Lecture 8
Multi-User MIMO

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Multi-User MIMO System

- So far we discussed how multiple antennas increase the capacity and reliability in point-to-point channels

- Question: how do multiple antennas help in multi-user uplink and downlink channels?

- Spatial-Division Multiple Access (SDMA):
    - Multiple antennas provide spatial resolvability for distinguishing different users' signals
    - More spatial degrees of freedom for multiple users to share

Plot

- First study uplink/downlink scenarios with single-antenna mobiles and a multi-antenna base station
- Achieve uplink capacity with MMSE and successive interference cancellation
- Achieve downlink capacity with uplink-downlink duality and dirty paper precoding
- Finally extend the results to MIMO uplink and downlink

Outline

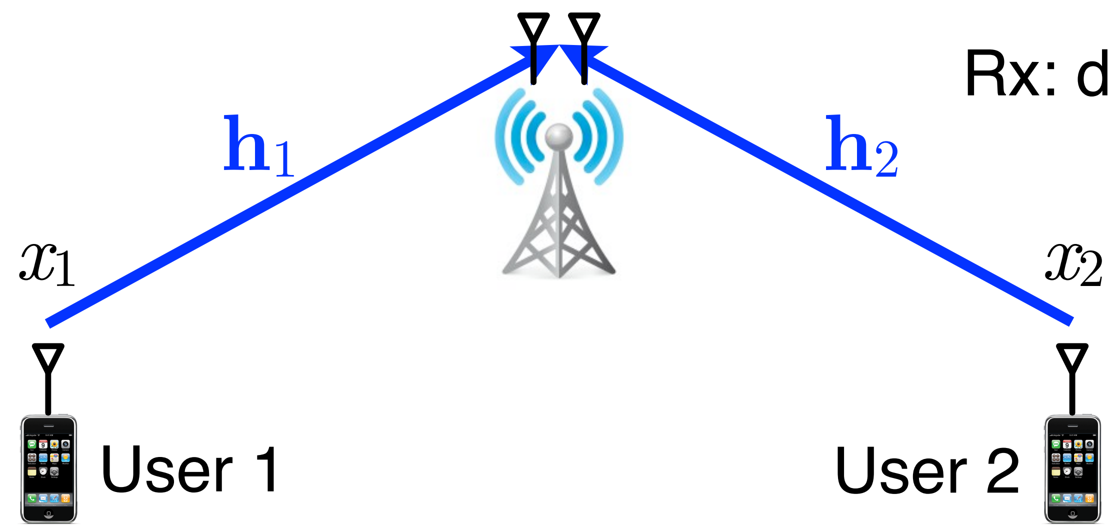
- Uplink with multiple Rx antennas
  - MMSE-SIC
- Downlink with multiple Tx antennas
  - Uplink-downlink duality
  - Dirty paper precoding
- MIMO uplink and downlink

Uplink with Multiple Rx Antennas

Spatial Division Multiple Access

$$\mathbf{y} = \mathbf{h}_1 x_1 + \mathbf{h}_2 x_2 + \mathbf{w}$$

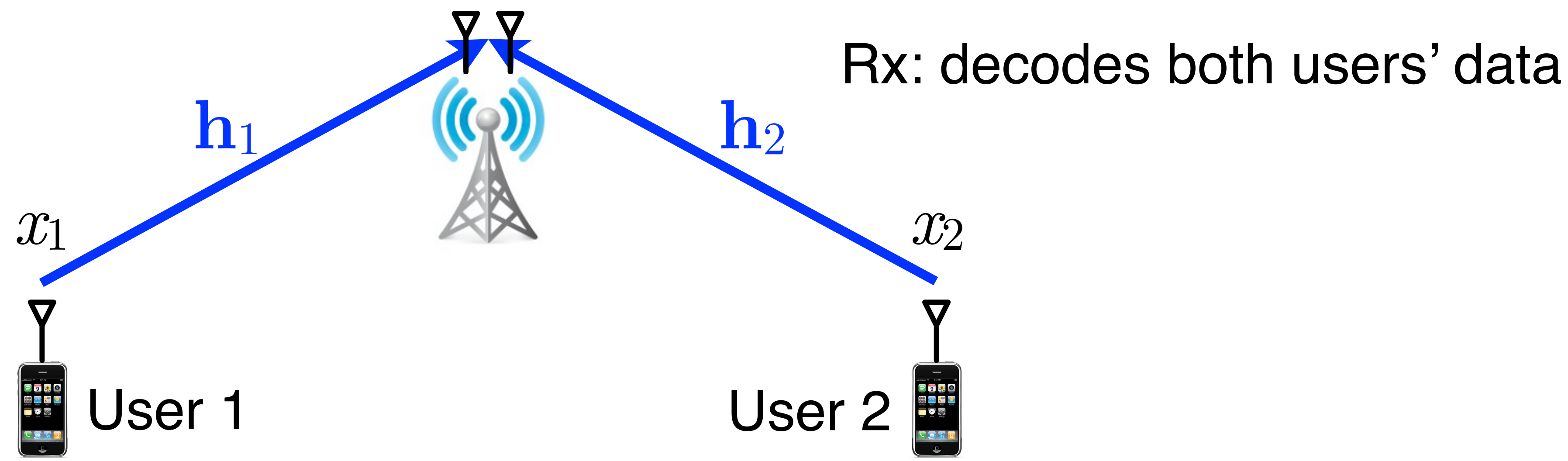
Rx: decodes both users' data



- Equivalent to the point-to-point MIMO using V-BLAST with identity precoding matrix
- Rx beamforming (linear filtering without SIC) distinguishes two users spatially (and hence the name spatial division multiple access (SDMA))
  - MMSE: the optimal filter that maximizes the Rx SINR
  - As long as the users are geographically far apart $\Longrightarrow \mathbf{H} := [\mathbf{h}_1 \ \mathbf{h}_2]$ is well-conditioned $\Longrightarrow$ 2 spatial DoF for the 2 users to share

Capacity Bounds

$$\mathbf{y} = \mathbf{h}_1 x_1 + \mathbf{h}_2 x_2 + \mathbf{w}$$



- Individual rates: each user is faced with a SIMO channel

$$\implies R_k \leq \log\left(1 + \frac{P_k}{\sigma^2}||\mathbf{h}_k||^2\right), \quad k = 1, 2$$

- Sum rate: viewed as a MIMO channel with V-BLAST and identity precoding matrix: ($\mathbf{H} = \begin{bmatrix}\mathbf{h}_1 & \mathbf{h}_2\end{bmatrix}, \ \Lambda = \mathrm{diag}\,(P_1, P_2)$)

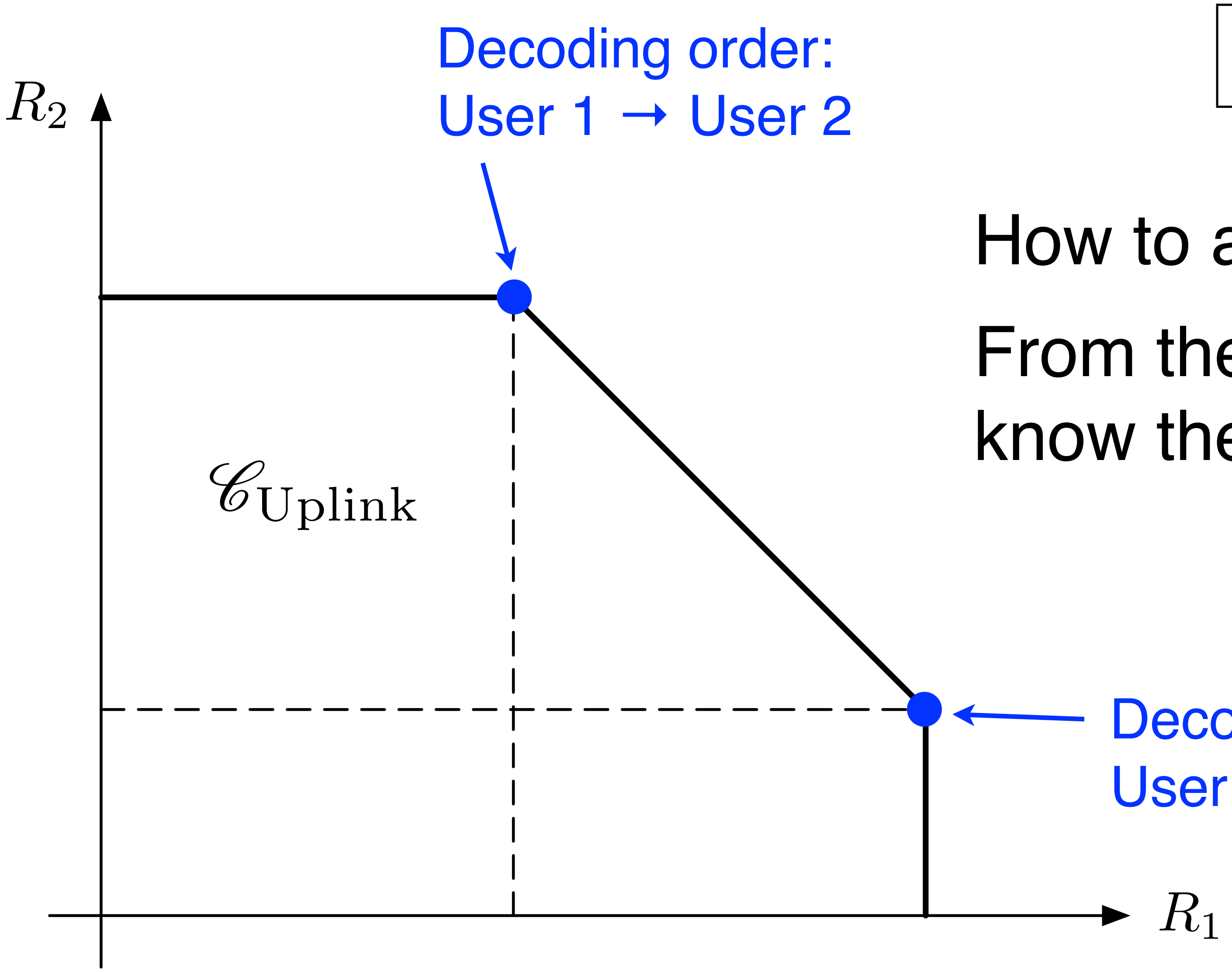
$$\implies R_1 + R_2 \leq \log\det\left(\mathbf{I}_{n_r} + \frac{\mathbf{H\Lambda H}^*}{\sigma^2}\right)$$

