Lecture 5: Model-Free Control

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Outline

Model-Free Reinforcement Learning

- Last lecture:
  - Model-free prediction
  - *Estimate* the value function of an *unknown* MDP
- This lecture:
  - Model-free control
  - *Optimise* the value function of an *unknown* MDP

Uses of Model-Free Control

Some example problems that can be modelled as MDPs

- Elevator
- Parallel Parking
- Ship Steering
- Bioreactor
- Helicopter
- Aeroplane Logistics
- Robocup Soccer
- Quake
- Portfolio management
- Protein Folding
- Robot walking
- Game of Go

For most of these problems, either:

- MDP model is unknown, but experience can be sampled
- MDP model is known, but is too big to use, except by samples

Model-free control can solve these problems

On and Off-Policy Learning

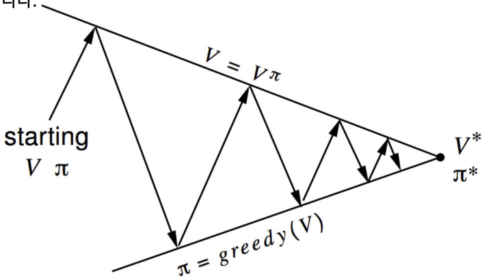
On-policy learning은 자신이 직접 시행착오를 겪으면서 스스로 배우는 것에 비유를 할 수 있습니다. 동일한 policy인 파이에 대하여 샘플링된 경험을 따르면서 이를 통해서 학습을 하는 방식을 의미합니다.
Off-policy learning은 다른이가 시행착오를 겪는것을 보면서 배우는 것에 비유를 할 수 있습니다. 다른 policy인 뮤에 대해서 샘플링된 경험을 따르면서 자신의 policy 파이를 학습하는 방식을 의미합니다.
시행착오를 하는 정책과 학습하는 정책이 다른 것입니다.

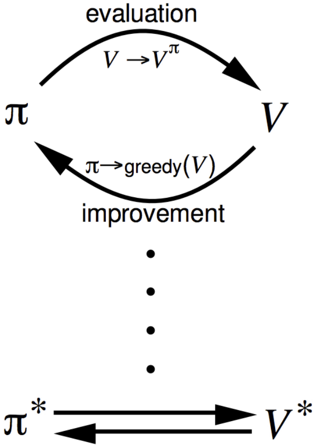
- On-policy learning
  - "Learn on the job"
  - Learn about policy π from experience sampled from π
- Off-policy learning
  - "Look over someone's shoulder"
  - Learn about policy π from experience sampled from μ

다른 policy

Generalised Policy Iteration (Refresher)

다이나믹 프로그래밍에서 본것과 같이 policy 가 반복적으로 진행이 되면서 evaluation과 improvement를 수행하면서 value function을 추정하고 최적화된 policy를 업데이트합니다. 이를 계속하면 최적화된 점으로 수렴을 하게 되고 이때가 가장 optimal policy를 찾을 수 있었습니다.





Policy evaluation Estimate v_π

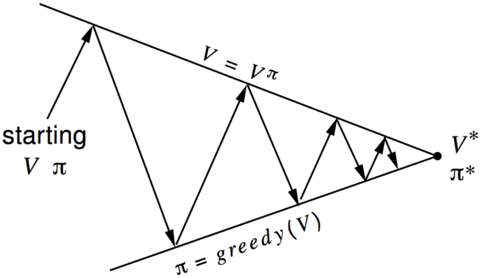
e.g. Iterative policy evaluation

Policy improvement Generate $\pi' \geq \pi$

e.g. Greedy policy improvement

Generalised Policy Iteration With Monte-Carlo Evaluation

이러한 업데이트 방식을 몬테카를로의 방식에 적용을 해볼 수 있습니다. 몬테카를로 방식은 자신이 경험한 경로에 평균값을 value로 업데이트를 했습니다. 이와 같은 방식에서 발생하는 문제점은 경로가 매우 긴 에피소드의 경우에 연산이 오래걸리는 단점이 있고, 또 greedy한 policy를 따르기 때문에 경험하지 못한 경로에 대해서는 알수 없게 됩니다.



greedy한 경로를 따르므로

exploration 문제가 생김.

