

Impact of Elevated Base Stations on the Ultra-Dense Networks

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Abstract—In this letter, we examine the impact of the elevated base stations (BSs) on the performance of ultra-dense networks (UDNs). To this end, we first model the channel between an elevated BS and typical user equipment as a 3-D probabilistic line-of-sight channel by considering the height of BSs. Using stochastic geometry, we then derive the expression for the coverage probability of our considered UDNs. Finally, through numerical results, we show how the coverage probability can be maximized by judiciously selecting the height and the density of BSs. The results reported here provide insights into the deployment of BSs in the UDNs.

Index Terms—Ultra-dense network, line-of-sight, non-line-of-sight, network interference, stochastic geometry.

I. INTRODUCTION

Due to the proliferation of the smart devices and the increasing demand for mobile-based high-quality and high-definition multi-media services, the amount of mobile traffic is exploding each year [1]. This explosive growth in traffic demand of mobile users has imposed critical requirements on the fifth generation (5G) wireless communications, such as ultra-high data rate, ultra-low latency, ultra-large number of devices, and ultra-wide radio coverage [2]. Against this backdrop, UDNs have been recognized as a promising solution to fulfill these requirements of 5G [3].

On the other hand, the densification of wireless networks introduces new and challenging problems. Specifically, the height of BSs cannot be ignored when the distance between BS and user equipment (UE) is small. Moreover, the possibility of having line-of-sight (LoS) propagation increases drastically in UDNs. Therefore, significant research efforts have been devoted to characterizing the impacts of the densification on the performance of wireless networks [4]–[7]. Using the probability of LoS propagations, modeled as a function of the horizontal distance between BS and UE [8], the coverage probability of UDNs was studied in [4], considering both closest and strongest BS association. The impact of different path loss exponents for LoS and non-line-of-sight (NLoS) propagation and the impact of elevated BSs were examined in [5] and [6].

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respectively. Following the same model, the coverage probability of UDNs was investigated considering both the LoS propagations and elevated BSs [7]. However, [5] and [6] implicitly assumed that the probability of LoS propagations decreases as the antenna height of BSs increases, which is unlikely to be valid in practice since BSs with a higher height are more likely to establish the LoS path towards the UEs. In addition, in [7], they considered the elevation of the BS height but assumed equal height for all buildings, which is not realistic. It is more interesting to consider the scenario where heights of buildings are different and random. This motivates us to examine the impact of elevated BSs on the performance of UDNs under realistic propagation models.

Compared to the aforementioned works [4]–[7], our contributions are as follows: 1) We model the probability of LoS propagations as a function of the antenna height of BSs and the horizontal distance between BS and UE, considering different and random heights of buildings. As such, the probability of LoS propagations increases as the antenna height of BS increases, while decreases as the horizontal distance between BS and UE increases. 2) Based on our model, we derive a new expression for the coverage probability in UDNs with elevated BSs. 3) Different from the conclusion in the existing works [5]–[7] that the coverage probability of UDNs decreases as the height of BSs increases, we show that the coverage probability of our considered system can be maximized by selecting the antenna height and the BS density.

II. SYSTEM MODEL

In this section, we present the system model of UDN with elevated BSs including the network description, the channel model, and the BS association rule. We also examine the signal-to-interference ratio (SIR) at a typical UE.

A. Network Description

We consider a UDN, where single-antenna BSs and single-antenna UEs are spatially randomly distributed, as shown in Fig. 1. The locations of BSs and UEs are modeled as independent 2-dimensional homogeneous Poisson point processes (PPPs) Φ_{BS} and Φ_{UE} , respectively. We denote λ_{BS} and λ_{UE} as the densities of Φ_{BS} and Φ_{UE} , respectively. In this work, we assume each BS has at least one UE associated within its coverage area for simplicity.¹ We assume that all UEs are on the ground, while all BSs have their antennas at the height l . We assume that each BS supports one UE using one resource (i.e., time and frequency) with the same transmission power.