$$= \log\det\left(\mathbf{I}_{n_r} + P_1\mathbf{h}_1\mathbf{h}_1^* + P_2\mathbf{h}_2\mathbf{h}_2^*\right)$$

Capacity Region of the UL Channel

$$\mathscr{C}_{\text{Uplink}} = \bigcup \left\{ (R_1, R_2) \geq 0 : \begin{cases} R_1 \leq \log\left(1 + \frac{P_1}{\sigma^2}||\mathbf{h}_1||^2\right) \\ R_2 \leq \log\left(1 + \frac{P_2}{\sigma^2}||\mathbf{h}_2||^2\right) \\ R_1 + R_2 \leq \log\det\left(\mathbf{I}_{n_r} + \frac{1}{\sigma^2}\mathbf{H}\boldsymbol{\Lambda}\mathbf{H}^*\right) \end{cases} \right\}$$

$$\mathbf{H} = \begin{bmatrix} \mathbf{h}_1 & \mathbf{h}_2 \end{bmatrix}, \quad \Lambda = \operatorname{diag}(P_1, P_2)$$



How to achieve the corner points?

From the study of V-BLAST we know the answer:

MMSE-SIC!

K-user Uplink Capacity Region

- The idea can be easily extended to the K-user case

$$\boxed{\mathscr{C}_{\text{Uplink}} = \bigcup \left\{ \begin{array}{l} (R_1, \ldots, R_K) \geq 0 : \\ \quad \forall \mathcal{S} \subseteq [1:K], \\ \quad \sum_{k \in \mathcal{S}} R_k \leq \log\det\left(\mathbf{I}_{n_r} + \frac{1}{\sigma^2}\mathbf{H}_{\mathcal{S}}\boldsymbol{\Lambda}_{\mathcal{S}}\mathbf{H}_{\mathcal{S}}^*\right) \\ \qquad\qquad\quad = \log\det\left(\mathbf{I}_{n_r} + \frac{1}{\sigma^2}\sum_{k \in \mathcal{S}} P_k \mathbf{h}_k \mathbf{h}_k^*\right) \end{array} \right\}}$$

$$\mathbf{H}_{\mathcal{S}} := \begin{bmatrix} \mathbf{h}_{l_1} & \mathbf{h}_{l_2} & \cdots & \mathbf{h}_{l_{|\mathcal{S}|}} \end{bmatrix}, \quad l_1, \ldots, l_{|\mathcal{S}|} \in \mathcal{S}$$

$$\boldsymbol{\Lambda}_{\mathcal{S}} := \operatorname{diag}\left(P_{l_1}, P_{l_2}, \ldots, P_{l_{|\mathcal{S}|}}\right), \quad l_1, \ldots, l_{|\mathcal{S}|} \in \mathcal{S}$$

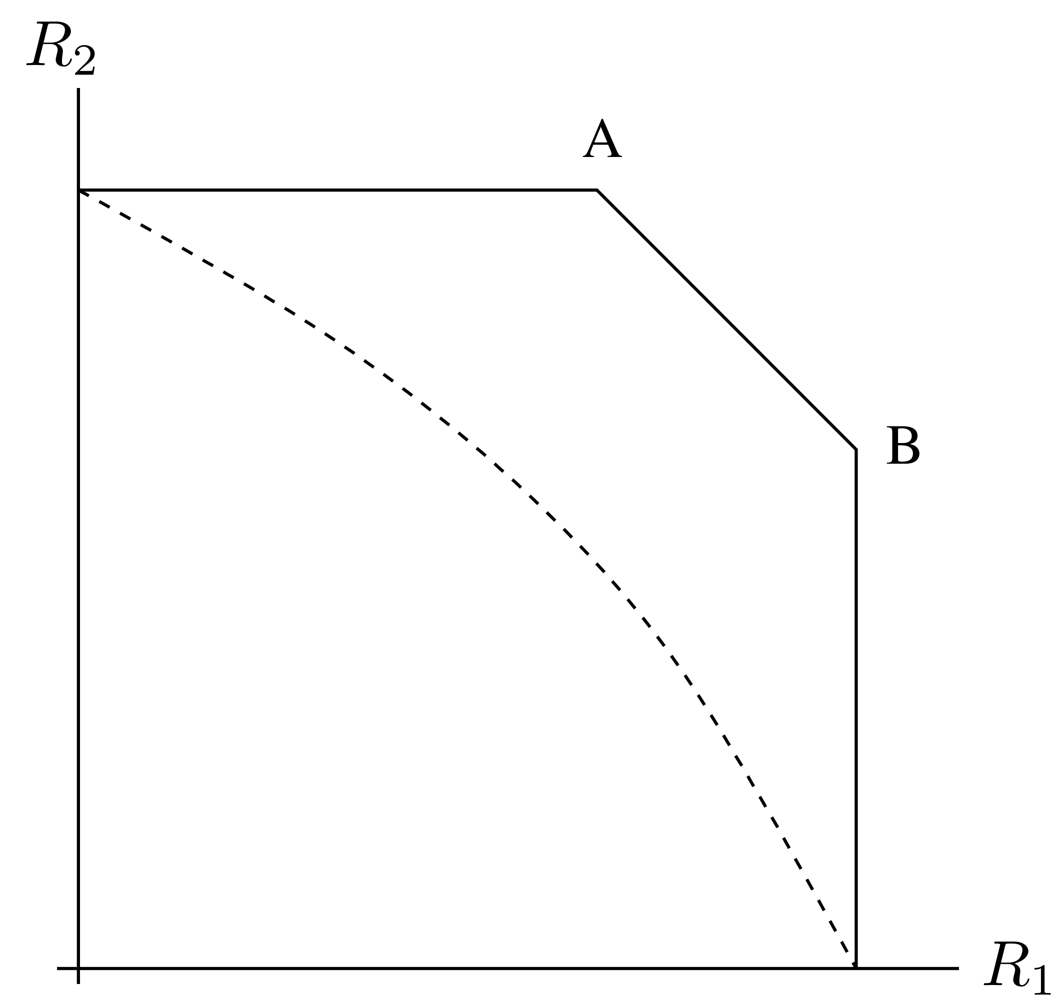
- Again, can be achieved using MMSE-SIC architectures

Comparison with Orthogonal Access

- Orthogonal multiple access can achieve

$$\begin{cases} R_1 = \alpha \log\left(1 + \frac{P_1||\mathbf{h}_1||^2}{\alpha\sigma^2}\right) \\ R_2 = (1-\alpha)\log\left(1 + \frac{P_2||\mathbf{h}_2||^2}{(1-\alpha)\sigma^2}\right) \end{cases} \qquad \alpha \in [0,1]$$

- Unlike the single-antenna case, it's cannot achieve the sum capacity



- In total only 1 spatial DoF

Because the rate expressions are the same as those in the single-antenna case!