Policy evaluation Monte-Carlo policy evaluation, $V = v_\pi$?

Policy improvement Greedy policy improvement?

Model-Free Policy Iteration Using Action-Value Function

상단에 V(s)는 state에 대한 value function을 추정해야하기 때문에 MDP 모델을 사용하게 되는 형태입니다.
하단에 Q(s,a)는 s state에 대하여 a action을 취했을때 가장 큰 Q 값을 갖는 best action을 사용하게 되므로 model-free에 해당하는 방식입니다.

- Greedy policy improvement over $V(s)$ requires model of MDP

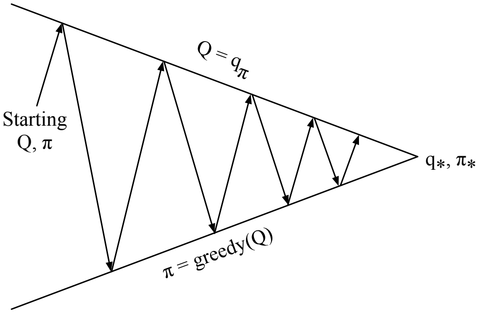
$$\pi'(s) = \underset{a \in \mathcal{A}}{\operatorname{argmax}}\ \mathcal{R}_s^a + \gamma \mathcal{P}_{ss'}^a V(s')$$

- Greedy policy improvement over $Q(s,a)$ is model-free

$$\pi'(s) = \underset{a \in \mathcal{A}}{\operatorname{argmax}}\ Q(s,a)$$

Generalised Policy Iteration with Action-Value Function

V 대신에 Q 를 사용해서 state-action value 를 업데이트 하도록 하여 policy evaluation과 improvement를 반복합니다. 최적의 q*를 찾게 되면 최적의 policy를 찾을 수 있습니다.



Policy evaluation Monte-Carlo policy evaluation, $Q = q_\pi$

Policy improvement Greedy policy improvement?

Example of Greedy Action Selection

"Behind one door is tenure - behind the other is flipping burgers at McDonald's."

- There are two doors in front of you.
- You open the left door and get reward 0
  $V(left) = 0$
- You open the right door and get reward +1
  $V(right) = +1$
- You open the right door and get reward +3
  $V(right) = +2$
- You open the right door and get reward +2
  $V(right) = +2$

$$\vdots$$

- Are you sure you've chosen the best door?

ϵ-Greedy Exploration

- Simplest idea for ensuring continual exploration
- All m actions are tried with non-zero probability
- With probability $1 - \epsilon$ choose the greedy action
- With probability ϵ choose an action at random

$$\pi(a|s) = \begin{cases} \epsilon/m + 1 - \epsilon & \text{if } a^* = \underset{a \in \mathcal{A}}{\operatorname{argmax}}\ Q(s,a) \\ \epsilon/m & \text{otherwise} \end{cases}$$

ϵ-Greedy Policy Improvement

Theorem

For any ϵ-greedy policy π, the ϵ-greedy policy π' with respect to q_π is an improvement, $v_{\pi'}(s) \geq v_\pi(s)$

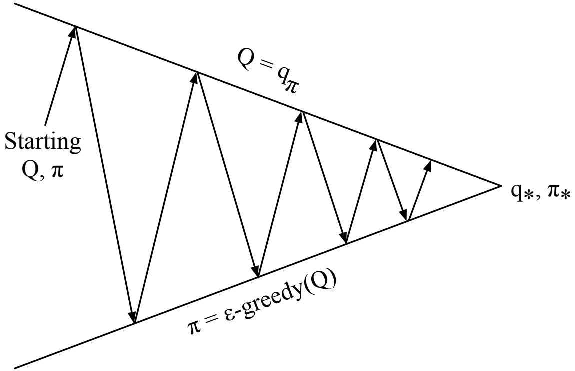
Bellman

$$\epsilon\left[\frac{1}{m}\sum_{a\in\mathcal{A}} q_\pi(s,a)\right]$$

$$\begin{aligned}
q_\pi(s, \pi'(s)) &= \sum_{a \in \mathcal{A}} \pi'(a|s) q_\pi(s, a) \\
&= \epsilon/m \sum_{a \in \mathcal{A}} q_\pi(s, a) + (1 - \epsilon) \max_{a \in \mathcal{A}} q_\pi(s, a) \\
&\geq \epsilon/m \sum_{a \in \mathcal{A}} q_\pi(s, a) + (1 - \epsilon) \sum_{a \in \mathcal{A}} \frac{\pi(a|s) - \epsilon/m}{1 - \epsilon} q_\pi(s, a) \\
&= \sum_{a \in \mathcal{A}} \pi(a|s) q_\pi(s, a) = v_\pi(s)
\end{aligned}$$

