Electricity price forecasting as optimization problem using discrete dynamic model

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Abstract-- **Deregulation of wholesale energy markets has brought many challenges and one of them is the accuracy of price forecasting methods. In the environment where price is set not only by matching supply (generation side) and demand (utilities and customer side) but is also influenced by various factors such as capacity, ancillary services and transmission congestion, the most popular forecasting tools are performing poorly. Developing the proper pricing strategies is becoming increasingly important to all market participants in the competitive electric power markets. The goal of this paper is to present a price - forecasting method which uses optimized set of parameters to predict the price for the next hour. The model consists of an objective function describing the minimum difference between real price for current hour and forecasted price for the same hour. It is constrained by four equations describing current demand, supply, price and also its forecasted values. The whole system comprised of dynamic discrete model of which fundamentals are described by the author in the previous paper. The forecast accuracy of the proposed method is evaluated with real data from the Californian ISO day-ahead energy market. The proposed method provides significant innovation in area of price – forecasting tools and is characterized as new approach compared to most popular techniques applied to energy markets.**

***Index Terms* — day-ahead electricity market, optimization, price forecasting, dynamic discrete model**

I. Introduction

AFTER regulation, the proper contracting, selling or buying power plays a crucial role as the costs of over or under contracting can cause a huge financial loss. Quantitative risk and its minimization is nowadays an issue of the greatest importance. Hence not surprisingly, there is a pursue among researchers to provide a forecasting tool with the lowest prediction error. Price forecasts have become the decisive factors for energy suppliers and it has to be underlined that volatility of electricity price can be even two orders of magnitude higher than for other commodities [1]. Price-forecasting tools are essential for all market participants and proper pricing is crucial to hedge against price volatility that also affects downstream customers

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A. Overview

The main techniques used so far in electricity price forecasting are time series models. These methods focuse on the past behavior of the dependent variable. Analyzing time series data in order to build models to predict future values based on previously observed values seems to be reasonable at first sight for energy markets as lots of historical data should provide fair approximations. Alas it occurs, this kind of market is characterized by huge volatility and the assumption of stationarity in time series model makes this method undependable. That has led the author to the proposition of the discrete dynamic model consisting of four recursive 1st order equations extensively described in [2]. The applied model used the idea of computing parameters from a specified period of time and dataset and later apply those computed parameters to forecast of the demand and the price in the next-hour prediction for the next period of time. For the simplicity of calculations, the average of every computed parameter was used. The outcome showed that the adaptation of the proposed dynamic, discrete model is possible to forecast prices on electricity markets. It was shown that the described model is universal and by applying different historical data can be used to describe various energy markets through computing parameters. Unfortunately, the ex-post errors comparing forecasted and real prices were not sufficiently satisfying and the chaotic nature of the system was observed. That resulted in forecasted price being prone to unstable behaviors.

B. Various Approaches

Ma and Feng [3] presented a model which was the base for the previous paper by the author [2]. Authors describe the application of the model to retailer's demand. Conclusions in [3] show that the model is chaotic and in later state remains unpredictable. It is caused by both: model's parameters and changes of initial state of the system. On the other hand, Nockowska-Rosiak et al [4] extended those studies and proved the stability of equilibrium points can be achieved by setting conditions for initial values of variables and parameters in order to avoid singularities.

This paper does not deal with problem of imposing stability in the finite time but focuses on optimization of parameters for the described model. Based on historical data from one – year window, the model will calculate the forecasted price for the next hour. As the described model is supposed to be used for the day – ahead market i.e. short time period, the perturbations of chaos and instability should not apply. The parameters in model are time - dependent and are calculated every time the algorithm to forecast the next-hour price is triggered. The idea is to optimize parameters used for price forecast for the previous hour and once they are optimized, apply them for a next-hour price forecast taking into account that all constraints of the systems are met.

