Chapter 4
Fading, Shadowing, and Link Budgets

Fading is a significant part of any wireless communication design and is important to model and predict accurately. There are two very different types of fading: small scale fading and large scale fading (or shadowing). Small scale fading is often handled in a wireless system with diversity schemes. Large scale shadowing, on the other hand, is very dependent on location with respect to obstacles; its modeling often consists in predicting the likelihood of outage.

4.1 Large-Scale Shadowing

Large scale fading, or shadowing is fading that occurs on several meters or more; it reflects conditions that may vary as one turns a corner, moves behind a large building, or enters a building.

4.1.1 Log-Normal Shadowing

Large-scale variations caused by shadowing of obstacles are shown to follow a log-normal distribution [17][18][19]; which means that, when shadowing power levels are measured in dB, they follow a Gaussian distribution. Consequently, shadowing effects they are usually incorporated into path loss estimates by the addition of a zero-mean Gaussian random variable, with standard deviation σ: N(0,σ), were σ is often estimated by empirical measurements. Commonly accepted values for σ are between 6 dB and 12 dB.

Measured values of σ itself seem to display Gaussian distribution as well, in their variations from one area to another, and depend on: the radio frequency, the type of environment (rural, suburban, or urban), base station and subscriber station height. Many measurement campaigns have been conducted and reported in the literature, some of which are summarized in table 4.1.

Table 4.1: Path loss exponent (n) and log-normal shadowing standard deviation (σ, in dB) — summary of values for various frequencies reported for suburban or residential areas.

Source	Frequency (GHz)	Path Loss Exponent n	σ (dB)	Comments
Seidel [69]	0.9	2.8	9.6	Suburban (Stuttgart)
Erceg [32]	1.9	4.0	9.6	Terrain-category B
Feuerstein [70]	1.9	2.6	7.7	Medium antenna height
Abhayawardhana [71]	3.5	2.13	6.7–10	[71] Table 2, 3.
Durgin [72]	5.8	2.93	7.85	[72] Fig. 7, residential
Porter [73]	3.7	3.2	9.5	Some denser urban
Rautiainen [74]	5.3	4.0	6.1	[74] Fig. 3, 4.
Schwengler [75]	5.8	2.0	6.9	LOS
	5.8	3.5	9.5	NLOS

	3.5	2.7	11.7	Near LOS
Average	3.5-5.8 0.9-1.9	3.0 3.1	8.7 9.0	
Approximation	1-6	$n = 3.0$	$\sigma =$	$9.29 - 1.58\log(f_{GHz})$

4.1.2 Coverage Reliability

To describe a wireless link, one we often seek to establish link budgets: they provide detailed power information from transmitter to receiver; but they provide only a median value for these power levels, actual levels vary with many other parameters, including shadowing.

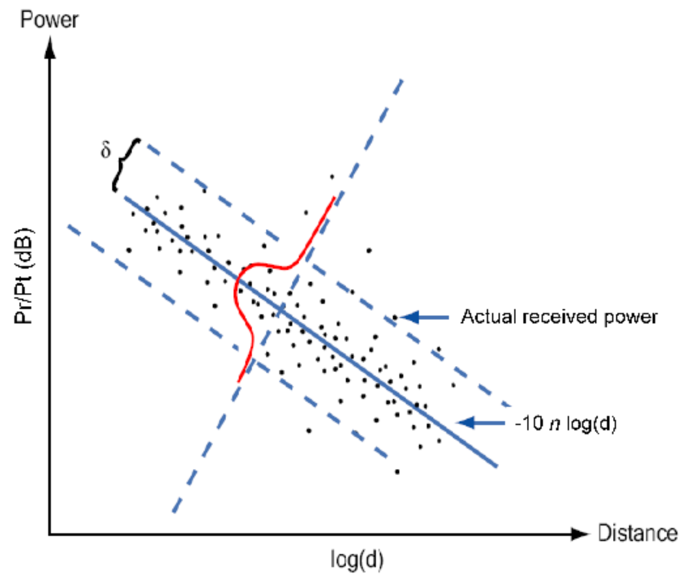

Figure 4.1: Large-scale shadowing can be pictured as a zero-mean Gaussian random variable around the median received power. An additional fade margin or excess margin F is chosen to model received power levels at a greater probability than the median 50 percent of the time.

Usually a minimum signal strength is required to maintain service. In order to maintain signal strength above that level, an additional margin is added to the link budget. Jakes' equation [76] gives an estimate of such excess margin, or fade margin. The assumption is that the shadowing statistic throughout the cell is log-normally distributed (i.e. values in dB are normally distributed):

$$p(x) = \frac{1}{\sigma\sqrt{2\pi}} \times \exp\frac{-(x-m)^2}{2\sigma^2} \tag{4.1}$$

The probability that x exceeds the threshold x_0 (the receiver threshold that provides an acceptable signal) at a given radius R is

$$P_0(R) = P[x \geq x_0] = \int_{x_0}^{\infty} p(x)\mathrm{d}x \tag{4.2}$$

By integrating the probability density function from x_0 to ∞, the edge reliability result is

(4.3)

$$P_0(R) = \frac{1}{2}\left(1 + \mathrm{erf}\left(\frac{m - x_0}{\sigma\sqrt{2}}\right)\right)$$

Typically the threshold of interest x_0 is lower than the median provided by the path loss model, and the value m-x_0 is an excess, positive amount (in dB), and is usually referred to as excess margin, or fade margin: $F = m - x_0$. Of course, for a fade margin of zero ($m - x_0 = 0$) at a given R, the error function (erf) equals zero, resulting in 50% edge reliability.

Alternative representations of that formula sometime make use of the complementary error function or the Q function. [1]

$$P_0(R) = \frac{1}{2}\mathrm{erfc}\left(-\frac{F}{\sigma\sqrt{2}}\right) = Q\left(-\frac{F}{\sigma}\right) \tag{4.4}$$

Instead of an edge reliability, the reliability in the entire cell area is often more useful: the fraction of useful service area, $F_A(R)$, within a circle of radius R where the received signal strength exceeds a threshold value x_0 is the integration of the probability function over the area as shown below.

$$F_A(R) = \frac{1}{\pi R^2}\iint P_0(r)\mathrm{d}A(r) \tag{4.5}$$

With the assumption that the mean value of the signal strength, m, behaves according to an r^{-n} propagation law, then $m = m_R - 10n\log_{10}\frac{r}{R}$, where n is the propagation exponent value; and m_R (in dB) is the mean signal strength at the edge distance $r = R$ (m_r is determined from the transmitter power, antenna heights, gain, and so on), thus the value m_R -x_0 in equation (4.6) corresponds to the excess margin at the edge). Substituting m into the probability density function gives the area reliability: (after substitution and integration by part):

$$F_A(x_0, \sigma) = \frac{1}{2}\left[1 - \mathrm{erf}(a) + \left(1 - \mathrm{erf}\frac{1 - ab}{b}\right)\exp\left(\frac{1 - 2ab}{b^2}\right)\right] \tag{4.6}$$

where

$$a = \frac{x_0 - m_R}{\sigma\sqrt{2}} \quad \text{and} \quad b = \frac{10n\log_{10}e}{\sigma\sqrt{2}} \tag{4.7}$$

where x_0 - m_R is simply the opposite of the excess margin at the edge.