We consider that each BS has a LoS path towards its associated UE with a given probability due to the random distribution of obstacles between BSs and UEs. This LoS

¹Note that our work can be readily extended to the case with idle BSs, which do not have any associated UE. In the case with idle BSs, the density of BSs, interfering the typical UE, in (14) can be changed to the density of active BSs, of which approximated density is provided in [9].

count function $\Phi(A) = \# \text{ of } \{\Phi \cap A\}$



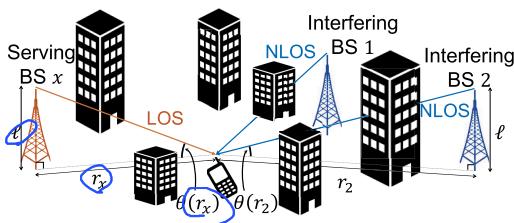


Fig. 1. Example of UDN with elevated BSs.

probability is determined by the environment parameters such as the locations of the BS and the UE and their elevation angle. As such, we present the LoS probability between the BS i in Φ_{BS} and a typical UE located at the origin as [10]

$$\mathcal{P}_L(r_i) = \frac{1}{1 + a \exp(-b(\theta(r_i) - a))}, \quad (1)$$

where r_i denotes the horizontal distance between the BS i and the typical UE, $\theta(r_i) = \frac{180}{\pi} \arctan\left(\frac{\ell}{r_i}\right)$, and a, b are the constants, determined by the environment. As such, the NLoS probability between the BS i and the typical UE can be obtained as $\mathcal{P}_{\text{NL}}(r_i) = 1 - \mathcal{P}_L(r_i)$.

We consider that the channel experiences both fading and path loss. Specifically, we assume that the channel for LoS propagation experiences Nakagami- m fading with path loss exponent α_L and the channel for NLoS propagation experiences Rayleigh fading with path loss exponent α_{NL} . As such, we have that $2 < \alpha_L \leq \alpha_{\text{NL}} \leq 4$. We denote the BSs forming LoS propagations and NLoS propagations to the typical UE as Φ_L and Φ_{NL} , respectively. The channel gain between the typical UE and the BS i is denoted by h_i . When the serving BS is in LoS propagation or NLoS propagation, we have $h_i = h_L$ or $h_i = h_{\text{NL}}$, respectively, where $h_L \sim \Gamma(m, 1/m)$ and $h_{\text{NL}} \sim \text{Exp}(1)$. Here, m denotes the Nakagami- m factor.

B. BS Association Rule

We consider a UE associating to the BS with the strongest average received power. Since the path loss exponents are different for LoS and NLoS propagations, the signal strength of the nearest BS is not always the strongest one, different to the conventional model. For example, as shown in Fig. 1, when the typical UE is associated to a BS with LoS propagation condition, there can exist an interfering BS with NLoS propagation condition, closer to the typical UE. As such, the probability density function (PDF) of the distance between the typical UE and its serving BS in conventional cellular networks cannot be directly applied to our system. To address this issue, in the sequel, we examine the distribution of the distances between a UE and its serving BS with LoS propagations and NLoS propagations, respectively.

The minimum link distances from the typical UE to the nearest BS among the ones with LoS and NLoS propagations are denoted by r_L and r_{NL} , respectively. For BSs in Φ_k , where $k = \{\text{L, NL}\}$, the conditional complementary cumulative distribution function (CCDF) and PDF of the horizontal distance between the typical UE and the nearest BS are, respectively, given by

$$\mathbb{P}[r_k > r] = \mathbb{P}[\Phi_k(B(0, r)) = 0] = \exp\left(-2\pi\lambda_{\text{BS}} \int_0^r t \mathcal{P}_k(t) dt\right), \quad (2)$$

$$f_{r_k}(r) = \frac{\partial}{\partial r} (1 - \mathbb{P}[r_k > r]) = 2\pi\lambda_{\text{BS}} r \mathcal{P}_k(r) \exp\left(-2\pi\lambda_{\text{BS}} \int_0^r t \mathcal{P}_k(t) dt\right), \quad (3)$$

where $B(0, r)$ denotes the ball of radius r centered at the origin. Using (2) and (3), the PDFs of the distance between the typical UE and its serving BS, r_o , in LoS and NLoS propagations are, respectively, obtained as