Total Available Spatial DoF

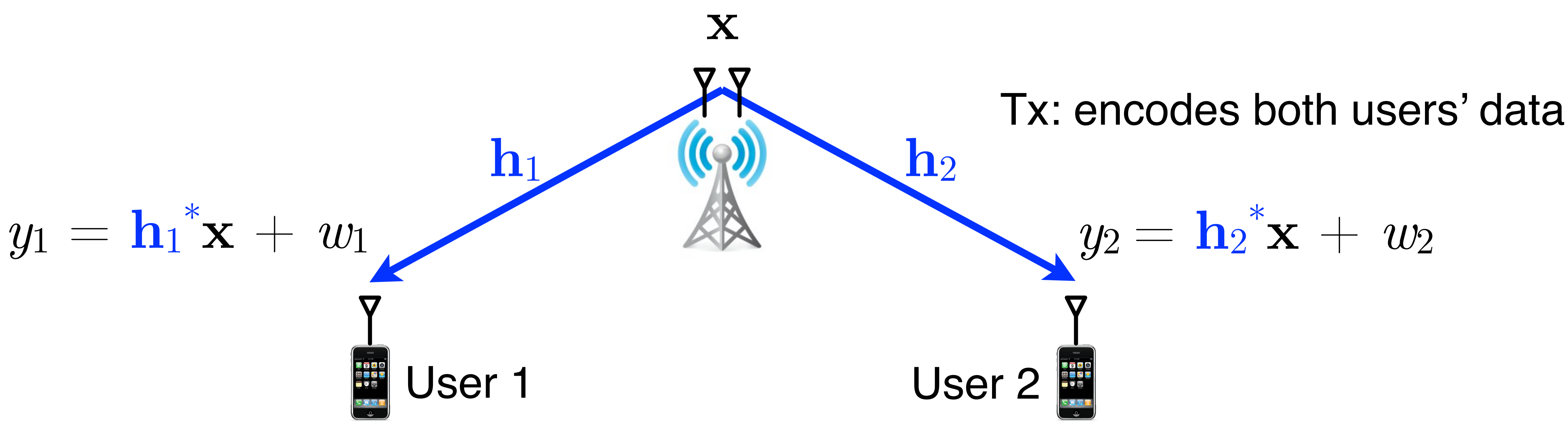
- With K single-antenna mobiles and n_r antennas at the base station, the total # of spatial DoF is $\min\{K, n_r\}$.

- When $K \leq n_r$, the multi-antenna base station is able to distinguish all K users with SDMA

- When $K > n_r$, the multi-antenna base station cannot distinguish all K users
- Instead, divide the users into n_r groups: in each group, users share the single DoF by orthogonalization

Downlink with
Multiple Tx Antennas

Downlink with Multiple Tx Antennas



- Superposition of two data streams: $\mathbf{x} = \mathbf{u}_1 x_1 + \mathbf{u}_2 x_2$
  - $\mathbf{u}_k$: Tx beamforming signature for user k
- Downlink SDMA:
  - Design goal: given a set of SINR's, find the power allocation & the beamforming signatures s.t. the total Tx power is minimized
- Achieve 2 spatial DoF with $\mathbf{u}_1 \perp \mathbf{h}_2$ & $\mathbf{u}_2 \perp \mathbf{h}_1$.
  - Similar to zero forcing (decorrelator) in point-to-point and uplink

Downlink SDMA: Power Control Problem

- Finding the optimal Tx signatures & power allocation:
  - SINR of each user depends on all the Tx signatures (and the power allocation); in contrast to the uplink case
  - Hence maximizing all SINR is not a meaningful design goal
- Our design goal is to solve a power control problem:
  - Given a set of SINR's, find the power allocation & a set of Tx signatures such that the total amount of Tx power is minimized
  - It turns out that the power control problem is dual to a power control problem in a dual uplink channel
- Through the uplink-downlink duality, the downlink problem can be solved

Uplink-Downlink Duality (1)

- Primal downlink:
  - Superposition of data streams: $\mathbf{x}_{\mathrm{dl}} = \sum_{k=1}^{K} \mathbf{u}_k x_k$
  - Received signals and SINR:

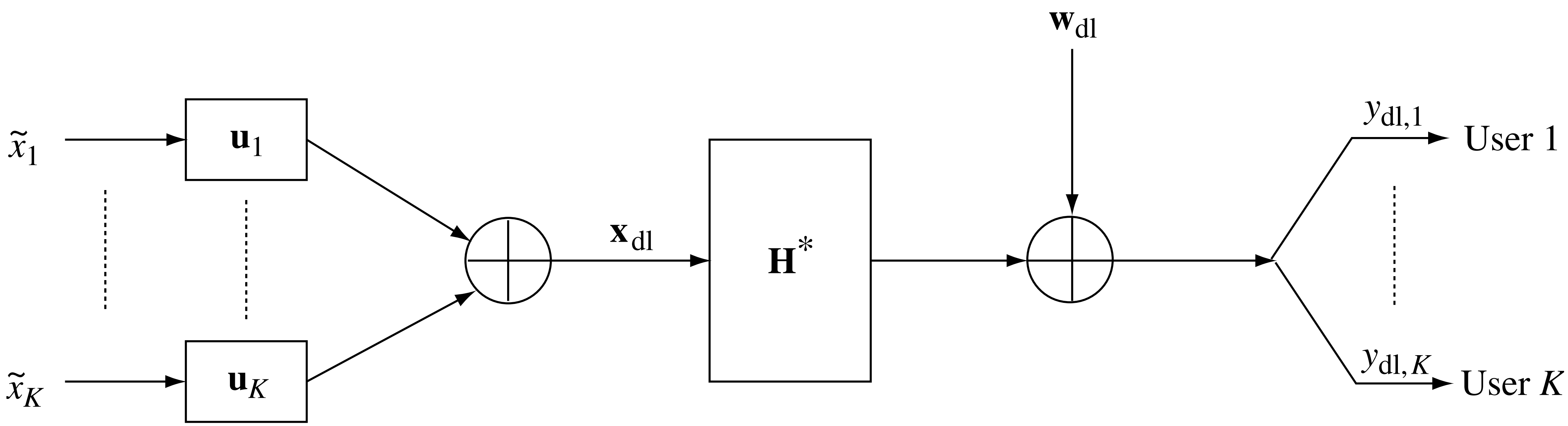
$$y_{\mathrm{dl},k} = \left(\mathbf{h}_k^* \mathbf{u}_k\right) x_k + \sum_{j \neq k} \left(\mathbf{h}_k^* \mathbf{u}_j\right) x_j + w_{\mathrm{dl},k}, \;\; k = 1, \ldots, K$$

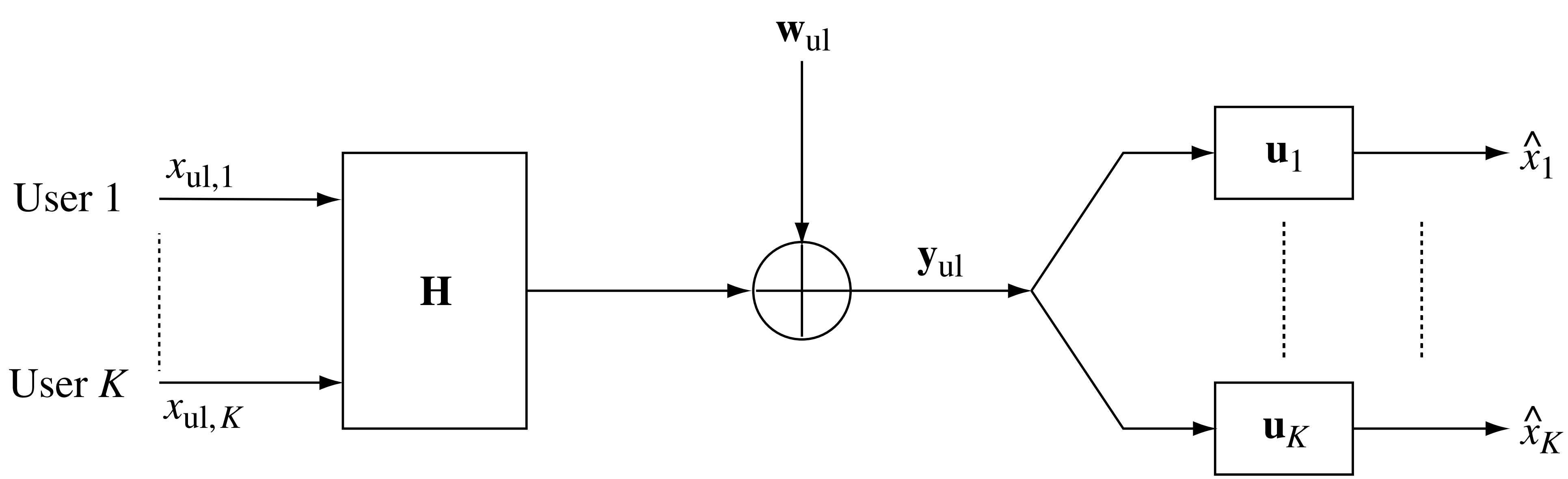
$$\mathsf{SINR}_{\mathrm{dl},k} = \frac{P_k |\mathbf{h}_k^* \mathbf{u}_k|^2}{\sigma^2 + \sum_{j \neq k} P_j |\mathbf{h}_k^* \mathbf{u}_j|^2}, \;\; k = 1, \ldots, K$$

