Direction Of Arrival Estimation Based On Smooth Support Vector Regression

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***Abstract*—In this paper, we propose a new approach on direction of arrival (DOA) estimation based on smooth support vector regression. The proposed method can achieve higher accurate estimates for DOA while avoiding the all-direction peak value searching technique used in other traditional DOA estimation methods. Meanwhile, this approach reduces the extensive computations required by conventional super resolution algorithms such as MUSIC and is easier to implement in real-time applications. The proposed method map among the outputs of the array and the DOAs by means of a family of support vector machines. Computer simulation results show the effectiveness of the proposed method.**

Keywords: Direction of Arrival(DOA) estimation Support Vector Machine(SVM) smooth support vector regression(SSVR)

I. INTRODUCTION

Smart antennas can significantly improve the efficiency of wireless communication systems [1]. Smart antennas using direction of arrival (DOA) estimation algorithms can considerably reduce the effects of interference and multi-path fading. MUSIC and ESPRIT [2] are the some of the popular conventional methods of DOA estimation based on signal subspace decomposition. The main drawback of these schemes is that they need a full-rank estimate of the correlation matrix, which requires enough temporal smoothing and may result in prolonged acquisitions.

To reduce the computation complexity and implement in real-time easily, neural networks have been successfully applied to the problem of DOA estimation[3,4,5]. But the main drawbacks of classical neural network approaches are that some of them lead to optimization problems which do not have a single solution and that the complexity of the used structures may produce over-fitting.

Support Vector Machine (SVM) [6] is a new type of learning machine which is based on statistical learnig theory and principle of structural risk minimization. It is a way to construct linear and non-linear processors (neural networks) which solve the problem of local minimum and over-fitting. The regression arithmetic of SVM is named as support vector regression (SVR). As a powerful learning machine, SVR has been successfully extended its application in time series prediction, system identification, antenna array processing and communication problems [7,8,9,10]. In [11], an efficient DOA estimation method based on SVR has been proposed, it performs the transformation from the outputs of the elements of the smart array to the direction of arrival.

But the greatest disadvantage of SVR is the minimization problem of the model being required to the increase of the problem's complexity. Lee Y J and Mangasarian together proposed smooth support vector machines [12], whose basic idea is to transform the constrained quadratic optimization problem into an non-constrained convex quadratic optimization problem using smoothing, in order to reduce training complexity effectivelly. SSVR not only have excellent generalization ability as SVR algorithm, but also have higher convergence rate and fitting precision compared with SVR.

In this paper, for the estimation of DOA, we present a smooth support vector machine algorithm, we treat it as a nonlinear mapping from a space to another space. After a training phase in which several known input/output mappings are used to determine the parameters of the SVM. They perform well in response to input signals that have not been initially included in the training set. This paper proposes algorithm based on smooth support vector regression to perform the estimation of DOA, and compares the performance with that of other robust algorithms mentioned above.

The results of simulation indicate that SSVR have excellent performance on DOA estimation. Compared with other DOA estimation method such as neural network, SSVR has faster convergence speed and higher fitting precision, which effectively extends the application of SVR.

II. THEORY

Statistical Learning Theory (SLT) is presented by Vapnik et al. [6], which gives structural risk minimum principle and Vapnik-Chervonenkis (VC) dimension concept. SLT provides support vector machine learning (SVM) method, and support vector regression (SVR) is SVM applied for regression approximation. SVR has become a hot area in the field of machine learning.

A. *Standard Support Vector Regression*

The traditional SVR is a linear or nonliear regression function, which makes the predictive value of the regression

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function closer to the data observation value and be able to predict funture information. Meanwhile, it can tolerate the error of observation value and predictive value within the error scope of ε, and through ε, the non-sensitive function, solve the value of support vector regression.

