Coordination and optimization of dynamic pricing and production decisions

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***Abstract*—This paper deals with the coordination of the production operations and the pricing strategies. The manager must determine the production planning and selling price at each decision period. The objective is to determine the production operations rationing (production quantities, inventory level and setup operations) as well as pricing policies in order to maximize the profit function of a firm that produces multiple products intended for multiple markets.**

This problem is formulated as a mixed integer nonlinear model with considering different constraints such as: production capacity, setup costs, and demand seasonality. The demand function of each product is assumed be continuous and strictly decreasing according to the prices. An optimization algorithm, based on the outer approximation methodology, is also presented.

The effectiveness of the constant and dynamic pricing policies is comparatively analyzed based on some numerical instances inspired by the literature. The numerical study also shows that the proposed algorithm outperforms the commercial solver in terms of solution quality and computational times.

I. Introduction

From a historical perspective, the interest on revenue management and pricing practices started with the pioneering research on airline and hotel overbooking. Pricing is an important topic that offers many challenges to companies and researchers. Initially, many firms and researchers focus on pricing alone as a tool to improve profits. However, for manufacturing industries, the coordination of price decisions with other aspects of the supply chain such as production and distribution is not only useful, but is essential. The coordination of these decisions means an approach that optimizes the system rather than individual elements, improving both the efficiency of the supply chain and of the firm [4].

Pricing strategies such as dynamically changing the pricing of products are an important revolution in retail and manufacturing industries driven in large part by the Internet and the Direct-to-Customer model. Elmaghraby and Keskinocak [6] presented a review of current practices in dynamic pricing as well as the dynamic pricing literature. Recently, Ahmadi et al. [1] studied a joint pricing and rationing in a production system. The authors considered production system with a single product and two classes of customers that are different in shortage cost with two levels of sales price. Their objective was to determine the inventory rationing as well as pricing policies in order to maximize the profit function of the system. They formulated the problem as a Markov decision problem and characterized the optimal policies for pricing and inventory rationing. Wu et al. [14] investigated the production-planning problem for new products with pricing decisions. They combined the dynamic lot-sizing model with the generalized Bass model that incorporates prices effect to provide optimal decisions for pricing and production planning problems.

Morgan et al. [12] considered a multi-products production model where a production line manufactures all products with the same frequency. They solved it using a vector of prices that maximize the profit over a fixed production interval. Dobson and Yano [5] generalized this model by distinguishing two decision contexts: make-to-stock and make-to-order. Kunreuter and Scharge [11] developed a procedure to solve the pricing problem coordinating with production for a single product case. The demand function is considered to be deterministic, seasonal and decreasing according to the prices. The authors assumed a fixed price order that varies throughout the time horizon. Their approach does not guarantee the optimality. Gilbert [8] and Van den Heuvel and Wagelmans Gilbert [13] developed a polynomial time algorithm to resolve this problem to optimality. These two research studies integrated the setup costs, but they did not consider the production capacity constraints. Later, Gilbert [9] extended his previous work to the case of constant pricing for several products characterized by seasonal demands. The author used a linear decreasing demand function. Guenes et al. [10] introduced some efficient models and optimization methods to deal with several variants of the pricing problem in the case of a single product. Guenes et al. [7] extended this work by considering concave revenue

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functions. They also showed that dynamic pricing and constant pricing problems are solvable in a polynomial time.

All the papers mentioned above assumed the linearity of the demand functions. However, this assumption is not realistic. In fact, for many application contexts, the real demand functions may have some non-linear correlations with different parameters suchs: prices, productions costs and marketing strategies. Modeling a non-linear demand function modifies the structure of the problem because some of its constraints are not linear. BhattaCharijee and Ramesh [3] used a constant function of demand elasticity for a single perishable product. They formulated the profit maximization problem for a monopoly retailer over multiple periods and presented two heuristic algorithms. They addressed some numerical examples to prove the efficiency of their algorithms.