  - Vector channel: $\mathbf{y}_{\mathrm{dl}} = \mathbf{H}^* \mathbf{x}_{\mathrm{dl}} + \mathbf{w}_{\mathrm{dl}}$
- Vector SINR: let $a_k := \frac{1}{|\mathbf{h}_k^* \mathbf{u}_k|^2} \frac{\mathsf{SINR}_{\mathrm{dl},k}}{1 + \mathsf{SINR}_{\mathrm{dl},k}}, \;\; k = 1, \ldots, K$
  - Let the matrix $\mathbf{A}$ have entry $A_{k,j} = |\mathbf{h}_k^* \mathbf{u}_j|^2$
  - Then we have $\left(\mathbf{I}_K - \mathrm{diag}\left(\mathbf{a}\right) \mathbf{A}\right) \mathbf{p} = \sigma^2 \mathbf{a}$
- For given $\{\mathbf{u}_k\}$, we can compute the power vector $\mathbf{p}$:

$$\mathbf{p} = \sigma^2 \left(\mathbf{I}_K - \mathrm{diag}\left(\mathbf{a}\right) \mathbf{A}\right)^{-1} \mathbf{a} = \sigma^2 \left(D_{\mathbf{a}} - \mathbf{A}\right)^{-1} \mathbf{1}$$

$$D_{\mathbf{a}} := \mathrm{diag}\left(1/a_1, \ldots, 1/a_K\right)$$


$\mathbf{w}_{\mathrm{dl}}$
$\tilde{x}_1$
$\mathbf{u}_1$
$y_{\mathrm{dl},1}$
User 1
$\mathbf{x}_{\mathrm{dl}}$
$\mathbf{H}^*$
$\tilde{x}_K$
$\mathbf{u}_K$
$y_{\mathrm{dl},K}$
User K


$\mathbf{w}_{\mathrm{ul}}$
$x_{\mathrm{ul},1}$
User 1
$\mathbf{u}_1$
$\hat{x}_1$
$\mathbf{H}$
$\mathbf{y}_{\mathrm{ul}}$
User K
$x_{\mathrm{ul},K}$
$\mathbf{u}_K$
$\hat{x}_K$

Uplink-Downlink Duality (2)

- Dual uplink:
  - Vector channel: $\mathbf{y}_{\mathrm{ul}} = \mathbf{H}\mathbf{x}_{\mathrm{ul}} + \mathbf{w}_{\mathrm{ul}}$
  - Filtered output SINR: $\mathsf{SINR}_{\mathrm{ul},k} = \frac{Q_k|\mathbf{u}_k^*\mathbf{h}_k|^2}{\sigma^2+\sum_{j\neq k} Q_j|\mathbf{u}_k^*\mathbf{h}_j|^2}, \; k=1,\ldots,K$
- Vector SINR: let $b_k := \frac{1}{|\mathbf{h}_k^*\mathbf{u}_k|^2}\frac{\mathsf{SINR}_{\mathrm{ul},k}}{1+\mathsf{SINR}_{\mathrm{ul},k}}, \; k=1,\ldots,K$
  - Let the matrix $\mathbf{B}$ have entry $B_{k,j} = |\mathbf{u}_k^*\mathbf{h}_j|^2$
  - Then we have $\left(\mathbf{I}_K - \mathrm{diag}\left(\mathbf{b}\right)\mathbf{A}^T\right)\mathbf{q} = \sigma^2\mathbf{b}$ since $\mathbf{B} = \mathbf{A}^T$
- For given $\{\mathbf{u}_k\}$, we can compute the power vector $\mathbf{q}$:

$$\mathbf{q} = \sigma^2\left(\mathbf{I}_K - \mathrm{diag}\left(\mathbf{b}\right)\mathbf{A}^T\right)^{-1}\mathbf{b} = \sigma^2\left(D_{\mathbf{b}} - \mathbf{A}^T\right)^{-1}\mathbf{1}$$

$$D_{\mathbf{b}} := \mathrm{diag}\left(1/b_1,\ldots,1/b_K\right)$$

Uplink-Downlink Duality (3)

- For the same $\{\mathbf{u}_k\}$, to achieve the same set of SINR ($\mathbf{a}=\mathbf{b}$), the total Tx power of the UL and DL are the same:

$$\sum_{k=1}^{K} P_k = \sigma^2 \mathbf{1}^T \left(D_{\mathbf{a}} - \mathbf{A}\right)^{-1} \mathbf{1} = \sigma^2 \mathbf{1}^T \left(D_{\mathbf{a}} - \mathbf{A}^T\right)^{-1} \mathbf{1} = \sum_{k=1}^{K} Q_k$$

- Hence, to solve the downlink power allocation and Tx signature design problem, we can solve the dual problem in the dual uplink channel
- Tx signatures will be the MMSE filters in the virtual uplink

Beyond Linear Strategies

- Linear receive beamforming strategies for the uplink map to linear transmit beamforming strategies in the downlink

- But in the uplink we can improve performance by doing successive interference cancellation at the receiver

- Is there a dual to this strategy in the downlink?

Transmit Precoding

- In downlink Tx beamforming, signals for different users are superimposed and interfere with each other
- With a single Tx antenna, users can be ordered in terms of signal strength
  - A user can decode and cancel all the signals intended for the weaker user before decoding its own
- With multiple Tx antennas, no such ordering exists and no user may be able to decode information beamformed to other users
- However, the base station knows the information to be transmitted to every user and can precode to cancel at the transmitter

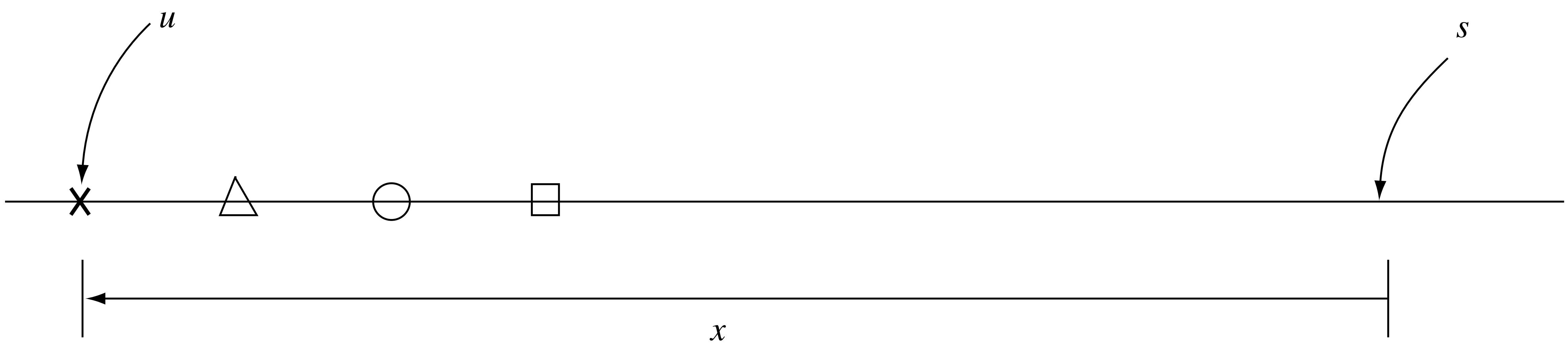
Symbol-by-Symbol Precoding

- A generic problem: $y = x + s + w$
  - x : desired signal
  - s : interference known to Tx but unknown to Rx
  - w : noise

- Applications:
  - Downlink channel: s is the signal for other users
  - ISI channel: s is the intersymbol interference

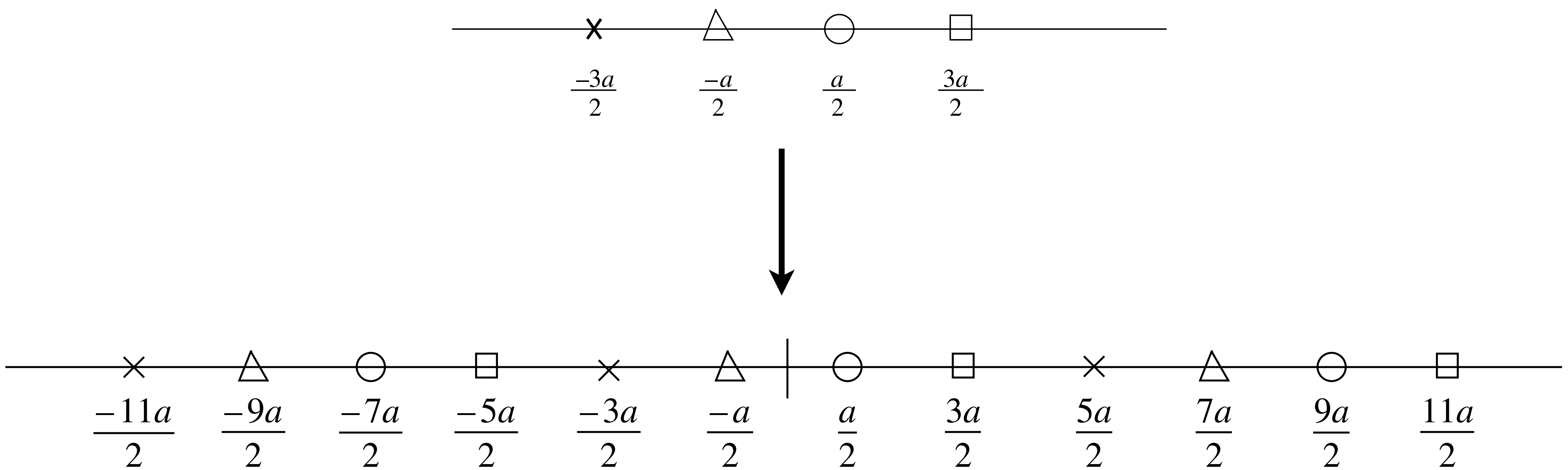
Naive Pre-Cancellation Strategy

- Want to send point u in a 4-PAM constellation
- Transmit $x = u - s$ to pre-cancel the effect of s
- But this is very power inefficient if s is large



