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Brief paper

An effective method to interval observer design for time-varying systems[☆]

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ABSTRACT

An interval observer for Linear Time-Varying (LTV) systems is proposed in this paper. Usually, the design of such observers is based on monotone systems theory. Monotone properties are hard to satisfy in many situations. To overcome this issue, in a recent work, it has been shown that under some restrictive conditions, the cooperativity of an LTV system can be ensured by a static linear transformation of coordinates. However, a constructive method for the construction of the transformation matrix and the observer gain, making the observation error dynamics positive and stable, is still missing and remains an open problem. In this paper, a constructive approach to obtain a time-varying change of coordinates, ensuring the cooperativity of the observer error in the new coordinates, is provided. The efficiency of the proposed approach is shown through computer simulations.

1. Introduction

The problem of unmeasurable system state estimation is challenging and its solution is required in many engineering applications. The problem of state estimation of systems has many solutions and has been widely investigated in the literature. Popular and well-known observers are mainly based on, for instance, Kalman/H_∞ filtering (Grip, Saberi, & Johansen, 2011; Särkkä, 2007) or Luenberger structure (Barmish & Galimidi, 1986). In situations where external disturbances and noises are assumed bounded without any additional assumption, interval observers can be an appealing alternative approach. Under some assumptions, these observers allow the designer to cope with uncertainties and evaluate the set of admissible values of the state vector, at any time instant.

Several approaches exist for designing interval observers (Bernard & Gouzé, 2004; Jaulin, 2002; Moisan, Bernard, & Gouzé, 2009), for linear systems (Ait Rami, Cheng, & de Prada, 2008; Bernard & Mazenc, 2010; Combastel & Raka, 2011) or when the system exhibits nonlinear behavior (Moisan et al., 2009; Raïssi, Efimov, & Zolghadri, 2012). The design of such observers is based on the monotone systems theory (Ait Rami, Tadeo, & Helmke, 2011; Bernard & Gouzé, 2004; Moisan et al., 2009). This approach has been recently extended to some nonlinear systems using LPV representations with known minorant and majorant matrices (Raïssi, Videau, & Zolghadri, 2010). One of the most restrictive assumptions for the interval observer design is the positivity (Smith, 1995) of the interval estimation error dynamics. It was relaxed for LTI systems in Bernard and Mazenc (2010), Combastel (2013), Combastel and Raka (2011) and Mazenc and Bernard (2011) by using a time-varying change of coordinates. Furthermore, a time-invariant transformation is proposed in Raïssi et al. (2012) to design a closed-loop observer for LTI systems where the transition matrix and the observer gain verify a Sylvester equation. This technique has also been extended to a class of nonlinear systems based on exact

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linearizations. Time-varying systems have been investigated in Efimov, Raïssi, Chebotarev, and Zolghadri (2013) where the case of time-varying transformation for periodic systems is considered and in Efimov, Raïssi, Chebotarev, and Zolghadri (2012) where the observer gain has to ensure stability of the observation error, and a static linear transformation of coordinates is proposed that provides the positivity of the observation error. The main limitation of the technique proposed in Efimov et al. (2013) is that the matrix $D(t) = A(t) - L_{\text{obs}}(t)C(t)$, where $L_{\text{obs}}(t)$ is the observer gain, should belong to a thin domain whose size is proportional to the inverse of the system dimension. Furthermore, no constructive methodology has been provided in Efimov et al. (2013) to prove the existence and to design a similarity transformation making $D(t)$ Metzler in the new coordinates.

The goal of this paper is to design a stable interval observer for LTV systems overcoming the previous limitations. The proposed interval observer is based on a time-varying change of coordinates which has been proposed in earlier works (Zhu & Johnson, 1989a,b, 1991). It should be noted that the proposed methodology does not require any additional assumption with respect to classical observers.

The paper is organized as follows. In Section 2, the problem is formulated and a previous result of an interval observer design for LTV systems is recalled. Section 3 is devoted to a procedure making the transformation of any time-varying matrix into a Metzler matrix. The result given in Section 3 is then used to design an interval observer for LTV systems in Section 4. Section 5 shows the efficiency of the interval observer through numerical simulations. To emphasize the improvement, a comparison with previous results reported in Efimov et al. (2013) and Thabet, Raïssi, Combastel, and Zolghadri (2013) is given.

2. Notations and problem statement

A square matrix $A = (A_{ij}) \in \mathbb{R}^{n\times m}$ is said to be Metzler if $A_{ij} \geq 0,\ \forall i \neq j$. For two vectors $x_1, x_2 \in \mathbb{R}^n$ or matrices $A_1, A_2 \in \mathbb{R}^{n\times n}$, the relations $x_1 \leq x_2$ and $A_1 \leq A_2$ are understood elementwise. The relation $P \prec 0$ ($P \succ 0$) means that the matrix $P \in \mathbb{R}^{n\times n}$ is negative (positive) definite.

Lemma 1 (*Smith, 1995*). *Given a non-autonomous system described by $\dot{x}(t) = Ax(t) + B(t)$ where A is a Metzler matrix and $B(t) \geq 0$. Then, $x(t) \geq 0,\ \forall t > 0$ provided that $x(0) \geq 0$.*

Note that the result of Lemma 1 is also valid for time-varying systems (i.e. $A(t)$ is time-varying). Now, consider an LTV system described by:

$$\begin{cases} \dot{x}(t) = A(t)x(t) + f(t) \\ y(t) = C(t)x(t) + \varphi(t) \\ x(0) \in [\underline{x}(0), \overline{x}(0)] \\ \quad \forall t,\ f(t) \in [\underline{f}(t), \overline{f}(t)] \subset \mathbb{R}^n,\ \varphi(t) \in [\underline{\varphi}(t), \overline{\varphi}(t)] \subset \mathbb{R}^p \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n, f(t) \in \mathbb{R}^n, y(t) \in \mathbb{R}^p$ and $\varphi(t) \in \mathbb{R}^p$ are respectively the state vector, an unknown but bounded input, the output vector and a bounded noise. The goal is to design an interval observer for systems described by (1).

