C9: Mining Association Rules:

Apriori and Its Related Issues

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Tentative Class Agenda

- Class 8 – (10/30) Introduction to data
- Class 9 – (11/6) Association (Project announced, HW#4)
- Class 10 – (11/13) More on data, OLAP , Intro. to Classification
- Class 11 – (11/20) More on Classification (HW#5)
- Class 12 – (11/27) Clustering and others; go over project abs.
- Class 13 –(12/4) R (HW#6)
- Class 14 – (12/11) Exam (in class, closed book)
- Class 15 – (12/18) Project presentation I
- Class 16 – (12/25) Project presentation II

Procedure of Data Mining

- Obtain and look over the data
- Decide your goal (usually a stretched and reachable one)
- Data cleaning/cleansing
- Choose data granularity, feature selection
- Apply mining methods
- Decide what to output and in what form
- Interpret your results (may have iterative refinements); convince your receiver/boss

Mining Capabilities

- Association
- Classification
- Clustering
- Sequential Pattern
- and more

Mining Association Rules

- Transaction data analysis: Mining association rules
  - Given: (1) a database of transactions
    (2) each tx has a list of items purchased
- Find all asso. rules: the presence of one set of items implies the presence of another set of items
  - people who purchased hammers also purchased nails

Two Parameters

- Confidence (how true)
  – the rule X&Y => Z has 90% conf. means 90% of customers who bought X and Y also bought Z
- Support (how useful is the rule)
  – useful rules should have some minimum tx support

Mining Association Rules in Transaction DBs

- Measurement of rule strength in a transaction DB.

$$A \rightarrow B \ \ [support,\ confidence]$$

$$support = Prob(A \cup B) = \frac{\#\_of\_trans\_that\_contain\_both\ A\ \ and\ \ B}{total\_\#\_of\_trans}$$

$$confidence = Prob(B|A) = \frac{\#\_of\_trans\_that\_contain\_both\ A\ \ and\ \ B}{\#\_of\_trans\_containing\ A}$$

- We are often interested in only strong associations, i.e.

  $support \ \geq \ min\_sup$ and $confidence \geq min\_conf.$

- Examples.

  milk $\rightarrow$ bread [5%, 60%].

  tire $\wedge$ auto_accessories $\rightarrow$ auto_services [2%, 80%].

Two Steps for Mining Asso.

- Determining "large itemsets"
  - the main factor for overall performance
- Generating rules

Two approaches for Large Itemset Counting

- Apriori-Based
  - R. Agrawal and R. Srikant
- FP-Tree-Based
  - J. Han and J. Pei, etc (SIGMOD 2000)

Methods for Mining Association Rules

- Apriori (Agrawal & Srikant'94).
- Itemset generation
  - derivation of large 1-itemsets L1: At the first iteration, scan all the transactions and count the number of occurrences for each item.
  - level-wise derivation: At the *k*-th iteration, the candidate set *Ck* are those whose every (*k* - 1)-item subset is in *Lk-1*. Scan DB and count the # of occurrences for each candidate itemset.
  - the cardinality of C2 is huge
  - the exe time for the first 2 iterations is the dominating factor to overall performance

Support=2 tx's (i.e., 50%)

Database D

TID	Items
100	A C D
200	B C E
300	A B C E
400	B E

Scan D →

C_1

Itemset	Sup.
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

L_1

Itemset	Sup.
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A B}
{A C}
{A E}
{B C}
{B E}
{C E}

Scan D →

C_2

Itemset	Sup.
{A B}	1
{A C}	2
{A E}	1
{B C}	2
{B E}	3
{C E}	2

L_2

Itemset	Sup.
{A C}	2
{B C}	2
{B E}	3
{C E}	2

C_3

Itemset
{B C E}

Scan D →

C_3

Itemset	Sup.
{B C E}	2

L_3

Itemset	Sup.
{B C E}	2

BE=>C conf:66%

Two Steps for Mining Asso. (cont'd)

- for each large itemset m do
  for each subset p of m do
  if (sup(m)/sup(m-p)>= minconf) then
  output the rule (m-p)=>p
  with conf= sup(m)/sup(m-p) and
  support=sup(m)
- m={a,c,d,e,f,g} 2000 tx's
  p={a,d} 5000 tx's
  {a,d}=>{c,e,f,g} conference: 40%, support: 2000 tx's

