HUB4045F assignment 4: 25% of final mark

[Solutions to all questions to be submitted on Amathuba] **[Total Marks: 54]**

Question 1 [8 marks]

When answering this question refer to Figures 1 and 2 below:

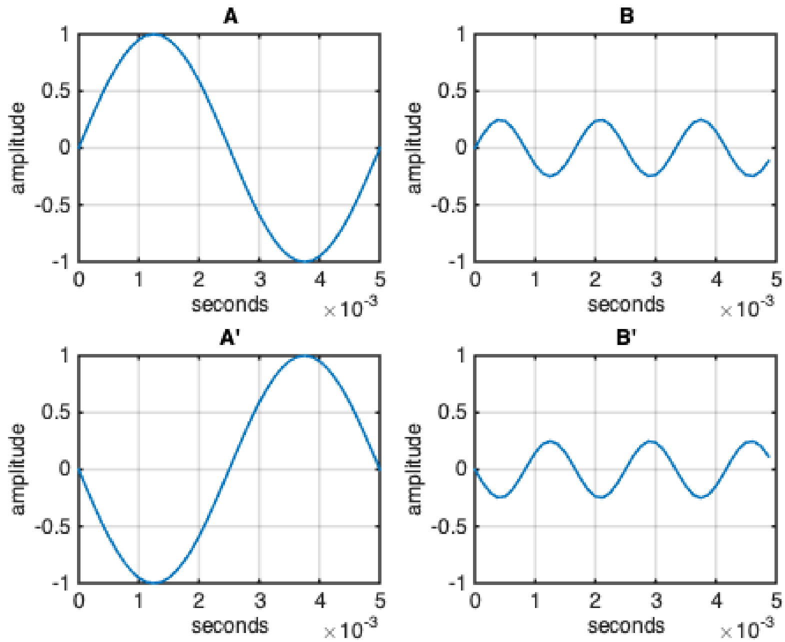


Figure 1

Inverse Fourier transforms of the MRI signals obtained when a frequency-encoding gradient (G_F) of 0.1 G/cm is applied, yield the components shown in Figure 1 for the AA' and BB' columns, respectively.

a) Calculate the distance (in cm) of the point in the tissue corresponding to pixel A from the reference point that is assigned the frequency 0Hz. (γ = 42.58 MHz/T for protons.) (3)

b) What is the distance between tissue structures corresponding to pixel A and pixel B? (2)

c) What is the proton density of pixel A relative to pixel B? (1)

d) In Figure 2 below, pixel A' is in the same frequency-encoding column as A, and pixel B' is in the same frequency-encoding column as B. A' and B' are in the same phase-encoding row of the image matrix along the y-axis and represent coordinates in the tissue that are 2cm from the AB row. How long would a phase-encoding gradient (G_P) of 0.2 G/cm need to be applied to cause the observed phase changes between pixel A and A' and pixel B and B'? (2)

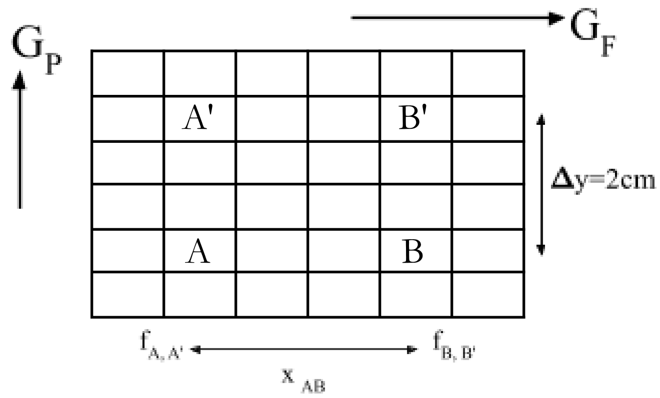


Figure 2

Question 2 [12 marks]

Given the table of tissue parameters below, investigate the pulse sequence parameters for T1- and T2-weighted pulse sequences using the relationship:

$$S = \rho(1 - e^{\frac{-TR}{T1}})(e^{\frac{-TE}{T2}})$$

where S is the MR signal strength, ρ is the effective spin density, TR is the repetition time and TE is the echo time.

