Homework 9

Due 5/24

Problem 1.

FInd SVD for the following matrices.

1-1.

$$A = \begin{bmatrix} 1101 \end{bmatrix}$$

1-2.

$$\begin{bmatrix} 110 \\ 011 \end{bmatrix}$$

1-3.

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

Problem 2.

Let $A = U\Sigma V^T$ be the SVD of $m \times n$ matrix A. Let $u_1, \cdots, u_m$ be the column vectors of U and $v_1, \cdots, v_n$ be the column vectors of V.

2-1.

Show that A can be expressed as

$$A = u_1\sigma_1 v_1^T + \cdots + u_r\sigma_r v_r^T$$

where $\sigma_1, \cdots, \sigma_r$ is the nonzero diagonal entries of Σ.

2-2.

Explain how r is determined in the equation above.

Problem 3.

3-1.

Prove that symmetric matrix is diagonalizable.

3-2.

Show that the number of nonzero entries in Σ in the SVD is the rank of A.

Problem 4.

In terms of basis change, explain the meaning following decompositions:

4-1.

LDU decomposition: $A = LDU$

4-2.

QR decomposition: $A = QR$

4-3.

Triangularization by Schur's lemma: $U^{-1}AU = T$

4-4.

Jordan form: $M^{-1}AM = J$

4-5.

SVD: $A = U\Sigma V^T$