Homework 8

Due 19/5/17

Problem 1

Let $\mathcal{B}_1 = \{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3\}$ be the canonical basis of $\mathbf{R}^3$. Let

$$\mathcal{B}_2 = \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$$

be another basis of $\mathbf{R}^3$.

1-1.

Let v be a vector in $\mathbf{R}^3$ such that $[v]_{\mathcal{B}_2} = \begin{bmatrix} -1 \\ 1 \\ 1 \end{bmatrix}$. Express $[v]_{\mathcal{B}_1}$.

1-2.

Find the **change of basis** $M = [Id]_{\mathcal{B}_1}^{\mathcal{B}_2}$.

1-3.

Let $T : \mathbf{R}^3 \to \mathbf{R}^3$ be a linear transformation such that

$$[T]_{\mathcal{B}_1}^{\mathcal{B}_1} = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

Write $[T]_{\mathcal{B}_2}^{\mathcal{B}_2}$ as 3×3 matrix.

Problem 2

2-1.

Find a unitary matrix U and a upper triangular matrix T such that $U^{-1}AU = T$ for

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 2 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

2-2.

Find a hermitian matrix U and a upper triangular matrix T such that $U^{-1}AU = T$ for

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Problem 3

3-1.

Find eigenvalues and Jordan blocks of the following Jordan form:

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

3-2.

Find a nonsingular matrix M and a Jordan form J such that $M^{-1}AM = J$ for

$$A = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & 1 & 0 \end{bmatrix}$$

2-2.

Find a nonsingular matrix M and a Jordan form J such that $M^{-1}AM = J$ for

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Problem 4

Explain reason if true, provide counter example if false.

1. If a $n \times n$ matrix A has n linearly independent eigenvectors, then A is diagonalizable.
2. If A is similar to a Jordan matrix with r Jordan blocks, then A has r distinct eigenvalues.
3. All $n \times n$ matrix is diagonalizable if we count all complex eigenvalues.