Therefore from policy improvement theorem, $v_{\pi'}(s) \geq v_\pi(s)$

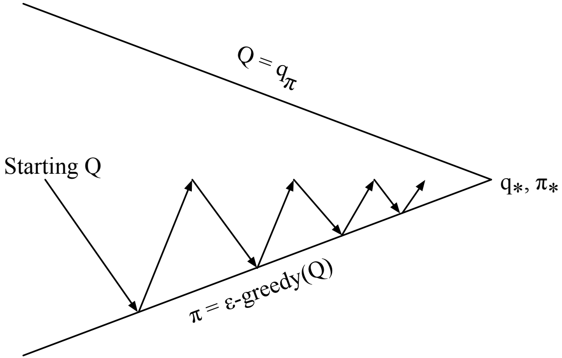
Monte-Carlo Policy Iteration



Policy evaluation Monte-Carlo policy evaluation, $Q = q_\pi$

Policy improvement ϵ-greedy policy improvement

Monte-Carlo Control



Every episode:

Policy evaluation Monte-Carlo policy evaluation, $Q \approx q_\pi$

Policy improvement ϵ-greedy policy improvement

GLIE

Definition

Greedy in the Limit with Infinite Exploration (GLIE)

- All state-action pairs are explored infinitely many times,

$$\lim_{k \to \infty} N_k(s, a) = \infty$$ about exploration

- The policy converges on a greedy policy,

about exploitation

$$\lim_{k \to \infty} \pi_k(a|s) = \mathbf{1}(a = \underset{a' \in \mathcal{A}}{\operatorname{argmax}} \; Q_k(s, a'))$$

- For example, ϵ-greedy is GLIE if ϵ reduces to zero at $\epsilon_k = \frac{1}{k}$

GLIE Monte-Carlo Control

- Sample kth episode using π: $\{S_1, A_1, R_2, ..., S_T\} \sim \pi$
- For each state S_t and action A_t in the episode,

$$N(S_t, A_t) \leftarrow N(S_t, A_t) + 1$$
$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \frac{1}{N(S_t, A_t)} (G_t - Q(S_t, A_t))$$

- Improve policy based on new action-value function

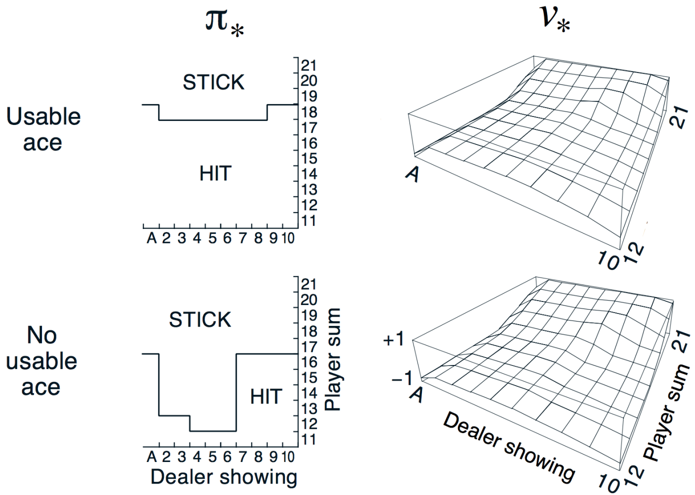
$$\epsilon \leftarrow 1/k$$
$$\pi \leftarrow \epsilon\text{-greedy}(Q)$$

Theorem

GLIE Monte-Carlo control converges to the optimal action-value function, $Q(s, a) \rightarrow q_*(s, a)$