Tomlinson-Harashima Precoding (1)

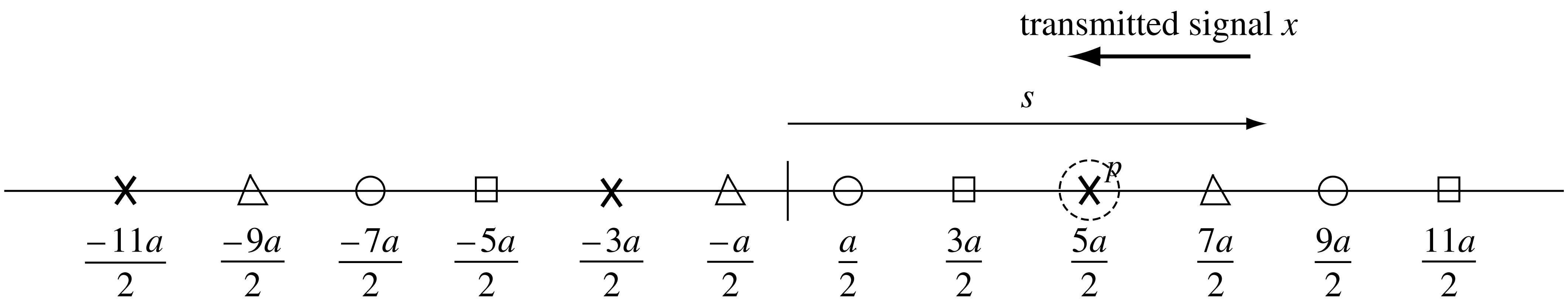
- Replicate the PAM constellation to tile the whole real line



- Represent information u by an equivalent class of constellation points instead of a single point

Tomlinson-Harashima Precoding (2)

- Given u and s, find the point in its equivalent class closest to s and transmit the difference



Writing on Dirty Paper

- Can extend this idea to block precoding
  - Problem is to design codes which are simultaneously good source codes (vector quantizers) as well as good channel codes
- Somewhat surprising, information theory guarantees that one can get to the capacity of the AWGN channel with the interference completely removed
- Applying this to the downlink, can perform SIC at the transmitter
- The pre-cancellation order in the downlink is the reverse order of the SIC in the dual uplink

MIMO Uplink and Downlink

MIMO Uplink

- Channel model: $\mathbf{y} = \mathbf{H}_1\mathbf{x}_1 + \mathbf{H}_2\mathbf{x}_2 + \mathbf{w}$
- Now the mobiles (Tx) have multiple antennas, and hence can form their own Tx covariance matrices
- For the two-user case, capacity bounds become
  - Individual rate bounds:

$$R_k \leq \log\det\left(\mathbf{I}_{n_r} + \tfrac{1}{\sigma^2}\mathbf{H}_k\mathbf{K}_{\mathbf{x}_k}\mathbf{H}^*\right), \quad k = 1, 2$$

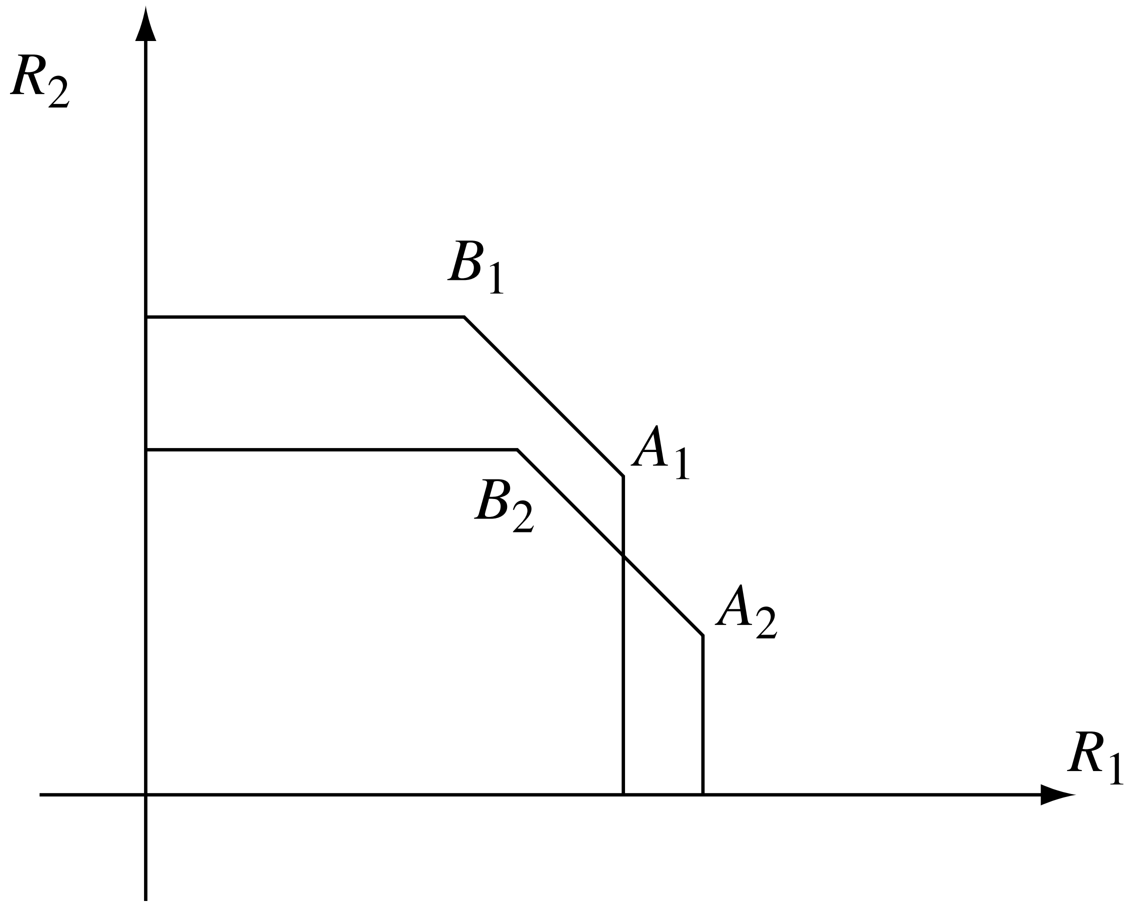
  - Sum rate bound:

$$R_1 + R_2 \leq \log\det\left(\mathbf{I}_{n_r} + \frac{1}{\sigma^2}\sum_{k=1}^{2}\mathbf{H}_k\mathbf{K}_{\mathbf{x}_k}\mathbf{H}^*\right)$$

- Note: in general there are no single $\mathbf{K}_{\mathbf{x}_1}$ and $\mathbf{K}_{\mathbf{x}_2}$ that can simultaneously maximize the three rate constraints

Capacity Region (Two Users)

$$\mathscr{C}_{\text{Uplink}} = \operatorname{conv} \bigcup_{\substack{k=1,2,\ \mathbf{K}_{\mathbf{x}_k} \succeq 0 \\ \operatorname{Tr}\left(\mathbf{K}_{\mathbf{x}_k}\right) \le P_k}} \left\{ \begin{array}{l} (R_1, R_2) \geq 0 : \\ \quad R_k \leq \log\det\left(\mathbf{I}_{n_r} + \frac{1}{\sigma^2}\mathbf{H}_k \mathbf{K}_{\mathbf{x}_k} \mathbf{H}_k^*\right), \quad k = 1, 2 \\ \quad R_1 + R_2 \leq \log\det\left(\mathbf{I}_{n_r} + \frac{1}{\sigma^2}\sum_{k=1}^{2} \mathbf{H}_k \mathbf{K}_{\mathbf{x}_k} \mathbf{H}_k^*\right) \end{array} \right\}$$