C. Importance of Price Forecasting

Price forecasting techniques on energy markets are still in the early phase and require a lot of research. Due to the more and more popular deregulations, the forecast of electricity price and demand are getting even more attention and many researchers work on improving forecasting algorithms. The complexity of a problems comes from the nature of the price curve which can be characterized by high volatility, multiple seasonality, non-constant mean and unusual price movement [13]. Transmission congestion prevents energy from low-cost generation from meeting all loads and clearing the market. In this case, when the congestion occurs, the locational marginal price (LMP) should be introduced. Locational marginal pricing is a mechanism for using market-based prices for managing transmission congestion. LMPs are determined by the bids/offers submitted by market participants [6]. This price is the sum of generation marginal cost and cost of marginal losses and those can differ within areas. If prices were fixed, then they would reflect the cost of production of energy, but in reality, the cost of electricity production depends mainly on energy demand. Therefore, it can be deducted that the curve of daily price should be similar to the typical daily curve of electricity consumption.

Weron [7] states that although demand is the factor with biggest influence, the price is dependent on a larger set of fundamental drivers, including system loads, weather variables (temperatures, wind speed, precipitation, solar radiation), fuel costs (oil and natural gas), surplus generation and the scheduled maintenance or forced outages of important power grid components.

Another aspect is risk minimisation of price volatility on the wholesale market and as a result, minimizing price for the final recipients in the retail market. At the same time, it should be noted that the price cannot be too low, as providing energy entities with basic revenues is necessary for development of technical and organizational infrastructure. In a free market economy, the electricity is no longer regarded as a "good" which is delivered as a public service, but it has rather become a commodity that is traded on the market. Developing the proper pricing strategies can help to maximize the achievement of power sector development goals.

D. Demand

In short, demand is the rate at which energy is being used by the customer and generally expressed in kilowatts (kW) or megawatts (MW). Load should not be confused with Demand, which is the measure of Power that a Load receives or requires.

As volume of demand continuously change, power plant operators need to adjust generation to meet the changing demand in order to maintain the acceptable levels of power quality and reliability [7]. Motamedi et al. [8] describes the importance of demand fluctuations and its impact to price is described. It is highlighted that the main root cause of this phenomenon are time – dependent consumer preferences. An increase of demand or immediate decrease of renewable energy generation can result in a higher energy price. Increase of price is also often the result of higher cost of generation during peak hours, grid constraints and overall energy dispatch. That is why, it is assumed that real time pricing can reflect current operating conditions and forecasting energy demand can reduce the cost and uncertainty as well as improve system efficiency which ultimately leads to lowering price volatility. As stated in [9] the dynamic pricing, especially real-time marginal cost pricing is widely assumed as number one priority before implementing the idea of wholesale electricity markets. In this paper, it is assumed that the pool predicts the demand using the simple exponential smoothing called the Brown Model.

E. Energy Markets

Wholesale energy market works as a pool and energy is sold and purchased for immediate delivery. Due to the nature of this commodity, it cannot be stored and must be consumed at the same time as it is produced. The price in this kind of environment is established by matching supply (what generators want to sell) and demand (what utilities and customers want to buy) [2]. The purchased energy can be further sold on retail market but this market is not in scope of this paper. This model is constructed for a pool type of market i.e. a place where customers and producers meet and the demand of customers is highly dependent on the price of energy. Customers order the required amount of energy based on the previous orders and forecasted demand. Given the impossibility of storage, suppliers generate only the ordered amount of energy. Customers have impact on price for the next period because in case of high level of stock, the discount price is offered by suppliers to dismantle the overstock. That happens when there is a surplus of energy in the market due to availability of wind or hydropower and in case when the energy generators cannot be turned off on demand. The detailed process was described in previous paper [2]. It is worth adding, this market is characterized by many uncertainties coming both from generation and demand sides.

These two above described markets are complementary and have different purposes. However, according to statistics roughly 98% of energy is scheduled in the day-ahead market [1]. The day-ahead market is a forward market in which hourly locational marginal prices (LMPs) are calculated for the next operating day based on generation offers, demand bids and scheduled bilateral transactions. In this paper, the price forecasting model is dedicated to that kind of market.

F. Data for modelling

To demonstrate the application of the proposed model, publicly available data acquired from the California Independent System Operator (CAISO) were used for testing. California ISO [10] is one of the largest and most modern power grids in the world. It deals with operation of bulk electric power system, transmission lines and electricity market and encompasses about 80% of California's electric flow. CASIO was created as soon as the barriers to completion in the wholesale electricity market was removed. It plays a role as market operator by balancing the needs of generators and wholesale market customers.