Back to the Blackjack Example

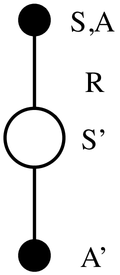
Monte-Carlo Control in Blackjack



MC vs. TD Control

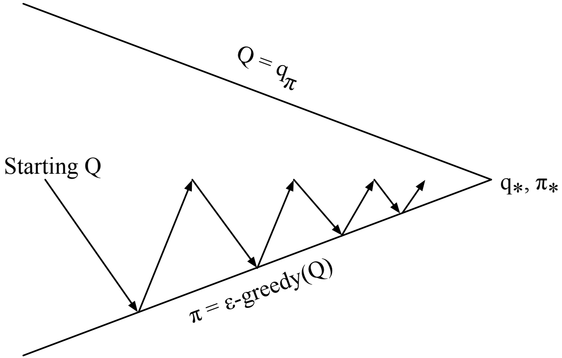
- Temporal-difference (TD) learning has several advantages over Monte-Carlo (MC)
    - Lower variance
    - Online
    - Incomplete sequences
- Natural idea: use TD instead of MC in our control loop
    - Apply TD to $Q(S, A)$
    - Use ϵ-greedy policy improvement
    - Update every time-step

Updating Action-Value Functions with Sarsa



$$Q(S,A) \leftarrow Q(S,A) + \underbrace{\alpha}_{\text{step size}} \left(R + \gamma Q(S',A') - Q(S,A)\right)$$

On-Policy Control With Sarsa



Every time-step:

Policy evaluation Sarsa, $Q \approx q_\pi$

Policy improvement ϵ-greedy policy improvement

Sarsa Algorithm for On-Policy Control

Initialize $Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
    Initialize S
    Choose A from S using policy derived from Q (e.g., ε-greedy)
    Repeat (for each step of episode):
        Take action A, observe R, S'
        Choose A' from S' using policy derived from Q (e.g., ε-greedy)
        $Q(S,A) \leftarrow Q(S,A) + \alpha\left[R + \gamma Q(S',A') - Q(S,A)\right]$
        $S \leftarrow S'$; $A \leftarrow A'$;
    until S is terminal

Convergence of Sarsa

Theorem

Sarsa converges to the optimal action-value function, $Q(s,a) \rightarrow q_*(s,a)$, under the following conditions:

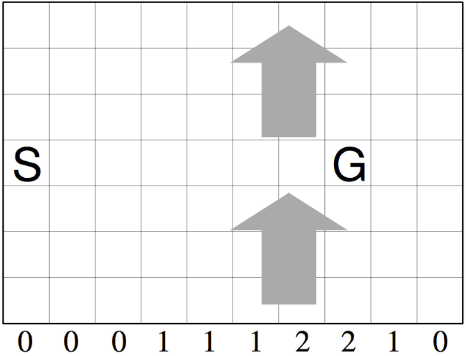
- *GLIE sequence of policies $\pi_t(a|s)$*
- *Robbins-Monro sequence of step-sizes α_t*

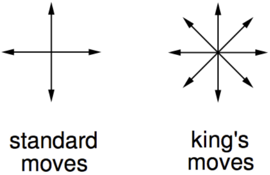
$$\sum_{t=1}^{\infty} \alpha_t = \infty$$

$$\sum_{t=1}^{\infty} \alpha_t^2 < \infty$$

실제적용에서는
고려하지 않아도 잘 찾는다.

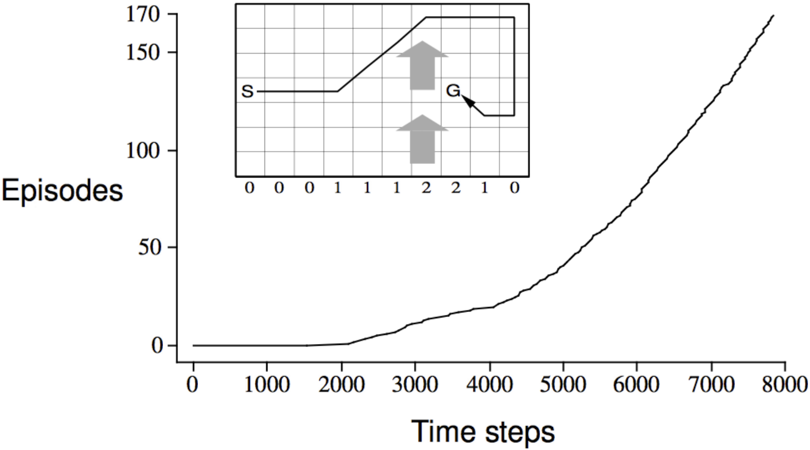
Windy Gridworld Example





- Reward = -1 per time-step until reaching goal
- Undiscounted

Sarsa on the Windy Gridworld



n-Step Sarsa

- Consider the following n-step returns for $n = 1, 2, \infty$:

$$\begin{array}{lll} n = 1 & (Sarsa) & q_t^{(1)} = R_{t+1} + \gamma Q(S_{t+1}) \\ n = 2 & & q_t^{(2)} = R_{t+1} + \gamma R_{t+2} + \gamma^2 Q(S_{t+2}) \\ \vdots & & \vdots \\ n = \infty & (MC) & q_t^{(\infty)} = R_{t+1} + \gamma R_{t+2} + ... + \gamma^{T-1} R_T \end{array}$$

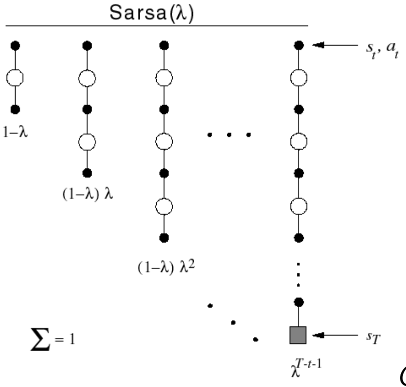
- Define the n-step Q-return

$$q_t^{(n)} = R_{t+1} + \gamma R_{t+2} + ... + \gamma^{n-1} R_{t+n} + \gamma^n Q(S_{t+n})$$

- n-step Sarsa updates $Q(s, a)$ towards the n-step Q-return

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left(q_t^{(n)} - Q(S_t, A_t) \right)$$

Forward View Sarsa(λ)



- The q^λ *return* combines all n-step Q-returns $q_t^{(n)}$
- Using weight $(1-\lambda)\lambda^{n-1}$

$$q_t^\lambda = (1-\lambda)\sum_{n=1}^{\infty} \lambda^{n-1} q_t^{(n)}$$

- Forward-view Sarsa(λ)

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \left(q_t^\lambda - Q(S_t, A_t) \right)$$

Backward View Sarsa(λ)

- Just like TD(λ), we use eligibility traces in an online algorithm
- But Sarsa(λ) has one eligibility trace for each state-action pair

$$
\begin{aligned}
E_0(s, a) &= 0 \\
E_t(s, a) &= \gamma \lambda E_{t-1}(s, a) + \mathbf{1}(S_t = s, A_t = a)
\end{aligned}
$$

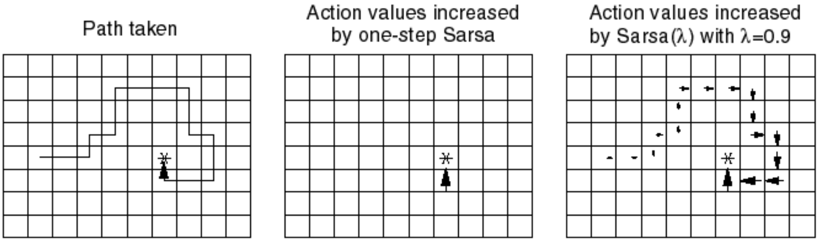
- $Q(s, a)$ is updated for every state s and action a
- In proportion to TD-error δ_t and eligibility trace $E_t(s, a)$

$$
\begin{aligned}
\delta_t &= R_{t+1} + \gamma Q(S_{t+1}, A_{t+1}) - Q(S_t, A_t) \\
Q(s, a) &\leftarrow Q(s, a) + \alpha \delta_t E_t(s, a)
\end{aligned}
$$