Tissue	**T1**	**T2**	ρ
Gray matter	1.4 s	90 ms	0.80
White matter	900 ms	70 ms	0.65
Fat	250 ms	60 ms	0.90
CSF	3.5 s	2 s	1.00

a) If TR=2.5s and TE=100ms, compute the signal for each tissue type. Which tissue will be brightest? What type of weighting is this?

(6)

b) If TR=200ms and TE=20ms, what type of weighting will the image have? State the tissue that will be brightest and give its signal.

(3)

c) If TR=5.0s and TE=20ms, what type of weighting will the image have? State the tissue that will be brightest and give its signal?

(3)

Question 3 [10 marks]

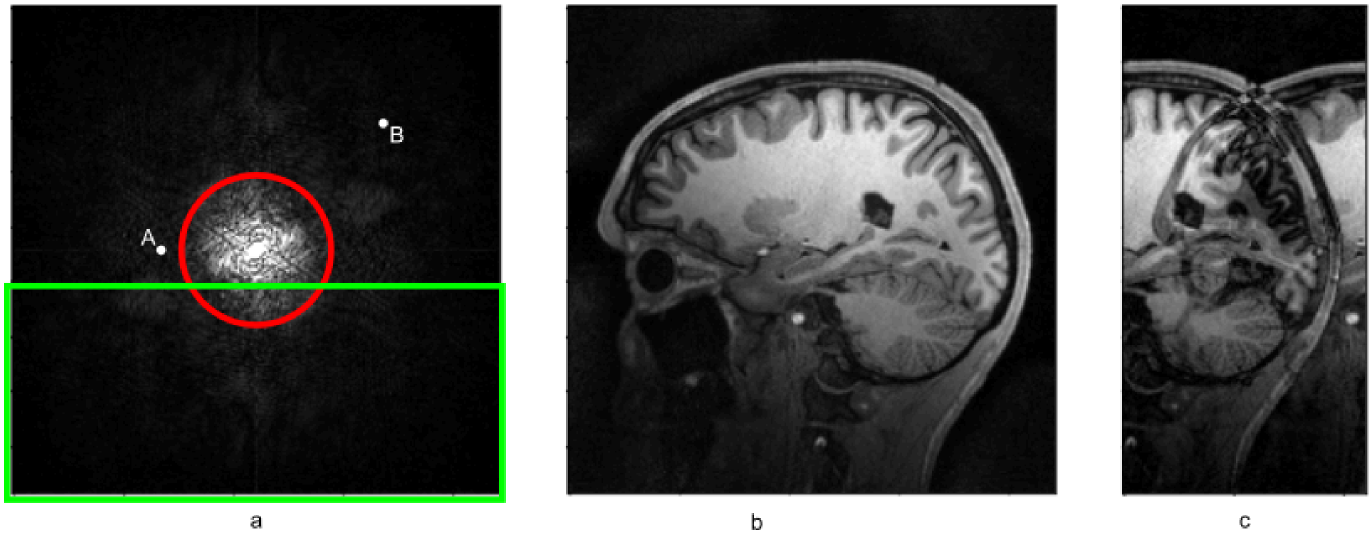


Figure 3

Figure 3 shows a) the absolute value of the raw k-space data of a sagittal slice through the brain, as well as b) its inverse FT i.e. the brain in image space.

The brain image is provided for you (brain.dcm). You may use Matlab's image processing toolbox or Python's pydicom package to experiment with k-space to obtain answers to the questions below. Read in the image and use fft2 to generate k-space. Note that you need to use fftshift to move the zero-frequency component to the centre of k-space. You can then manipulate k-space and use ifft2 to see the result on the image. Include images of the results for each question.

