# Homework 7

Due 5/3

## Problem 1.

Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

### 1-1.

Confirm that $AB = BA$.

### 1-2.

Diagonalize $A$ and confirm that $\mathbf{e}_1 = (1, 0, 0), \mathbf{e}_2 = (0, 1, 0)$ are eigenvectors for eigenvalue $1$.

### 1-3.

Let $V = \mathrm{span}\{\mathbf{e}_1, \mathbf{e}_2\}$. Let $B|_V$ be the restriction of $B$ onto $V$. Confirm that $B|_V$ is written as a $2 \times 2$ matrix

$$B|_V = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

with respect to the basis $\{\mathbf{e}_1, \mathbf{e}_2\}$, and the eigenvalues are $1, 3$.

### 1-4.

Find the eigenvector matrix $S$ such that $S^{-1}AS, S^{-1}BS$ are simultaneously diagonalized.

## Problem 2.

Find the stead-state solution of

$$A = \begin{bmatrix} .4 & .2 \\ .6 & .8 \end{bmatrix}$$

## Problem 3.

Decide the stability of the system

$$A = \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix},$$

and find the solution of $u' = Au$ for $u_0 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$.