$$f_{r_0}^L(r) = \mathbb{P}\left[r_{\text{NL}} > r^{\frac{\alpha_L}{\alpha_{\text{NL}}}}\right] f_{r_L}(r) = 2\pi\lambda_{\text{BS}} r \mathcal{P}_L(r) \exp\left(-2\pi\lambda_{\text{BS}} \int_0^r t \mathcal{P}_L(t) dt\right) \times \exp\left(-2\pi\lambda_{\text{BS}} \int_0^{r^{\frac{\alpha_L}{\alpha_{\text{NL}}}}} t \mathcal{P}_{\text{NL}}(t) dt\right), \quad (4)$$

$$f_{r_0}^{\text{NL}}(r) = \mathbb{P}\left[r_L > r^{\frac{\alpha_{\text{NL}}}{\alpha_L}}\right] f_{r_{\text{NL}}}(r) = 2\pi\lambda_{\text{BS}} r \mathcal{P}_{\text{NL}}(r) \exp\left(-2\pi\lambda_{\text{BS}} \int_0^r t \mathcal{P}_{\text{NL}}(t) dt\right) \times \exp\left(-2\pi\lambda_{\text{BS}} \int_0^{r^{\frac{\alpha_{\text{NL}}}{\alpha_L}}} t \mathcal{P}_L(t) dt\right). \quad (5)$$

Since the further derivation of distance PDFs in (4) and (5) are not tractable, mainly due to the LoS probability function in (1), we propose the following approximation

$$\mathcal{P}_L(r) \approx \exp(-p_\ell r^2), \quad (6)$$

where p_ℓ is a tunable value determined by the BS height ℓ .² Based on (4)–(6), the approximated PDFs of r_o are derived as

$$\tilde{f}_{r_0}^L(r) = 2\pi\lambda_{\text{BS}} r (1 - \exp(-p_\ell r^2)) \exp\left(-\pi\lambda_{\text{BS}} r^{\frac{2\alpha_L}{\alpha_{\text{NL}}}}\right) \times \exp\left[\frac{\pi\lambda_{\text{BS}}}{p_\ell} (\exp(-p_\ell r^2) - \exp(-p_\ell r^{\frac{2\alpha_L}{\alpha_{\text{NL}}}}))\right], \quad (7)$$

$$\tilde{f}_{r_0}^{\text{NL}}(r) = 2\pi\lambda_{\text{BS}} r \exp(-p_\ell r^2) \exp(-\pi\lambda_{\text{BS}} r^2) \times \exp\left[\frac{\pi\lambda_{\text{BS}}}{p_\ell} (\exp(-p_\ell r^2) - \exp(-p_\ell r^{\frac{2\alpha_{\text{NL}}}{\alpha_L}}))\right]. \quad (8)$$

C. Signal-to-Interference Ratio

In this work, we focus on the interference-limited environment (as in [4]–[7]). As such, the instantaneous SIR at the typical UE located at the origin can be expressed as

$$\text{SIR}_o = \frac{h_o (r_o^2 + \ell^2)^{-\frac{\alpha_o}{2}}}{I_L + I_{\text{NL}}}, \quad (9)$$

where for the link between the typical UE and the BS located at x , h_x is the channel fading, r_x is the horizontal distance, and α_x is the path loss exponent. When the serving BS is in LoS propagation or NLoS propagation, we have $h_o = h_L$ or $h_o = h_{\text{NL}}$, respectively. In (9), I_L and I_{NL} are the total interference power received from the BSs in Φ_L and the

²The LoS probability function in (1) is approximated to (6) with tuned parameter p_ℓ from the cftool function in MATLAB for various BS height.

BSs in Φ_{NL} , given by $I_L = \sum_{y \in \Phi_L \setminus \{o\}} h_y (r_y^2 + \ell^2)^{-\frac{\alpha_L}{2}}$ and $I_{NL} = \sum_{y \in \Phi_{NL} \setminus \{o\}} h_y (r_y^2 + \ell^2)^{-\frac{\alpha_{NL}}{2}}$, respectively.

