HOMEWORK 02

Due 19/3/22 (11:59)

Homework must be uploaded to lms (https://lms.dgist.ac.kr).

Problem 1

1-1.

Find a basis for the orthogonal complement of the row space of A.

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & 1 & 4 \end{bmatrix}$$

1-2.

Construct a matrix A with each property or say why that is impossible.

1. Column space $C(A)$ contains $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix}$, nullspace contains $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$
2. Row space $R(A)$ contains $\begin{bmatrix} 1 \\ 2 \\ -3 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -3 \\ 5 \end{bmatrix}$, nullspace contains $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$
3. $Ax = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$ has a solution and $A^T \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$.
4. Every row of A is orthogonal to every column (A is not the zero matrix.)
5. The columns of A add up to a column of 0s, the rows of A add to a row of 1s.

Problem 2

2-1.

By choosing the correct vector b in the Schwarz inequality, prove that

$$(a_1 + \cdots + a_n)^2 \le n(a_1^2 + \cdots + a_n^2)$$

and explain when does equality hold.

2-2.

Compute the projection matrices P_1, P_2 onto the lines through $a_1 = (1, 2, 2)$ and $a_2 = (2, 2, -1)$. Explain what does the multiple $P_1 P_2$ mean.

Problem 3

The following is to show that if a $n \times n$ matrix A has a left-inverse (that is, there is a $n \times n$ matrix B such that $BA = I_n$ where I_n is the identity matrix), then A has a right-inverse (that is, there is a $n \times n$ matrix C such that $AC = I_n$).

3-1.

Suppose that $A = LDU$ is the LDU-decomposition of A. Prove that the LDU-decomposition of A^T is $U^T DL^T$. (Use the uniqueness of LDU-decomposiiton)

3-2.

Show that A^T has a left inverse if and only if A has a right inverse.

3-3.

Let $A = LDU$ is the LDU-decomposition of A. Prove that A has a left-inverse if and only if all diagonal entries of D are not zero.

The conclusion (no need to prove, just read)

Let A be a $n \times n$ matrix with left inverse B. Then by 3-3, we know that the diagonal matrix D in LDU-decomposition of A has nonzero diagonal entries. By 3-1, we also know that the diagonal matrix in LDU-decomposition of A^T also has nonzero diagonal entries. Using 3-3 again (replace A by A^T), we know that A^T has a left inverse. Finally, by 3-2, we know that A has a right inverse.

REMARK

The definition of a inverse matrix requires that the inverse must be both left and right-inverse. The argument above shows that if a left-inverse exist, then a right-inverse exists. (The converse is also true: if a right-inverse exists, then a left-inverse also exists.) As we have shown in the class, the left-inverse and right-inverse are the same. Thus to check if a matrix is an inverse of a given matrix, we only need to show that it is either left inverse or right inverse.