Estimating Aircraft Orientation with EKF

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Abstract

In recent years, Bayesian filtering and smoothing have become a hot research topic due to its range of application. Time-varying systems can be found, for example, in navigation, aerospace engineering, remote surveillance and telecommunications. Often, the underlying phenomena are non-linear, requiring more attention in the computations. One filter that handles non-linear models by linearizing them is the Extended Kalman Filter (EKF). It combines the prediction of the state with noisy measurements to update the solution. In this work, we present the EKF algorithm for estimating aircraft orientation using gyroscope and accelerometer measurements.

1 Introduction

In many engineering applications, such as the ones described in [4], the tracking problem has become widespread. It consists of determining position, velocity, acceleration of one or more moving targets. Sensors such as radars, cameras, infrared cameras, and sonars are used to collect measurements, however depending on the sensor type, noise is inherently present. The term *filter* is used because one wants to *filter* out the noise from the measurements. The Kalman filter algorithm, developed more than 60 years ago in [2], is a well-known filter used on linear Gaussian problems.

Our aim is to use its non-linear version, the Extended Kalman Filter (EKF), as described in [4], to estimate the trajectory of the motion of an aircraft. In particular, we are interested in estimating the Euler-Cardan angles which represent the orientation of the aircraft with respect to the world frame. To do so, we use gyroscope and accelerometer measurements to refine the predicted solution.

2 Model

Aircraft system identification is the process of developing a dynamic model of an aircraft using measured input and output data collected using informative experiments. In this section we present a standard aircraft motion model, which incorporates several simplifying assumptions expressed in [3], such as symmetry and the avoiding of critical angles.

The Euler-Cardan angles roll (ϕ), pitch (θ), and yaw (ψ) fully describe the rotation in the body frame of the flying aircraft, as shown in Figure 1. We define

$$\mathbf{x} = \begin{bmatrix} \phi \\ \theta \\ \psi \end{bmatrix} \tag{1}$$

and with respect to the world reference frame, the rotational kinematics equations are

$$\begin{aligned}\dot{\phi} &= p + (q\sin\phi + r\cos\phi)\tan\theta,\\ \dot{\theta} &= q\cos\phi - r\sin\phi,\\ \dot{\psi} &= (q\sin\phi + r\cos\phi)\sec\theta,\end{aligned} \tag{2}$$

where p, q, r are the body-axis angular velocities in deg/s (all the computations are carried out in degrees). In matrix form we have

$$\dot{\mathbf{x}} = M\mathbf{w}, \qquad \text{with } \mathbf{w} = [p, q, r]^{\mathrm{T}},\ M = \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi/\cos\theta & \cos\phi/\cos\theta \end{bmatrix}. \tag{3}$$

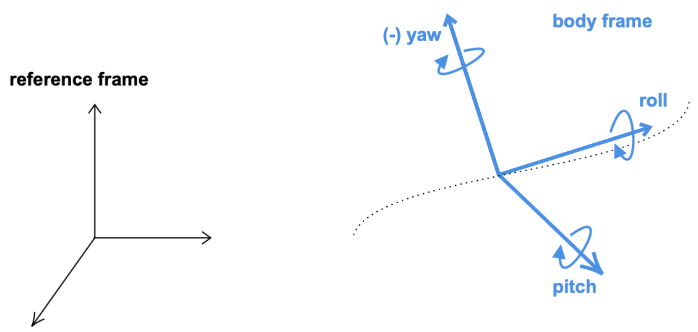

Figure 1: Fixed reference frame, trajectory of an aircraft and possible rotations with respect to its body frame.

We consider such a motion during the time interval $[0, T = 10\text{s}]$ and we discretize Equation 3 using Euler's method with a time step $dt = 0.1$s, obtaining

$$\mathbf{x}_k = \mathbf{x}_{k-1} + dtM\mathbf{w}_{k-1}. \tag{4}$$

Equation 4 represents the dynamic function $f = f(\mathbf{x}_{k-1}, \mathbf{w}_{k-1})$, which describes our process model. In other words, we use the angular velocities detected by the gyroscope to predict the new state $\mathbf{x}_k$.

For the measurement model, we consider the equation

$$a = -g\sin\theta,\ b = g\sin\phi\cos\theta,\ c = g\cos\phi\cos\theta, \tag{5}$$

where g is the gravitational acceleration and a, b, c are the body-axis accelerations. From Equation 5 we derive that

$$\phi = \arctan\left(\frac{b}{c}\right),\ \theta = \arctan\left(\frac{-a}{\sqrt{b^2 + c^2}}\right). \tag{6}$$

Therefore, thanks to the data $\mathbf{a} = [a, b, c,]^{\mathrm{T}}$ detected by the accelerometer sensor we are able to retrieve information about the roll and pitch angles:

$$\mathbf{y}_k = h(\mathbf{x}_k) = \begin{bmatrix} \arctan\left(\frac{g\sin\phi\cos\theta}{g\cos\phi\cos\theta}\right) \\ \arctan\left(\frac{g\sin\theta}{\sqrt{(g\sin\phi\cos\theta)^2 + (g\cos\phi\cos\theta)^2}}\right) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \begin{bmatrix} \phi & \theta \end{bmatrix}. \tag{7}$$

We do not obtain any information about the yaw angle ψ since rotations with respect to the vertical axis are gravity-independent; therefore we cannot correct the yaw prediction. It is common practice in aerospace engineering to combine an additional sensor, such as magnetometer or GPS. For a complete explanation, see [1].

