

# Optimal Pricing in Social Networks (Extended Abstract)

OZAN CANDOĞAN

Laboratory for Information and Decision Systems, MIT

and

KOSTAS BIMPIKIS

Graduate School of Business, Stanford University

and

ASUMAN OZDAGLAR

Laboratory for Information and Decision Systems, MIT

---

We consider the pricing strategies of a monopolist selling a divisible good (service) to consumers who are embedded in a social network. We assume that each consumer's usage level depends directly on the usage of her neighbors in the social network, and investigate the optimal pricing policies of the monopolist. We show that if the monopolist can perfectly price discriminate the agents, then the price offered to each agent has three components: a nominal price, a discount term due to the agent's influence on her neighbors, and a markup term due to the influence of her neighbors on the agent. We also characterize the optimal pricing strategies in settings where the monopolist is constrained to offering a single price, and where she can choose two distinct prices (a discounted and a full price). For the former setting we provide a polynomial time algorithm for the solution of the pricing problem. On the other hand, we show that in the latter setting the optimal pricing problem is NP-hard, and we provide an approximation algorithm, which, under some conditions, achieves at least 88% of the maximum profit.

Categories and Subject Descriptors: K.4.4 [Computers and Society]: Electronic Commerce

General Terms: Economics, Algorithms, Theory.

Additional Key Words and Phrases: Optimal pricing, social networks, externalities.

---

## 1. INTRODUCTION

Social networks underpin most human interactions and have traditionally been the main medium through which individuals obtain their information and form their opinions. The torrent of new communication technologies have transformed their structure and increased our reliance on them for social interactions and access to global information. Social, business and political decisions are now, to an unparalleled extent, shaped by information provided by the networks in which they are situated. This area presents a unique opportunity for academic research mainly for two reasons. First, social networks play a central role in a variety of real world environments. For example, it is well documented that they are an integral component of the labor market (see [Montgomery 1991], [Granovetter 1974]) and they are crucial for the successful marketing of consumer products and the adoption of new technologies (see [Ellison and Fudenberg 1993]).

---

Authors' addresses: [candogan@mit.edu](mailto:candogan@mit.edu), [kostasb@stanford.edu](mailto:kostasb@stanford.edu), [asuman@mit.edu](mailto:asuman@mit.edu).

Second, due to the overwhelming success of online social networking communities, such as Facebook and Twitter, and the ubiquity of the Internet based services that are built around social networks, there is a wealth of data available on the interaction between individuals and its influence on their decision making. So far, this data has been used mostly as part of research efforts descriptive in nature. Specifically, the focus has been on understanding the data and introducing models on how individuals interact with each other. However, we feel that there is a need for prescriptive research, i.e., research that mainly focuses on the following broad question: how can we use our understanding of the structure and function of social networks to improve outcomes? In our work [Candogan et al. 2011], we take a prescriptive point of view, and investigate whether and how a monopolist that possesses knowledge regarding the social network of her consumers, can derive a pricing policy that uses this information and maximize profits.

More concretely, we focus on products that exhibit a local (positive) network effect: increasing the usage level of a consumer has a positive impact on the usage levels of her peers. Concrete examples of such goods include online games (e.g., World of Warcraft, Second Life) and social networking tools and communities (e.g., online dating services, employment websites etc.). More generally, the local network effect can capture the word of mouth communication among agents. In this setting, we consider a monopolist, who has access to data on network interactions of individuals, and investigate how she could improve her pricing strategies using the relevant data.

## 2. MAIN RESULTS

We investigate the pricing problem of the monopolist in three different settings. In the first setting, we assume that the monopolist can use perfect price discrimination, i.e., it can offer a different unit price to each of the individuals in the network. In this setting, we show that the optimal prices admit an interesting decomposition to three components: (i) a nominal term which is independent of the network structure, (ii) a discount term proportional to the influence that this agent exerts over the rest of the social network, (iii) and a markup term proportional to the influence that the network exerts on the agent. Both the markup and the discount are proportional to the Bonacich centrality of the agent's neighbors in the social network structure, which is a measure of network influence introduced and used widely in the sociology literature. Thus, our work provides a micro-founded model that motivates use of Bonacich centrality as a measure of influence. Informally, our result suggests that agents get a discount proportional to the amount they influence their peers to purchase the product, and they receive a markup if they are strongly influenced by other agents in the network.

Perfect price differentiation is typically hard to implement. Thus, we also discuss the optimal strategy of a monopolist who can offer a single uniform price for the good. Since the monopolist can use a single price, at the optimal price some of the agents may prefer not to purchase the product. In this setting, the monopolist should choose the price such that price is low enough to induce high total consumption, and large enough to result in large revenues. We develop an algorithm that finds the optimal single price in time polynomial in the number of agents. The

algorithm relies on a ranking of agents according to a novel measure of centrality in the underlying network. Using this ranking, the algorithm considers different subsets of the consumers and finds the optimal price provided that only the consumers in the given subset purchase a positive amount of the good. We show that it is sufficient to consider  $N$  such subsets (for a network of  $N$  agents), and the algorithm computes the optimal price in time polynomial in  $N$ .

Third, we consider an intermediate setting, where the monopolist can offer a full price or a discounted price to each agent. In this setting, the decision problem of the monopolist involves deciding which subset of agents should receive the discounted prices. This problem is related to the well-known MAX-CUT problem, which involves finding a graph cut that maximizes the total weight of the edges between the two partitions, obtained as a result of the cut. Exploiting this relation, we establish that the optimal pricing problem is NP-hard. In this setting, we provide an approximation algorithm that recovers (in polynomial time) at least 88 % of the optimal revenue for the case of two prices.

Finally, we study the impact of the availability of network information on monopolist's profits. In particular, we compare profits of a monopolist that does not have access to this information to those of a monopolist that uses this information optimally. We show that when the influence structure is asymmetric (when agent  $i$  influences agent  $j$ , significantly more than  $j$  influences  $i$ ), the monopolist can significantly improve her profits by using pricing rules which exploit the underlying network structure.

### 3. CONCLUSIONS

Our recent work [Candogan et al. 2011] shows that firms can significantly improve their pricing decisions by using the available social network data. In particular, we characterize the optimal pricing strategies of a monopolist who has access to the social network information of her consumers. This work leads to a number of interesting future questions. One interesting direction is to see how the optimal pricing strategies change in presence of competition between firms. A second interesting direction is characterizing the pricing strategies in dynamic settings, where the players learn their preferences or the underlying quality of the good. More generally, we feel that the field is in a great need for more prescriptive research that mostly focuses on how to use our, mature by now, understanding of the structure of social networks to improve managerial decision making and policy recommendations. We believe that this is both an extremely challenging and rewarding area for future research.

### REFERENCES

- CANDOGAN, O., BIMPIKIS, K., AND OZDAGLAR, A. 2011. Optimal pricing in networks with externalities. *Submitted*.
- ELLISON, G. AND FUDENBERG, D. 1993. Rules of thumb for social learning. *Journal of Political Economy*.
- GRANOVETTER, M. 1974. Getting a job: A study of contacts and careers. *Harvard University Press*.
- MONTGOMERY, J. 1991. Social networks and labor-market outcomes: Toward an economic analysis. *The American Economic Review*.