

# Study on Fault Diagnosis of Rolling Bearing Based on K-L Transformation and Lagrange Support Vector Regression

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**Abstract**—On the basis of vibration signal of rolling bearing, a new method of fault diagnosis based on K-L transformation and Lagrange support vector regression is presented. Multidimensional correlated variable is transformed into low dimensional independent eigenvector by the means of K-L transformation. The pattern recognition and nonlinear regression are achieved by the method of Lagrange support vector regression. Lagrange support vector regression can be used to recognize the fault after be trained by the example data. Theory and experiment shows that the recognition of fault diagnosis of rolling bearing based on K-L transformation and Lagrange support vector regression theory is available to recognize the fault pattern accurately and provides a new approach to intelligent fault diagnosis.

**Keywords**—K-L transformation; Lagrange support vector regression; Rolling bearing; Fault diagnosis

## I. INTRODUCTION

At present, Rotating Machinery is developing in the direction of large-scale, high-speed, light type, automation and large load, at the same time, the accuracy of diagnosis for a variety of complex fault is the key of system performance guarantee. According to statistics, the 30% of Rotating Machinery fault is caused by rolling bearing fault. Bearing vibration signals carry a wealth of information, when the vibration signal is transiting from stationary to non-stationary, the highlight reflection is the impact, vibration, structural changes and changes in the gap or crack which is caused by rolling bearing fault. The graphics of vibration signals is different in different conditions, the mission of pattern recognition for rolling bearing fault is to extract, dispose and classify vibration signals in the vibratory signal, in order to deduce the state of rolling bearings operation[1-2]. Due to the complexity of the rolling bearing system and diversity of fault style, it does not exist definite function between vibration signals and state information, there is a complex non-linear mapping between signals set and the states set, which determines the difficulty and complexity of rolling bearing fault recognition pattern.

Support vector machine which is originated by Dr Vapnik is new machine learning technique. Being different from traditional neural network, it is based on structure risk minimization principle, while the latter on empirical risk minimization principle. A large number of experiments have

shown that, comparing with traditional neural network, support vector machine has not only simpler structure, but also better performances, especially better generalization ability [3-6]. At same time, it transforms optimization problems to convex quadratic programming problems, the solution is the sole global optimum. Lagrange support vectors regression is proposed by O.L.Mangasarian, which is an effective algorithm of Support vector classification machine in the case of solving linear planning problems. The Lagrange method for solving linear complementary problem is introduced in the non-linear support vectors regression machine in this paper, the efficient iteration algorithm is obtained [7-8].

In this paper, according to existed problems of rolling bearing fault diagnosis, for example: difficult to obtain a large number of faults data samples; difficult to obtain diagnosis knowledge; weakness of reasoning ability; feature extraction is difficult, and so on, a new method of fault diagnosis based on K-L transformation and Lagrange support vector regression is presented. Multidimensional correlated variable is transformed into low dimensional independent eigenvector by the means of K-L transformation. The pattern recognition and nonlinear regression are achieved by the method of Lagrange support vector regression. Theory and experiment shows that it can successfully achieve a bearing failure mode identification and classification with satisfactory results.

## II. CALCULATING THE PRINCIPAL EIGENVALUE OF K-L TRANSFORM

Suppose that the vibration signals of rolling bearing can be expressed by n-dimensional vector model  $X = \{x_i\}$ ,  $i = 1, 2, \dots, n$ , it corresponds to a point in the original space, and exist an orthogonal function set  $A = \{A_j(i), i, j = 1, 2, \dots, n\}$  satisfied that:

$$Y = \sum_{j=1}^n x_j A_j = AX \quad (1)$$

The eigenvector after the transformation is  $Y = (y_1, y_2, \dots, y_n)^T$ .