G. Initial Approach

The steps to construct a reliable price forecasting model were taken before as shown in [2]. The proposed earlier model had the form of four recursive, 1st order equations. It should be noted that the model firstly proposed by Ma-Feng [3] and later applied by Hachuła-Schmeidel [11] has been changed and the 4th equations has been added. The final model had the following form:

$$\begin{cases} D_{t+1} = \dfrac{(qaT)^{k}\left(\frac{p_{t+1}}{p_t}\right)^{-2c}}{[(qa+1)T-S_{t+1}]^{k}} D_t \\ S_{t+1} = \breve{D}_{t+1} + S_t - D_t \\ \breve{D}_{t+1} = \alpha D_t + (1-\alpha)\breve{D}_t \\ D_{t+1} = \breve{D}_{t+1} + c p_{t+1} \end{cases} \tag{1.0}$$

This model describes the equations for computing the demand forecast $\breve{D}_{t+1}$, demand D_{t+1}, as well as an amount of energy being produced by generators S_{t+1}. Parameters (a, c , q, k, T and α) had fixed values which were computed based on the historical data from the first 6 months and then used to forecast the hourly price for the period of next 6 months. Parameters had the following meaning:

α - constant coefficient which describes how fast the real data are being counted. For α =1 the forecasted demand is the real demand.

T - controlling parameter for generators to decide whether to lower the price or not.

q - parameter used to describe the relationship between excessive production and rate of discounted price.

a - the upper limit of a discount.

k - equivalent of price elasticity c.

The precise descriptions of all variables and parameters can be found in [2]. Although it was shown that such mathematical model can predict not only the demand for the next hour but most importantly, the price for the next hour (p_{t+1}), the results and ex-post errors were not satisfying. The goal of this paper is to optimize parameters and by changing the 4th equation, to improve correctness of the model.

H. New Approach

The dynamic discrete model is proposed to solve the optimization problem. The optimization is introduced to compute parameters (T, c , q, k and α) so that the objective function is minimized. Only then, parameters are set in such a way, that using historical data, for an hour before our target time, the forecasted price for that hour ($\breve{p}_M$) is as close as possible to the real energy price, already known for that time (p_M). The parameters obtained in the described way are optimal for optimizing price for (p_M) and those parameters will be used to forecast the price for the next hour – the target time (p_{M+1}). A one year window of historical data is used to obtain the best possible parameters and they will be computed for each consecutive hour based on information from the hour before and the corresponding one year window.

The fourth equation was changed as compare to the first proposed model and now it describes the forecasted price using the relationship between demand for time t and forecasted demand for the same time, multiplied by coefficient of price elasticity and current price:

$$p_{t+1} = \frac{D_t}{\breve{D}_t} c\, p_t \tag{1.1}$$

II. System and its Components

A. Timing

Time plays a crucial role in this model as different versions of the constraints are used for particular steps. As the hourly data from one-year window are used for calculations, every parameter and variable has different time index. As one year has 365 days and one day has 24 hours the index t can be expressed as:

$$t \in [0, 365\times24]$$

Assuming $365\times24 = M$, then $t \in [0, M]$

In that way, the whole one-year window is captured but it has to be noted that the actual forecast happens one hour in the future i.e. the index t should have the following final form:

$$t \in [0, M+1],\ t \in N$$

B. Historical data

Data collected from Californian energy market encompasses:

D_t - the actual demand at the time t

D_{t+1} - the actual demand at the time $t+1$

$\breve{D}_t$ - the forecasted demand for the time t

$\breve{D}_{t+1}$- the forecasted demand for the time $t+1$

S_t - the amount of energy produced at time t (i.e. supply)

S_{t+1}- the amount of energy produced at time $t+1$

p_t- price of electricity at time t

C. Final form of the model

Assuming the aim is to forecast the price for the next hour (p_{M+1}), the final system should take the following form:

Objective function is:

$$|\,\check{p}_M - p_M\,| \rightarrow min. \tag{1.2}$$

It has to be noted that here the difference between the computed price for current hour and actual price for current hour is minimized. It is done to find the optimal parameters from dynamic system (1.3), (1.4) and use them to compute the forecasted price for next hour ($\check{p}_{M+1}$).