III. COVERAGE PROBABILITY ANALYSIS

In this section, we analyze the coverage probability of our system, considering both the antenna height of BSs and the LoS propagations. The coverage probability is defined as the probability that the received SIR at the typical UE is larger than a certain threshold ζ . According to (9), we derive the coverage probability of our system as

$$\begin{aligned} \text{SIR}_o &= \mathbb{P}[\text{SIR}_o > \zeta] \\ &= \int_0^\infty \mathbb{E} \left[\mathbb{P} \left[h_L > \frac{\zeta I}{(r^2 + \ell^2)^{-\frac{\alpha_L}{2}}} \mid r, I \right] \right] f_{r_0}^L(r) dr \\ &\quad + \int_0^\infty \mathbb{E} \left[\mathbb{P} \left[h_{NL} > \frac{\zeta I}{(r^2 + \ell^2)^{-\frac{\alpha_{NL}}{2}}} \mid r, I \right] \right] f_{r_0}^{NL}(r) dr, \end{aligned} \quad (10)$$

where $I = I_L + I_{NL}$. In (10), we have

$$\begin{aligned} &\mathbb{E} \left[\mathbb{P} \left[h_L > \frac{\zeta I}{(r^2 + \ell^2)^{-\frac{\alpha_L}{2}}} \mid r, I \right] \right] = \mathbb{E} [\mathbb{P}[h_L > zI \mid r, I]] \\ &\stackrel{(a)}{=} \mathbb{E}_I \left[e^{mzI} \sum_{k=0}^{m-1} \frac{(mz)^k}{k!} I^k \right] = \sum_{k=0}^{m-1} \frac{(mz)^k}{k!} \mathbb{E}_I [I^k e^{-mzI}] \\ &\stackrel{(b)}{=} \sum_{k=0}^{m-1} \left[\frac{(-s)^k}{k!} \frac{d^k}{ds^k} \mathcal{L}_I(s) \right]_{s=m\zeta/(r^2+\ell^2)^{-\frac{\alpha_L}{2}}}, \quad (11) \\ &\mathbb{E} \left[\mathbb{P} \left[h_{NL} > \frac{\zeta I}{(r^2 + \ell^2)^{-\frac{\alpha_{NL}}{2}}} \mid r, I \right] \right] \\ &= \mathcal{L}_I \left(\frac{\zeta}{(r^2 + \ell^2)^{-\frac{\alpha_{NL}}{2}}} \right). \quad (12) \end{aligned}$$

In (11), (a) is obtained from the fact that $h_L \sim \Gamma(m, 1/m)$ and (b) is obtained by applying the property of the Laplace transform. Since $h_{NL} \sim \text{Exp}(1)$, (12) is obtained.

Substituting (11) and (12) into (10), we obtain the coverage probability of our system in (13) (shown at the bottom of the page). In (13), the Laplace transform of the interference, $\mathcal{L}_I(s)$, can be derived as [7]

$$\begin{aligned} \mathcal{L}_I(s) &= \mathbb{E}_\Phi \left[\prod_{y \in \Phi_L \setminus \{o\}} \mathbb{E}_{h_y} \left[e^{-sh_L(r_y^2 + \ell^2)^{-\frac{\alpha_L}{2}}} \mid h_y = h_L \right] \right. \\ &\quad \times \left. \prod_{y \in \Phi_{NL} \setminus \{o\}} \mathbb{E}_{h_y} \left[e^{-sh_{NL}(r_y^2 + \ell^2)^{-\frac{\alpha_{NL}}{2}}} \mid h_y = h_{NL} \right] \right] \\ &\stackrel{(a)}{=} \mathbb{E}_{\Phi_L} \left[\prod_{y \in \Phi_L \setminus \{o\}} \frac{1}{\left(1 + \frac{s}{m}(r_y^2 + \ell^2)^{-\frac{\alpha_L}{2}}\right)^m} \right] \end{aligned}$$