Assumption 1. There exist bounded matrix functions $L_{\text{obs}} : \mathbb{R} \to \mathbb{R}^{n\times p}$, $M : \mathbb{R}_+ \to \mathbb{R}^{n\times n}$, $M(\cdot) = M(\cdot)^T \succ 0$ such that for all $t \geq 0$,

$$\begin{cases} \dot{M}(t) + D(t)^T M(t) + M(t)D(t) \prec 0, \\ D(t) = A(t) - L_{\text{obs}}(t)C(t). \end{cases}$$

Assumption 1 is a conventional requirement for LTV systems (Amato, Pironti, & Scala, 1996). Under this assumption, the observer gain $L_{\text{obs}}(t)$ and the matrix function $M(t)$ are such that the stability of the LTV system $\dot{x}(t) = D(t)x(t)$ can be proven by taking $V(t) = x(t)^T M(t)x(t)$ as Lyapunov function. It determines the output stabilization conditions of the system dynamics (1) which can be rewritten as:

$$\begin{cases} \dot{x}(t) = D(t)x(t) + \tilde{\phi}(t) \\ y(t) = C(t)x(t) + \varphi(t) \end{cases} \tag{2}$$

where $\tilde{\phi}(t) = f(t) - L_{\text{obs}}(t)\varphi(t) + L_{\text{obs}}(t)y(t)$. Linear Parameter-Varying or polytopic system results (Anstett, Millrioux, & Bloch, 2009; Bara, Daafouz, Ragot, & Kartz, 2000) can be used to compute an observer gain $L_{\text{obs}}(t)$ satisfying Assumption 1. In addition, if the matrix $D(t) = A(t) - L_{\text{obs}}(t)C(t)$ is Metzler, an interval observer for the LTV system (2) can be easily designed. Nevertheless, the Metzler condition is not usually satisfied without applying some model transformations. This problem has been investigated in recent work (Efimov et al., 2013) where the goal was to find a time-invariant change of coordinates and an observer gain in order to obtain a positive observation error at each time. The design of the gain and the existence of the static transition matrix ensuring the Metzler property remains a difficult task. Assumption 2 (Assumption 4 in Efimov et al., 2013) was used to design interval observers.

Assumption 2. Let $D(t) \in \Xi$ for all $t \geq 0$, $\Xi = \{D \in \mathbb{R}^{n\times n} : D_a - \Delta \leq D \leq D_a + \Delta\}$ for some $D_a^T = D_a \in \mathbb{R}^{n\times n}$ and $\Delta \in \mathbb{R}_+^{n\times n}$. Let for some constant $\mu > n\|\Delta\|_{\max}$ (where $\|\Delta\|_{\max} = \max_{i=\overline{1,n}, j=\overline{1,n}} |\Delta_{i,j}|$ the elementwise maximum norm) and a diagonal matrix $\Upsilon \in \mathbb{R}^{n\times n}$ the Metzler matrix $R = \mu E_n - \Upsilon$, where $E_n \in \mathbb{R}^{n\times n}$ denotes the matrix with all elements equal to 1, have the same eigenvalues as the matrix D_a.

Note that the case of $\Delta = 0$ corresponds to LTI systems for which several solutions exist (Combastel, 2013; Mazenc & Bernard, 2011; Raïssi et al., 2012). Under Assumption 2, (Efimov et al., 2013) shows that there is an orthogonal matrix $S \in \mathbb{R}^{n\times n}$ such that the matrices $S^T D(t)S$ are Metzler for all $D(t) \in \Xi$. By introducing the new state variable $z = S^T x$, (1) can be rewritten in the new coordinates:

$$\dot{z} = S^T A(t)Sz + \phi(t),$$

where $\phi(t) = S^T f(t)$. The proposed interval observer for the system (1) in the new coordinates is:

$$\begin{cases} \dot{\underline{z}} = S^T D(t)S\underline{z} + \underline{\phi}(t) + \underline{\Psi}(t) + K_{\text{obs}}(t)y \\ \dot{\overline{z}} = S^T D(t)S\overline{z} + \overline{\phi}(t) + \overline{\Psi}(t) + K_{\text{obs}}(t)y \end{cases} \tag{3}$$

where $\underline{\phi}(t) = (S^+)^T \underline{f}(t) - (S^-)^T \overline{f}(t)$, $\overline{\phi}(t) = (S^+)^T \overline{f}(t) - (S^-)^T \underline{f}(t)$, $K_{\text{obs}} = S^T L_{\text{obs}}(t)$, $\underline{\Psi}(t) = K_{\text{obs}}^-(t)\underline{\varphi} - K_{\text{obs}}^+(t)\overline{\varphi}$, $\overline{\Psi}(t) = K_{\text{obs}}^-(t)\overline{\varphi} - K_{\text{obs}}^+(t)\underline{\varphi}$. Given a matrix $N \in \mathbb{R}^{m\times n}$, N^+ and N^- are defined as: $N^+ = \max\{0, N\}, N^- = \max\{0, -N\}$. Then, S^+, S^-, $K_{\text{obs}}^+(t)$ and $K_{\text{obs}}^-(t)$ can be deduced. In the original coordinates, applying Lemma 2 given below to the relation $x = Sz$, the bounds of the state vector x are given by:

$$\underline{x} = S^+\underline{z} - S^-\overline{z}, \qquad \overline{x} = S^+\overline{z} - S^-\underline{z}. \tag{4}$$

Lemma 2 (*Efimov et al., 2013*). *Let $x \in \mathbb{R}^n$ be a vector variable, $\underline{x} \leq x \leq \overline{x}$ for some $\underline{x},\ \overline{x} \in \mathbb{R}^n$, and $S \in \mathbb{R}^{m\times n}$ be a matrix, then*

$$S^+\underline{x} - S^-\overline{x} \leq Sx \leq S^+\overline{x} - S^-\underline{x}. \tag{5}$$

According to Assumption 2, the main limitation of the technique proposed in Efimov et al. (2013) is that the matrix $D(t)$ should belong to a thin domain whose size is proportional to the inverse of the system dimension ($\|\Delta\|_{\max} < \frac{\mu}{n}$). Then, the bigger the system dimension is, the thinner the domain enclosing $D(t)$ must be. Overcoming such limitations motivates this work. In the next section, a new constructive method, based on a time-varying change of coordinates ensuring the cooperativity property of the system (2), is proposed in order to design an interval observer for (1) in a fully LTV framework.

3. Transformation procedure of time-varying matrices into a Metzler form

In this section, it is shown that any time-varying matrix can be transformed into a Metzler time-varying one through a linear time-varying change of coordinates. This result, based on the earlier works (Zhu & Johnson, 1989a,b, 1991), is used in the next section to design an interval observer. For brevity of presentation, the proofs of the theorems of this section are omitted, the reader can refer to Zhu and Johnson (1989a,b, 1991) and references therein. In the following, a constructive procedure transforming any time-varying matrix $D(t)$ into an upper triangular Metzler matrix $\Gamma(t)$ is given through two successive transformations. The first transformation of a time-varying matrix $D(t)$ into a companion matrix $\tilde{C}(t)$ is given by Theorem 3. Then, Theorem 4 deals with the transformation of $\tilde{C}(t)$ into a Metzler matrix $\Gamma(t)$. The description of the main steps of the procedure requires the following lemmas and definitions.