4.2 Link Budgets

Link budgets are a convenient tool to compare power levels between different technologies and different systems. Terrain conditions for instance may cause great variations in how far a wireless system reaches; link budgets allows for good system comparison while removing some of these variations.

We will examine link budgets for popular new wireless access standards such as cdmaOne (IS-95), cdma2000 (IS-2000), EV-DO (IS-856) [81], and fixed WiMAX. We will also discuss coverage and capacity tradeoffs, increasing throughput vs. capacity or coverage, soft handoff benefits and cost.

4.2.1 Important Parameters

Link budgets from different radio manufacturers are sometimes difficult to compare because they use different terms and definitions (without always clearly specifying them). Always compare them to a common definition, and try to identify the following parameters.

EIRP (or ERP):

Defines the maximum transmit power. Effective isotropic radiated power (EIRP) is the power radiated relative to a perfect isotropic antenna; it is obtained by adding available transmit power and antenna gain in dBi, and removing any loss (due to cable, inefficiency, angle away from boresight, etc.) Sometimes (though rarely) manufacturers give an equivalent parameter called effective radiated power (ERP), which is the power radiated relative to a dipole; it is obtained by adding available power to antenna gain in dBd (instead of dBi) and again removing any transmission loss.

ERP is smaller than EIRP by the amount of gain difference between an isotropic antenna and a dipole, that is 2.15 dB. (Indeed a dipole gain is 0 dBd=2.15 dBi, so any antenna gain G may be expressed in either unit with the simple conversion $G(dBi) = G(dBd) + 2.15\ dB$, and EIRP=ERP+2.15 dB.)

SNR or Eb/No:

A minimum signal to noise ration (SNR) is required to achieve at the receiver a certain error probability given a signal modulation. The SNR is sometimes expressed in term of energy per bit over noise power spectral density noted E_b/N_0 or E_b/N_t.

Receiver sensitivity:

On the receiving side some measure of sensitivity must be given: it is usually expressed in terms of a signal to noise ratio required above a certain noise floor.

Some care must be taken to define the receiver sensitivity, which is the lowest power level at which the received signal may be decoded, it is usually defined as a power level above ambient noise and interferences, and depends on several parameters such as bit rate, coding, error rate. It uses the following parameters:

- Boltzman's constant: $k = 1.38 \cdot 10^{-23}$ J/K,
- Reference temperature $T_0 = 290$ K, (63°F, 17°C), hence $kT_0 = -174$ dBm/Hz,
- Thermal noise is the noise caused by components in the receiving chain (of bandwidth B): $N_0 = kT_0B$ dBm,
- Total noise is the sum of thermal noise and any other noise sources, usually noted N_t. Hystorically it was sometimes identified with N_0.
- Total noise and interference also adds to N_t any source of interferences, and is usually noted I_0 or I_t.
- Noise figure F of the receiver, the noise added by the receiver system.
- Receiver sensitivity is the minimal signal for successful decoding. If a certain SNR is required the receiver sensitivity is simply: $S = F \cdot E_b \cdot R$, where E_b is the energy per user bit, and R is that user bit rate (e.g. 9.6 kb/s):

$$S = \frac{E_b}{N_t} \cdot R \cdot F \cdot k \cdot T_0 \tag{4.8}$$

Other similar and equivalent expressions my be derived for system sensitivity using the minimum SNR required in an RF channel or bandwidth B, in which case $S = SNR \cdot F \cdot N_t$ or:

$$S = \mathrm{SNR} \cdot F \cdot kT_0B \tag{4.9}$$

(as seen in link budget on figure 4.5).

Excess or fademargin:

Any number of reasons can be required to add an excess margin to the link budget: increased service availability, in-building penetration, etc. When comparing radio systems, it is important to verify that the same excess margin conditions are used. In radio systems deep fades occur and an excess margin is used to achieve a given success rate of the receiver, it is therefore referred to as fade margin.

Maximum allowable path loss:

In the end the goal of the link budget it to derive the difference between transmitted radiated power and received power (possibly removing any margin required). That result is what quantifies the radio system performance. It provides a technology independent value that can be used for coverage, capacity, or other estimates.

Different manufacturers present link budgets differently, and some analyses are required to reduce them to a common format. Still, transmitted EIRP, receiver sensitivity, excess margin, and maximum allowable path loss can usually be found.

4.2.2 Reverse Link Budget

Equipment manufacturers typically claim a certain reverse link budget, which is studied by potential operator buyers in order to predict performance, coverage, capacity, and compare them with other equipment vendors. The reverse link lends itself well to a straightforward power budget, based on the mobile maximum transmit power and the base sensitivity level and the industry commonly admits that reverse link budgets are the basis for radio design, and the forward link is studied subsequently, simply in order to verify that it provides enough resources to be balanced with the reverse link.

4.2.3 Forward Link Budget

Equipment manufacturers sometimes do not provide forward link budgets, and argue that systems are usually reverse-link limited. For voice systems, both reverse and forward link budgets should be balanced; the forward link budget should insure that a power allocation between devices within the cell is sufficient to provide enough capacity. For data systems, the revers link is typically used to define a maximum range, and the forward link allows to determine the corresponding download speeds.

Unlike in the reverse link, the entire power is not necessarily allocated to one remote client device: either a portion of orthogonal channels (CDMA or OFDMA) are allocated to it, or a certain percentage of the time (as in TDMA, or IS-856 EV-DO). The link budget should reflect that fact as shown on forward link budget figures of this section. (The details of derivation of these percentages are not trivial and depend heavily on standards, system efficiency, and suppliers implementations.)

4.2.4 Voice vs. Data Link Budgets

There are several differences between voice an data link budgets. Voice SNR requirements are typically higher, as higher data rates can benefit from increased coding efficiency (like turbo-coding) and is less delay-sensitive and can afford some retransmit. For instance a cdma2000 link budget for EVRC voice (9.6kbps) requires an Eb/No value around 4 to 5dB, which I can achieve a data rate around 38.4kbps (thanks to more efficient coding and retransmissions).

Data link budgets vary greatly with data rates, so it is important to specify the data rate expected given a certain channel bandwidth, modulation, and coding. Finally body loss (usually 3 dB) associated with the handset near the head (or on the belt with a hands-free device) are usually omitted (of course this is somewhat debatable as the data device is still near the human body).

4.2.5 Variable Bandwidth Link Budgets

Wireless standards that allow multiple bandwidth, such as LTE, have to specify the bandwidth used for the link budget (either in MHz, of in number of resource elements detailed further in �8.3). It is always important to specify how much spectrum is used (or how many resource blocks) for what data rate.