As we can conclude from the review of the literature, there is a lack of decision-support systems that truly integrate the production/inventory and pricing decisions, especially for multi-product and multi-market problems. However, there is a growing research interest focusing on joint marketing decision-making and production planning in order to develop approaches that avoid conflicting marketing and operations planning decisions.

The remainder of the paper is presented as follow. Section 2 presents the problem statement and formulates the mathematical model. Section 3 describes the solution procedure with a detailed description of the proposed optimization method. Section 4 provides some preliminary numerical experimentations. Finally, section 5 concludes by summarizing the contribution of this research and discusses future extensions.

II. Problem formulation

This paper develops a multi-product, discrete-time model with capacity constraints and setup costs. Specially, we consider the case of a manufacturer, who produces different products using the same equipment. These products can be sold in several markets. It is also assumed that the demand for each product is a function of its current selling price only. The demand functions are assumed to be continuous, differentiable, strictly decreasing according to the prices and may vary for each period.

The objective is to determine prices such that the producer maximizes his profit over a finite planning horizon. While selecting the prices, the manufacturer may follow one of the two strategies: choose a constant price, for each product, which then remains the same throughout all the periods or dynamically change the prices from one period to another.It is to notice that the constant pricing strategy can be considered as a special case of the dynamic one.

The notation and the mathematical formulation of this problem are described below.

- j index of products, $j \in \{1, ..., J\}$
- t index of periods, $t \in \{1, ..., T\}$
- k index of markets, $k \in \{1, ..., K\}$
- c_{jt} unit variable cost for producing product j in period t
- b_t production capacity available in period t
- h_{jt} unit inventory carrying cost for product j in period t
- v_j capacity used per unit of product j
- a_{jt} setup cost for product j in period t t.
- P_{jt}^k price of product j in period t in market k
- $d_{jt}^k(P_{jt}^k)$ demand function of product j in period t in market k
- S_{jt}^k units of product j sold in period t in market k.
- I_{jt} inventory of product j at end of period t.
- X_{jt} quantity of product j produced in period t
- Y_{jt} binary variable of setup for product j in period t

$$\max z_p = \sum_{j=1}^{J}\sum_{t=1}^{T}\sum_{k=1}^{K} S_{jt}^k P_{jt}^k - \sum_{j=1}^{J}\sum_{t=1}^{T}(c_{jt}X_{jt} + h_{jt}I_{jt} + a_{jt}Y_{jt})$$

Subject to:

$$S_{jt}^k - d_{jt}^k(P_{jt}^k) \le 0; \quad \forall j, t, k \tag{1}$$
$$\textstyle\sum_{j=1}^{J} v_j X_{jt} - b_t \le 0; \quad \forall t \tag{2}$$
$$\textstyle\sum_{k=1}^{K} S_{j1}^k + I_{j1} - X_{j1} = 0; \quad \forall j \tag{3}$$
$$\textstyle\sum_{k=1}^{K} S_{jT}^k + I_{jT-1} - X_{jT} = 0; \quad \forall j \tag{4}$$
$$\textstyle\sum_{k=1}^{K} S_{jt}^k + I_{jt} - I_{jt-1} - X_{jt} = 0; \quad \forall j, t \in \{2, ..., T-1\} \tag{5}$$
$$v_j X_{jt} - b_t Y_{jt} \le 0; \quad \forall j, t \tag{6}$$
$$S_{jt}^k \ge g^k \textstyle\sum_{k=1}^{K} S_{jt}^k; \quad \forall j, t \tag{7}$$
$$S_{jt}^k, P_{jt}^k \ge 0; \quad \forall j, t, k \tag{8}$$
$$I_{jt}, X_{jt} \ge 0; \quad \forall j, t \tag{9}$$
$$Y_{jt} \in \{0, 1\}; \quad \forall j, t \tag{10}$$

The first constraint set ensures that the sales quantity for each product in each period and each market must not exceed the demand for that product in that period and that market. The second constraint set limits the total quantity produced in a period by the production capacity. Constraint sets (3 to 5) are the inventory balance constraints, and Constraint set (6) ensures that a setup cost is incurred whenever a product is produced. Constraint (7) ensures a minimum sales level required for each market. Finally, constraints (8), (9) and (10) are the non-negativity and binary variable constraints respectively.