Lemma 3 (*Zhu & Johnson, 1989a* and *Zhu & Johnson, 1991*). *Two time-varying matrices $A_1(t) \in \mathbb{R}^{n\times n}$ and $A_2(t) \in \mathbb{R}^{n\times n}$ are said to be D-similar if there exists a transformation matrix $\Sigma(t) \in \mathbb{R}^{n\times n}$ such that $\det(\Sigma(t)) \equiv$ constant $\neq 0$ and*

$$A_2(t) = \Sigma^{-1}(t)[A_1(t)\Sigma(t) - \dot{\Sigma}(t)]. \tag{6}$$

$A_1(t)$ and $A_2(t)$ are D-similar if $\mathrm{tr}(A_1(t)) = \mathrm{tr}(A_2(t))$. $\det(\cdot)$ and $\mathrm{tr}(\cdot)$ respectively denote the determinant and the trace of a square matrix.

For brevity of presentation, the interested reader can refer to Zhu and Johnson (1989a,b, 1991) to obtain more details about D-similarity transformations which can be shown to be transitive.

Definition 1. A matrix $\hat{C}(t) \in \mathbb{R}^{n\times n}$ is under a companion canonical form if it is given by:

$$\hat{C}(t) = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & & \ddots & \ddots & 0 \\ 0 & \cdots & \cdots & 0 & 1 \\ -\beta_1(t) & -\beta_2(t) & \cdots & \cdots & -\beta_n(t) \end{bmatrix}, \\ = \mathrm{Comp}\left([-\beta_1(t), -\beta_2(t), \ldots, -\beta_n(t)]\right). \tag{7}$$

Definition 2 (*Definition 5.4 in Zhu and Johnson (1989a)*). A matrix $Q(t) \in \mathbb{R}^{n\times n}$ is said to be under a Quasi-Companion form if there exists a companion canonical matrix $\hat{C}(t) \in \mathbb{R}^{n\times n}$ and a scalar function $\xi(t)$ such that $Q(t) = \hat{C}(t) - \xi(t)I$.

The first step is the transformation of the time-varying matrix $D(t)$ into a companion canonical form $\tilde{C}(t)$. This transformation is ensured through two successful transformations namely the transformation of $D(t)$ into a Quasi-Companion matrix $Q(t)$ and then the transformation of $Q(t)$ into $\tilde{C}(t)$. Therefore, the similarity transformation (6) firstly established between the matrices $D(t)$ and $Q(t)$, which is defined in Definition 2, is given by Lemma 4.

Definition 3. Given $D(t) \in \mathbb{R}^{n\times n}$ and $b(t) \in \mathbb{R}^n$. The operator $\wp_{D(t)}$ and the orbit of the pair $(\wp_{D(t)},\ b(t))$, denoted by $\mathrm{orb}(\wp_{D(t)}, b(t))$, are defined as in (8) where $[\ldots|\ldots]$ refers to the horizontal concatenation operator.

$$\wp_{D(t)} = D(t) - I\delta \quad \text{with } \delta = d/dt,$$

$$\mathrm{orb}(\wp_{D(t)}, b(t)) = [b(t)|\wp_{D(t)}\{b(t)\}|\wp_{D(t)}\wp_{D(t)}\{b(t)\}| \\ \cdots |\wp_{D(t)}^{n-1}\{b(t)\}]. \tag{8}$$

Lemma 4. *Let $K(t) = \mathrm{orb}(\wp_{D(t)}, b(t))$ where $b(t) \in \mathbb{R}^n$ is a vector such that*

$$\det(K(t)) = \rho(t) \neq 0 \tag{9}$$

on a subinterval of $\mathbb{R}^+$ with positive measure; let e_n be the nth column vector of the $(n \times n)$ identity matrix I and $\hat{C}(t) = \mathrm{Comp}([-\beta_1(t), -\beta_2(t), \ldots, -\beta_n(t)])$, then $D(t)$ can be transformed into a Quasi-Companion matrix $Q(t)$ given by:

$$Q(t) = L_0^{-1}(t)(D(t)L_0(t) - \dot{L}_0(t)) = \hat{C}(t) - \xi(t)I \tag{10}$$

where

$$L_0(t) = \rho^{-1/n}(t)K(t)H^{-1}(\beta(t)), \tag{11}$$

$$\beta(t) = [\beta_1(t), \beta_2(t), \ldots, \beta_n(t)], \tag{12}$$

$$H(\beta(t)) = \mathrm{orb}(\wp_{\hat{C}(t)}, e_n), \tag{13}$$

$$\xi(t) = -\frac{1}{n}\dot{\rho}(t)\rho^{-1}(t) \tag{14}$$

and the functions $\beta_1, \ldots, \beta_n$ are computed in Proposition 1.

Proof. This lemma follows from Definition 2 and the proof of the parts (i)–(iii) in Theorem 5.4 in Zhu and Johnson (1989a).

As shown in Lemma 4, every matrix can be transformed into a Quasi-Companion one using the transformation matrix $L_0(t)$ under the condition (9). Note that the vector b can be chosen constant or time-varying and that the expression of $L_0(t)$ is deduced by replacing in (11) the obtained $\beta(t)$ given by Proposition 1.

Proposition 1 (*From Theorem 5.4 in Zhu and Johnson (1989a)*). *The scalar functions $\beta_i(t)$ in $\hat{C}(t)$ and $L_0(t)$ are computed by solving Eq. (15), where $\wp_{\hat{C}}^n\{e_n\}$ is the operator $\wp_{\hat{C}}$ defined in (8) and applied n times to e_n.*

$$H(\beta(t))K^{-1}(t)[D(t)K(t) - \dot{K}(t)]e_n = \wp_{\hat{C}}^n\{e_n\}. \tag{15}$$

Now, the similarity (6) is secondly established between the Quasi-Companion matrix $Q(t)$ and the companion matrix $\tilde{C}(t)$ given by Proposition 2.