a) Show the brain image that is produced when you zero out the centre of k-space (inside the red circle) before performing the inverse FT. Describe what you see and explain why.
(2)

b) Show the brain image that results if there is an RF spike at the position marked by A. Comment on the orientation and appearance of the artefact.
(2)

c) Show the brain image that results if there is an RF spike at the position marked by B. Explain how and why this artefact differs from position A.
(2)

d) Show the brain image that is produced when you zero out just under half of k-space (the green rectangle) and give an explanation why.
(2)

e) How would you produce the image in Figure 3c by manipulating the k-space of Figure 3a? Explain what you did and why it has this effect. Show the resultant k-space.
(2)

Question 4 [12 marks]
Radial sampling is an alternative method of covering k-space, as opposed to Cartesian sampling, which uses radial spokes rotated around the centre of k-space. Here we will design a pulse sequence for a single slice radial readout as shown in Figure 4.

Draw a pulse sequence diagram that will produce each of the 4 radial spokes in k-space marked a, b, c and d, showing the timings and relative amplitude of the frequency and phase encoding gradients, as well as the ADC timing. You can use either a spin echo or gradient echo sequence and assume the phase and frequency encoding gradients are oriented along the x- and y-axes, respectively.

(4× 3)

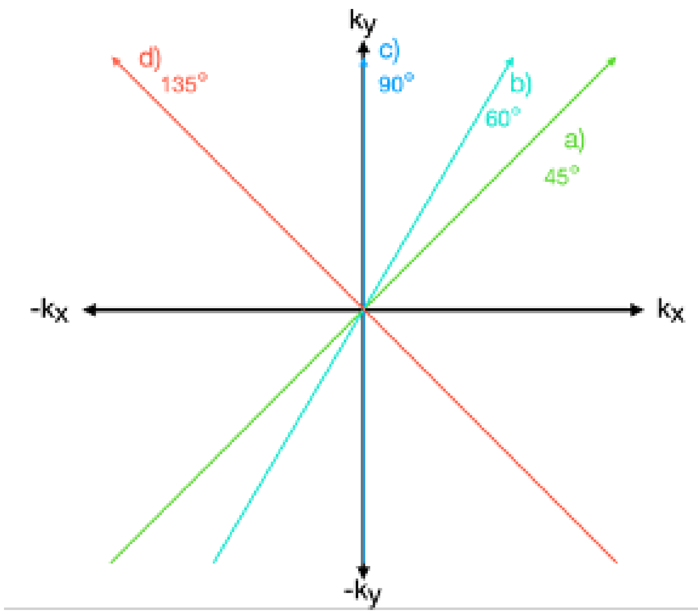
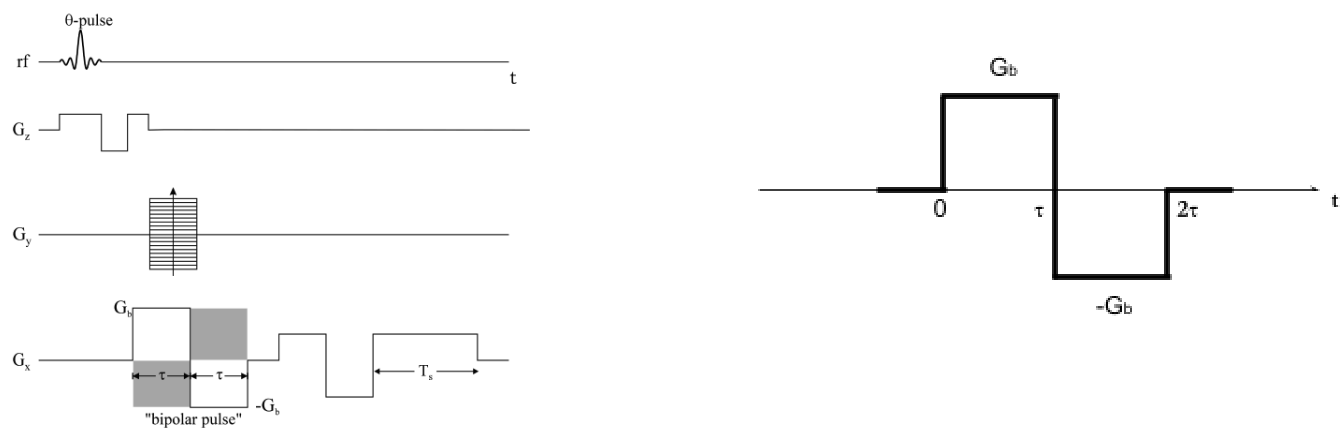