Sarsa(λ) Algorithm

Initialize $Q(s,a)$ arbitrarily, for all $s \in \mathcal{S}, a \in \mathcal{A}(s)$
Repeat (for each episode):
  $E(s,a) = 0$, for all $s \in \mathcal{S}, a \in \mathcal{A}(s)$
  Initialize S, A
  Repeat (for each step of episode):
    Take action A, observe R, S'
    Choose A' from S' using policy derived from Q (e.g., ε-greedy)
    $\delta \leftarrow R + \gamma Q(S', A') - Q(S, A)$
    $E(S,A) \leftarrow E(S,A) + 1$
차이 For all $s \in \mathcal{S}, a \in \mathcal{A}(s)$:
      $Q(s,a) \leftarrow Q(s,a) + \alpha \delta E(s,a)$
      $E(s,a) \leftarrow \gamma \lambda E(s,a)$
    $S \leftarrow S'$; $A \leftarrow A'$
  until S is terminal

Sarsa(λ) Gridworld Example



Off-Policy Learning

- Evaluate target policy $\pi(a|s)$ to compute $v_\pi(s)$ or $q_\pi(s,a)$
- While following behaviour policy $\mu(a|s)$

$$\{S_1, A_1, R_2, ..., S_T\} \sim \mu$$

- Why is this important?
  - Learn from observing humans or other agents
  - Re-use experience generated from old policies $\pi_1, \pi_2, ..., \pi_{t-1}$
  - Learn about *optimal* policy while following *exploratory* policy
  - Learn about *multiple* policies while following *one* policy

Importance Sampling

- Estimate the expectation of a different distribution

확률분포

$$
\begin{aligned}
\mathbb{E}_{X \sim P}[f(X)] &= \sum P(X) f(X) \\
&= \sum Q(X) \frac{P(X)}{Q(X)} f(X) \\
&= \mathbb{E}_{X \sim Q}\left[\frac{P(X)}{Q(X)} f(X)\right]
\end{aligned}
$$

$P(X) \sim \pi$
$Q(X) \sim \mu$

이 알고리즘은 variance가 커서 현실적으로 동작하지 않는다.
$\mu = 0$ 일때에도

Importance Sampling for Off-Policy Monte-Carlo

- Use returns generated from μ to evaluate π
- Weight return G_t according to similarity between policies
- Multiply importance sampling corrections along whole episode

$$G_t^{\pi/\mu} = \frac{\pi(A_t|S_t)}{\mu(A_t|S_t)} \frac{\pi(A_{t+1}|S_{t+1})}{\mu(A_{t+1}|S_{t+1})} \ldots \frac{\pi(A_T|S_T)}{\mu(A_T|S_T)} G_t$$

- Update value towards *corrected* return

$$V(S_t) \leftarrow V(S_t) + \alpha \left(G_t^{\pi/\mu} - V(S_t) \right)$$

- Cannot use if μ is zero when π is non-zero
- Importance sampling can dramatically increase variance

Importance Sampling for Off-Policy TD

- Use TD targets generated from μ to evaluate π
- Weight TD target $R + \gamma V(S')$ by importance sampling
- Only need a single importance sampling correction

$$V(S_t) \leftarrow V(S_t) + \alpha \left(\frac{\pi(A_t|S_t)}{\mu(A_t|S_t)} \left(R_{t+1} + \gamma V(S_{t+1})\right) - V(S_t) \right)$$

- Much lower variance than Monte-Carlo importance sampling
- Policies only need to be similar over a single step

Q-Learning

- We now consider off-policy learning of action-values $Q(s, a)$
- No importance sampling is required
- Next action is chosen using behaviour policy $A_{t+1} \sim \mu(\cdot|S_t)$
- But we consider alternative successor action $A' \sim \pi(\cdot|S_t)$
- And update $Q(S_t, A_t)$ towards value of alternative action

$$Q(S_t, A_t) \leftarrow Q(S_t, A_t) + \alpha \underbrace{\left(\underbrace{R_{t+1} + \gamma Q(S_{t+1}, A')}_{\text{TD target}} - Q(S_t, A_t)\right)}_{\text{error}}$$

