Spring 19 SE201 (Hyosang Kang) **Due 3/15 23:59**

Homework Assignment #1

Problem 1.

1-1.

Find a third equation that can't be solved if $x + y + z = 0$ and $x - 2y - z = 1$. Explain how three planes are aligned in the 3-dimensional space.

1-2.

Apply elimination to the system

$$\begin{aligned} u + v + w &= -2 \\ 3u + 3v - w &= 6 \\ u - v + w &= -1 \end{aligned}$$

Problem 2.

2-1.

Prove that the product of two lower triangular matrices is lower triangular.

2-2.

Let $A = \begin{bmatrix} 1 & 1 & 0 \\ 4 & 6 & 0 \\ -2 & 2 & 0 \end{bmatrix}$. Find constants a, b, c such that

$$E_{32}(a)E_{31}(b)E_{21}(c)A = U$$

where U is a lower triangular matrix.

Problem 3.

3-1.

Factor $A = \begin{bmatrix} 2 & 3 & 3 \\ 0 & 5 & 7 \\ 6 & 9 & 8 \end{bmatrix}$ into LU, and solve

$$Ax = A\begin{bmatrix} u \\ v \\ w \end{bmatrix} = b = \begin{bmatrix} 2 \\ 2 \\ 5 \end{bmatrix}$$

using series of two equations $Lc = b$ and $Ux = c$.

3-2.

Find L, U for the symmetric matrix

$$A = \begin{bmatrix} a & a & a & a \\ a & b & b & b \\ a & b & c & c \\ a & b & c & d \end{bmatrix}$$

and find four conditions on a, b, c, d to get $A = LU$ with four pivots.

Problem 4.

4-1.

Use Gauss-Jordan method to invert

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$$

4-2.

Prove that A is invertible if $a \neq 0$ and $a \neq b$, and find the pivots and A^{-1}.

$$A = \begin{bmatrix} a & b & b \\ a & a & b \\ a & a & a \end{bmatrix}$$

4-3.

True or false (with a counterexample if false and a reason if ture):

1. A 4-by-4 matrix with a row of zeros is not invertible.
2. A matrix with 1s down the main diagonal is invertible.
3. if A is invertible then A^{-1} is invertible.
4. If A^T is invertible then A is invertible.

Problem 5

5-1.

Answer the six questions for

$$A = \begin{bmatrix} 2 & 4 & 6 & 4 \\ 2 & 5 & 7 & 6 \\ 2 & 3 & 5 & 2 \end{bmatrix}, \quad b = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 3 \\ 5 \end{bmatrix}$$

1. Reduce $[A\ \ d]$ to $[U\ \ c]$ where U is the row echelon form of A.
2. Find the condition on b_1, b_2, b_3 to have a solution.
3. Describe the column space of A in $\mathbf{R}^3$.
4. Describe the null space of A and find special solutions in $\mathbf{R}^4$.
5. Find a paricular solution.
6. Reduce $[U\ \ c]$ to $[R\ \ d]$ where R is the reduced row echelon form, and derive special solutions from R and particular solution from d.

5-2.

Prove that for $n \times m$ matrix A and $m \times l$ matrix B, the rank of AB is less than or equal to the rank of A and B.

Problem 6

6-1.

Find the dimension of column space and row space of

$$A = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 3 & 1 \\ 3 & 1 & -1 \end{bmatrix}$$

6-2.

Find a basis for each of these subspaces of $\mathbf{R}^4$.

1. All vectors whose components are equal.
2. All vectors whose components add to zero.
3. All vectors that are perpendicular to $(1, 1, 0, 0)$ and $(1, 0, 1, 1)$.
4. The null space of $U = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{bmatrix}$