For given s groups of samples: $\{x_i, y_i\}$, $i = 1,2,\ldots,s$, $x_i \in R^m$, $y_i \in R$, a non-linear mapping Φ is used to map data x into a higher dimensional feature space G. In this feature space, the mapping f is found out to linearly approach the given data [6]. in the method of SVR, ε-insensitive as loss function was selected:

$$\left|y - f(x)\right|_{\varepsilon} = \max\{0, \left|y - f(x)\right| - \varepsilon\} \tag{1}$$

Then the problem of regression changes to optimal problem as follows:

$$\min J = \frac{1}{2}\|\omega\|^2 + C\sum_{i=1}^{s}(\xi_i^* + \xi_i)$$
$$s.t.\begin{cases} y_i - (\omega, \Phi(x_i)) - b \le \varepsilon + \xi_i^* \\ (\omega, \Phi(x_i)) + b - y_i \le \varepsilon + \xi_i \\ \xi_i^*, \xi_i \ge 0 \end{cases} \tag{2}$$

Where ξ_i, ξ_i^* are slack terms to make the solution of the problem existing, C is regular factor which is a constant number to get tradeoff complexity of modelling and learning ability. Normally, the bigger is the value of C, the higher is the degree of approximation.

B. The SSVR Algorithm

The method of SVM is involved in solving quadratic programming (QP) problem under constrained condition, which is practical for small datasets. However for lager datasets, it is time-consuming and needs complex computation, which results in the limitation usability in practice. Lee Y J and Mangasarian have both proposed smooth support vector machine (SSVM), whose idea is transform constrained quadratic optimization problem into non-constrained convex quadratic optimization problem using smoothing method, which effectively reduces the computational complexity of support vector machines.

SSVR mainly introduce two concepts in the SVR algorithm

1) Smoothing methods: using smoothing techniques and a higher differential smooth function to replace ε, the non sensitive function, it can solve the complex operation issue that was caused by the unlimited minimization of SVR

2) Newton-Armijo Algorithm and Reduced Kernel: Introducing the Newton-Armijo Algorithm and Reduced Kernel to reduce the required memory space and computation time, and increase the efficiency of smooth support vector regression.

SSVM adopts quadratic ε-insensitive function as loss function [4, 5], then support vector regression can be denoted as follows:

$$\min J = \frac{1}{2}(\|\omega\|^2 + b^2) + C\sum_{i=1}^{s}\xi_i^2 + C\sum_{i=1}^{s}(\xi_i^*)^2$$
$$s.t.\begin{cases} y_i - (\omega, \Phi(x_i)) - b \le \varepsilon + \xi_i^* \\ (\omega, \Phi(x_i)) + b - y_i \le \xi_i \\ \xi_i^*, \xi_i \ge 0 \end{cases} \tag{3}$$

In smoothing method, substitute $C/2$ for C in the equation (3), and add $b^2/2$ to it, then the equation (3) turns into:

$$\min J = \frac{1}{2}(\|\omega\|^2 + b^2) + \frac{C}{2}\sum_{i=1}^{s}\xi_i^2 + \frac{C}{2}\sum_{i=1}^{s}(\xi_i^*)^2$$
$$s.t.\begin{cases} y_i - (\omega, \Phi(x_i)) - b \le \varepsilon + \xi_i^* \\ (\omega, \Phi(x_i)) + b - y_i \le \xi_i \\ \xi_i^*, \xi_i \ge 0 \end{cases} \tag{4}$$

Let

$$\begin{aligned} z_i &= (\omega, \Phi(x_i)) + b - y_i - \varepsilon \\ z_i^* &= y_i - (\omega, \Phi(x_i)) + b - \varepsilon \end{aligned} \tag{5}$$

Define function $(u)_+ = \max\{u, 0\}$, and make

$\xi_i = (z_i)_+$, and $\xi_i^* = (z_i^*)_+$, then the equation (4) is transformed into the following:

$$\begin{aligned} \min J = {} & 1/2(\|\omega\|^2 + b^2) + C/2\sum_{i=1}^{s}(z_i)_+^2 \\ & + C/2\sum_{i=1}^{s}(z_i^*)_+^2 \end{aligned} \tag{6}$$

The equation (6) is a convex quadratic optimization problem without constraint, which has unique solution. However, target function is not quadratic differentiable. So a strict convex and infinite differentiable smoothing function, described as follows, is used to substitute for $(u)_+$:

$$p(u, \alpha) = 1/\alpha \ln(1 + e^{\alpha u}) \tag{7}$$

Where $\alpha > 0$.