4.2.6 Licensed vs. Unlicensed Radios

Whenever possible, operators use licensed spectrum for wireless communications. For instance CDMA systems are commonly used at PCS frequency (1.9 GHz), and fixed WiMAX systems operate at 3.5 to 3.7 GHz. We summarize parameters for these licensed radio systems with the link budgets shown on figures 4.2 to 4.4.

Link budgets in unlicensed bands are similar to the above but are usually limited by a lower maximum allowed EIRP set by government regulations (FCC in the US) as shown in a separate table on figure 4.5.

Parameter	Unit	Equation	cdmaOne voice	CDMA2000 voice	IS-856 data
Data rate (EVRC for voice)	bps	r	9600	9600	19200
MS Tx Power	dBm	A	23.0	23.0	23.0
MS Antenna Gain	dBi	B	0.0	0.0	0.0
Body Loss	dB	C	3.0	3.0	0.0
Transmitted EIRP	**dBm**	**D=A+B**	**23.0**	**23.0**	**23.0**
BTS Rx Antenna Gain	dBi	E	16.0	16.0	16.0
BTS Cable Loss	dB	F	2.0	2.0	2.0
Thermal noise	dBm/Hz	G=10*log(kT)+30	-174.0	-174.0	-174.0
BTS Noise Figure	dB	H	5.0	5.0	5.0
BTS Thermal Noise	dBm/Hz	I=G+H	-169.0	-169.0	-169.0
Symbol Rate	dB Hz	J=10*log(r)	39.8	39.8	42.8
Eb/No Required	dB	K	7.0	4.0	4.3
Receiver Interference Margin	dB	L	3.0	3.0	3.0
BTS Rx Sensitivity	**dBm**	**M=I+J+K+L**	**-119.2**	**-122.2**	**-118.9**
Soft Hand-off Gain	dB	N	4.1	4.1	4.1
Log-normal fading std dev	dB		8.0	8.0	8.0
Log-normal Fade Margin	dB	O	10.3	10.3	10.3
Penetration Loss	dB	P	10.0	10.0	10.0
Maximum Reverse Path Loss	**dB**	**Q=D-C-M+E-F+N-O-P**	**137.0**	**140.0**	**139.7**

Figure 4.2: Reverse link budgets for cdmaOne, cdma2000, and IS-856 (1.9 GHz).

Parameter	Unit	Equation	cdmaOne voice	CDMA2000 voice	IS-856 data
Data rate (EVRC for voice)	bps	r	9600	9600	32175
Average Burst Rate	bps	rb	9600	9600	*225000*
BTS Tx Power	dBm		42.0	42.0	42.0
Maximum Power per channel	%		*5.3%*	*4.4%*	100.0%
Maximum Power per channel	dBm	A	29.3	28.5	42.0
BTS Antenna Gain	dBi	B	16.0	16.0	16.0
BTS Cable Loss	dB	C	2.0	2.0	2.0
Transmitted EIRP in channel	**dBm**	**D=A+B-C**	**43.3**	**42.5**	**56.0**
Time in channel	%	dt	100.0%	100.0%	*15.0%*
MS Rx Antenna Gain	dBi	E	0.0	0.0	0.0
MS Body Loss	dB	F	3.0	3.0	3.0
Thermal noise	dBm/Hz	G=10*log(kT)+30	-174.0	-174.0	-174.0
MS Noise Figure	dB	H	9.0	9.0	9.0
BTS Thermal Noise	dBm/Hz	I=G+H	-165.0	-165.0	-165.0
Symbol Rate	dB Hz	J=10*log(rb/dt)	39.8	39.8	61.8
Eb/No Required	dB	K	12.3	9.3	2.5
Receiver Interference Margin	dB	L	0.0	0.0	0.0
BTS Rx Sensitivity	**dBm**	**M=I+J+K+L**	**-112.9**	**-115.9**	**-100.7**
Soft Hand-off Gain	dB	N	5.6	5.6	5.6
Log-normal fading std dev	dB		8.0	8.0	8.0
Log-normal Fade Margin	dB	O	10.3	10.3	10.3
Penetration Loss	dB	P	10.0	10.0	10.0
Maximum Forward Path Loss	**dB**	**Q=D-M+E-F+N-O-P**	**138.4**	**140.7**	**139.0**

Figure 4.3: Forward link budgets for cdmaOne, cdma2000, and IS-856 (1.9 GHz).

Parameter	Unit	Equation	BPSK 1/2	QPSK 3/4	64QAM 3/4
Data rate	Mbps	r	1.4	4.2	12.7
SS Tx Power	dBm	A	23.0	23.0	23.0
SS Antenna Gain	dBi	B	18.0	18.0	18.0
SS Cable Loss	dB	C	0.0	0.0	0.0
Transmitted EIRP	**dBm**	**D=A+B-C**	**41.0**	**41.0**	**41.0**
BTS Rx Antenna Gain	dBi	E	17.0	17.0	17.0
BTS Cable Loss	dB	F	1.0	1.0	1.0
Thermal noise	dBm/Hz	10*log(kT)+30	-174.0	-174.0	-174.0
Channel Width	MHz	G	3.5	3.5	3.5
Thermal noise in Channel	dBm	H=10*log(kTG)+90	-108.6	-108.6	-108.6
BTS Noise Figure	dB	I	4.0	4.0	4.0
BTS Thermal Noise	dBm/Hz	J=H+I	-104.6	-104.6	-104.6
SNR Required	dB	K	6.4	11.2	24.4
Receiver Interference Margin	dB	L	0.0	0.0	0.0
BTS Rx Sensitivity	**dBm**	**M=J+K+L**	**-98.2**	**-93.4**	**-80.2**
Diversity Gain	dB	N	0.0	0.0	0.0
Log-normal fading std dev	dB		9.6	9.6	9.6
Log-normal Fade Margin	dB	O	12.3	12.3	12.3
Building Penetration Loss	dB	P	0.0	0.0	0.0
System Gain	dB	Q=D+E-F-M+N	155.2	150.4	137.2
Maximum Reverse Path Loss	**dB**	**R=D+E-F-M+N-O-P**	**142.9**	**138.1**	**124.9**

Figure 4.4: Reverse link budget for fixed WiMAX (3.5 GHz).