3 Methods

Equations 4 and 7 describe the dynamics of the system, but when considering real sensors we need to account for measurement noise. Under the Gaussian assumption, we obtain

$$\begin{aligned} \mathbf{x}_k &= f(\mathbf{x}_{k-1}, \mathbf{w}_{k-1}) + \mathbf{q}, \\ \mathbf{y}_k &= h(\mathbf{x}_k) + \mathbf{r}, \end{aligned} \tag{8}$$

where $\mathbf{q} \sim \mathcal{N}(0, Q)$ and $\mathbf{r} \sim \mathcal{N}(0, R)$. The filtering model in Equation 8 is fully analyzed in [4]; therefore we present here the EKF solution consisting of two steps:

- prediction step:

$$\begin{aligned} \mathbf{m}_k^- &= f(\mathbf{m}_{k-1}, \mathbf{w}_{k-1}) \\ P_k^- &= F_\mathbf{x}(\mathbf{m}_{k-1}, \mathbf{w}_{k-1}) P_{k-1} F_\mathbf{x}^\mathrm{T}(\mathbf{m}_{k-1}, \mathbf{w}_{k-1}) + Q \end{aligned} \tag{9}$$

- update step:

$$\begin{aligned} \mathbf{v}_k &= \mathbf{y}_k - h(\mathbf{m}_k^-) \\ S_k &= H_\mathbf{x}(\mathbf{m}_k^-) P_k^- H_\mathbf{x}^\mathrm{T}(\mathbf{m}_k^-) + R \\ K_k &= P_k^- H_\mathbf{x}^\mathrm{T}(\mathbf{m}_k^-) S_k^{-1} \\ \mathbf{m}_k &= \mathbf{m}_k^- + K_k \mathbf{v}_k \\ P_k &= P_k^- - K_k S_k K_k^\mathrm{T} \end{aligned} \tag{10}$$

Above, $F_\mathbf{x}(\mathbf{m}, \mathbf{w})$ and $H_\mathbf{x}(\mathbf{m})$ respectively denote the Jacobian matrices of functions $f(\cdot)$ and $h(\cdot)$ with respect to the state variable $\mathbf{x}$.

In the following section, we present how we generated data for our aircraft motion and what results we achieved by implementing Equations 9 and 10 on `Matlab`.

4 Numerical Examples

We set $\mathbf{q} \sim \mathcal{N}(0, \mathrm{I}_3)$ and $\mathbf{r} \sim 0.2\,\mathcal{N}(0, \mathrm{I}_2)$ and the three true Cardan angles ϕ, θ, ψ are modeled as sinusoidal functions. Thus, the true angular velocities p, q, r are known, and we add $\mathbf{q}$ to obtain the gyroscope measurements; the same applies to the accelerometer data. Figure 2 illustrates the ground truth and the noisy measurements, as well as the trajectory of a fixed point on the aircraft under the rotational motion.

Figure 3 shows the EKF estimates. We underline that the yaw estimated is not accurate due to missing data. For the pitch and roll angle, we obtained RMSE values of 0.2085 and 0.2549 respectively.

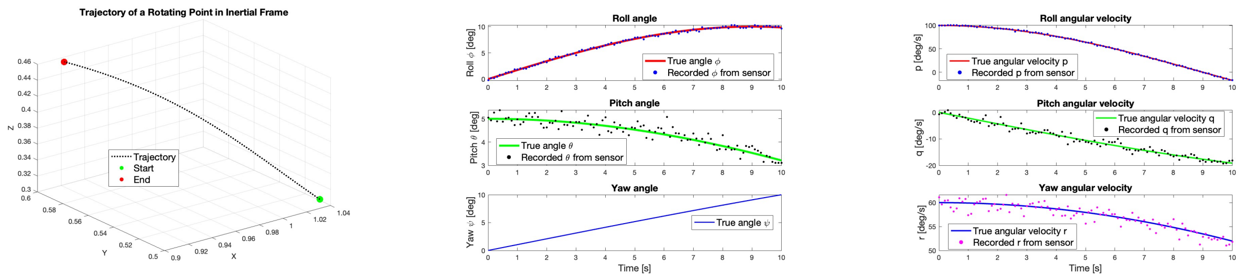

Figure 2: Left: Trajectory of a rotating point in the inertial frame, supposing $(0, 0, 0)$ as the body-centre. Middle: ground truth and noisy measurements of the angles (no data for the yaw). Right: ground truth and noisy measurements of the angular velocities.

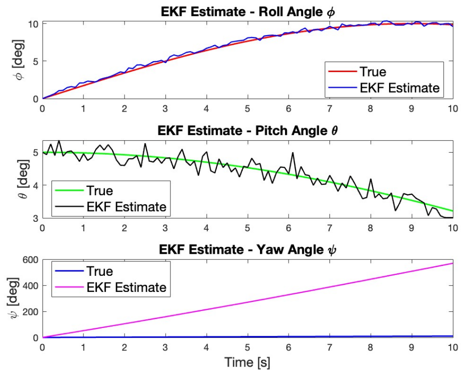

Figure 3: EKF estimates of roll, pitch and yaw angles.

5 Conclusions

We have demonstrated how simple sensors such as gyroscopes and accelerometers can be used to correct the estimate of the orientation of an aircraft under rotational motion, in particular estimating the Euler-Cardan angles ϕ and θ. Future work could focus on incorporating magnetometer data to estimate the ψ angle, or transitioning to a quaternion-based model to handle critical angle analysis.

References

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[3] Simmons, B. M. (2023). *Advances in Aero-Propulsive Modeling for Fixed-Wing and eVTOL Aircraft Using Experimental Data*. Faculty of the Virginia Polytechnic Institute and State University.

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