The smooth support vector regression is

$$\begin{aligned} \min J = {} & 1/2(\|\omega\|^2 + b^2) + C/2\sum_{i=1}^{s} p(z_i, \alpha)^2 \\ & + C/2\sum_{i=1}^{s} p(z_i^*, \alpha)^2 \end{aligned} \tag{8}$$

It is proven that $p(u, \alpha)$ discretionarily approximates $(u)_+$ when smoothing parameter $\alpha = 10$, and when $\alpha \to \infty$, the solution of the equation (8) in convergent to original problem solution.

III. SSVR AND DOA ESTIMATION

A. Signal Modal

A ULA composed by N elements with inter-element spacing d is used to intercept M narrowband plane waves transmitted by sources located at angels $\theta_m, m=1,\cdots,M$. Assuming isotropic array elements, the response of the ULA to the incident waves can be written as

$$X(k) = A(\theta)S(k) + N(k) \tag{9}$$

Where $X=[x_1,x_2,\cdots,x_N]^T$ is the vector of the baseband signals seen at the output of each element, A is the $N\times M$ steering matrix of the array toward the direction of the incoming signals defined as $A=[a(\theta_1)\cdots a(\theta_l)\cdots a(\theta_M)]$, the $a(\theta_l)$ corresponds to $a(\theta_l)=[1 \quad e^{-jk_l}\ldots e^{-j(N-1)k_l}]^T$, $S=[s_1,s_2,\cdots s_M]^T$ is the vector containing the complex amplitudes of the transmitted signals, and $N=[n_1,n_2,\cdots,n_N]^T$ is the noise vector with the power of each entry equal to σ_n^2.

Array processing algorithms use the correlation matrix, the $N\times N$ array covariance matrix can be written as

$$R_X \triangleq \mathrm{E}\{x(k)x^H(k)\} = AR_SA^H + \sigma_n^2 I \tag{10}$$

Where $R_S=\mathrm{E}\{s(k)s^H(k)\}$ is the source covariance matrix, σ_n^2 is the sensor noise variance, I is the identity matrix, $\mathrm{E}(\cdot)$ is the statistical expectation, and $(\cdot)^H$ is the Hermitian transpose.

R_X is symmetric, only the upper triangular part (composed by $N(N+1)/2$ complex values) is considered. As[3], these matrix elements are organized in an array b, given by

$$\begin{aligned}\mathrm{b}=[&r_{11},r_{12},\ldots,r_{1N},r_{22},\ldots r_{2N},\ldots,\\ &r_{(N-1)(N-1)},r_{(N-1)N},r_{NN}]\end{aligned} \tag{11}$$

Where $r_{ij}=[R_X]_{ij}, i,j=1,\ldots N$.

The array b is then normalized in order to obtain the input data z of the SVR,

$$\mathrm{z}=\frac{\mathrm{b}}{\|\mathrm{b}\|} \tag{12}$$

The antenna array can be thought of as performing a mapping $G: R^M \to C^N$ from the space of the DOA $\{\Theta=[\theta_1,\theta_2,\cdots,\theta_M]^T\}$ to the space of sensor output $\{X=[x_1,x_2,\cdots,x_N]^T\}$. For post processing, the elements of the matrix R are rearranged in a normalized array $\mathrm{z}\in\Sigma\subset C^{M(M-1)}$. A support vector machine is used to perform the inverse mapping $F: C^N \to R^M$. The algorithm described in this paper for the problem of direction finding is based on using SSVR to approximate the inverse mapping F.

Then, the steps for DOA estimation can be summarized as follows:

Procedure of the algorithm proposed:

- Generate the set of DOA is uniformly distributed in the range $[-90^\circ, 90^\circ]$.
- Generate the observed signals using (9).
- Estimate the covariance matrix of the array output vector R_{ij}.
- Form the vectors b and the normalized vectors z.
- Form the training set $\{\mathrm{b},\theta\}$ and train an appropriate SVM.
- Form the test set $\{\mathrm{b}_t\}$ to evaluate the SVM.