Figure 4

Question 5 [12 marks]
Commonly when visualising MRI acquisitions, only the magnitude (or amplitude) part of the signal (or echo) is used to produce an image. However, the other part of the signal is often discarded as it usually does not contain any 'useful' information: the phase.
Here we will explore an MRI sequence that allows us to use the phase of the signal to encode useful information, specifically velocity.
Figure 5a shows a sequence diagram for a velocity-encoded phase contrast MRI sequence.
Figure 5b shows the component of the sequence diagram that is responsible for encoding velocity, a bipolar gradient.



a b

Figure 5

Given $f(t) = \gamma G x(t)$. Assume at t=0, a spin is located at $x(t)=x_0$.

a) Use Figure 4b to derive the equation for phase, Φ, for a square monopolar gradient pulse (i.e. $0 \le t \le \tau$) if,
   i) the spin is stationary (1)
   ii) the spin is moving with constant velocity (v_x) (2)

b) Extend the equation to cover the full range of the bipolar gradient pulse (i.e. $0 \le t \le 2\tau$) for,
   i) the stationary spin (1)
   ii) the spin with constant velocity (v_x) (2)

Two images are collected with opposite polarity gradients (as shown by the grey blocks in Figure 5a) allowing background phase contributions to be subtracted out leaving only contributions due to velocity.

$$\Phi_{vel} = \Phi_{+-} - \Phi_{-+} = 2G\gamma v_x \tau^2$$

Consider a cylindrical tube in which there is laminar flow as shown in Figure 6.

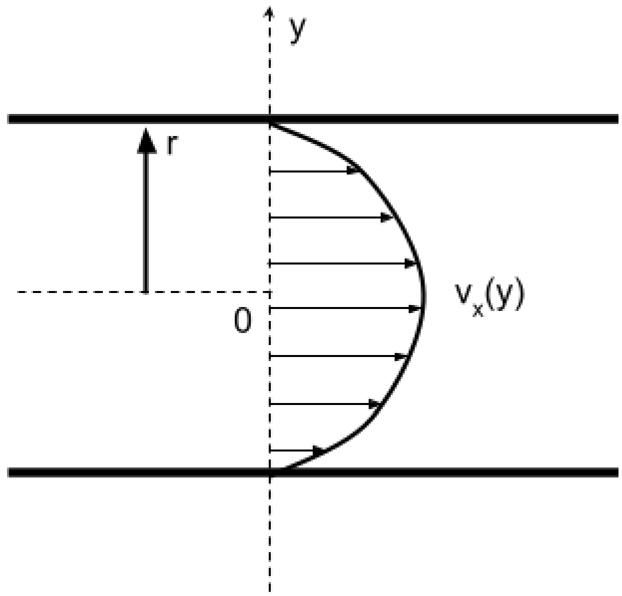


Figure 6

The equation for the velocity at a given position along the y-axis is given by:

$$v_x(y) = v_{max}\left(1 - \left(\frac{y}{r}\right)^2\right)$$

c) Calculate the phase for three points along the slice of the tube. With the given bipolar pulse: τ=1.5 ms, G_x=0.5 G/cm, and laminar flow profile: v_{max}=65 cm/s

   i) $y = r$

   (1)

   ii) $y = \frac{1}{2}r$

   (1)

   iii) $y = 0$

   (1)

d) Consider that phase can only be measured between $-\pi < \Phi \leq \pi$.

   i) Calculate the maximum velocity that can be uniquely encoded (known as the v_{enc} parameter) for the laminar flow example above.

   (1)

   ii) Calculate the position along the y-axis where $v_x(y)=v_{enc}$ if the radius of the tube is 1 cm.

   (1)

   iii) Draw a diagram of velocity vs. measured phase for the range v_x:[$-2v_{enc}$; $2v_{enc}$]

   (1)