The minimization problem is **under constraints:**

<u>Case I</u> : for $t \in [0, M-2]$

$$\begin{aligned} &1. \quad D_{t+1} = \frac{(qaT)^k \left(\frac{p_{M-1}}{p_t}\right)^{-2c}}{[(qa+1)T - S_{t+1}]^k} D_t \\ &2. \quad S_{t+1} = \breve{D}_{t+1} + S_t - D_t \\ &3. \quad \breve{D}_{t+1} = \alpha D_t + (1-\alpha)\breve{D}_t \\ &4. \quad p_{M-1} = \frac{D_t}{\breve{D}_t}\, c\, p_t \end{aligned} \tag{1.3}$$

<u>Case II</u> : for $t = M-1$

$$\begin{aligned} &1. \quad D_M = \frac{(qaT)^k \left(\frac{\check{p}_M}{p_M}\right)^{-2c}}{[(qa+1)T - S_M]^k} D_{M-1} \\ &2. \quad S_M = \breve{D}_M + S_{M-1} - D_{M-1} \\ &3. \quad \breve{D}_M = \alpha D_{M-1} + (1-\alpha)\breve{D}_{M-1} \\ &4. \quad \check{p}_M = \frac{D_{M-1}}{\breve{D}_{M-1}}\, c\, p_{M-1} \end{aligned} \tag{1.4}$$

where:

$\check{p}_M$ is the calculated forecasted price for current hour computed by adjusting parameters based on one – year window

p_M is the known price for the last available hour on the market (i.e. one hour before the time we want the forecasted price for - namely p_{M+1})

α is a parameter which describes how fast the real data are being counted,
For α=1 the forecasted demand will be equal the real demand D_{t-1} in the previous period and if α=0 then $\breve{D}_t = \breve{D}_{t-1}$ which means the amount of real demand does not have any influence on the forecasted value

$$\alpha \in (0,1) \tag{1.5}$$

T is a controlling parameter for generators to decide whether to lower the price or not. It is important that this decision has impact only in one period, in the next one it is calculated again based on the new amount of produced energy

$$T \in [0, max(S_t)], \; T \in N \tag{1.6}$$

q is a parameter used for price adjustments

$$q \in (0,1) \tag{1.7}$$

c is a coefficient of price elasticity

$$c \in (0,1) \tag{1.8}$$

k is a parameter indicating demand in case of discounted price

$$k \in (0,1) \tag{1.9}$$

$a = \overline{p_t}$ is mean of all prices – calculated every hour, parameter a changes accordingly to the time window but it will not be in the optimized set of parameters

$$a = \overline{p_t}\;,\; t \in [0, M] \tag{1.10}$$

All parameters (T, a, c, q, α and k) change every hour so that one can differentiate:

$$t \longrightarrow (T_t, a_t, c_t, q_t, \alpha_t, k_t) \tag{1.11}$$

$$t+1 \longrightarrow (T_{t+1}, a_{t+1}, c_{t+1}, q_{t+1}, \alpha_{t+1}, k_{t+1}) \tag{1.12}$$

$$t \in [0, M] \;,\; t \in N$$

It should be highlighted that the optimized set of parameters (T, c, q, k and α) has to be computed using discrete not continues sets as the data changes every hour.

The aim of this paper is to find the optimized set of these parameters. Optimization is calculated using (1.3), (1.4) and historical data from one-year window (a time series). Taking (1.3), (1.4) with these computed optimal parameters, they approximate the dynamics of time series behavior. That approximated dynamics is used to calculate the price for the next time period.

In short, the idea is to find the optimal sequence of parameters out of all sequences of parameters using (1.3) and (1.4) that gives the price ($\check{p}_M$) as close as possible to the current price *(*p_M). Use this optimal sequence of parameters to calculate the price for the next hour *i.e. (*$\check{p}_{M+1}$).