Parameter	Unit	Equation	BPSK 1/2	QPSK 3/4	64QAM 3/4
Data rate	Mbps	r	2.0	6.0	18.2
SS Tx Power	dBm	A	18.0	18.0	18.0
SS Antenna Gain	dBi	B	16.0	16.0	16.0
SS Cable Loss	dB	C	0.0	0.0	0.0
Transmitted EIRP	**dBm**	**D=A+B-C**	**34.0**	**34.0**	**34.0**
BTS Rx Antenna Gain	dBi	E	16.0	16.0	16.0
BTS Cable Loss	dB	F	1.0	1.0	1.0
Thermal noise	dBm/Hz	10*log(kT)+30	-174.0	-174.0	-174.0
Channel Width	MHz	G	10.0	10.0	10.0
Thermal noise in Channel	dBm	H=10*log(kTG)+90	-104.0	-104.0	-104.0
BTS Noise Figure	dB	I	4.0	4.0	4.0
BTS Thermal Noise	dBm/Hz	J=H+I	-100.0	-100.0	-100.0
SNR Required	dB	K	6.4	11.2	24.4
Receiver Interference Margin	dB	L	0.0	0.0	0.0
BTS Rx Sensitivity	**dBm**	**M=J+K+L**	**-93.6**	**-88.8**	**-75.6**
Diversity Gain	dB	N	0.0	0.0	0.0
Log-normal fading std dev	dB		9.6	9.6	9.6
Log-normal Fade Margin	dB	O	12.3	12.3	12.3
Building Penetration Loss	dB	P	0.0	0.0	0.0
System Gain	dB	Q=D+E-F-M+N	142.6	137.8	124.6
Maximum Reverse Path Loss	**dB**	**R=D+E-F-M+N-O-P**	**130.3**	**125.5**	**112.3**

Figure 4.5: Reverse link budget for unlicensed fixed WiMAX (5.8 GHz).

4.3 Small Scale Fading

We've seen in �3.3 the impact of multiple rays on propagation models: this effect of multipath causes deep fades within small distances and is referred to as small-scale fading. Another important yet different cause of fading is that of small frequency variations such as Doppler effect. Both of these small-scale fading effects are studied in this section. [2]

4.3.1 Multipath Fading

Multipath fading is significant for both mobile and fixed wireless systems. Intuitively that type of fading varies with surrounding scatterers which reflect differently the wavefront between transmitter and receiver. Practically, it is very important to quantify that aspect of the propagation environment, and even to taylor the standard to perform well in such an environment: for instance we'll see later that the length of a transmitted symbol will be depending on the multipath situation in which it has to perform well.

In the time domain, multipath parameters can be seen as the spread of the arriving waves. In the frequency domain, the concept is less intuitive and relates to a coherence bandwidth, that is the width of the spectrum that is attenuated by a fade. The main parameters are summarized in table 4.2.

Table 4.2: Multipath fading parameters to measure and quantify.

Multipath	Parameter	Symbol
Time domain	Channel impulse response	$\underline{H}(t)$
	Mean excess delay	τ
	RMS delay spread	σ_τ
	Excess delay spread (for x dB threshold)	τ_X
Frequency domain	Channel impulse response	$H(f)$
	Coherence bandwidth (90%)	$B_c \approx \frac{1}{50\sigma_\tau}$
	Channel impulse response (50%)	$B_c \approx \frac{1}{5\sigma_\tau}$

Flat or frequency-selective fading: Depending on the values of these above parameters and how they compare to the speed of transmitted symbol, the wireless channel will have flat fading (over the entire bandwidth used) or frequency-selective fading. This is of course a frequency domain interpretation describing what happens to the signal: it is either faded over the entire spectrum, or selectively only over a portion of it.

High or low delay spread: Again depending on the values of the above parameters and how they compare to the length of transmitted symbol, the wireless channel is said to have high delay spread (or heavy multipath), or low delay spread (low multipath). This simply says in the time domain what flat vs. frequency-selective fading said in the frequency domain.

4.3.2 Doppler Spread

Another aspect of wireless communication, different from the above, is the concept of how fast things are changing in the wireless channel. In the time domain, that aspect is referred to as time dispersion and is measured by coherence time; the coherence time describes how fast the wireless channel is changing. That aspect is important for estimating the quality of communication; for instance if the channel has a certain property, how long can we count on it? This defines for instance how often training sequences should be sent to estimate the wireless channel.

In the frequency domain the effect is best described by the Doppler spread: it describes how fast transmitter, receiver, and scatterers in-between are moving; the faster they are moving, the faster the wireless channel changes, and the more Doppler shift will be present. The instantaneous Doppler shift depends on the wave frequency and the maximum radial speed of the many scatters within the wireless channel; with a radial speed v_r and wave incident at an angle θ:

$$f_d = \frac{v_r \cdot \cos\theta}{\lambda} \tag{4.10}$$

Doppler shift distribution varies, but an approximation of the maximum shift is simply: $f_m = v_m/\lambda$, where v_m is the maximum speed of the mobile (and λ the wavelength of the signal). And the Doppler spread is defined as twice the maximum Doppler shift: $B_D = 2f_m$, since the shift can be positive or negative depending on the direction of the motion.

Table 4.3: Doppler fading parameters to measure and quantify.

Doppler	Parameter	Symbol
Frequency domain	Channel impulse response	$H(f)$
	Doppler spread	$B_D = 2f_m$
Time domain	Channel impulse response	$H(t)$
	Coherence time (50%)	$T_c \approx \frac{9}{16\pi f_m}$
	Practical rule of thumb	$T_c \approx \frac{0.423}{f_m}$

Fast or slow fading Depending on the values of these above parameters and how they compare to the length of transmitted symbol, the wireless channel will have fast fading (faster than a transmitted symbol) or slow fading (one or several transmitted symbol during a fade). This is of course a time domain interpretation describing how fading time compares to transmitted symbol time.

High or low Doppler Again depending on the values of the above parameters and how they compare to the speed of transmitted symbol, the wireless channel is said to have high or low Doppler spread. This simply expresses in the frequency domain what fast vs. slow fading said in the time domain.

It is important to understand the non-intuitive equivalence: for a given transmission rate, slow fading means long fades, meaning high coherence time, therefore low Doppler spread.

4.3.3 Fading Summary

Wireless engineers might talk about fast fading meaning all of the above types of small-scale fading, simply because its variations are large over a small distance; this however should be avoided, instead always refer to it as small-scale fading (as it might be fast or slow).

It is important to reiterate the difference between the types of fading presented in the previous two section, and to understand that the characteristics presented in these two sections are completely uncorrelated. A wireless channel can be fast or slow and flat or frequency-selective. In either case, parameters above have to be compared to the transmitted symbol period (in the time domain) or to the data symbol baseband frequency (in the frequency domain).

To summarize, remember the following:

- The amount of multipath is related to the delay spread and inversely to the coherence bandwidth; it produces flat or frequency-selective fades.
- The variability of the transmit media is related to the coherence time and inversely to the Doppler spread; it produces fast or slow fades.

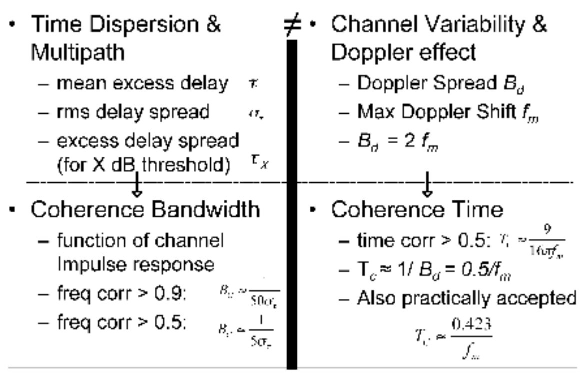

Figure 4.6: Summary of small-scale parameters: multipath parameters and time variance (Doppler) parameters are different concepts; but each have parameter interpretation in the frequency domain or time domain.