Off-Policy Control with Q-Learning

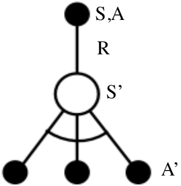
- We now allow both behaviour and target policies to improve
- The target policy π is greedy w.r.t. $Q(s, a)$

$$\pi(S_{t+1}) = \underset{a'}{\operatorname{argmax}}\, Q(S_{t+1}, a')$$

- The behaviour policy μ is e.g. ϵ-greedy w.r.t. $Q(s, a)$
- The Q-learning target then simplifies:

$$\begin{aligned}
& R_{t+1} + \gamma Q(S_{t+1}, A') \\
=& R_{t+1} + \gamma Q(S_{t+1}, \underset{a'}{\operatorname{argmax}}\, Q(S_{t+1}, a')) \\
=& R_{t+1} + \max_{a'} \gamma Q(S_{t+1}, a')
\end{aligned}$$

Q-Learning Control Algorithm



$$Q(S,A) \leftarrow Q(S,A) + \alpha \left(R + \gamma \max_{a'} Q(S',a') - Q(S,A) \right)$$

Theorem

Q-learning control converges to the optimal action-value function, $Q(s,a) \rightarrow q_*(s,a)$

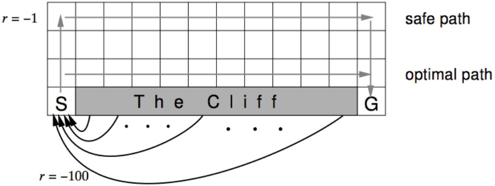
Q-Learning Algorithm for Off-Policy Control

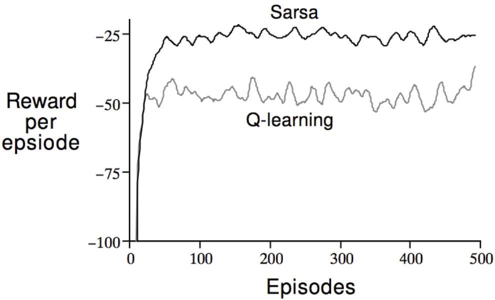
Initialize $Q(s,a), \forall s \in \mathcal{S}, a \in \mathcal{A}(s)$, arbitrarily, and $Q(\textit{terminal-state}, \cdot) = 0$
Repeat (for each episode):
  Initialize S
  Repeat (for each step of episode):
    Choose A from S using policy derived from Q (e.g., ε-greedy)
    Take action A, observe R, S'
    $Q(S,A) \leftarrow Q(S,A) + \alpha\big[R + \gamma \max_a Q(S',a) - Q(S,A)\big]$
    $S \leftarrow S'$;
  until S is terminal

Q-Learning Demo

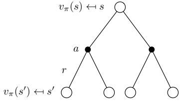
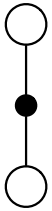
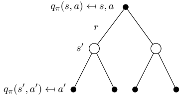
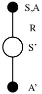
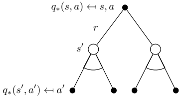
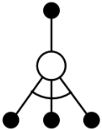
Q-Learning Demo

Cliff Walking Example





Relationship Between DP and TD

	Full Backup (DP)	*Sample Backup (TD)*
Bellman Expectation Equation for $v_\pi(s)$	 Iterative Policy Evaluation	 TD Learning
Bellman Expectation Equation for $q_\pi(s,a)$	 Q-Policy Iteration	 Sarsa
Bellman Optimality Equation for $q_*(s,a)$	 Q-Value Iteration	 Q-Learning

Relationship Between DP and TD (2)

Full Backup (DP)	*Sample Backup (TD)*
Iterative Policy Evaluation $V(s) \leftarrow \mathbb{E}\left[R + \gamma V(S') \mid s\right]$	TD Learning $V(S) \xleftarrow{\alpha} R + \gamma V(S')$
Q-Policy Iteration $Q(s,a) \leftarrow \mathbb{E}\left[R + \gamma Q(S',A') \mid s,a\right]$	Sarsa $Q(S,A) \xleftarrow{\alpha} R + \gamma Q(S',A')$
Q-Value Iteration $Q(s,a) \leftarrow \mathbb{E}\left[R + \gamma \max_{a' \in \mathcal{A}} Q(S',a') \mid s,a\right]$	Q-Learning $Q(S,A) \xleftarrow{\alpha} R + \gamma \max_{a' \in \mathcal{A}} Q(S',a')$

where $x \xleftarrow{\alpha} y \equiv x \leftarrow x + \alpha(y - x)$

Questions?