Two pentagon regions are achieved by different choices of $\mathbf{K}_{\mathbf{x}_1}$ and $\mathbf{K}_{\mathbf{x}_2}$

Hence the capacity region is NOT a pentagon region anymore

MMSE-SIC can achieve all corner points in each pentagon region

MIMO Uplink with Fast Fading (1)

- Full CSI: need to solve a joint optimization problem regarding power allocation and precoding matrix design
  - Cannot use SVD because the two channel matrices may not have the same factoring left matrix $\mathbf{U}$
  - The problem can be solved by iterative water-filling efficiently
  - Reference: W. Yu *et al*, "Iterative Water-Filling for Gaussian Vector Multiple-Access Channels," IEEE Transactions on Information Theory, vol.50, no.1, pp.145 – 152, January 2004

MIMO Uplink with Fast Fading (2)

- Receiver CSI:
  - Capacity region is the convex hull of the collection of rate pairs (R_1, R_2) that satisfy the following inequalities for some covariance matrix $\mathbf{K}_{\mathbf{x}_1}$ and $\mathbf{K}_{\mathbf{x}_2}$ with $\mathrm{Tr}(\mathbf{K}_{\mathbf{x}_1}) < P_1$ and $\mathrm{Tr}(\mathbf{K}_{\mathbf{x}_2}) < P_2$:

$$R_k \leq \mathbb{E}\left[\log\det\left(\mathbf{I}_{n_r} + \frac{1}{\sigma^2}\mathbf{H}_k\mathbf{K}_{\mathbf{x}_k}\mathbf{H}_k^*\right)\right], \quad k = 1, 2$$

$$R_1 + R_2 \leq \mathbb{E}\left[\log\det\left(\mathbf{I}_{n_r} + \frac{1}{\sigma^2}\sum_{k=1}^{2}\mathbf{H}_k\mathbf{K}_{\mathbf{x}_k}\mathbf{H}_k^*\right)\right]$$

- For i.i.d. Rayleigh $\{\mathbf{H}_k\}$, it is straightforward to see that uniform power allocation and identity precoding matrices maximize all bounds simultaneously

  $\Longrightarrow$ the capacity region is a pentagon with $\mathbf{K}_{\mathbf{x}_k} = \frac{P_k}{n_{t,k}}\mathbf{I}_{n_t,k}$

Nature of Performance Gains

- For the uplink MIMO, regardless of CSIT, the total # of spatial DoF is $\min\left\{\sum_{k=1}^{K} n_{t,k}\ ,\ n_r\right\}$
- CSIT is NOT crucial in obtaining multiplexing gain in the uplink, as long as receiver CSI is available
- Power gain is increased with CSIT
- Multi-user diversity gain is limited

MIMO Downlink

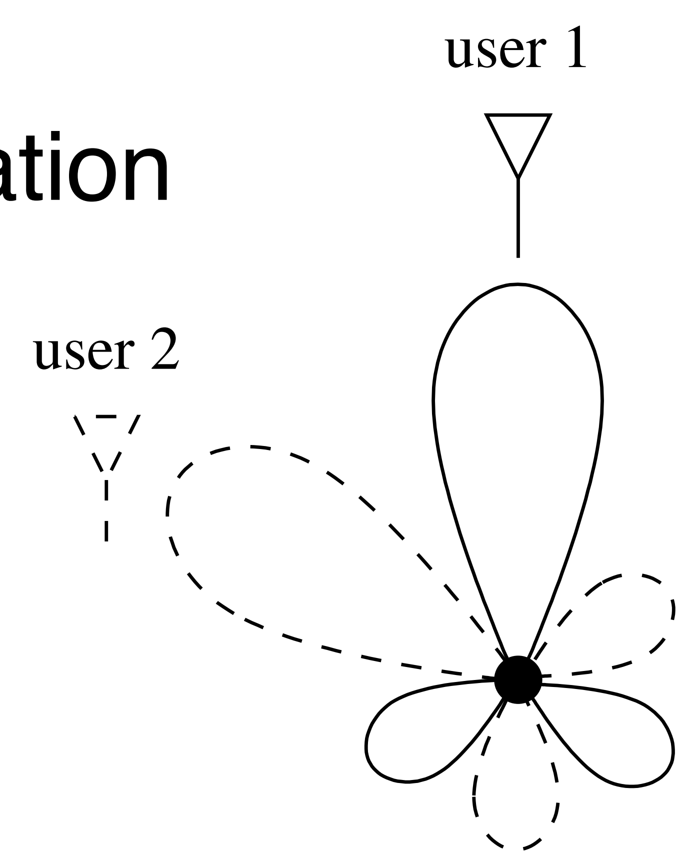
- Compared to the case with single-antenna users, one needs to further design the receive filters at the users
- Uplink-downlink duality can be naturally extended to the case with multiple Rx antennas:
  - The Rx linear filters are the Tx linear precoding filters in the dual uplink channel
- Hence in the case without fading, the sum capacity of the MIMO downlink channel is equal to that of the dual uplink channel (with total power constraint)

MIMO Downlink with Fast Fading

- With full CSI one can assort to the uplink-downlink duality to solve the joint optimization problem
- However, with receiver CSI:
  - Not possible for the base station to carry out Tx beamforming
  - In the symmetric downlink channel with CSIR, time-sharing is optimal in achieving the capacity region
  - DoF drops significantly from $\min\{n_t, K\}$ to 1
- Some partial CSI at Tx could recover the spatial DoF:
  - Channel quality of its own link rather than the entire channel
  - No phase information

Opportunistic Beamforming

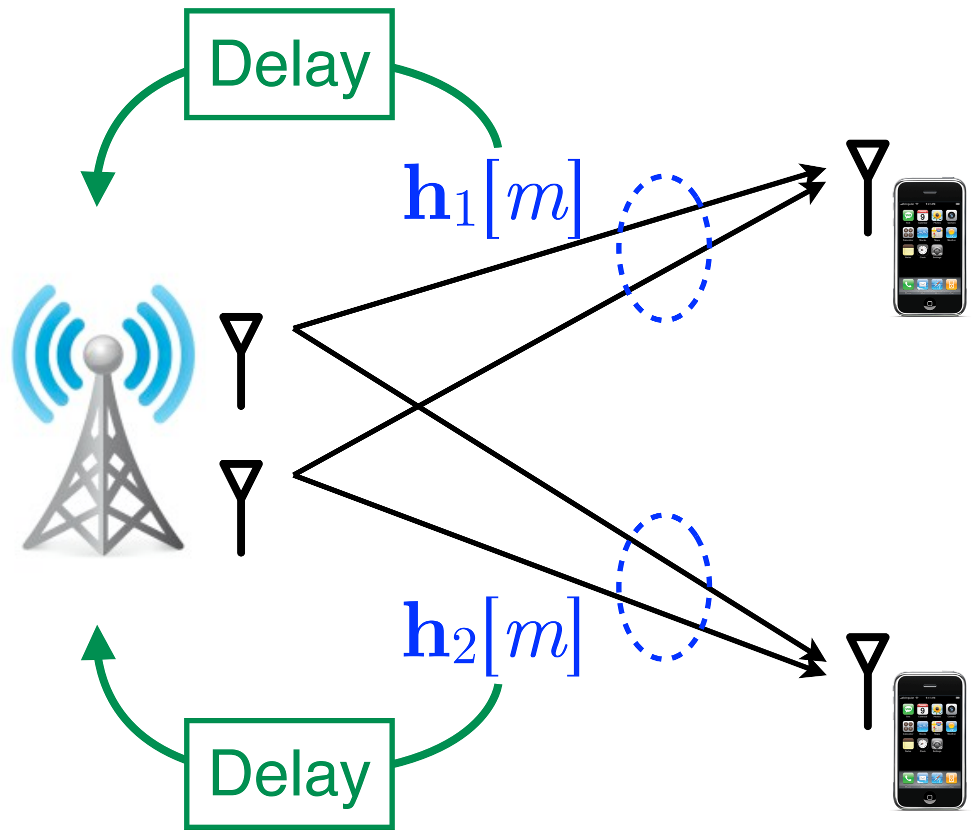
- Opportunistic beamforming with multiple beams:
    - Form n_t orthogonal beams
    - Whenever a user falls inside the beam, it feedback this piece of information back to the base station
    - As the number of users get large, one is able to find a user in each beam with high probability
- Hence the full DoF could be recovered
- Still, need instantaneous channel information



Transmitter CSI Affects DoF Drastically

- Behaviors of power gain and multiuser diversity gain are similar to those in uplink
- For the downlink MIMO, CSIT is critical for obtaining spatial multiplexing gain (assuming i.i.d. Rayleigh below)
  - Full CSI: Total DoF = $\min\left\{\sum_{k=1}^{K} n_{r,k}\ ,\ n_t\right\}$
  - CSIR: Total DoF = $\min\left\{\max_{k\in[1:K]} n_{r,k}\ ,\ n_t\right\}$
  - For the case of single-antenna users, DoF with CSIR is merely 1
- Hence it might be beneficial to spend some resource for estimating the channel and predict the current channel from the past observations
- Under i.i.d. Rayleigh, prediction is not possible.
- Question: can past CSI at Tx still help?