Proposition 2 (*From Theorem 5.3 in Zhu and Johnson (1989a)*). *The Quasi-Companion matrix $Q(t)$ can be transformed into a companion form $\tilde{C}(t)$ (i.e. Q and $\tilde{C}$ are D-similar) given by:*

$$\tilde{C}(t) = R_n^{-1}(\xi(t))[Q(t)R_n(\xi(t)) - \dot{R}_n(\xi(t))] \\ = \mathrm{Comp}\left([-\alpha_{n,1}(t), -\alpha_{n,2}(t), \ldots, -\alpha_{n,n}(t)]\right) \tag{16}$$

where

$$R_n(\xi(t)) = \begin{bmatrix} R_{n-1}^{-1}(\xi(t)) & \mathbf{0}_{n-1,1} \\ -\gamma_{n-1}R_{n-1}^{-1}(\xi(t)) & 1 \end{bmatrix} \in \mathbb{R}^{n\times n}, \tag{17}$$

$$\begin{cases} R_1^{-1} = [1] \quad \forall n \geq 2,\ \gamma_{n-1} = [\gamma_{n-1,1}|\gamma_{n-1,2}|\cdots|\gamma_{n-1,n-1}] \\ \quad \text{where } \gamma_{n-1,k} = \dot{\gamma}_{n-2,k} - \xi(t)\gamma_{n-2,k} \\ \quad + \gamma_{n-2,k-1} \text{ for } k = 1, \ldots, n-1 \\ \quad \text{with } \gamma_{j,0} = 0,\ \gamma_{j,j+1} = 1 \text{ for } j = 0, \ldots, n-1 \end{cases} \tag{18}$$

and $\mathbf{0}_{n-1,1}$ is a column vector with zero elements.

As a result, the transformation of $D(t)$ into $\tilde{C}(t)$ is given by the following theorem with $R(\xi(t)) = R_n(\xi(t))$:

Theorem 3. *The matrix ensuring the transformation of $D(t)$ to $\tilde{C}(t)$ is defined by*

$$L(t) = L_0(t)R(\xi(t)), \tag{19}$$

and the companion matrix is given by:

$$\tilde{C}(t) = L^{-1}(t)(D(t)L(t) - \dot{L}(t)). \tag{20}$$

Once $\tilde{C}(t)$ is obtained, the next step is to transform this matrix into a Metzler matrix $\Gamma(t)$ described by:

$$\Gamma(t) = \begin{bmatrix} \lambda_1(t) & 1 & \cdots & 0 \\ 0 & \lambda_2(t) & \ddots & \vdots \\ \vdots & \ddots & \ddots & 1 \\ 0 & \cdots & 0 & \lambda_n(t) \end{bmatrix}, \tag{21}$$

where $\lambda_i(t)$ are Essential D-eigenvalues (or ED-eigenvalues, see Definition 3.1 in Zhu and Johnson (1989a)) of $D(t)$. The elements $\alpha_{n,i}(t)$ of the matrix $\tilde{C}(t)$ can be recursively computed using the ED-eigenvalues $\lambda_i(t)$ $(i = 1, \ldots, n)$ by the relation (22), with $\alpha_{k,0} = 0$ and $\alpha_{k,k+1} = 1$ for $0 \leq k \leq n-1$ (Zhu & Johnson, 1989a,b, 1991).

$$\alpha_{n,j}(t) = \dot{\alpha}_{n-1,j}(t) - \lambda_n(t)\alpha_{n-1,j}(t) + \alpha_{n-1,j-1}(t) \quad \text{for } j = 1, \ldots, n. \tag{22}$$

The example (23) gives the general expressions for $\alpha_{n,i}(t)$ in terms of $\lambda_i(t)$ for $n = 1$ and 2:

$$\begin{aligned} n = 1 \quad & \alpha_1(t) = \alpha_{1,1}(t) = -\lambda_1(t) \\ n = 2 \quad & \alpha_2(t) = [\alpha_{2,1}(t) \quad \alpha_{2,2}(t)] \\ j = 1 \quad & \alpha_{2,1}(t) = -\dot{\lambda}_1(t) + \lambda_1(t)\lambda_2(t) \\ j = 2 \quad & \alpha_{2,2}(t) = -(\lambda_1(t) + \lambda_2(t)). \end{aligned} \tag{23}$$

The transition matrix ensuring the similarity between $\tilde{C}(t)$ and $\Gamma(t)$ is given by the following theorem.

Theorem 4 (*Theorem 4.1 in Zhu and Johnson (1989a)*)**.** *Every Companion matrix $\tilde{C}(t)$ can be transformed into a matrix $\Gamma(t)$ (21) after a change of variables given by:*

$$\Gamma(t) = (P(t)\tilde{C}(t) + \dot{P}(t))P^{-1}(t) \tag{24}$$

where the transition matrix $P(t)$ is constructed recursively by the following algorithm:

$$\begin{cases} P_1 = [1] \\ \vdots \\ P_n(t) = \begin{bmatrix} P_{n-1}(t) & \mathbf{0}_{n-1,1} \\ \alpha_{n-1}(t) & 1 \end{bmatrix} \in \mathbb{R}^{n \times n} \end{cases} \tag{25}$$

where the jth element $\alpha_{n-1,j}(t)$ in the row vector $\alpha_{n-1}(t)$ is given by (22) as an explicit function of $\lambda_1(t), \lambda_2(t), \ldots, \lambda_{n-1}(t)$ and their appropriate derivatives.

Proposition 3. *The transformation of $D(t)$ into a Metzler matrix $\Gamma(t)$ (21) is ensured by the two transition matrices $L(t)$ and $P(t)$, given respectively by (19) and (25), through the following relation:*

$$\Gamma(t) = T(t)\left(D(t)T^{-1}(t) - d(T^{-1}(t))/dt\right) \quad \text{where } T(t) = P(t)L^{-1}(t). \tag{26}$$

The proof of Proposition 3 is a direct consequence of Theorems 3 and 4 respectively defining the change of variables given in (20), (24). This result (Zhu & Johnson, 1989a,b, 1991) relies on the transitivity of D-similarities and appears to be very useful to design interval observers in a full LTV framework after noticing that $\Gamma(t)$ as in (21) is a Metzler matrix. Therefore, any time-varying matrix $D(t)$ can be transformed into a Metzler matrix $\Gamma(t)$ through a time-varying change of variables resulting from a constructive procedure. The different steps mentioned in this procedure will be illustrated through the numerical example given in Section 5. In the following, the generic transformation procedure is used to design an interval observer for LTV systems described by (2).

4. Interval observer for LTV systems

4.1. Structure of the proposed interval observer

To design a stable interval observer for the LTV system (2), the only choice of an observer gain $L_{\text{obs}}(t)$ ensuring the stability of $D(t) = A(t) - L_{\text{obs}}(t)C(t)$ is not sufficient. A usual interval observer design would also require $D(t)$ to be Metzler. As this condition is difficult to satisfy, a time-varying change of coordinates is used. More precisely, using Theorems 3, 4 and Proposition 3, a time-varying change of coordinates, $z(t) = T(t)x(t)$ with $T(t) = P(t)L^{-1}(t)$, applied to the system (2) ensures the cooperativity of the observation error. The resulting structure of the interval observer for the LTV system is provided by Theorem 6. Since it is always possible to build a domain $[\underline{x}(0), \overline{x}(0)]$ for the initial state x_0, then the domain $[\underline{z}(0), \overline{z}(0)]$ of $z(0)$ is computed as: $\underline{z}(0) = T^+(0)\underline{x}(0) - T^-(0)\overline{x}(0)$ and $\overline{z}(0) = T^+(0)\overline{x}(0) - T^-(0)\underline{x}(0)$. Furthermore, the following assumption (usual in the interval observers literature) is required for the design of the interval observer.