D. Use case.

Supposing the price forecast for the day of 01.01.2017 at 6 pm is required, the following steps need to be completed:

1. Take dataset – a one-year window- ending with 5 pm of 1st January 2017 (i.e. data encompasses time between 01.01.2016 at 5 pm and 01.01.2017 at 5 pm)

2. Using mathematical software, optimize the parameters so that for the 5 pm of 01.01.2017 they are in optimal state.

3. Having the computed parameters, apply them to the model in order to get the forecasted price for 6 pm of 01.01.2017

E. Trajectory of the changing parameters.

The role of the proposed algorithm is to deliver the most optimal price for the next hour by using the moving dataset from one – year window i.e. it takes data from the last 365 days, in regard to the date and hour of the forecasted price. In other words, the horizon keeps being shifted forward. For this purpose, an iterative, finite-horizon optimization is implemented. That means the algorithm has similar characteristics to NMPC - *model predictive control* method. It is a method of controlling dynamic systems which solves cyclically tasks of optimal control, where the initial conditions are equal to the actual estimation of the current state of the object. The whole procedure is repeated for the new, current state. It takes one iteration towards the solution of the most current problem, before proceeding to the next one, which is suitably initialized.

The main advantage of this kind of systems is the fact that it allows the current timeslot to be optimized, while keeping future timeslots in account. This is achieved by optimizing a finite time-horizon, but only implementing the current timeslot [12]. As a result, for every step – for every hour the forecasted price is wanted – the algorithm computes new parameters and the trajectory of parameters throughout the researched period can be investigated.

III. Analysis

A. Computing parameters

The first approach to compute parameters and obtain the forecasted price was done using data from year 2016 for the PGE-TAC area in California ISO. Calculations were done using MATLAB, version R2016b. The goal was to predict prices for the first serveral days of January 2017. In order to obtain the forecasted price, it is necessary to compute trajectories for paramerers for the last 365 days and find out the optimal one. The algorithm is as follows:

- Take one year window preceding the time you want the forecasted price for.
- For every hour except the last available in this window, compute parameters using (1.3) and historical data for $t \epsilon [0, M]$.
- Having several sets of parameters for every hour, once again use (1.3) but this time insert the calculated parameters to obtain p_{M-1} . Repeat this step for every sequence of parameters.
- For the last available hour in time window, calculate the $\check{p}_M$ using the computed earlier sets of p_{M-1} . Number of $\check{p}_M$ values should be equal to: no. of sets of parameters for every hour times 24 times 365.
- Having the price for current hour compare it with the just calculated prices computed for current hour as stated in the objective function (1.2).
- Find the optimal sequence of parameters out of the computed sequence of parameters that gives the price ($\check{p}_M$) as close as possible to the current price in time *t: i.e.(* p_M).
- Use this optimal sequence of parameters to calculate the price for the next hour *i.e. (* $\check{p}_{M+1}$).

B. Findings

In order to show the effectiveness of the model, graphs comparing the forecasted and real prices were constructed for selected day of the researched period.

Fig. 1 Forecasted and real price for the day 01.01.2017

Also, the particular parameters chosen by computer software as optimal are shown throughout the one – year time window.

