Homework 7

Due 5/10

Problem 1.

Let $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$.

1-1.

Confirm that $AB = BA$.

1-2.

Diagonalize A and confirm that $\mathbf{e}_1 = (1, 0, 0), \mathbf{e}_2 = (0, 1, 0)$ are eigenvectors for eigenvalue 1.

1-3.

Let $V = \operatorname{span}\{\mathbf{e}_1, \mathbf{e}_2\}$. Let $B|_V$ be the restriction of B onto V. Confirm that $B|_V$ is written as a 2×2 matrix

$$B|_V = \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

with respect to the basis $\{\mathbf{e}_1, \mathbf{e}_2\}$, and the eigenvalues are $1, 3$.

1-4.

Find the eigenvector matrix S such that $S^{-1}AS, S^{-1}BS$ are simultaneously diagonalized.

Problem 2.

Find the stead-state solution of

$$A = \begin{bmatrix} .4 & .2 \\ .6 & .8 \end{bmatrix}$$

Problem 3.

Decide the stability of the system

$$A = \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix},$$

and find the solution of $u' = Au$ for $u_0 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$.