III. Optimization approach

As a resolution methodology, we proposed an algorithm based on the outer approximation approach. This method is an adaption of the methodology proposed by Bajwa et al. [2] to deal with particular case of a single market. The general solution approach is presented in Figure III. The Outer Approximation method formulates two different problems whose solutions provide upper and lower bounds on the original problem.

The first problem, called the primal problem, is a nonlinear program (NLP) formulated by fixing the binary variables Y_{jt} to some feasible set to form a restriction of the original problem. Its solution is also feasible for the original problem, and its objective value is a lower bound.
The second problem called the reduced master problem is a mixed integer linear program (MILP) formulated by introducing outer approximations of the objective function and that of nonlinear constraints to obtain a relaxation of the original problem.

IV. Numerical experiments

To illustrate the application of the proposed method, we propose some numerical examples inspired from the literature [2]. These instances consider the case with two types of products, three possible markets.

For each product, a linear relationship is used to approximate demand as function of its price. The different instance parameters are summarized in Table I. This linear relationship, as assumed in the literature, is given by: $d^k_{jt}(P^k_{jt}) = \gamma(\alpha - \beta P^k_{jt})$ such as γ represents the seasonality factor for each product, β is the price effect of the demand and α is the maximum demand for each product.

Product(j)	a_{jt}	v_j	h_{jt}	c_{jt}
A	7.5	0,86	0.043	2.85
B	2.0	0.60	0.017	1.10

TABLE I. Example instance parameters

This case examines four scenarios for the demand variations: demand for both the products is stationary, demand for both the products is increasing and peaks at the end of the season, demand for both the products is decreasing and peaks at the beginning of the season, and demand for one of the product peaks at the beginning of the season while the demand for the other product peaks at the end of the season. The different parameters are given in table II.

We solved the same instances for both constant pricing strategy and dynamic pricing strategy and summarized the optimal profits for each strategy in table III. We compared the impact of the two strategies without explicitly incorporating the cost of price changes. The results give an idea about the interest of applying dynamic pricing strategy. This table presents also the percentage improvement in terms of profit that can be achieved by using a dynamic pricing strategy. The obtained results show that changing dynamically the prices is especially more interesting in the case of lower production capacities or when the demands follow both scenario-3 and scenario-4. This improvement is just marginally better for scenarios 1 and 2.

The optimal dynamic prices for each market and for the four scenarios are illustrated in Figure IV.

We notice that the prices variations are not monotonous for scenarios 1 and 4, while they are monotonous in the cases of scenario 2 and 3 (increasing for scenario 2 and decreasing for scenario 3). That can be explained by the products seasonality factors for each scenarios. We also notice that the optimal pricing strategies have similar tendencies for the different markets. These results are predictable since the instances considered allow the same importance and the same characteristics for the different markets. The next step of our tests is to focus on the characterization of different markets.

V. Conclusion

In this paper, we investigated the problem of joint production and pricing decisions optimization with considering capacity constraint limitation, multi-period time horizon, multi-product and multi-market. The objective is to solve this problem for maximizing the profit over the decision horizon.

As first contribution, we propose a non-linear mathematical formulation solved by LINGO Software. The second contribution consists of an optimization method based on outer approximation approach for solving the problem. We also analyze the difference between constant and dynamic pricing strategies among different instances and scenarios inspired by the literature.

We tested both mathematical model and outer approximation algorithm using the same demand functions proposed by Bajwa et al. [2]: linear and exponential functions. The proposed solution approaches are efficient for both linear and nonlinear demand functions

A direct extension of this work is to test other kind of demand functions and other instances. The proposed model assumes that demand is a function of price only. So, incorporating other parameters such customers and markets behaviors could be an interesting future extension of this preliminary work.