4.3.4 Angular Spread

One more important aspect of the wireless channel and its small-scale fading deals with the distribution of angle of incidence to the receiving antenna. It is also referred to as angle of arrival. The RMS spread for angle of arrival has an impact on the statistical diversity of the received signal, which is important for instance in MIMO systems. The dual of angular spread is the coherence distance (see for instance reference [[140]], p. 68).

4.4 Small Scale Fading Distribution

Small-scale fading is caused by different reflections of the signal (delayed, frequency shifted, constructive or destructive) and is usually modeled by a random variable with a certain probability distribution.

Rayleigh Rayleigh fading channels are widely used in theoretical approaches as well as in empirical urban studies. They are generally accepted to model multipath environments with no direct line of sight (LOS).

Given two random variable x and y Gaussian and zero-mean, that represent some central limit theorem of a large number of multipaths (practically more than six), it is shown that the signal envelope or amplitude $\alpha = \sqrt{x^2 + y^2}$ is Rayleigh distributed [8]. This means that the channel amplitude follows the Rayleigh distribution:

(4.11)

$$P_\alpha(\alpha) = \frac{2\alpha}{\Omega} \exp\left(-\frac{\alpha^2}{\Omega}\right)$$

where $\Omega = \overline{\alpha^2}$ is the mean square value of the random variate α.

The signal power is then related to α^2. When the noise spectral density N_0 is assumed to be one-sided Gaussian, the SNR has exponential distribution [78]: let us use as a measure of signal to noise ratio, SNR: $\gamma = \alpha^2 E_s/N_0$

$$P_\gamma(\gamma) = \lambda \exp(-\lambda(\gamma - \mu)) \tag{4.12}$$

with $\gamma \geq 0$, and $\lambda = 1/\mathrm{E}(\gamma)$.

We can estimate the parameter λ for the exponential distribution in some way, such as by matching the first two moments with sample data $\{X_i\}$, of mean m_X and standard deviation σ_X.

$$\tilde{\lambda} = 1/s_X \quad , \quad \tilde{\mu} = m_X - s_X \tag{4.13}$$

Rice The amplitude of a fading channel may have a dominant component; the faded amplitude is now given by $\alpha = \sqrt{(K+x)^2 + y^2}$. Its probability distribution is given by the Ricean distribution:

$$P_\alpha(\alpha) = \frac{\alpha}{\sigma^2} \exp\left(-\frac{\alpha^2 + K^2}{2\sigma}\right) I_0\left(\frac{\alpha K}{\sigma^2}\right) \tag{4.14}$$

where $I_0(z)$ is the modified Bessel function of the first kind of order zero. That channel model offers the advantage of having an additional parameter K that has a physical meaning; but the Bessel function makes its computationally difficult, and has no straightforward form for its power or SNR.

Nakagami-m Similarly a Nakagami-m fading channel is often used for fade channels:

$$P_\alpha(\alpha) = \frac{2m^m \alpha^{2m-1}}{\Omega^m \Gamma(m)} \exp\left(-\frac{m\alpha^2}{\Omega}\right) \tag{4.15}$$

$\alpha \geq 0$, the SNR then follows the distribution [8][78]:

$$P_\gamma(\gamma) = \frac{1}{\Gamma(m)} \left(\frac{m}{\bar{\gamma}}\right)^m \gamma^{m-1} \exp\left(-\frac{m\gamma}{\bar{\gamma}}\right) \tag{4.16}$$

$\gamma \geq 0$, which is gamma distributed. The problem of estimating parameters is more complicated in this case (as discussed in [79]ch. 17.7). Still, moment matching estimates lead to:

$$\tilde{m} = (m_X/s_X)^2 \quad , \quad \tilde{\gamma} = s_X^2/m_X \tag{4.17}$$

Gaussian Although not usually used for fading, the normal (or Gaussian) distribution is given for comparison:

$$P_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right) \quad , \quad x \geq 0 \tag{4.18}$$

for which we may use the following simple estimates:

$$\tilde{\mu} = m_X \quad , \quad \tilde{\sigma} = s_X \tag{4.19}$$

The estimate for $\tilde{\mu}$ is unbiased and corresponds to moment matching and maximum likelihood, and with large enough sample size the estimate for $\tilde{\mu}$ although biased is usually a good estimate.

Log-normal There is a general consensus that large-scale fading may be approximated by lognormal distributions [17]. Its probability distribution is:

$$P_X(x) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\ln x - \mu)^2}{2\sigma^2}\right) \quad , \quad x > 0 \tag{4.20}$$

for which, the best estimates are simply obtained by change of variable $Y = \ln X$ and referring to the Gaussian case. A more complex approach would be to investigate $Z = ln(X - \Theta)$ but these estimations are more difficult and in many cases rather inaccurate ([79] ch. 14).

Weibull The flexibility and relative simplicity of Weibull distribution may also be convenient and leads to good data fit [80]:

$$P_X(x) = \frac{a}{b^a} x^{a-1} \exp\left(-(x/b)^a\right) \quad , \quad x \geq 0 \tag{4.21}$$

To estimate parameters, the simplest approach is to follow Weibull's method based on the first two moments about the smallest sample value [79]ch. 21.

4.4.1 Case Study: Dropped Calls and Setup Failures

The above probability distributions are useful for modeling fading in wireless channels. In some cases empirical measurements are taken, and finding the fading situation may be a difficult problem. We look in this section at a specific case of fading in urban cores, and measure different events.

Test drives are conducted throughout major US cities in which mobile handsets continually place calls on several major cellular service providers. For that purpose a van is outfitted with several handsets, each cabled to a roof antenna, these antennas are placed as far as physically possible from one another to limit interferences. A system is setup to place a 90-second call on every handset, then remain idle for 30 seconds, and repeat the cycle. A wealth of data may be analyzed and compared; in particular we focus presently on the occurrence of dropped calls and call setup failures.

Dropped Calls:
We peg a dropped call every time a handset is in active talk mode and for some reason loses that call. In such case, the handset remains in idle mode for 30 seconds before attempting to place another call.

Setup Failures:
We peg a setup failure every time an idle handset attempts to originate a call to a fixed test number. Such an attempt may fail for many reasons including trunk blocking, network resource allocation failures, RF resource failures, handoff failures, etc.

In this example we collect the rates of dropped calls and setup failures for major cities and service providers by driving between 1000 miles and 1500 miles (depending on the size of the city) on every major road and a portion of secondary roads. The data collected for dropped calls and setup failures is summarized in table 4.4.

Table 4.4: Measured Dropped Calls and Setup Failures in several major US cities on different cellular networks.

Moment of sample	Dropped Calls	Setup Failures

Mean (m)	m_X(DC)= 1.32%	m_X(SF)= 2.32%
Standard Dev. (s)	s_X(DC)= 1.44%	s_X(SF)= 2.67%

Moment matching for the above probability density distributions lead to table 4.5.

