Experiment Report 2: Interval Observer

I. Theory deduction

Interval observer is a type of state observer based on interval theories.

For a linear time invariant discrete system, we can design its interval observer by calculating the lower bounds and upper bounds of state variables, denoted as $\underline{x}$ and $\underline{x}$. Therefore, we have two observers estimating the bounds of state variables, which comprising the whole interval observer.

Given a system as shown in formula (1), we can design the interval observer as follows.

$$\begin{aligned} x(t+1) &= Ax(t)+Bu(t) \\ y(t) &= Cx(t)+v(t) \end{aligned} \tag{1}$$

We have several assumptions about this system:

1. The input signal u is bounded within certain upper bound and lower bound;
2. The parameter matrices (A, C) are detectable and $A-LC$ is a nonnegative matrix;
3. The noise signal v is also bounded in the form of $\|v(t)\|_\infty \le V$.

Suppose the above assumptions are satisfied and we have the initial value of state variables within lower and upper bound. Then we have the following equations:

$$\begin{aligned} \underline{x}(t+1) &= A\underline{x}(t)+B\underline{u}(t)+L\left(y(t)-C\underline{x}(t)\right)-\overline{L}V \\ \overline{x}(t+1) &= A\overline{x}(t)+Bu(t)+L\left(y(t)-C\overline{x}(t)\right)-\overline{L}V \end{aligned} \tag{2}$$

where $\overline{L}=(L^{+}+L^{-})E_{p+1}$.

The former equation in formula (2) is the lower bound observer and the later one is upper bound observer. There two observers comprise one interval observer for the system.

II. Implementation

First, we choose the parameters and establish a system as follow:

$$\begin{aligned} x(t+1) &= \begin{bmatrix} 0.5 & 1 \\ 0 & -0.5 \end{bmatrix} x(t)+\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u(t) \\ y(t+1) &= \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} x(t)+v(t) \\ u(t) &= \begin{bmatrix} 2+\sin(2\pi t) \\ 2+\cos(2\pi t) \end{bmatrix} \end{aligned} \tag{3}$$

where *v(t)* is the white noise with *mean* = 0.5 and *variance* = 0.1. The white noise is shown below.

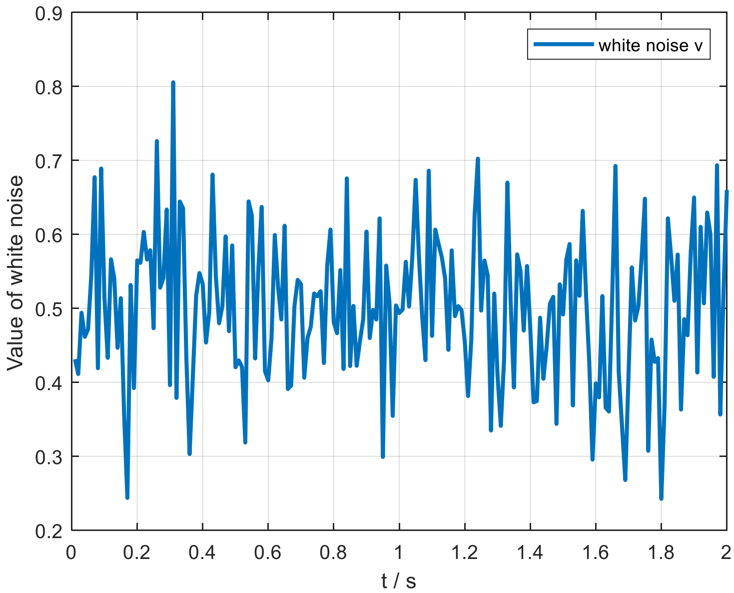

Figure.1 White noise *v(t)*

In order to make sure that $A - LC$ is nonnegative, we calculate a proper L = (0.25, 0.25; 0, -0.75).

However, the simulation result is not very satisfactory as shown in figure 1.

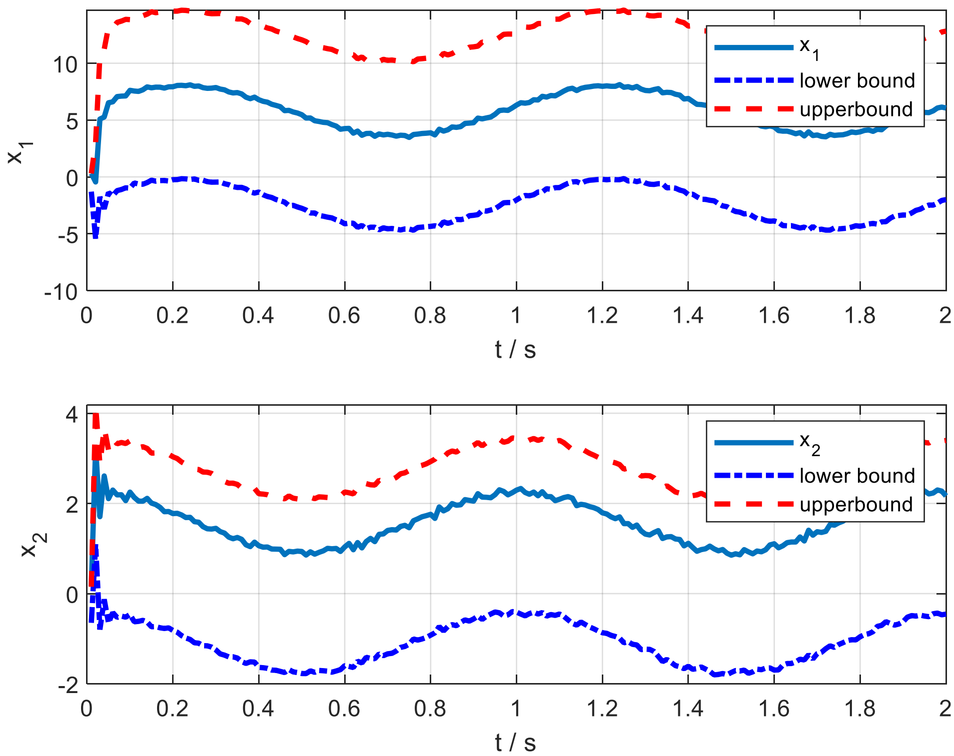

Fig.2 Simulation results of interval observer

As is shown in Figure.2, the performance of designed interval observer for the system is not very good. Specifically, the upper bound and lower bound for x_1 is too far away from the true value, though showed the same trend. As for state variable x_2, the upper bound shows a little better performance than that of x_1, closer to the true value, but lower bound shows worse performance, which makes me quite upset.

I have checked the deduction of the equations. The values of matrices can satisfy the requirements of stability and the observe design. The codes can run without any errors. I don’t know where is the problem.

III. Conclusion

Though the simulation result is not very satisfactory and I didn’t find out where the problem is, I gained deeper understanding of the theory of interval observer. In fact, the thought of upper bound and lower bound is very broadly used in nonlinear systems.

In my bachelor thesis, I dealt with the nonlinearity of the time-varying system model of the soft robot in the same way. Through such processing method, the model is well simplified and can be analyzed more easily.

Moreover, the interval theory alongside set theory is one different way of analyzing the uncertainty of systems from the Bayesian method, or more generally, the statistical method such as Kalman filter. But the interval method is also very useful and worth much time digging into it.