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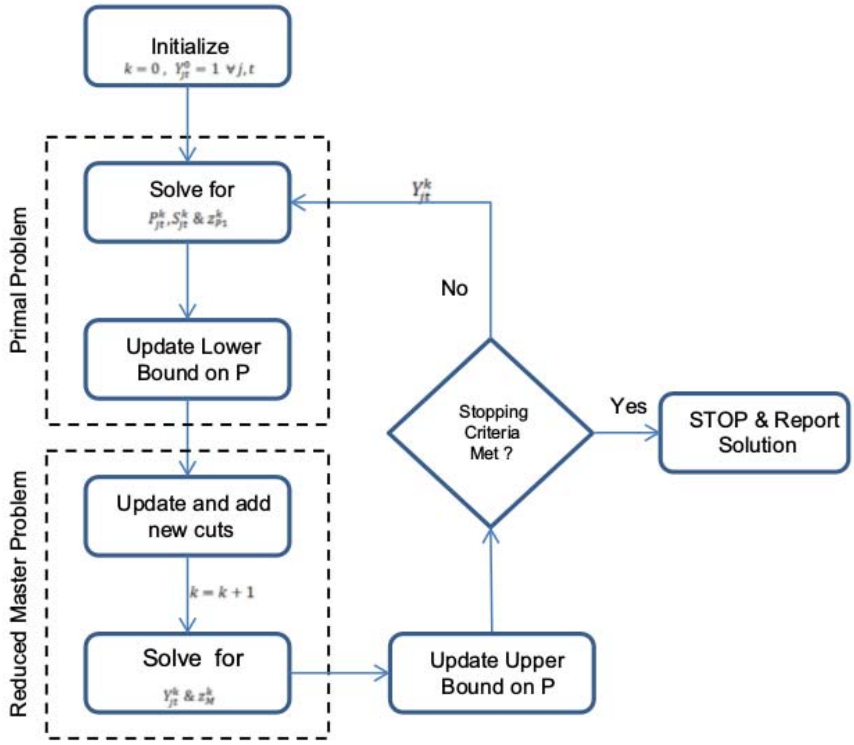

Fig. 1. Optimization method based on Outer Approximation approach Bajwa et al. [2]

scenarios	j/t	1	2	3	4	5	6
1	1	0.167	0.167	0.167	0.167	0.167	0.167
	2	0.167	0.167	0.167	0.167	0.167	0.167
2	1	0.1	0.1	0.1	0.2	0.2	0.3
	2	0.1	0.1	0.1	0.2	0.2	0.3
3	1	0.3	0.2	0.2	0.1	0.1	0.1
	2	0.3	0.2	0.2	0.1	0.1	0.1
4	1	0.1	0.1	0.1	0.2	0.2	0.3
	2	0.3	0.2	0.2	0.1	0.1	0.1

TABLE II. DIFFERENT SCENARIOS PARAMETERS

	constant pricing	dynamic pricing	iterations	% gap
scenario 1	519.052	519.614	73	0.1 %
scenario 2	509.084	519.311	24	2 %
scenario 3	479.141	493.399	44	2.96 %
scenario 4	499.52	522.381	26	4.58 %

TABLE III. OPTIMAL SOLUTIONS FOR CONSTANT AND DYNAMIC PRICING STRATEGIES

	OA-sol	Lingo-sol	gap (%)	OA-iter	Lingo-iter	OA-t (s)	Lingo-t (s)
scenario 1	519.614	519.05	0.10 %	73	159	8.00	0.09
scenario 2	519.311	504.60	2.91 %	24	507	1.10	0.09
scenario 3	493.399	491.61	0.36 %	44	193	3.20	0.05
scenario 4	522.381	512.25	1.97 %	26	469	1.61	0.09

TABLE IV. OUTER APPROXIMATION SOLUTIONS VS LINGO SOLUTIONS

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Fig. 2. Optimal Dynamic Prices for the different scenarios (price vs period).

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