Assumption 5. Assume that $f(t) \in [\underline{f}(t), \overline{f}(t)]$, $\varphi(t) \in [\underline{\varphi}(t), \overline{\varphi}(t)]$, $\|y(t)\| \leq Y$ $\forall t \geq t_0$, where the constant $Y > 0$ is given and $\underline{f}(t)$, $\overline{f}(t)$, $\underline{\varphi}(t)$, $\overline{\varphi}(t)$ are bounded. In addition, $\exists\, M_1 \in \mathbb{R}_+$ such that $\forall\, t \geq t_0$, $\|T(t)\| \leq M_1$, $\|T^{-1}(t)\| \leq M_1$.

Theorem 6. *Given a system described by (1) and consider a matrix T constructed as in Section 3, let Assumptions 1 and 5 hold (notice that the restrictive Assumption 2 is not required). Then, the system (27) is a stable interval observer for (2) in the sense of (28).*

$$\begin{cases} \dot{\underline{z}}(t) = \Gamma(t)\underline{z}(t) + \underline{\phi}_{\text{obs}}(t) + \underline{\Psi}_{\text{obs}}(t) + T_{\text{obs}}(t)y(t) \\ \dot{\overline{z}}(t) = \Gamma(t)\overline{z}(t) + \overline{\phi}_{\text{obs}}(t) + \overline{\Psi}_{\text{obs}}(t) + T_{\text{obs}}(t)y(t) \end{cases} \tag{27}$$

$$\underline{z}(t) \leq T(t)x(t) \leq \overline{z}(t), \quad \forall t \geq t_0, \tag{28}$$

where $\underline{\phi}_{\text{obs}}(t) = T^+(t)\underline{f}(t) - T^-(t)\overline{f}(t)$, $T_{\text{obs}}(t) = T(t)L_{\text{obs}}(t)$, $\underline{\Psi}_{\text{obs}}(t) = T^-_{\text{obs}}(t)\underline{\varphi}(t) - T^+_{\text{obs}}(t)\overline{\varphi}(t)$, $\overline{\phi}_{\text{obs}}(t) = T^+(t)\overline{f}(t) - T^-(t)\underline{f}(t)$, $\overline{\Psi}_{\text{obs}}(t) = T^-_{\text{obs}}(t)\overline{\varphi}(t) - T^+_{\text{obs}}(t)\underline{\varphi}(t)$. $\Gamma(t)$ is defined in (21) and (26); the time-varying transformation matrices $L(t)$ and $P(t)$ are given respectively by (19) and (25).

Proof. Using the time-varying change of coordinates $z(t) = T(t)x(t)$ with $T(t) = P(t)L^{-1}(t)$, we have $\dot{z}(t) = \dot{T}(t)x(t) + T(t)\dot{x}(t)$. As $x(t) = T^{-1}(t)z(t)$, $\dot{x}(t) = D(t)x(t) + \tilde{\phi}(t)$, $dT^{-1}(t)/dt = -T^{-1}(t)\dot{T}(t)T^{-1}(t)$ and $\dot{T}T^{-1}(t) = -T(t)dT^{-1}(t)/dt$, then the state equation in the new coordinates is given by:

$$\dot{z}(t) = \Gamma(t)z(t) + T(t)\tilde{\phi}(t) \tag{29}$$

where $\Gamma(t) = T(t)D(t)T^{-1}(t) - T(t)dT^{-1}(t)/dt$. Now, denote by $\bar{\tilde{z}}(t) = \overline{z}(t) - z(t)$ the upper error. Then,

$$\dot{\bar{\tilde{z}}}(t) = \Gamma(t)\overline{z}(t) + \overline{\phi}_{\text{obs}}(t) + \overline{\Psi}_{\text{obs}}(t) + T_{\text{obs}}(t)y(t) - \Gamma(t)z(t) - T(t)\tilde{\phi}(t). \tag{30}$$

By construction, the term $\overline{\phi}_{\text{obs}}(t)+\overline{\Psi}_{\text{obs}}(t)+T_{\text{obs}}(t)y(t)-T(t)\tilde{\phi}(t)$ is nonnegative and the matrix $\Gamma(t)$ is Metzler. Then, by using Lemma 1, the upper observation error verifies $\bar{\tilde{z}}(t) \geq 0,\ \forall t \geq t_0$ provided that $\bar{\tilde{z}}(t_0) \geq 0$. Thus, we can conclude that $\bar{z}(t) \geq z(t),\ \forall t \geq t_0$. The same methodology can be used to prove that $z(t) \geq \underline{z}(t),\ \forall t \geq t_0$. Therefore, we have:

$$\underline{z}(t) \leq z(t) \leq \bar{z}(t) \quad \forall t \geq t_0.$$

In order to analyze stability of the proposed interval observer note that both equations in (27) are decoupled, thus stability of each variable $\underline{z}$ and $\bar{z}$ can be investigated separately. Let us consider the variable $\bar{z}$ (for $\underline{z}$ the same arguments can be applied) whose dynamics can be rewritten as follows:

$$\dot{\bar{z}} = \Gamma(t)\bar{z} + \bar{d}(t),$$
$$\bar{d}(t) = \overline{\phi}_{\text{obs}}(t) + \overline{\Psi}_{\text{obs}}(t) + T_{\text{obs}}(t)y(t).$$