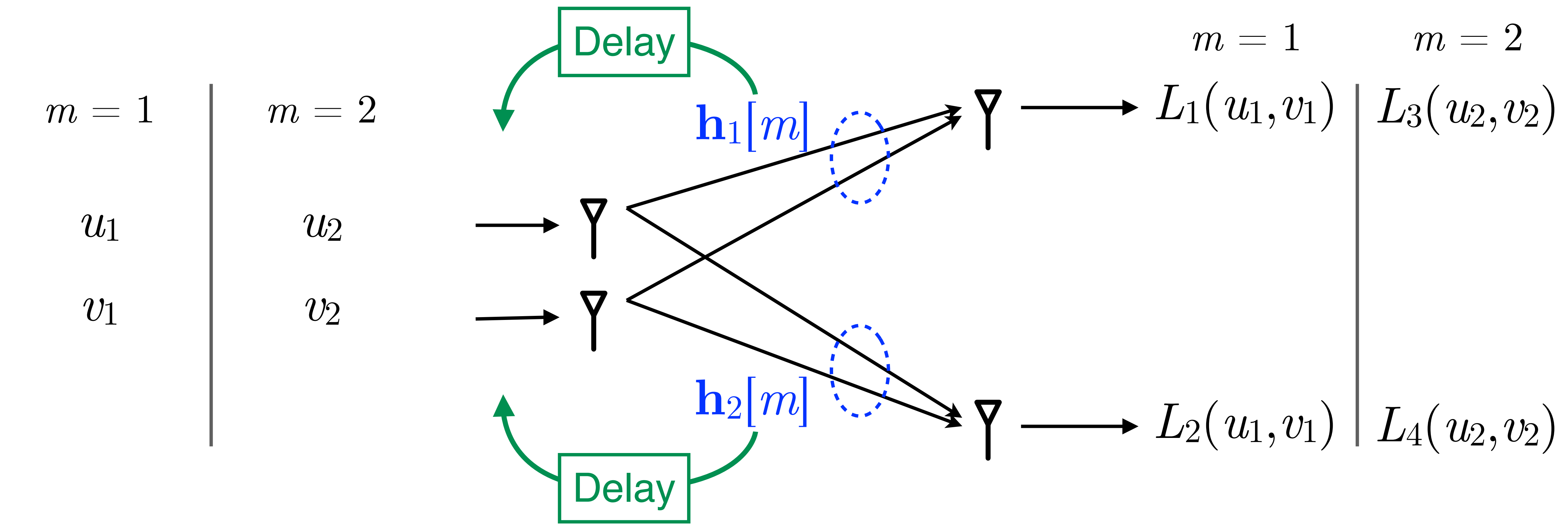
Two-user MISO Downlink

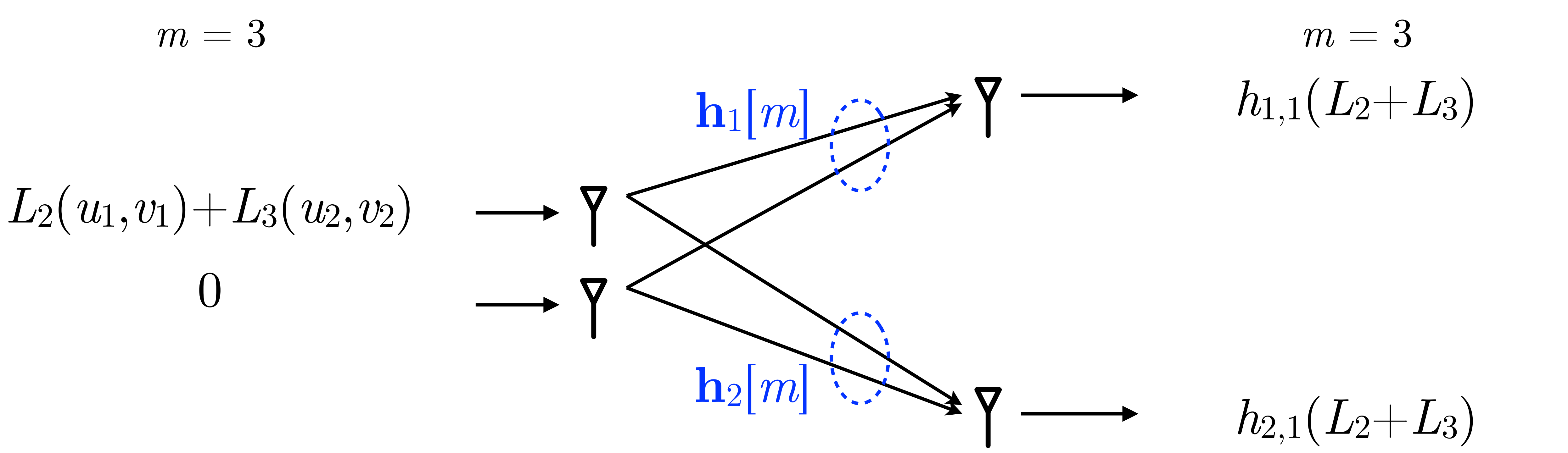


$\mathbf{h}_1[m]$, $\mathbf{h}_2[m]$:
i.i.d. Rayleigh

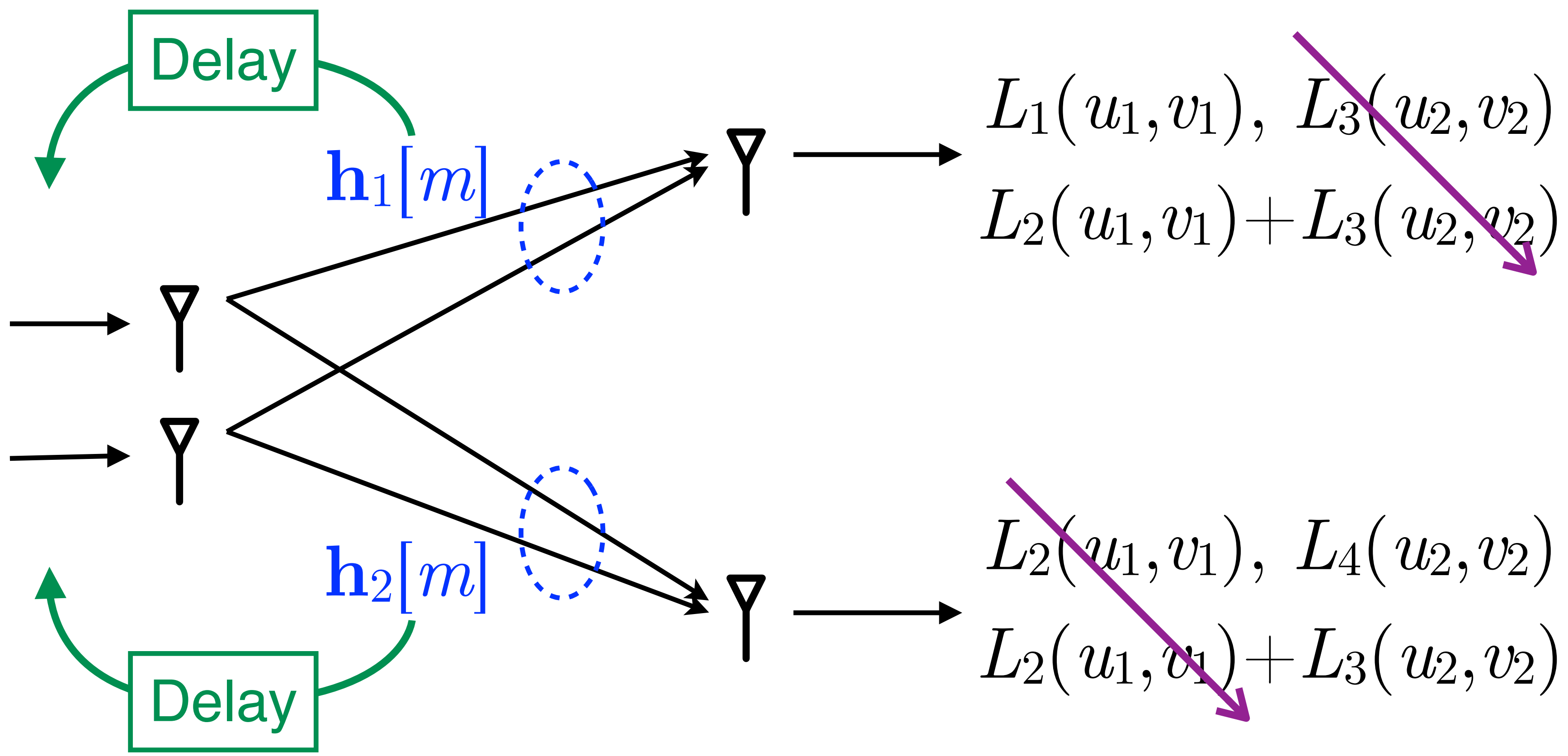
- Without transmitter CSI (CSIT), DoF = 1
- With instantaneous CSIT $\mathbf{H}[m]$ at time m, DoF = 2
- With delayed CSIT $\mathbf{H}[1{:}m{-}1]$ at time m, DoF = ?
- Prediction is useless $\Longrightarrow$ delayed CSIT is useless?