$$\mathcal{P}_{\text{cov}}(\zeta) = \int_0^\infty f_{r_0}^L(r) \underbrace{\sum_{k=0}^{m-1} \left[\frac{(-s)^k}{k!} \frac{d^k}{ds^k} \mathcal{L}_I(s) \right]_{s=m\zeta(r^2+\ell^2)^{-\frac{\alpha_L}{2}}} dr}_{\varphi(\zeta)} + \int_0^\infty f_{r_0}^{NL}(r) \mathcal{L}_I \left(\frac{\zeta}{(r^2 + \ell^2)^{-\frac{\alpha_{NL}}{2}}} \right) dr \quad (13)$$

$$\begin{aligned} &\times \mathbb{E}_{\Phi_{NL}} \left[\prod_{y \in \Phi_{NL} \setminus \{o\}} \frac{1}{1 + s(r_y^2 + \ell^2)^{-\frac{\alpha_{NL}}{2}}} \right] \\ &\stackrel{(b)}{=} \exp \left(-2\pi\lambda_{\text{BS}} \left[\int_r^\infty \mathcal{P}_L(t) t \right. \right. \\ &\quad \times \left(1 - \frac{1}{\left(1 + \frac{s}{m}(t^2 + \ell^2)^{-\frac{\alpha_L}{2}}\right)^m} \right) dt \\ &\quad \left. \left. + \int_{r^{\frac{\alpha_L}{2}}}^\infty (1 - \mathcal{P}_L(t)) t \left(1 - \frac{1}{1 + s(t^2 + \ell^2)^{-\frac{\alpha_{NL}}{2}}} \right) dt \right] \right), \end{aligned} \quad (14)$$

where (a) is due to the fact that Φ_L and Φ_{NL} are independent, and (b) is obtained by applying the probability generating functional (PGFL).

Based on (14), by applying the Faà di Bruno's formula [11], we further obtain $\varphi(\zeta)$ in (13) as

$$\varphi(\zeta) = \sum_{k=0}^{m-1} (-s)^k \sum \frac{f^{(m_1+\dots+m_n)}(g(s))}{\prod_{n=1}^k m_n! n!^{m_n}} \prod_{j=1}^k \left(g^{(j)}(s) \right)^{m_j}, \quad (15)$$

where $f(s) = \exp(s)$ and $f(g(s)) = \mathcal{L}_I(s)$ in (14). The second summation in (15) is over all nonnegative integers (m_1, \dots, m_n) satisfying the constraint

$$1 \cdot m_1 + 2 \cdot m_2 + 3 \cdot m_3 + \dots + k \cdot m_k = k. \quad (16)$$

To obtain $g^j(s)$, we apply the Leibniz integral rule [12], which is given by

$$\frac{d}{dx} \left(\int_a^b f(x, t) dt \right) = \int_a^b \frac{\partial}{\partial x} f(x, t) dt. \quad (17)$$

By applying (17) and $f^{(n)}(s) = f(s)$, (15) is represented as

$$\begin{aligned} \varphi(\zeta) &= \mathcal{L}_I(s) \sum_{k=0}^{m-1} (-s)^k \sum \frac{1}{\prod_{n=1}^k m_n! n!^{m_n}} \\ &\quad \times \prod_{j=1}^k \left[\pi \lambda_{\text{BS}} e^{p_\ell \ell^2} \prod_{u=0}^{j-1} (-m-u) \right. \\ &\quad \times \int_{r^2+\ell^2}^\infty \left(1 + \frac{s}{m} x^{-\frac{\alpha_L}{2}} \right)^{-m-j} \\ &\quad \times \left. \left(\frac{x^{-\frac{\alpha_L}{2}}}{m} \right)^j e^{-p_\ell x} dx \right. \\ &\quad + \pi \lambda_{\text{BS}} \prod_{v=0}^{j-1} (-1-v) \int_{r^{2\alpha'}+\ell^2}^\infty \left(x^{-\frac{\alpha_{NL}}{2}} \right)^j \\ &\quad \times \left. \left(1 + sx^{-\frac{\alpha_{NL}}{2}} \right)^{-1-j} \left(1 - e^{-p_\ell(x-\ell^2)} \right) dx \right]^{m_j}. \end{aligned} \quad (18)$$

