# Homework 5

Due 19/4/12

# Problem 1.

## 1-1.

Find the determinant of

$$\begin{bmatrix} 1 \\ 4 \\ 2 \end{bmatrix} \begin{bmatrix} 2 & -1 & 2 \end{bmatrix}, \quad \text{and} \begin{bmatrix} 4 & 4 & 8 & 8 \\ 0 & 1 & 2 & 2 \\ 0 & 0 & 2 & 6 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

## 1-2.

Find the determinant of

$$\begin{bmatrix} 101 & 201 & 301 \\ 102 & 202 & 302 \\ 103 & 203 & 303 \end{bmatrix}, \text{ and} \begin{bmatrix} 1 & t & t^2 \\ t & 1 & t \\ t^2 & t & 1 \end{bmatrix}$$

# Problem 2.

## 2-1.

True or false, with proof if true and counterexample if false:

1. If $A$ and $B$ are identical except that $B_{11} = 2A_{11}$, then $\det B = 2 \det A$.
2. The determinant is the product of the pivots.
3. If $A$ is invertible and $B$ is singular, then $A + B$ is invertible.
4. If $A$ is invertible and $B$ is singular, then $AB$ is singular.
5. The determinant of $AB - BA$ is zero.

## 2-2.

Find the determinant of

$$\begin{bmatrix} 11 & 12 & 13 & 14 \\ 21 & 22 & 23 & 24 \\ 31 & 32 & 33 & 34 \\ 41 & 42 & 43 & 44 \end{bmatrix}, \text{ and} \begin{bmatrix} 1 & t & t^2 & t^3 \\ t & 1 & t & t^2 \\ t^2 & t & 1 & t \\ t^3 & t^2 & t & 1 \end{bmatrix}$$

# Problem 3.

## 3-1.

Let $\sigma$ be the permutation $\sigma = (5, 4, 1, 2, 3)$.

1. Compute $\sigma^2$.
2. Compute $\sigma^{-1}$.

### 3-2.

Find the inverse matrices of

$$\begin{bmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{bmatrix}, \text{ and} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$$

## Problem 4.

In this problem, we assume that the determinant of $n \times n$ matrix $A$ is defined by

$$\det A = \sum_{\sigma} \operatorname{sgn}(\sigma) A_{1\sigma(1)} \cdots A_{n\sigma(n)}.$$

### 4-1.

Let $\sigma$ be a permutation on $\{1, \cdots, n\}$. Show that

$$\operatorname{sgn}(\sigma) = \operatorname{sgn}(\sigma^{-1})$$

### 4-2.

Using 4-1, show that

$$\det A = \sum_{\sigma} \operatorname{sgn}(\sigma) A_{\sigma(1)1} \cdots A_{\sigma(n)n}.$$

### 4-3. ¶

Using a similar idea in class, show that

$$\det A = \sum_{k=1} A_{k1} C_{k1}$$

### Question (no need to submit)

How will you generalize the argument made in problem 4-2 to prove

$$\det A = \sum_{k=1} A_{kj} C_{kj}, \qquad \text{for all } j = 1, \cdots, n$$