Table 4.5: Moment matching of typical small-scale distributions for measured Dropped Calls and Setup Failures, in several major US cities on different cellular networks.

Estimated Distribution	Parameters for Dropped Calls	Parameters for Setup Failures
Uniform	b - a=0.0500	b - a=0.0925
Exponential	λ=69.2761, μ=-0.0012	λ=37.4559, μ=-0.0035
Gamma	a=0.8383, b=0.0158	a=0.7542, b=0.0307
Gaussian	μ=0.0132, σ=0.0144	μ=0.0232, σ=0.0267
Lognormal	μ=0.0130, σ=0.0140	μ=0.0230, σ=0.0255
Weibull	a=0.38, b=0.0022	a=0.53, b=0.0150

A simple error estimate may be used to estimate differences between measured data and the different probability distribution functions covered above. The best fit (minimal error) is that of the gamma distribution; graphical representations also show a good fit.

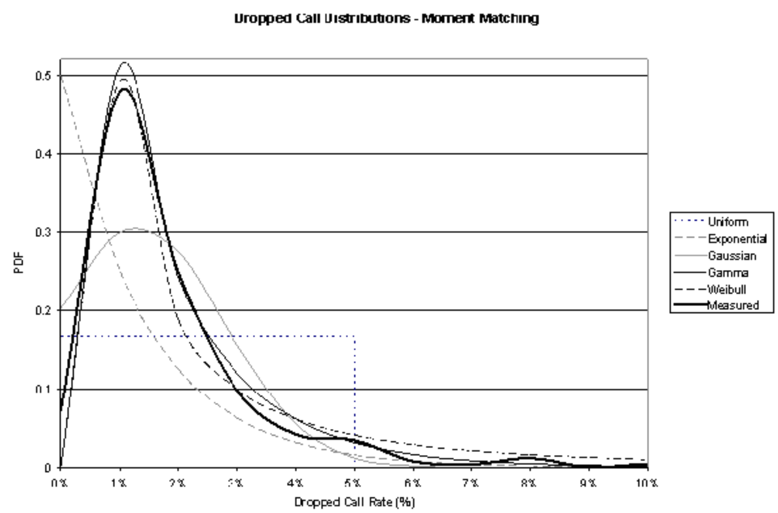

Figure 4.7: Dropped call rates and various distributions estimated with moment matching; best fit is that of gamma distribution.

4.5 Correlation

Important correlation properties of fading deserve a closer look, both for large scale, and for small scale fading.

4.5.1 Shadowing Autocorrelation

We've seen that Shadowing is modeled by a lognormal, zero-mean random variable; and that its standard deviation (in dB) relates with Jakes' equation to service availability. But shadowing is largely caused by large obstacles, and is therefore correlated over fairly large distances. This is usually modeled with an autocorrelation function of the received signal $r(d)$, a random variable varying (in space) with distance d from the transmitter;

the autocorrelation function $\varrho(\delta)$ is defined as the moment of the product of signals separated by δ. Sometimes the definition of the autocorrelation function divides by the variance (square of the standard deviation) in order to normalize the function between zero (uncorrelated) and one (perfectly correlated) signals:

$$\rho(\delta) = E[r(d) \cdot r(d+\delta)]/\sigma^2 \tag{4.22}$$

(We use this normalized definition in this text.)

Shadowing autocorrelation often uses Gudmundson's model [82], which approximates autocorrelation with an exponential decay, leading to the expression:

$$\rho(\delta) = \exp\left(\frac{-\delta \ln 2}{d_{50\%}}\right) \tag{4.23}$$

where $d_5 0$ is the 50% correlation distance. The paper also cites measured values for suburban macrocell: normalized correlation is $\varrho = 0.82$ at 100m for measurements made at $f_c = 900$ MHz; and urban microcell: $\varrho = 0.3$ at 10m and $f_c = 1.7$ GHz.

Other models offer a two-dimensional approach to model the correlated shadowing that may occur in any direction of motion [83].

Further correlations exist between shadowing and other parameters:

- Log-normal shadowing itself increases with path distance, [84] derives a distance model for shadowing:

  $$\sigma(d) = a \cdot 10 \log\left(\frac{d}{d_0}\right) + b \tag{4.24}$$

  where d_0=1m is a reference distance, a = 1.48 dB, b = -14.44 dB.
- A (negative) correlation exists between shadowing and delay spread ($\varrho \approx 0.75$), both log-normal distributed [18]. RMS delay spread follows a model Td^{ϵ} logN(0,s) at a distance d, where T is the median value of the RMS delay spread at d=1km, ϵ is an exponent between 0.5 (in urban, suburban, rural areas) and 1.0 (in mountainous areas), and logN(0,s) is a log-normal random variable of zero mean, and s between 2 and 6dB, correlated to the log normal shadowing with correlation around -0.75.
- Azimuth spread was shown to be log-normal as well, correlated to delay spread ($\varrho \approx 0.5$) and negatively correlated to shadowing ($\varrho \approx -0.6$) [85].

4.5.2 Diversity and Autocorrelation

Diversity is related to de-correlation between paths. To measure path correlation, we define the autocorrelation of a signal, and examine how it changes in space. A classic autocorrelation model by Clarke or Jakes is often used to illustrate how a received signal changes at a different location (say at a distance D away) – [3] ch. 3 p. 65, [76]. With a number of assumptions, including a large (infinite) amount of scattering near the receiver, Jakes develops a convenient and realistic approximation model: $\varrho(\tau) = J_0(2\pi\tau f_D)$, where J_0 is the Bessel function of zero order, and f_D is the Doppler shift. This leads to a correlation between paths to diversity antennas of: $\varrho(d) = J_0(2\pi d/\lambda)$, where d is the antenna separation (λ is the wavelength).

The optimal antenna separation is at the Bessel function's first zero, $J_0(2.40) = 0$ at $D = 0.38\lambda$, which is often approximated to half a wavelength. Also note that correlation decreases with (and is better than) the square root of the separation D/λ, since for large arguments $J_0(x) \sim \sqrt{\frac{2}{\pi x}} \cos(x - \pi/4)$.

Table 4.6: Correlation values reported for different antenna separations values relative to wavelength (d/λ). (When several experiments are conducted, NLOS cases and the lowest antenna heights were chosen – typically 10 to 15m – in this comparison table, in order to best relate to the many scatterer hypothesis of the Clarke model.

D/λ	**0.5**	**1**	**1.5**	**2**	**3**	**5**	**10**	**20**
$J_0(2\pi D/\lambda)$	-0.30	0.22	-0.18	0.16	0.13	0.10	0.07	0.05
[87] NLOS1 3.7GHz	0.25	0.23	0.20					
[87] NLOS2 3.7GHz	0.23	0.17	0.19					
[88] route1 1.9GHz				0.19		0.12	0.05	
[88] route2 1.9GHz				0.24		0.10	0.03	
Private 1.9GHz					0.18			
[89] suburban								0.08-0.16

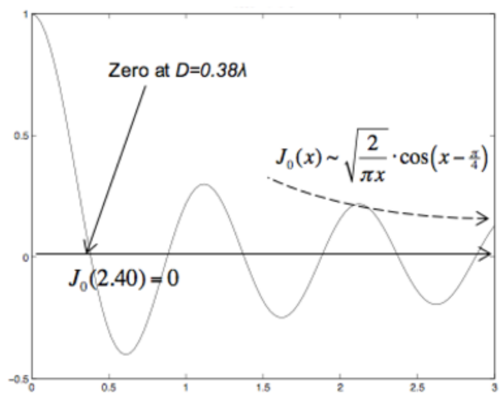

Figure 4.8: Small scale correlation function and Bessel function modeling from the Clarke-Jakes model.