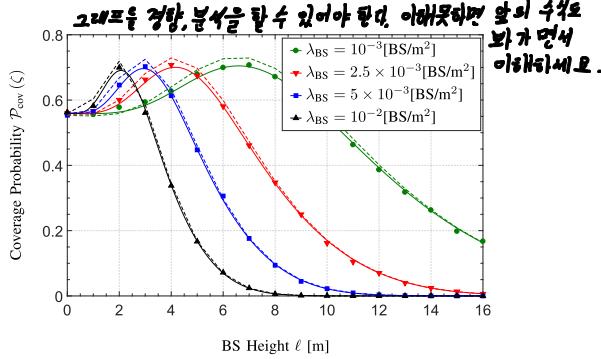


Fig. 2. $\mathcal{P}_{\text{cov}}(\zeta)$ versus ℓ for difference values of λ_{BS} with $\zeta = 1$.

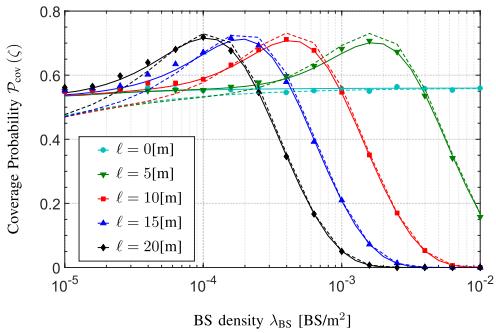


Fig. 3. $\mathcal{P}_{\text{cov}}(\zeta)$ versus λ_{BS} for difference values of ℓ with $\zeta = 1$.

IV. NUMERICAL RESULTS

In this section, we present numerical results to validate our analysis on the coverage probability of UDNs with elevated BSs and examine the impacts of the antenna height of BSs and the density of BS on the coverage probability of our system. Without loss of generality, throughout this section, we consider that the antenna height of BSs ranges from 0m to 20m, the typical UE is located at the origin of the cell and the Nakagami- m factor $m = 3$. Moreover, we consider a dense urban environment in [10] with $\alpha_L = 2.5$, $\alpha_{NL} = 4$.

Figures 2 and 3 shows the coverage probability in UDNs as a function of the BS height ℓ and the BS density λ_{BS} . Analysis results with and without the approximation in (6) are represented by the dotted and solid lines, respectively. Simulation results, obtained from Monte Carlo simulations, are also provided with filled markers, and they show a good agreement with both analyses, especially for dense networks.

From Fig. 2, we can see that $\mathcal{P}_{\text{cov}}(\zeta)$ first increases and then decreases with ℓ , and there exists a unique ℓ that maximizes $\mathcal{P}_{\text{cov}}(\zeta)$ for each value of λ_{BS} . For the small ℓ , increasing ℓ leads to a higher LoS probability and a better link quality between the typical UE and its serving BS. Consequently, the coverage probability improves. However, as ℓ keeps increasing, the link quality degrades as the main link distance increases. As a result, the coverage probability decreases. We also see that the optimal ℓ that maximizes the coverage probability increases as λ_{BS} decreases, indicating that the BSs with higher antenna height need to be deployed when the network is less densified.

Similarly, from Fig. 3, we see that there exists a unique λ_{BS} that maximizes the coverage probability for each ℓ . We then see that the optimal λ_{BS} that maximizes the coverage probability increases as the antenna height of BSs reduces. This is because the maximization of coverage probability requires the establishment of LoS propagation condition. Hence, the density of BSs that maximizes the coverage probability should be larger when the antenna height of BSs is small. However, as the antenna height of BSs increases, the probability of LoS propagation condition is already high, increasing the density of BSs introduces more interference. Subsequently the coverage probability is degraded.

V. CONCLUSION

In this letter, we investigated and derived the coverage probability of UDNs with elevated BSs under more realistic channel propagation model. Using the derived expression, we examined the impact of the antenna height and the density of BSs on the coverage probability. We showed that elevated BSs could actually improve the coverage probability, and a higher BSs antenna height is required for lower BS density to maximize the coverage probability. For future works, the results provided here can be extended to more sophisticated UDNs, such as a millimeter wave (mmWave)-based UDN where the LoS link severely affects the network performance.

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