IV. SIMULATION

Simulated signal are generated based on the signal model (9). Suppose the received signal are uncorrelated, the training and testing signals are the complex outputs from the antenna array with eight elements, therefore the training and testing signals were 8×1 vectors. The received complex signal is modeled with a zero mean distribution with unit variance, the additive noise includes a zero mean distribution with a 0.2 variance.

The DOA for the set of testing signals is unknown to the system. Both the training and testing signals consisted of 1500 samples and the window length of the sample covariance matrix was set to 5. Therefore the training and testing sets were composed of 300 samples of each 8×1 projection vector.

The kernel function used in this work is a radial kernel[13], defined as

$$\psi(\mathrm{z}^i,\mathrm{z}) = \exp(-\|\mathrm{z}\text{-}\mathrm{z}^i\|^2/\sigma^2) \tag{13}$$

SSVR parameters C, ε and σ are 0.82, 10^{-3} and 0.5 respectively.

In the first experiment, fig.1 shows the probability of resolution of four algorithms and the range of signal-to-noise ratio was set from -30dB to 10dB. Resolution is regarded as successful as long as the error between estimate and true value is less than half 3dB beam width at the center frequency. From the result, We note that four methods have almost equal probability of resolution when SNR is high, but when SNR is low, the performance of the proposed method is better.

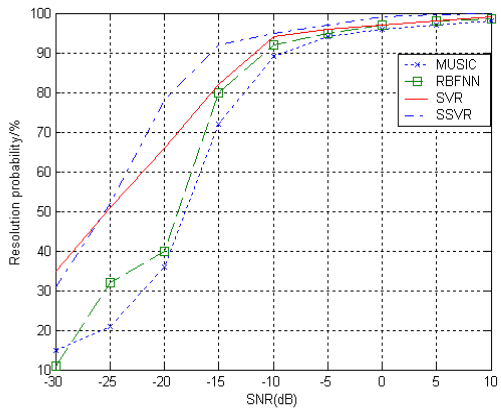

Figure 1. Resolution probability

In the second experiment, We simulated sources impinging from -90° to 90° .The range of signal-to-noise ratio was set from -10dB to 10dB. Fig.2 shows the RMSE of the DOA estimates versus input SNR. For simplicity, we assume that the numbers of sources and signals are known in all simulations. Define the root-mean-square error (RMSE) of the DOA estimates from 200 Monte Carlo trials as

$$\mathrm{RMSE} = \sqrt{\sum_{n=1}^{200} \sum_{k=1}^{N_s} (\hat{\theta}_k(n) - \theta_k)^2 / (200 N_s)} \tag{14}$$

Where $\hat{\theta}_k(n)$ is the estimate of θ_k for the n th Monte Carlo trial, and N_s is the number of all the signals.

The result illustrates that the performance of the proposed method is better than traditional method and other machine learning methods, especially when SNR is low.

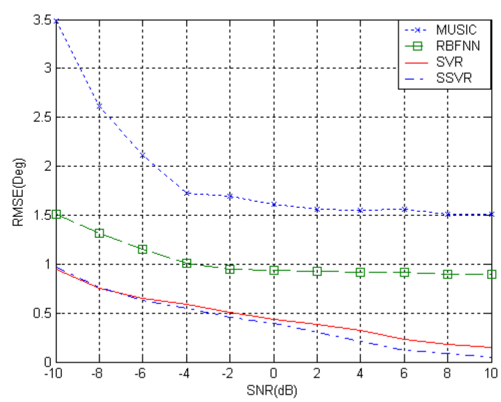

Figure 2. RMSE of the DOA estimates versus input SNR

V. CONCLUSION

In this paper, we have presented a novel method for DOA estimation based on smooth support vector regression. Compared with traditional DOA estimation algorithm, (MUSIC or ESPRIT), machine learning based DOA estimation algorithm reduce the compute complexity and easy to implement. The dimension of signal subspace, eigenvector of signal and noise subspace need to be estimated accurately by MUSIC and ESPRIT algorithm. SSVR-based algorithm needs use SVR as a nonlinear mapping from the outputs of the elements of the smart array to the direction of arrival. In this paper, we compare the performance of four algorithms, In some aspect, SSVR-based method performs better. Finally some simulation results have been shown to demonstrate the proposed algorithm is effective.

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