The transposed matrix of formula (1) is

$$Y^T = X^T A^T \quad (2)$$

Multiply the formula (1) and formula (2), we can obtain mathematical expectation is:

$$E[YY^T] = AE[XX^T]A^T \quad (3)$$

$$C_y = AC_n A^T \quad (4)$$

$C_x$ 、 $C_y$  are separately the covariance matrix of  $X$  and  $Y$ . We can use K-L Linear transformation method to select appropriate transformation matrix  $A$ , making each component  $y_i$  independent and making  $C_y$  diagonal matrix, that is

$$C_y = \text{diag}(\lambda_1, \lambda_2, \dots, \lambda_n) \quad (5)$$

In order to achieve the relevant eigenvector  $X$  converting into an independent eigenvector  $Y$  to complete the analysis process of main eigenvalue  $(\lambda_1, \lambda_2, \dots, \lambda_n)$ .

Because  $C_x$  is real symmetric matrix,  $C_y$  is diagonal matrix, it is composed of  $n$  positive characteristic root  $\lambda_i (i=1, 2, \dots, n)$  of  $C_x$ , and  $\lambda_1 > \lambda_2 > \dots > \lambda_n$ . According to the concept of covariance,  $\lambda_i$  equal to the variance of the  $i$ th

weight in  $Y$ ,  $\sum_{j=1}^n \lambda_i$  represents the overall variance of vibration signals.

In general, selecting the main characteristics which is corresponding the front of  $m$  largest eigenvalues constituting  $m$ -dimensional feature space in the  $n$ -dimensional vector space, making:

$$\sum_{i=1}^m \lambda_i / \sum_{i=1}^n \lambda_i > 85\%$$

At this time, the eigenvector composed by the  $m$  principal eigenvalue still retains enough information of the original signal, it can study the different state of the main bearings eigenvector  $\lambda_i$  to achieve the purpose of fault pattern recognition.

### III. LAGRANGE SUPPORT VECTOR REGRESSION MACHINE

Support vector regression machines method was beginner with solving classification problem, support vector regression machine problem is general described as:

Given training samples set  $(x_1, y_1), (x_2, y_2), \dots, (x_m, y_m)$ ,  $x_i \in R^n$  is input values,  $y_i \in R$  is corresponding target values, the aim is to find the relevant regression function  $y = f(x)$  through these training samples. Under linear circumstances, suppose  $f(x) = wx + b$ , here,  $w$  is normal radial of the optimal hyperplanes;  $b$  is threshold, it decides the distance

from the optimal hyperplanes to the origin. Under non-linear circumstances, Support vector regression machines can subtly bring the samples into high-dimensional space by introducing the concept of kernel function, thus linear regression analysis can be carried out in high-dimensional space, avoiding the "dimension disaster."

The Common kernel functions are:

(1) Ordinary polynomial kernel function:

$$K(x, y) = (x^T y + c)^p, \quad p \in N, c \geq 0 \quad (6)$$

(2) Gaussian radial basis kernel function

$$K(x, y) = \exp\left(-\frac{\|x - y\|^2}{2\sigma^2}\right) \quad (7)$$

(3) Sigmoid kernel function

$$K(x, y) = \tan(K(x^T y) + c), \quad K > 0, c < 0 \quad (8)$$

In this paper, we use Lagrange support vector regression machines, it is shown as follow, considering linear circumstance:

$$\min \frac{1}{2} (\|w\|^2 + b^2) + \frac{C}{2} (\bar{\xi}' \bar{\xi} + \hat{\xi}' \hat{\xi}) \quad (9)$$

$$Aw + be - y \leq \varepsilon + \bar{\xi} \quad (10)$$

$$y - Aw - be \leq \varepsilon + \hat{\xi} \quad (11)$$

Here:  $\bar{\xi}$ 、 $\hat{\xi}$  are allowable error is penalty parameter or regularization parameters it reflects compromise between the complexity and training error of model.  $A$  is a  $m \times n$  matrix whose row is  $x_i^T$ ;  $e$  is Arbitrary-dimensional vector whose component is 1;  $\varepsilon$  is free parameters of insensitive loss function  $L(y, f(x)) = L(|y - f(x)|)$