Through Assumptions 1 and 5, there is a constant $\bar{\delta} > 0$ such that $|\bar{d}(t)| \leq \bar{\delta}$ for all $t \geq 0$. Define the matrix $\Xi(t) = [T^{-1}(t)]^T M(t)T^{-1}(t)$, which is bounded if T^{-1} and M are so, then $\xi_1 I \preceq \Xi(t) \preceq \xi_2 I$ for some $\xi_1, \xi_2 > 0$. Consider a Lyapunov function $W(t, \bar{z}) = \bar{z}^T \Xi(t)\bar{z}$ whose derivative can be written as (after developing and simplifying the derivative $\dot{W}$ by introducing the expression $d(T^{-1})/dt = DT^{-1} - T^{-1}\Gamma$ obtained from (26)):

$$\dot{W} = \bar{z}^T \Upsilon(t)\bar{z} + 2\bar{z}^T \Xi(t)\bar{d}(t),$$

with

$$\Upsilon(t) = [T^{-1}(t)]^T\{\dot{M}(t) + D^T(t)M(t) + M(t)D(t)\}T^{-1}(t).$$

By Assumption 1, there exists a constant symmetric positive definite matrix Θ such that $\Upsilon(t) \preceq -\Theta$. Then,

$$\dot{W} \leq -\bar{z}^T\Theta\bar{z} + 2\bar{z}^T\Xi(t)\bar{d}(t).$$

Since Θ is symmetric, $\Theta = \Theta^{0.5}\Theta^{0.5}$ and

$$\begin{aligned} 2\bar{z}^T\Xi(t)\bar{d} &= 2\bar{z}^T\Theta^{0.5}\frac{\sqrt{2}}{\sqrt{2}}\Theta^{-0.5}\Xi(t)\bar{d} \\ &\leq 0.5\bar{z}^T\Theta\bar{z} + 2\bar{d}^T\Xi^T(t)\Theta^{-1}\Xi(t)\bar{d}. \end{aligned}$$

Therefore,

$$\begin{aligned} \dot{W} &\leq -0.5\bar{z}^T\Theta\bar{z} + 2\bar{d}^T\Xi^T(t)\Theta^{-1}\Xi(t)\bar{d} \\ &\leq -0.5\lambda_{\min}(\Theta)\xi_2^{-1}W + \frac{2}{\lambda_{\min}(\Theta)}\xi_2^2\bar{\delta}^2. \end{aligned}$$

Next, applying a standard comparison principle result we obtain that W is bounded, that implies the same properties for the variable $\bar{z}$.

Corollary 1. *Under the same assumptions as in Theorem 6, the upper and the lower state bounds given by (32) in the original coordinates ($x(t)$) verify the inclusion:*

$$\underline{x}(t) \leq x(t) \leq \bar{x}(t), \quad \forall t \geq t_0 \tag{31}$$

$$\begin{cases} \bar{x}(t) = (T^{-1}(t))^+\bar{z}(t) - (T^{-1}(t))^-\underline{z}(t) \\ \underline{x}(t) = (T^{-1}(t))^+\underline{z}(t) - (T^{-1}(t))^-\bar{z}(t). \end{cases} \tag{32}$$

Proof. Using the change of coordinates, $z(t) = T(t)x(t)$, $x(t) = T^{-1}(t)z(t)$. In fact, it is readily verified that $\det(P(t)) = 1 \neq 0$ and $\det(L(t)) = (-1)^{n-1} \neq 0$. Then, $T(t)$ is invertible. Applying Lemma 2 to $x(t) = T^{-1}(t)z(t)$ and since (28) is proved, then (32) and (31) are verified.

When the dimension of the state vector x is low, it is possible to compute the "parameters" α_i, β_i and λ_i by solving the ordinary differential equations (15) and (22). Then, Theorems 3–4 and Proposition 3 give a constructive procedure to build a time-varying D-similarity transformation presenting a time-varying state matrix into a Metzler one. Therefore, no additional assumption with respect to conventional observers is required. Thus, the limitations in Efimov et al. (2013) are overcome using the proposed approach. When the dimension of x is high, it is necessary to use computer solvers and the expression of T could be complex.

4.2. A methodology for a practical implementation

One of the steps of the transformation procedure described in Section 3 is to find a vector b which satisfies the condition (9). As it is mentioned, this vector can be constant or time-varying. For simplicity of practical implementation, b can be chosen constant and it can be computed by the following methodology:

Given a scalar function $\rho(t) = \det(\text{orb}(\wp_{D(t)}, b)) = \rho(D(t), b)$, and let $D(t) = D(\theta(t))$ where $\theta(t) \in [\underline{\theta},\ \bar{\theta}]$, with known upper $\bar{\theta}$ and lower $\underline{\theta}$ bounds. The condition $\rho(t) = \rho(\theta(t), b) \neq 0$ can be expressed as $\rho(\theta(t), b) > 0 \vee \rho(\theta(t), b) < 0$. Taking into account the upper $\bar{\theta}$ and the lower $\underline{\theta}$ bounds of $\theta(t)$, these conditions lead to the following constraints:

$$\underline{\rho}(\bar{\theta}, \underline{\theta}, b) > 0 \vee \bar{\rho}(\bar{\theta}, \underline{\theta}, b) < 0 \tag{33}$$

where $\bar{\rho}$ and $\underline{\rho}$ are respectively an upper and a lower bound of $\rho(t)$. The characterization of the set of all b values satisfying (33) can be performed in a guaranteed way using a paving algorithm like SIVIA (Set Inversion Via Interval Analysis) (Jaulin, Kieffer, Didrit, & Walter, 2001) which uses interval analysis tools to solve constraint satisfaction problems. In the case of $n = 2$:

$$D(\theta(t)) = \begin{bmatrix} d_{11}(\theta(t)) & d_{12}(\theta(t)) \\ d_{21}(\theta(t)) & d_{22}(\theta(t)) \end{bmatrix}, \qquad b = [b_1,\ b_2]^T.$$

$\rho(\theta(t), b) = \det(\text{orb}(\wp_{D(t)}, b(t))) = d_{21}(\theta(t))b_1{}^2 + (d_{22}(\theta(t)) - d_{11}(\theta(t)))b_1b_2 - d_{12}(\theta(t))b_2^2$. RealPaver (Granvilliers & Benhamou, 2006) is used in this case for solving the nonlinear constraints (34) by interval computation:

$$\begin{aligned} &\underline{\rho}(\theta(t), b) = \underline{d}_{21}b_1{}^2 + (\underline{d}_{22} - \bar{d}_{11})b_1b_2 - \bar{d}_{12}b_2^2 > 0 \\ &\vee \bar{\rho}(\theta(t), b) = \bar{d}_{21}b_1{}^2 + (\bar{d}_{22} - \underline{d}_{11})b_1b_2 - \underline{d}_{12}b_2^2 < 0 \end{aligned} \tag{34}$$

where $\underline{d}_{ij} = d_{ij}(\underline{\theta})$ and $\bar{d}_{ij} = d_{ij}(\bar{\theta})$ for $i, j = \{1, 2\}$. Note that a similar methodology enables to obtain b for $n > 2$.