Properties of Apriori

- Downward closure for large (also called frequent) itemset generation
- The bottleneck is usually in C2
- Database scan is expensive
- The setting of "support" and "confidence"
- Using "top-k" itemsets instead of support
  - How to do itemset generation

Follow-ups of Apriori

- Data Stream mining
  - W.-G. Teng, M.-S. Chen and P. S. Yu, ``A Regression-Based Temporal Pattern Mining Scheme for Data Streams," *Proc. of the 29th Intern'l Conf. on Very Large Data Bases (VLDB-2003)*, September 9-12, 2003.
- Upper bound on the number of large itemsets
  - F. Geerts and B. Goethals and J. V. D. Bussche, "Tight upper bounds on the number of candidate patterns", TODS 2005)
- "closed large itemset"
- Spawned many works to improve its efficiency and also to explore its variations

Closed Itemsets and Maximal Itemsets

- An itemset X is called closed if there does not exist a larger itemset Y, s.t. Y contains X and s(Y)=s(X)
- A large itemset X is called maximal if there does not exist a large itemset Y, s.t., Y contains X

- Q1: if a large itemset X is closed, is X always maximal?
- Q2: if a large itemset X is maximal large itemset, is X always closed?

Redundant Rules

- For the same support and confidence, if we have a rule {a,d}=>{c,e,f,g}, do we have
  - {a,d}=>{c,e,f}
  - {a}=>{c,e,f,g}
  - {a,d,c}=>{e,f,g}
  - {a}=>{d,c,e,f,g}

Scan Reduction

- Use candidate sets to generate candidate sets
  e.g., Instead of Ci→ Li→Ci+1 (dbscan)→Li+1
  We use Ci→Ci+1'→Ci+2' (dbscan)→Li+1,Li+2..
- Save the runs of database scans
- May get back to use large itemsets to generate candidate sets if so necessary

Improvement for Aprior

- DHP
  – J. Park, M.-S. Chen, and P. Yu. "*An effective hash based algorithm for mining association rules*. " *Proceedings of ACM SIGMOD*, May 1995. A complete version in Using A Hash-Based Method with Transaction Trimming for Mining Association Rules," IEEE Trans. on Knowledge and Data Eng., vol. 9, no. 5, pp. 813-825, Sept./Oct. 1997.
- Hash table scheme
  – Eliminate infrequent candidate itemsets in the early phase
- Transaction items pruning
  – Eliminate infrequent items from the database

Candidate Itemsets Pruning

TID	Items
100	A,C,D
200	B,C,E
300	A,B,C,E
400	B,E

L_1=<A,B,C,E> Minimum Support Frequency=2

<AC>,<AD>,<CD>
<BC>,<BE>,<CE>
<AB>,<AC>,<AE>,<BC>,<BE>,<CE>
<BE>

$h(x,y)=((ord(x)*10+ord(y)) \bmod 7$

<CE>				<BE>		<AC>
<CE>		<BC>		<BE>		<CD>
<AD>	<AE>	<BC>		<BE>	<AB>	<AC>
3	1	2	0	3	1	3
0	*1*	*2*	*3*	*4*	*5*	*6*

H_2

$C_2=L_1*L_1$

Itemset
<AB>
<AC>
<AE>
<BC>
<BE>
<CE>

→

C_2'

Itemset
<AC>
<BC>
<BE>
<CE>

pruning <AB>, <AE>

Hash table building

Candidate pruning

Transaction Items Pruning (from L2 to L3)

$L_2 = <<AC>, <BC>, <BE>, <CE>>$

TID	Items
100	A,C,D
200	B,C,E
300	A,B,C,E
400	B,E

Before trimming

a[0]	a[1]	a[2]	a[3]
1	3	0	
2	3	2	
1	2	3	2
2	2		

Counter array

Trim A,C,D

Trim A

Trim B,E

TID	Items
100	
200	B,C,E
300	B,C,E
400	

After trimming

Trimming information collecting (A appears in L2 once, C in L2 3 times, D not in L2)

Transaction trimming

A misleading “strong” association rule

- 10000 transactions
  - 6000 of them included computer games.
  - 7500 of them included video.
  - 4000 of them included computer games and video.
- Minimum support: 30%, minimum confidence: 60%

$$buys(\text{computer games}) \Rightarrow buys(\text{videos})$$
$$[support = 40\%, confidence = 66\%]$$

- However, P({video})=0.75

From Association Analysis to Correlation Analysis

- The support and confidence measures are insufficient at filtering out uninteresting association rules.

$$A \Rightarrow B[support, confidence, correlation]$$

Lift

- The **lift** between the occurrence of A and B can be measured by computing

  The probability of a transaction contains the *union* of sets A and B.