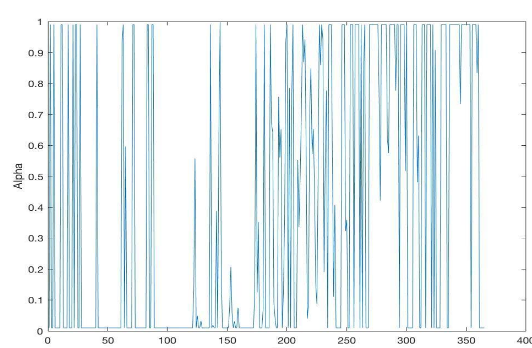

Fig. 2 Values of parameter α throughout one-year time widnow

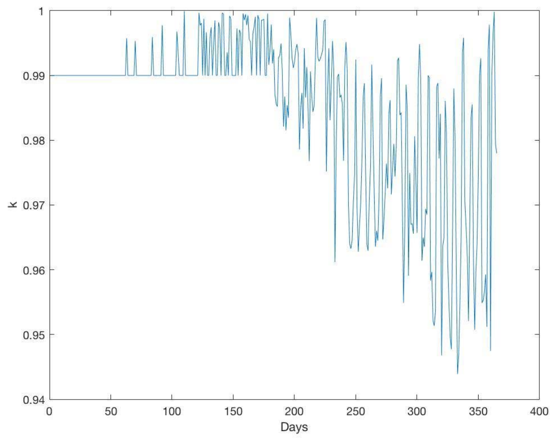

Fig. 3 Values of parameter k throughout one-year time widnow

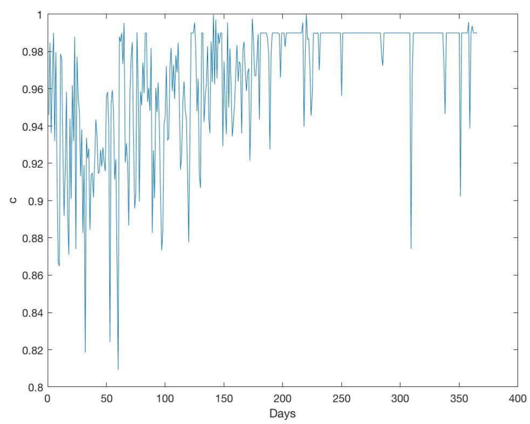

Fig. 3 Values of parameter c throughout one-year time widnow

IV. CONCLUSIONS

This paper shows application and adjustments of the earlier described dynamic discrete model used for electricity price forecasting. This time, the emphasis was put on describing this as an optimization problem and highlighting the importance of choosing the optimal parameters to improve the accuracy of the forecast. Final version of the model enables to predict energy price for the next hour by choosing the most optimal set of parameters from available trajectory.

Given the fact that all calculations are heavily based on historical data, the model is very data sensitive and it is exceptionally important to gather the proper dataset to achieve a greater accuracy. Of course the model can be applied to different wholesale energy markets and growing transparency of such markets and data availability will make application of this model much more useful.

Introducing differential equations should also improve the correctness of the model which is going to be done in the forthcoming papers.

V. REFERENCES

[1] Weron, R. (2008). Forecasting wholesale electricity prices: A review of time series models.

[2] L. Nowakowska "Dynamic discrete model for electricity price forecasting." *Energy (IYCE), 2015 5th International Youth Conference on*. IEEE, 2015.

[3] J. Ma, Y. Feng ”The Study of the Chaotic Behavior in Retailer’s Demand Model” *Discrete Dynamics in Nature Society* (2008), Article ID 792031

[4] M. Nockowska-Rosiak, P. Hachuła, E. Schmeidel. "Stability of equilibrium points of demand-inventory model with stock-dependent demand." *Journal of Difference Equations and Applications* (2016): 1-11.

[5] S. K. Aggarwal, L. M. Saini, Ashwani Kumar “Electricity price forecasting in deregulated markets: A review and evaluation”, *Electrical Power and Energy Systems*, 31 (2009), pp. 13-22

[6] R. Weron "Electricity price forecasting: A review of the state-of-the-art with a look into the future." *International Journal of Forecasting* 30.4 (2014): 1030-1081.

[7] R.Weron “Modeling and Forecasting Electricity Loads and Prices. A Statistical Approach” *John Wiley &Sons, Ltd* 2006

[8] A. Motamedi, Z. Hamidreza, R. William D. "Electricity price and demand forecasting in smart grids." *IEEE Transactions on Smart Grid* 3.2 (2012): 664-674.

[9] J.N. Tsitsiklis, Y. Xu, Pricing of fluctuations in electricity markets, European Journal of Operational Research (2015), http://dx.doi.org/10.1016/j.ejor.2015.04.020

[10] CAISO [Online] Available: http://www.caiso.com , March 2017

[11] P.Hachuła, E.Schmeidel “Analiza modelu sterowania popytem i zapasem pod kątem możliwych zastosowań”

[12] Z. Brijet, N. Bharathi, G.Shanmugapriya, “Design of Model Predictive Controller for Fluid catalytic Cracking Unit”, © International Science Press, Fluid- I J C T A*,* 8(5), 2015, pp. 1917-1926

[13] H. S. Hippert, C. E. Pedreira, R. C. Souza, “Neural networks for short-term load forecasting: A review and evaluation,” *IEEE Trans. Power Syst.*, (2001), pp. 44–55