5. Numerical example

To illustrate the efficiency of the proposed interval observer and for the sake of comparison, consider the LTV example given in Efimov et al. (2013):

$$\begin{aligned} &\dot{x} = A(t)x + f(t), \qquad y = x_2, \\ &A(t) = \begin{bmatrix} -0.632 - 0.8\sin(t) & 0.5\cos(3t) \\ -0.7\cos(2t) & 0.3\sin(t) \end{bmatrix}, \\ &\begin{bmatrix} -0.1 \\ -0.4 \end{bmatrix} = \underline{f} \leq f(t) \leq \bar{f} = \begin{bmatrix} 0.3 \\ 0.6 \end{bmatrix}. \end{aligned} \tag{35}$$

The system (1) is simulated with the unknown but bounded input $f(t) = [0.1+0.2\sin(0.5t),\ 0.1+0.5\cos(1.5t)]^T$. Nevertheless, it is assumed that only the bounds $\underline{f}$ and $\bar{f}$ are available in the evaluation of the interval observer. The system (35) is unstable and not cooperative. By introducing the observer gain $L_{\text{obs}} = [0, 4.368]^T$ the matrix $D(t) = A(t) - L_{\text{obs}}C$ is stabilized. Then, Theorems 3 and 4 and Proposition 3 are successively applied to transform the ma-

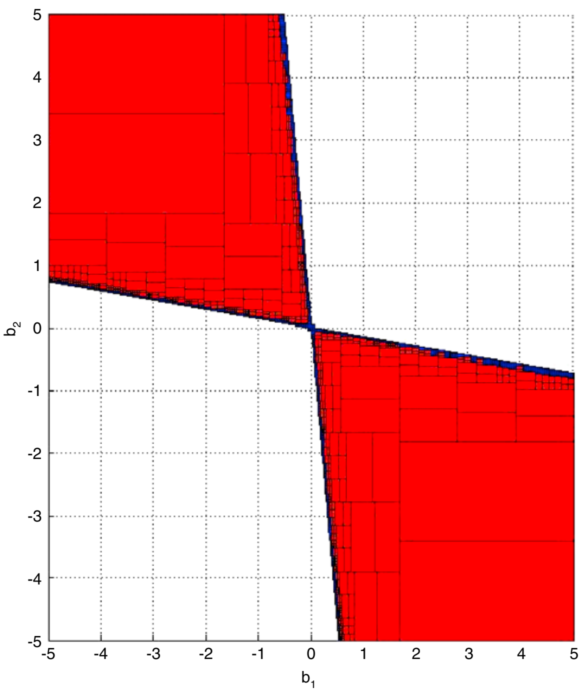

Fig. 1. Domain of possible values for b (red area: guaranteed solutions).

trix $D(t)$ into $\Gamma(t)$. Let us compute change of coordinates (19). First we find a vector $b = [b_1, b_2]^T$ which satisfies (9) using the methodology given in Section 4.2. The orbit of $(\wp_{D(t)}, b)$ is given by:

$$\mathrm{orb}(\wp_{D(t)}, b) = K(t) = \begin{bmatrix} b_1 & d_{11}b_1 + d_{12}b_2 \\ b_2 & d_{21}b_1 + d_{22}b_2 \end{bmatrix}, \tag{36}$$

where $d_{11} = -0.632 - 0.8\theta_1(t)$, $d_{12} = 0.5\theta_2(t)$, $d_{21} = -0.7\theta_3(t)$ and $d_{22} = -4.368 + 0.3\theta_1(t)$, with $[\theta_1(t),\ \theta_2(t),\ \theta_3(t)]^T = \theta(t) = [\sin(t),\ \cos(3t),\ \cos(2t)]^T$. $\underline{\theta} = [-1, -1, -1]$ and $\overline{\theta} = [1, 1, 1]$. The constraint to be satisfied is $\rho(\theta(t), b) = d_{21}{b_1}^2 + (d_{22} - d_{11})b_1b_2 - d_{12}b_2^2 > 0$ which returns, as mentioned in (33), to $\underline{\rho}(\underline{\theta}, \overline{\theta}, b) = -0.7b_1^2 - 4.836b_1b_2 - 0.5b_2^2 > 0$. Using a solver implementing a SIVIA like algorithm (here, RealPaver Granvilliers & Benhamou, 2006), the domain to which b should belong is obtained. It is given by Fig. 1. The value $b = [-0.2133 \quad 1]^T$ is chosen, it satisfies the condition (9). From (36), we have:

$$K(t) = \begin{bmatrix} -0.2133 & \frac{1}{2}\cos(3t) + 0.1706\sin(t) + 0.1348 \\ 1 & 0.1493\cos(2t) + 0.3\sin(t) - 4.368 \end{bmatrix}. \tag{37}$$

Given $\hat{C}(t)$ and e_2 (38), then $H(t)$ and $H^{-1}(t)$ can be expressed as in (39).

$$\hat{C}(t) = \begin{bmatrix} 0 & 1 \\ -\beta_1(t) & -\beta_2(t) \end{bmatrix}, \qquad e_2 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}. \tag{38}$$

$$H(t) = \begin{bmatrix} 0 & 1 \\ 1 & -\beta_2(t) \end{bmatrix}, \qquad H^{-1}(t) = \begin{bmatrix} \beta_2(t) & 1 \\ 1 & 0 \end{bmatrix}. \tag{39}$$

Using (37) and (39), $L_0(t)$ and $\xi(t)$, given respectively in (11) and (14), are computed. To find the functions $\beta_1(t)$ and $\beta_2(t)$, Eq. (15) is rewritten as:

$$\begin{aligned} \begin{bmatrix} E_1(t) \\ E_2(t, \beta_2(t)) \end{bmatrix} &= \begin{bmatrix} -\beta_2(t) \\ \beta_2^2(t) - \beta_1(t) + \dot{\beta}_2(t) \end{bmatrix}, \\ &= H(t)K^{-1}(t)(D(t)K(t) - \dot{K}(t))e_2. \end{aligned} \tag{40}$$