  It doesn't mean P(A or B).

$$lift(A,B) = \frac{P(A \cup B)}{P(A)P(B)} = \frac{P(B|A)}{P(B)} = \frac{conf(A \Rightarrow B)}{\sup(B)}$$

  < 1, negatively correlated

  > 1, positively correlated

  = 1, no correlation (A and B are independent)

- **Lift** assesses the degree to which the occurrence of one "lifts" the occurrence of the other.

Interestingness Measure: Correlations (Lift)

- *play basketball* $\Rightarrow$ *eat cereal* [40%, 66.7%] is misleading
  - The overall % of students eating cereal is 75% > 66.7%.
- *play basketball* $\Rightarrow$ *not eat cereal* [20%, 33.3%] is more accurate, although with lower support and confidence
- Measure of dependent/correlated events: lift

$$lift = \frac{P(A \cup B)}{P(A)P(B)}$$

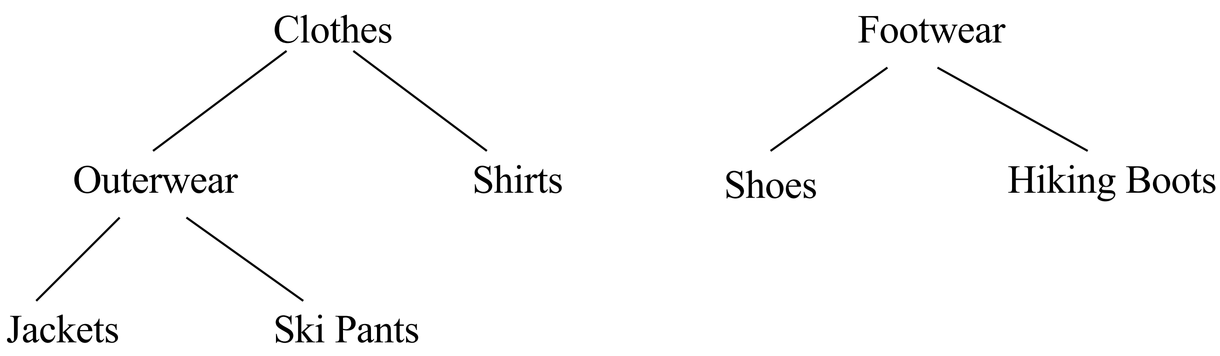
	Basketball	Not basketball	Sum (row)
Cereal	2000	1750	3750
Not cereal	1000	250	1250
Sum(col.)	3000	2000	5000

$$lift(B,C) = \frac{2000/5000}{3000/5000*3750/5000} = 0.89$$

$$lift(B,\neg C) = \frac{1000/5000}{3000/5000*1250/5000} = 1.33$$

Generalized Association Rules

- Given the class hierarchy (taxonomy), one would like to choose proper data granularities for mining.
- Different confidence/support may be considered.



Freg. Itemset	Itemset support
Jacket	2
Outerwear	3
Clothes	4
Shoes	2
Hiking Boots	2
Footwear	4
Outerwear, Hiking Boots	2
Clothes, Hiking Boots	2
Outerwear, Footwear	2
Clothes, Footwear	2

Database

Tx	Items bought
100	Shirt
200	Jacket, Hiking Boots
300	Ski Pants, Hiking Boots
400	Shoes
500	Shoes
600	Jacket

Rule	sup(30%)	conf(60%)
Outerwear → Hiking Boots	33%	66%
Outerwear → Footwear	33%	66%
Hiking Boots → Outwear	33%	100%
Hiking Boots → Clothes	33%	100%
However,		
Jacket → Hiking Boots	16%	50%
Ski Pants → Hiking Boots	16%	100%