \, 34'$. This means that as ϕ varies from 0° to 0° 34′, the relative intensities of the various orders wander according to (2) till when $\phi = 0° \, 34'$, all the orders disappear except the central one which attains maximum intensity. This does not mean that the intensities of all the orders except the central one decrease monotonically to zero but they *may possess* several maxima and minima before they become zero. The intensity of the nth order depends on the behaviour of $J_n^2\left[v_0 \sec\phi \dfrac{\sin(\pi L \tan\phi/\lambda^*)}{(\pi L \tan\phi/\lambda^*)}\right]$ under the above numerical conditions as ϕ varies from 0° to 0° 34′. As ϕ just exceeds 0° 34′, all the orders are reborn one by one till a definite value of ϕ after which they again fall one by one and when $\phi = 1° 8'$, all the orders disappear except the central one.

The numerical example in the above paragraph shows the delicacy of the diffraction phenomenon. If the wave-length is quite small, the diffraction phenomenon will be present in the case of the strictly normal *incidence* as the relative intensity expression (3) does not depend on λ^* but will soon considerably change even for slight variations of ϕ as the relative intensity expression (4) depends on λ^*. One should be very careful in carrying out the intensity measurements in the case of normal incidence, for even an error of a few minutes of arc in the incidence will affect the intensities of the various orders.

4. *Comparison with the experimental results of Debye and Sears.*

Debye and Sears make the following statement in their paper: "Fixing the attention on one of the spectra *preferably of higher order*, one can observe that it attains its maximum intensity if the trough is turned through a small angle such that the primary rays are no longer parallel to the planes of the supersonic waves. Different settings are required to obtain highest intensities in different orders. If the trough is turned continuously in one direction, starting from a position which gave the highest intensity to one of the orders, the intensity decreases steadily, goes through zero, increases to a value much smaller than the first maximum, decreases to zero a second time and goes up and down again through a still smaller maximum." This statement very aptly describes the behaviour of the function

$$J_n^2\left[v_0 \sec\phi \frac{\sin(\pi L \tan\phi/\lambda^*)}{(\pi L \tan\phi/\lambda^*)}\right]$$

as ϕ alters under the conditions imposed in the above statement. The zeroes and the maxima of the intensity of the nth order, as a function of ϕ, correspond to the zeroes and the maxima of the above function.

5. *Summary.*

The theory of the diffraction of light by sound waves of high frequency developed in our earlier paper is extended to the case when the light beam is incident at an angle to the sound wave-fronts, both from a geometrical point of view and an analytical one. It is found that the maxima of intensity of the diffracted light occur in directions which make definite angles, denoted by θ, with the direction of the incident light given by

$$\sin(\theta+\phi) - \sin\phi = \pm \frac{n\lambda}{\lambda^*},\ n \text{ (an integer)} \geqslant 0$$

where λ and λ^* are the wave-lengths of the incident light and the sound waves in the medium. The relative intensity of the mth order to the nth order is given by

$$J_m^2\left(v_0 \sec\phi \frac{\sin t}{t}\right) \Big/ J_n^2\left(v_0 \sec\phi \frac{\sin t}{t}\right)$$

where $v_0 = \frac{2\pi\mu L}{\lambda}$, $t = \frac{\pi L \tan\phi}{\lambda^*}$, ϕ is the inclination of the incident beam of light to the sound waves, μ is the maximum variation of the refractive index in the medium when the sound waves are present and $L \sec\phi$ is the distance of the light path in the medium. These results explain the variations of the intensity among the various orders noticed by Debye and Sears for variations of ϕ in a very gratifying manner.

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