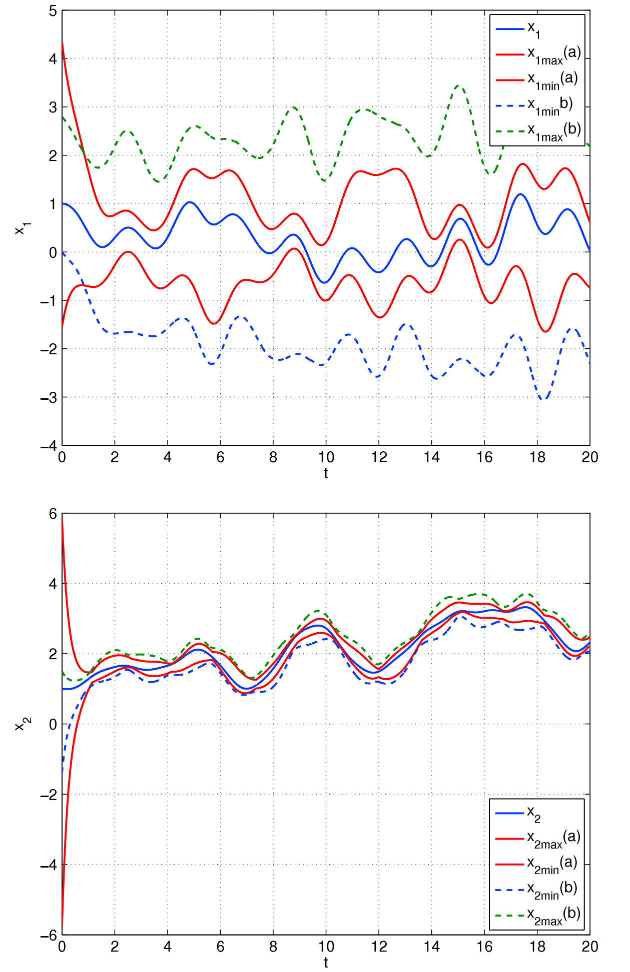

Fig. 2. Simulation results: LTV system state and time-varying interval observer ((a): proposed approach, (b): method given in Thabet et al., 2013).

Solving successively (40) yields:

$$\begin{aligned} \beta_2(t) &= -E_1(t), \\ \beta_1(t) &= \beta_2^2(t) + \dot{\beta}_2(t) - E_2(t, \beta_2(t)). \end{aligned} \tag{41}$$

Therefore, the expression of $L_0(t)$ is obtained by evaluating (11), and from (18) we obtain

$$R_2(\xi(t)) = \begin{bmatrix} R_1^{-1} & 0 \\ -\gamma_1 R_1^{-1} & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \xi(t) & 1 \end{bmatrix}.$$

Using the values of $L_0(t)$ and $R(\xi(t))$, the transition matrix transforming $D(t)$ into $\tilde{C}(t)$ is computed with (19). The companion matrix $\tilde{C}(t)$ is computed by the relation (20) and the expressions of $\alpha_1(t)$ and $\alpha_2(t)$ are obtained. As a next step, the transition matrix $P(t)$ ensuring the transformation of the companion matrix $\tilde{C}(t)$ to the Metzler one $\Gamma(t)$ is calculated as follows:

$$P(t) = \begin{bmatrix} 1 & 0 \\ \alpha_1(t) & 1 \end{bmatrix}$$

where $\alpha_1(t) = \alpha_{1,1}(t) = -\lambda_1(t)$ (using relation (22) with $n = j = 1$). Next, using Eq. (23) for $n = 2$, we have $\dot{\lambda}_1(t) = -\alpha_{2,1}(t) + \lambda_1(t)\lambda_2(t)$. Or $\lambda_2(t) = -\lambda_1(t) - \alpha_{2,2}(t)$. Then, $\dot{\lambda}_1(t) = -\lambda_1^2(t) - \alpha_{2,2}(t)\lambda_1(t) - \alpha_{2,1}(t)$. Using a numerical integration of this ODE (Ordinary Differential Equation), the value of $\lambda_1(t)$ is calculated at each time t as well as the matrix $P(t)$. It should be noted that $\lambda_2(t)$

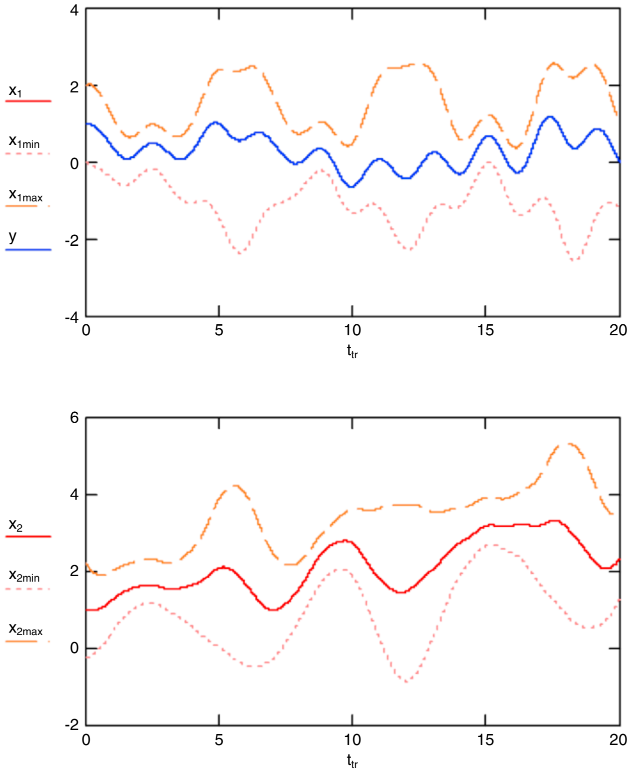

Fig. 3. Simulation results: LTV system state and time-varying interval observer from Efimov et al. (2013).

is deduced from the following equation $\lambda_2(t) = -\lambda_1(t) - \alpha_{2,2}(t)$. Then, the required Metzler matrix $\Gamma(t)$ is obtained.

Note that it has been numerically checked that the transition matrices $T(t)$ and $T^{-1}(t)$ are bounded for the time simulation horizon. The simulation results between $t = 0$ and $t = 20$ of the designed interval observer, in the original coordinates $(x(t))$ are given in Fig. 2 for both states $x_1(t)$ and $x_2(t)$ and superposed to the results obtained by applying the approach proposed in Thabet et al. (2013) where an interval observer, based on a time-varying change of coordinates, is designed. Furthermore, the simulation results of the interval observer given in Efimov et al. (2013) are also presented in Fig. 3 in order to compare the observers dynamics obtained by the different methods. As shown in Figs. 2 and 3, the inclusion property as well as the stability of the observers are verified. But it is clear that the approach proposed in this work gives tighter bounds than those resulting from the works of Efimov et al. (2013) and Thabet et al. (2013). Indeed, the proposed transformation matrices are time-varying, which is not the case in Efimov et al. (2013). Moreover, the obtained observer dynamics is an LTV one. The results are clearly improved.

6. Conclusion

The design of interval observers for LTV systems is investigated in this work. By using a time-varying change of coordinates obtained through a constructive method, this paper proposes a novel technique overcoming some of the main limitations of previous works devoted to this problem. The required cooperativity property is ensured after the introduction of an observer gain providing stability of the studied system in the original coordinates. An extension of the approach to address the design of interval observer for some classes of time-varying nonlinear systems will be the subject of further work.

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