

# spring19-se201-hw1

March 2, 2019

Spring 19 SE201 (Hyosang Kang) Due 3/15 23:59

## 1 Homework Assignment #1

### 1.1 Problem 1.

#### 1.1.1 1-1.

Find a third equation that can't be solved if  $x + y + z = 0$  and  $x - 2y - z = 1$ . Explain how three planes are aligned in the 3-dimensional space.

#### 1.1.2 1-2.

Apply elimination to the system

$$u + v + w = -2$$

$$3u + 3v - w = 6$$

$$u - v + w = -1$$

### 1.2 Problem 2.

#### 1.2.1 2-1.

Prove that the product of two lower triangular matrices is lower triangular.

#### 1.2.2 2-2.

Let  $A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 0 \\ -2 & 2 & 0 \end{bmatrix}$ . Find constants  $a, b, c$  such that

$$E_{32}(a)E_{31}(b)E_{21}(c)A = U$$

where  $U$  is a lower triangular matrix.

### 1.3 Problem 3.

#### 1.3.1 3-1.

Factor  $A = \begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{bmatrix}$  into  $LU$ , and solve

$$Ax = A \begin{bmatrix} u \\ v \\ w \end{bmatrix} = b = \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix}$$

using series of two equations  $Lc = b$  and  $Ux = c$ .

#### 1.3.2 3-2.

Find  $L, U$  for the symmetric matrix

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$$

and find four conditions on  $a, b, c, d$  to get  $A = LU$  with four pivots.

### 1.4 Problem 4.

#### 1.4.1 4-1.

Use Gauss-Jordan method to invert

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

#### 1.4.2 4-2.

Prove that  $A$  is invertible if  $a \neq 0$  and  $a \neq b$ , and find the pivots and  $A^{-1}$ .

$$A = \begin{bmatrix} a & b & b \\ a & a & b \\ a & a & a \end{bmatrix}$$

#### 1.4.3 4-3.

True or false (with a counterexample if false and a reason if true):

1. A 4-by-4 matrix with a row of zeros is not invertible.
2. A matrix with 1s down the main diagonal is invertible.
3. if  $A$  is invertible then  $A^{-1}$  is invertible.
4. If  $A^T$  is invertible then  $A$  is invertible.

## 1.5 Problem 5

### 1.5.1 5-1.

Answer the six questions for

$$A = \begin{bmatrix} 2 & 4 & 6 & 4 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$$

1. Reduce  $[A \ d]$  to  $[U \ c]$  where  $U$  is the row echelon form of  $A$ .
2. Find the condition on  $b_1, b_2, b_3$  to have a solution.
3. Describe the column space of  $A$  in  $\mathbf{R}^3$ .
4. Describe the null space of  $A$  and find special solutions in  $\mathbf{R}^4$ .
5. Find a particular solution.
6. Reduce  $[U \ c]$  to  $[R \ d]$  where  $R$  is the reduced row echelon form, and derive special solutions from  $R$  and particular solution from  $d$ .

### 1.5.2 5-2.

Prove that for  $n \times m$  matrix  $A$  and  $m \times l$  matrix  $B$ , the rank of  $AB$  is less than or equal to the rank of  $A$  and  $B$ .

## 1.6 Problem 6

### 1.6.1 6-1.

Find the dimension of column space and row space of

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 1 \\ 3 & 1 & -1 \end{bmatrix}$$

### 1.6.2 6-2.

Find a basis for each of these subspaces of  $\mathbf{R}^4$ .

1. All vectors whose components are equal.
2. All vectors whose components add to zero.
3. All vectors that are perpendicular to  $(1, 1, 0, 0)$  and  $(1, 0, 1, 1)$ .
4. The null space of  $U = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$