4.5.3 Spectral density

Recall here that the autocorrelation function is related to power spectral density by Fourier transform. The spectral density of the Clarke faded signal is therefore the Fourier transform of the Bessel function seen above:

$$\mathcal{F}(\rho(\tau)) = S_r(f) = \frac{1}{2\pi f_D} \frac{1}{\sqrt{1-(f/f_D)^2}} \tag{4.25}$$

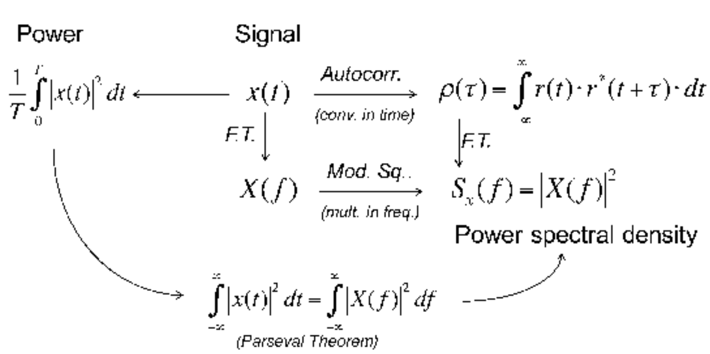

Figure 4.9: A signal autocorrelation function has for Fourier transform the power spectral density.

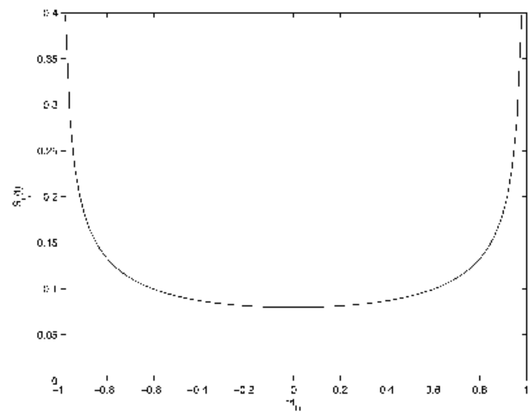

Figure 4.10: Power spectral density for the Clarke-Jakes model.

4.5.4 Diversity Gain

The value of de-correlation between paths lies in the diversity gain achieved when combining these paths. [76] mentions an often quoted result that correlation coefficient should be below 0.7 to achieve good diversity gain. [89] shows that 20λ horizontal separation (or 15λ vertical) leads to de-correlation greater than 0.7 for 93% of the measurements, and so recommends that as acceptable spacing between antennas (though it is much less for lower base station antenna heights as reported above). The paper goes on to estimate diversity gain as a function of correlation, for different diversity schemes:

$$\begin{aligned} G &= 5.71\exp(-0.87\rho - 0.16\Delta) \quad \text{for selection} \\ G &= -8.98 + 15.22\exp(-0.20\rho - 0.04\Delta) \quad \text{for equal-gain} \\ G &= 7.14\exp(-0.59\rho - 0.11\Delta) \quad \text{for max-ratio} \end{aligned} \tag{4.26}$$

where G is the diversity gain at 90% signal reliability (in dB), Δ is the mean signal level difference between paths, and ϱ their cross-correlation. Diversity gain varies with mean signal difference (the closer the signal levels, the better the diversity gain), cross-correlation, and of course combining schemes.

4.6 Homework

1. Assume an environment where lognormal shadowing is defined by standard deviation σ = 6 dB. Using �4.1.2, answer the following questions:
   a. What fade margin is required to achieve 50% edge reliability?
   b. What fade margin is required to achieve 90% edge reliability?
   c. Further assume Hata's model for propagation, which has path loss exponent: $n = 4.49 - 0.655\log(h_{BTS}(m))$, and a 20 m high average base station height. What is the usable surface area reliability with the previous two fade margins?
2. In this problem we consider some simple handoff rules and derive the impact of handoff on system capacity.
   a. A mobile is moving from base station 1 (B_1) to base station 2 (B_2) at speed v. We assume for simplicity that the system has no shadowing and can compensate for all fast fading, so the power received at the base station can be written: $P_r = P_0 - 10n\log_{10}(d/d_0)$ where P_0 and d_0 are reference values (P_0 = 0dBm, d_0 = 1m), and n = 3.6 is the path loss exponent.

      Assume that a call is dropped if the power received by all base stations is below a minimal power P_{min} = -110dBm. Assume that the system initiates handoff when base station B_1 power drops below

P_{HO} = -108dBm. The time required to complete the handoff is Δt=4s.

Question: above what speed v_{max} of the mobile would the call be dropped?

b. With the above distances, in what percentage of the cell coverage is a mobile in handoff situation? (Simply assume circular cells).

3. The popularity of Wi-Fi systems is obvious for small residential or small office applications; but the standard is getting so popular that attempts are being made at widening its use to much larger areas, almost like a cellular system. This problems aims at applying a few concept seen so far to study advantages and disadvantages of such possible wide area Wi-Fi systems.
   a. Some cities are trying to cover entire urban areas with Wi-Fi mesh. What are the main reasons why a service provider would consider using Wi-Fi mesh for major coverage areas rather than a 3G system (like EV-DO)?
   b. What are the main disadvantages of a wide coverage wireless system using Wi-Fi? (consider unlicensed aspects, frequency reuse, link budgets, and other parameters).
   c. An 802.11a System proposes to use 20 MHz channels (TDD) in the new 5.4 GHz frequency block. What advantages / disadvantages does this system have over a 2.4 GHz Wi-Fi system?
   d. Assume you're allowed 4 Watt EIRP for a Wi-Fi system, assume a typical receiver sensitivity of -90dBm for a 20 MHz Wi-Fi channel. Try to build a link budget for such a typical system. Give your answer in a typical link budget table; use any estimates that make sense to you – justify or discuss where you are unsure.
   e. What is the maximum allowable path loss for good outdoor coverage? (Use 90% coverage reliability.)
   f. How many access point per square miles would you recommend to provide good outdoor coverage (Assume a simple one-slope model with path-loss exponent n = 3.)
4. The following table shows a simplified LTE link budget (for data use).

