Homework 6

Due 4/26(Fri) 11:59

Problem 1

1-1.

Find the eigenvalues of A, B, and C

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 4 & 5 \\ 0 & 0 & 6 \end{bmatrix}, \quad B = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 2 & 0 \\ 3 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 2 & 2 \\ 2 & 2 & 2 \\ 2 & 2 & 2 \end{bmatrix}$$

1-2.

From the unit vector $u = \left(\frac{1}{6}, \frac{1}{6}, \frac{3}{6}, \frac{5}{6}\right)$, construct the rank-$1$ projection matrix $P = uu^T$. And then find three linearly independent eigenvectors of P all with eigenvalue $\lambda = 0$.

Problem 2

2-1.

Let $A = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$. Compute the matrix A^∞ is the limt of A^k as $k \to \infty$, and explain why $A^2 = \frac{1}{2}(A + A^\infty)$.

2-2.

Find all eigenvalues of

$$A = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 2 \\ 4 & 2 & 4 \\ 2 & 1 & 2 \end{bmatrix}$$

Problem 3

3-1.

If $A = \begin{bmatrix} 4 & 3 \\ 1 & 2 \end{bmatrix}$, find A^{100}.

3-2.

Let $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$. Find the eigenvalues of AB and BA.

Problem 4.

4-1.

Diagonalize B and compute $S\Lambda^k S^{-1}$ to prove the formula of B^k:

$$B = \begin{bmatrix} 3 & 1 \\ 0 & 2 \end{bmatrix}, \quad B^k = \begin{bmatrix} 3^k & 3^k - 2^k \\ 0 & 2^k \end{bmatrix}$$

4-2.

Let $A = \begin{bmatrix} 0.6 & 0.4 \\ 0.4 & 0.6 \end{bmatrix}$, $B = \begin{bmatrix} 0.6 & 0.9 \\ 0.1 & 0.6 \end{bmatrix}$. Comput A^∞ and B^∞.