To solve the dual model of model (9)-(11), we can obtain:

$$\begin{aligned} \min & \frac{1}{2} (\hat{\alpha} - \bar{\alpha})' (AA' + ee') (\hat{\alpha} - \bar{\alpha}) - y' (\hat{\alpha} - \bar{\alpha}) \\ & + \varepsilon e' (\hat{\alpha} + \bar{\alpha}) + \frac{1}{2C} (\hat{\alpha}' \hat{\alpha} + \bar{\alpha}' \bar{\alpha}) \end{aligned} \quad (12)$$

$$\hat{\alpha}, \bar{\alpha} \geq 0 \quad (13)$$

Here,  $\hat{\alpha}$ ,  $\bar{\alpha}$  are corresponding Lagrange multipliers of inequality constraints model (9)-(11).

We can see from model (12)-(13), Similar to the classification case, the model contains only a simple non-zero bound, there is no equality constraints and the constraints on the sector, so the simple problem is easy to be solved. At the same time, we can directly derive variable from the solution of dual problems.

$$w = A'(\hat{\alpha} - \bar{\alpha}), b = e'((\hat{\alpha} - \bar{\alpha})) \quad (14)$$

So solution of the problems (12) ~ (13) is very important as long as the issue is resolved, the optional regression surface  $f(x) = wx + b$  can be easily expressed by formula (14).

Suppose the optional solution of the problem is  $(\hat{\alpha}^*, \bar{\alpha}^*)$ , then  $w^* = A'(\hat{\alpha}^* - \bar{\alpha}^*)$ , the point corresponding to  $\hat{\alpha}^* \neq 0$  or  $\bar{\alpha}^* \neq 0$  is support vector

The above-mentioned problems is quadratic programming problem with only briefly bound by the lower bound, it can be solved by linear complementarily problem solving methods Lagrange, in order to get a simple and fast iterative algorithm.

$$\hat{y} = f(x) = x' A'(\hat{\alpha} - \bar{\alpha}) + e'(\hat{\alpha} - \bar{\alpha}) \quad (15)$$

As mentioned earlier, for non-linear regression function, it can be expressed by introduction of forms of the same kernel functions, for arbitrary  $x \in R^n$ :

$$\hat{y} = K((x_1')', (Ae'))(\hat{\alpha} - \bar{\alpha}) \quad (16)$$

Because the support vector regression machines is proposed for the two types of issues, it is needed to extend to a wide range of issues division in practical applications, because the real problems are generally multiple classification problems. For example, classification of the rolling bearing includes many kinds, such as the normal, the outer ring fault, inner ring faults, rolling-fault. Different combination rules have different classification algorithm. In this paper, we adopt "one-to-many" classification which is composed of multi-fault classifier by 4 Lagrange support vector regression machines (LSVR).

#### IV. FAULT DIAGNOSIS OF ROLLING BEARING

In order to verify the reasonableness and the feasibility of this method in the rolling bearing fault diagnosis, we take a rolling bearing for example to test in the rolling bearing vibration experiments platform. In the experiment, we take 40 bearings samples, including 10 normal bearings, 10 outer ring fault bearings, 10 inner ring fault bearings, 10 rolling element fault bearings, each bearing gather 10 times sample, and gather 1024 data each time.

In the samples, the number of normal bearings, outer ring fault bearings, inner ring fault bearings and rolling element fault bearings for training LSVR are 6, the remaining bearings is used as test samples after trained LSVR. The work speed of bearing is 10000 r/min, the input sampling frequency is 30kHz, axial load is 500N, radial load is 100N. Through experiment, when  $m=12$ , it satisfies:

Table 1 Parts of main characteristic vector

Main characteristic vector					
Eigenvalue	1	2	3	4	5
$\lambda_1$	0.78899	0.62807	0.04546	0.21667	0.32123
$\lambda_2$	0.77762	0.62415	0.03899	0.21353	0.28399

$$\sum_{i=1}^{12} \lambda_i / \sum_{i=1}^{40} \lambda_i > 85\%$$

So we choose 12 as principal eigenvalue, we use 12 principal eigenvalues to construct a eigenvector  $x = (\lambda_1, \lambda_2, \dots, \lambda_{23})$ , this eigenvector is used to be input element of four classification function  $f_1(x)$ ,  $f_2(x)$ ,  $f_3(x)$ ,  $f_4(x)$  of support vector regression machine. Classifier is composed of four Lagrange support vector machines. LSVR 1 is used to judge normal bearings; LSVR 2 is used to judge outer ring fault bearings, LSVR 3 is used to judge inner ring fault bearings, LSVR 4 is used to judge rolling element fault bearings,

First use the selected samples to train support vector machines, training LSVR 1 to determine the samples belonged to normal, the training samples belonging to normal bearings as a category, expressed as 1, the remaining samples will be marked as -1, according to the LSVR order, to calculate optimize coefficient and establishing corresponding normal LSVR1.

To apply the same approach to train LSVR2, LSVR3, LSVR4. Let these classifications four function of classifiers  $f_1(x)$ ,  $f_2(x)$ ,  $f_3(x)$  and  $f_4(x)$  make judge to input samples.

If the output of LSVR1 is 1, it belongs to normal bearings; if the output of LSVR2 is 1, it belongs to outer ring fault bearings; If the output of LSVR3 is 1, it belongs to inner ring fault bearings; If the output of LSVR4 is 1, it belongs to rolling element fault bearings; if the output of LSVR1 is 1 and one output of LSVR2、LSVR3、LSVR4 is 1, then it belongs to misjudgment.

We take the 40 samples input classifiers, the calculated values of part main eigenvectors is shown by Table 1.

The accuracy rate of using the trained LSVR classifiers to carry out fault diagnosis for rolling bearing, normal bearings, outer ring fault bearings, inner ring fault bearings and rolling element fault bearings is beyond 95%. Comparing the method of this paper with BP neural network, this method needs less fault diagnosis samples, needn't empirical knowledge of bearing fault classification in advance and data preprocessing.

#### V. CONCLUSIONS

In this paper, a new method of fault diagnosis based on K-L transformation and Lagrange support vector regression machine is presented. This paper introduced K-L transformation method and Lagrange support vector

$\lambda_3$	0.57662	0.58034	0.0384	0.19452	0.28027
$\lambda_4$	0.56486	0.55428	0.03233	0.19011	0.25998
$\lambda_5$	0.56368	0.54585	0.03135	0.17717	0.24812
$\lambda_6$	0.55869	0.51096	0.02949	0.17649	0.24313
$\lambda_7$	0.55369	0.50665	0.02312	0.16277	0.24244
$\lambda_8$	0.51517	0.48029	0.02243	0.15963	0.22059
$\lambda_9$	0.51508	0.4748	0.02047	0.1368	0.21814
$\lambda_{10}$	0.50596	0.46578	0.01978	0.12611	0.17776
$\lambda_{11}$	0.50439	0.4602	0.01831	0.12121	0.17345
$\lambda_{12}$	0.48979	0.45588	0.01782	0.1169	0.17266
Fault type	Normal	Normal	Outer ring fault	Inner ring fault	Rolling element fault

regression machine is presented. This paper introduced K-L transformation method and Lagrange support vector regression machine model, the latest achievements of artificial intelligence is applied to fault diagnosis of rolling bearing. Reliability and accuracy of this method is proved by examples of fault diagnosis, we can obtain the follow results:

- (1) Constructed main characteristic vector after K-L transformation signals is decomposed can accurately reflect the situation that fault bearing vibration signal energy changing with the state information. These main characteristic vectors are imputed to fault classifier which is composed by four Lagrange support vector regression machines; this algorithm is simple and high efficiency.
- (2) Reliability and accuracy of this method is proved by examples of fault diagnosis, it can be used for other fault diagnosis systems.

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