DoF = 4/3 with Delayed CSIT



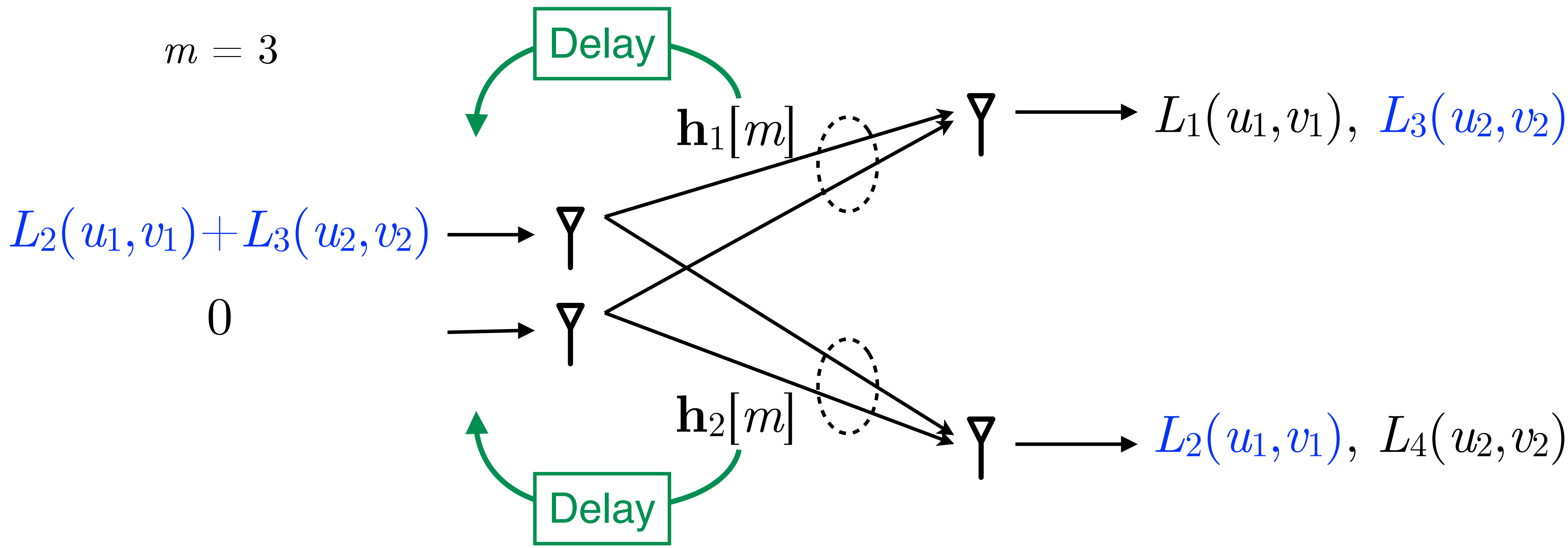


DoF = 4/3 with Delayed CSIT



- $L_1(u_1,v_1) \not\parallel L_2(u_1,v_1)$ & $L_3(u_2,v_2) \not\parallel L_4(u_2,v_2)$ with prob. 1
- Both decode their desired 2 symbols over 3 time slots
- DoF = (2+2)/3 = 4/3

Exploiting Side Information



- $L_2(u_1, v_1)$ is useful for user 1 but shipped to user 2
- $L_3(u_2, v_2)$ is useful for user 2 but shipped to user 1
- With delayed CSIT, Tx forms $u_{12} := L_2(u_1, v_1) + L_3(u_2, v_2)$
  - User 1 can extract L_2 because it has L_3 as useful side info.
  - User 2 can extract L_3 because it has L_2 as useful side info.

Hierarchy of Messages

- One can view $u_{12} := L_2(u_1, v_1) + L_3(u_2, v_2)$ as a common message for both users
- Order-1 message: aimed at only 1 user
  - User 1: u_1 , v_1 ; User 2: u_2 , v_2
- Order-2 message: aimed at 2 users
  - User {1,2}: u_{12}
- Define DoF_k := the DoF for sending all order-k messages
- Then we see that

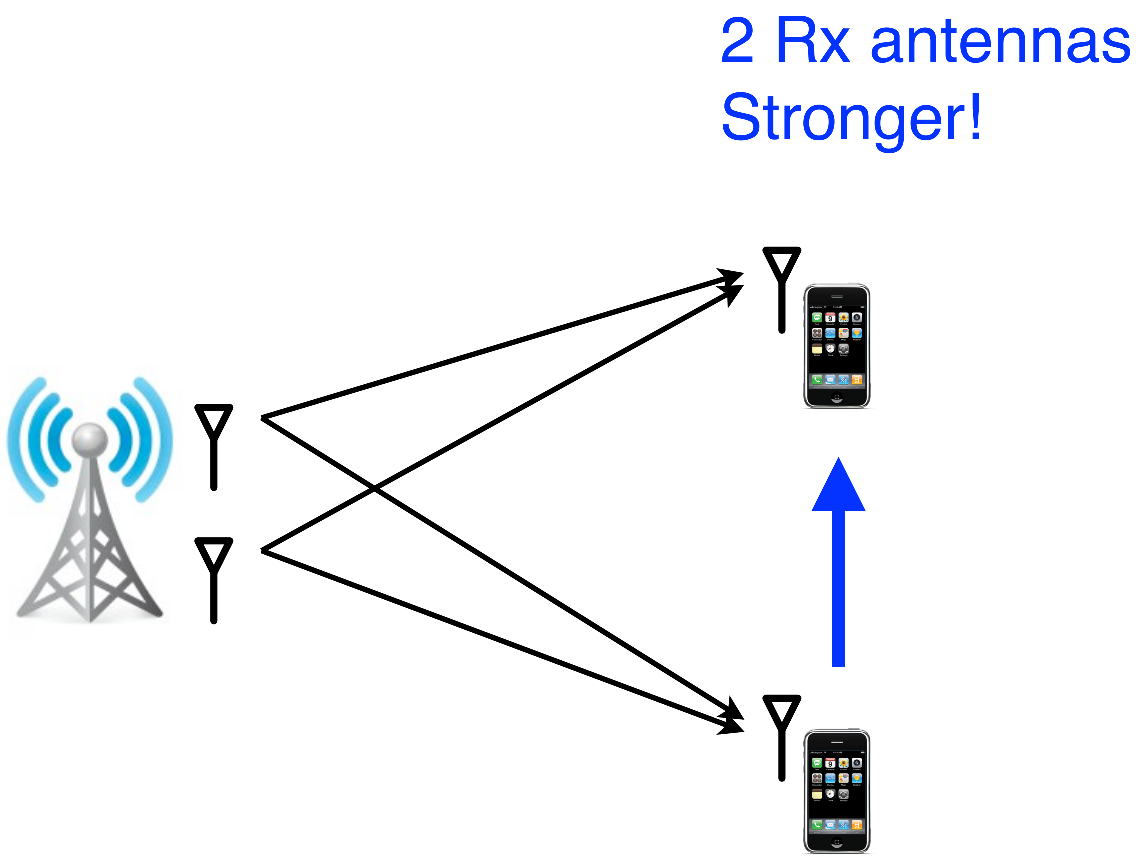
$$\mathsf{DoF}_1 = \frac{4}{2 + \frac{1}{\mathsf{DoF}_2}} = \frac{4}{2 + \frac{1}{1}} = \frac{4}{3}$$

Two Transmission Phases

- Transmission is divided into two phases
- Phase 1: time $m = 1{,}2$:
  - Transmit two order-1 messages using two time slots
- End of Phase 1:
  - From the delayed CSI, Tx is able to form one order-2 message
- Phase 2: time $m = 3$:
  - Transmit this order-2 message using one time slot
- Only phase 1 sends fresh data; the rest is to refine the reception by exploiting delayed CSIT and Rx side info.
- The idea can be extended to scenarios with more users and more Tx antennas, where higher-order messages have to be formed to achieve optimality

Optimality of 4/3

- It is remarkable that delayed CSIT is useful and we can achieve DoF = 4/3 > 1
- Can we do better?
- The answer is no



Enhance user 1 by feeding user 2's signal to user 1

Now we have a natural ordering of the users and we find again time-sharing is DoF optimal

Hence $d_1 + 2d_2 \leq 2$

Similarly $d_2 + 2d_1 \leq 2$

$\Longrightarrow 3\text{DoF} \leq 4$