spring19-se201-week-02

March 2, 2019

1 Week 02

- Period: 19/3/4 - 19/3/8
- Lecture
- Vectors and vector spaces
- Solving $Ax = b$
- Basis and dimensions
- Recitation
- Python package `numpy`
- Python package `matplotlib`

1.1 Lecture 03 (19/3/4)

1.1.1 To Do

- Review previous lecture

$$\begin{bmatrix} 2 & 1 & 1 \\ 4 & -6 & 0 \\ -2 & 7 & 2 \end{bmatrix} = \begin{bmatrix} 1 & & \\ 2 & 1 & \\ -1 & -1 & 1 \end{bmatrix} \begin{bmatrix} 2 & 1 & 1 \\ & -8 & -2 \\ & & 1 \end{bmatrix}$$

- Check roster
- Announce recitation classroom E3-317, 318 (Assign students while checking attendency)

1.1.2 Contents (1.5 - 1.6)

- LU decompoisition
- Example:

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 3 \\ 2 & 5 & 8 \end{bmatrix}, \quad P_{23}E_{31}(-2)E_{21}(-1)A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 3 & 6 \\ 0 & 0 & 2 \end{bmatrix}$$

- Permutation matrices
- **Q** Let A be a nonsingular matrix and P be the multiple of all permutation matrices during the reduction of A to row echelon form (REF). Show that PA can be decomposed to LU.
- Uniqueness of LDU decomposition

- Transpose and symmetric matrices
- Property : $(AB)^T = B^T A^T$, $(A^{-1})^T = (A^T)^{-1}$
- LDU factorization of symmetric matrix: LDL^T
- **Q** Re-visit the question: if a left (or right) inverse exists, then a right (or left) inverse exists.
- Gauss-Jordan elimination
- Reduced row echelon from (RREF)
  - All pivots are 1 and pivot columns are placed in order.
  - 0 entries above and below the pivots.
- Finding the inverse of

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 3 \\ 2 & 5 & 8 \end{bmatrix}$$

1.1.3 Timeline

- (0 ~ 5 min) Review
- (5 ~ 10 min) Questions, roster
- (10 ~ 55 min) Lecture on contents (English)
- LU decomposition (15 min)
- Transpose and symmetric matrices (15 min)
- Gauss-Jordan elimination (15 min)
- (55 ~ 75 min) Discussion (Korean)
- $PA = LU$. (Hint: see how elementary row operation matrix E 'commutes' with P.)
- For a nonsingular matrix A, if a matrix B satisfying $BA = I$ exists, then there is a matrix C such that $AC = I$. (Hint: use the uniqueness of the LDU decomposition.)

1.2 Lecture 04 (19/3/7)

1.2.1 To Do

- Review previous lecture
- Check roster

1.2.2 Contents (2.1 - 2.2)

- Vectors
- Vector addition, scalar multiplications
- Inner product
  - **Q** Show that for any two vectors v, w with internal angle θ, the following holds.

$$v \cdot w = \|v\| \|w\| \cos\theta$$

- linear combination, linear independence
- Vector spaces

- Difference between Euclidean space
- Working example: $R = \begin{bmatrix} 1 & 3 & 0 & -1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$
- Column space $C(A)$.
- Null space $N(A)$ and null matrix.
  - **Q** Why is $N(A)$ a vector space?
- Solving $Ax = b$
- Working example: $A = \begin{bmatrix} 1 & 3 & 3 & 2 \\ 2 & 6 & 9 & 7 \\ -1 & -3 & 3 & 4 \end{bmatrix}, \quad b = \begin{bmatrix} 1 \\ 5 \\ 5 \end{bmatrix}$
- Six steps to find solution of $Ax = b$.
  - Reduce $[A \ b]$ to REF.
  - Find condition on b.
  - Describe column space of A.
  - Describe null space of A.
  - Reduce REF to RREF and find special and particular solutions.
- The rank

1.2.3 Timeline

- (0 ~ 5 min) Review
- (5 ~ 10 min) Questions, roster
- (10 ~ 55 min) Lecture on contents (English)
- Vectors (10 min)
- Vector spaces (10 min)
- Solving $Ax = b$ (25 min)
- (55 ~ 75 min) Discussion (Korean)
- $v \cdot w = \|v\| \|w\| \cos\theta$ (Hint: show for 2-dimensional case, and generalize.)
- $N(A)$ is a vector space. (Hint: what is the definition of a vector space?)