line	Parameter name	value	unit	Comments
a	eNB transmit power	17.4	dBm	per Resource block
b	eNB antenna gain	10.0	dBi	
c	EIRP	**?**	dBm	per RB
d	Body loss	0.0	dB	
e	UE antenna gain	0.0	dBi	For three sectors
f	kT_0B	-122.2	dBm	Noise floor per RB
g	UE noise figure	8.0	dB	Total system NF
h	Required SNR	21.0	dB	For 64QAM
i	System sensitivity	**?**	dBm	?
j	Shadow margin	8.0	dB	Increases edge coverage
k	Interference margin	2.0	dB	Due to load
l	**Maximum path loss**	**?**	dB	?

   a. Is this a forward or reverse link budget? Justify.
   b. What is the EIRP of this system?
   c. What is the system sensitivity? (give a formula in terms of lines in the table, as well as a value in dBm).
   d. What is the maximum allowable path loss? Give a formula in terms of line numbers in the table and a value in dB.
   e. What is a quick justification that body loss for data use is 0dB as opposed to 3dB for voice?
   f. The above link budget SNR achieves high data rate with a 64QAM modulation. The modulation can be lowered to QPSK for instance by lowering SNR requirements to 11dB. Assume a $1/r^{3.6}$ propagation model, what coverage area gain would be achieved? (Calculate the ratio of area coverage A_{QPSK}/A_{64QAM})

5. We consider a very simple atmospheric attenuation model on a link from an earth station to a LEO satellite orbiting earth. The link works at 300GHz, and because satellites are not geostationary, the angles of elevation from the ground to the satellite vary continuously. The total atmospheric attenuation A_z at zenith (that is at an elevation angle of 90 degrees) can be estimated from figure 5.7 from chapter 1, assuming a 2km thick atmosphere.

   When the elevation angle α changes, slant path attenuation varies since the thickness of the atmosphere traversed by the radio link increases. That slant path attenuation is usually approximated by the cosecant law:

$$A(\alpha) = \frac{A_z}{\sin(\alpha)}$$

   (Values are not in dB in the above equation).

   a. Estimate total link attenuation for the satellite at zenith.
   b. Estimate total link attenuation at α=5 degrees (near horizon).
   c. Assume that one satellite is always visible in the sky (between 5 and 90 degrees of elevation), and that the system can handover from one satellite to another. What link budget variation does the system have to deal with to maintain a link continuously?
   d. Does the link budget variation from the previous question change if the frequency of operation is 20GHz?

6. Wireless satellite systems sometimes use a constellation of LEO satellites revolving around the earth. We will focus in this problem on the Doppler effect analysis of LEO satellite systems.
   Let us consider a system like Globalstar, operating at 1.6GHz, with satellites 1400km above ground, and with a rotation period of approximately 2 hours. (Also remember the average earth radius is approximately 6350km).

   a. What are LEO satellites? What are their main advantages? What are their main disadvantages?
   b. Calculate the speed of such a satellite. (In the remainder we assume this speed constant, and we assume that the rotation of earth is much smaller and therefore negligible).
   c. We now suppose that we have an earth station (such as a satellite handset) placed at location R. When the satellite appears at the horizon, at H_1 on figure 4.11, what is the angle α?
   d. What is the Doppler shift when the satellite appears at the horizon (H_1)? (Note: for each of these Doppler shift questions, specify if the shift is positive or negative).
   e. What is the Doppler shift at zenith (Z)?
   f. What is the Doppler shift when the satellite disappears at the horizon (H_2)?
   g. Conclude on the total amount of Doppler spread that such a LEO satellite system has to handle.
   h. If we now consider Iridium instead of Globalstar, the system operates at 1.6GHz also, with satellites 800km above ground, and at 27,000km/hr. What is that systems total Doppler spread?

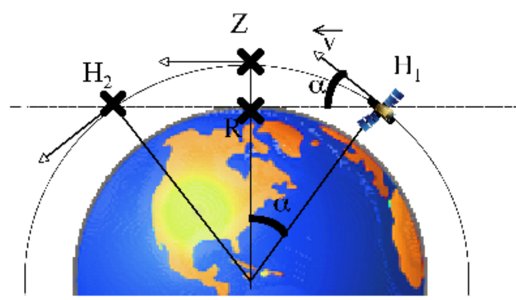

Figure 4.11: Doppler geometry for LEO satellites.

7. We examine the advantages of inserting a tower-top low-noise amplifier (LNA) at a cell site.
    a. For a wireless base station, the noise floor of a system may be expressed as $kT_{eff}B$, where T_{eff} is an effective temperature that may be calculated as follows: (Be careful: values in formulas are not in dB).
    For a conventional CDMA base station:
    $$T_{eff} = T_{ant} + T_c + T_{BTS} \cdot L_c \tag{4.27}$$
    Assume the following values:
        - Antenna temperature: $T_{ant} = 50K$,
        - Cable effective temperature and loss: $T_c = 300K$,$L_c = 2dB$,
        - BTS effective temperature: $T_{BTS} = 1200K$ (equivalent to a 7dB noise figure),
        - cdmaOne channel width: B=1.25MHz

    What is the noise floor of that conventional cdmaOne base station?
    b. In some case, that noise floor is considered too high, and one tries to reduce it by inserting a low-noise amplifier. For a base station with an added low-noise amplifier (LNA) the effective temperature of the system is calculated by:
    $$T_{eff+LNA} = T_{ant} + T_{LNA} + \frac{T_c + T_{BTS} \cdot L_c}{G_{LNA}} \tag{4.28}$$
    Assume the same values as above, and:
        - LNA effective temperature $T_{LNA} = 150K$,
        - LNA gain: $G_{LNA} = 20dB$,

    What is the noise floor of that system with LNA?
    c. What is the sensitivity improvement (noise floor improvement) of the receiving system when the LNA is inserted? Give the result in dB.
    d. This improvement translates into a direct gain in the total link budget. That gain may be seen as an improvement in coverage. Assume a $1/r^3$ propagation model (that is a one-slope model, with path loss exponent $n = 3$); what is the cell radius improvement?
    e. Yet another use may be in terms of service reliability. If we assume a log-normal shadowing of σ = 8dB, and if we have an 6dB fade margin before installing the LNA, what is edge coverage before installing the LNA?
    f. What is the improvement in edge coverage probability achieved by the LNA? Give the result $P(LNA)$ - $P(noLNA)$.
8. A wireless system operates at frequency f_c = 1GHz, for each case below, determine what type of small-scale fading occurs (fast or slow; flat or frequency-selective).
    a. User browses at data rate R = 1Mbps, walking around in an urban environment.
    b. User is on a voice call at data rate R = 5kbps, driving on a highway.
9. You need to design a radio system for public safety that operates at 700MHz and has to be able to handle a certain amount of Doppler spread (defined as twice the maximum Doppler shift).
    a. What maximum Doppler spread do you have to handle if you want fast-moving emergency vehicle to be able to communicate while driving at 120 miles per hour?
    b. Public safety can also use 4.9GHz, and you propose to use your same design. Assuming the same Doppler spread limitation, at what maximum speed can you communicate at 4.9GHz?
    c. Conclude on the importance of the carrier frequency in your choice of wireless standard and design.