Report.1_Comparison of Luenberger Observer and Unknown Input Observer

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First, given a plant in expression of state space model:

$$\begin{aligned} x_{k+1} &= Ax_k + Bu_k + E\omega_k \\ y_k &= Cx_k \end{aligned} \tag{1}$$

where ω_k is the unknown input and cannot be observed by Luenberger observer, we can design these two different observers respectively:

$$Luenberger\ \ Observer: \begin{aligned} \hat{x}_{k+1} &= (A-LC)\hat{x}_k + Bu_k + Ly_k \\ \hat{y}_k &= C\hat{x} + k \end{aligned} \tag{2}$$

$$Unknown\ \ Input\ \ Observer: \begin{aligned} z_{k+1} &= Nz_k + Tu_k + Ky_k \\ \hat{x}_k &= z_k + Hy_k \end{aligned} \tag{3}$$

In order to assure the errors of these two observers will converge in limited steps, each observer has to meet certain conditions. For a Luenberger observer, $A - LC$ has to be a Schur matrix. As for the **Unknown Input Observer**, conditions for convergence are shown below:

$$\begin{gathered} A - HCA - K_1C - N = 0 \\ (A - HCA - K_1C)H - K_2 = 0 \\ B - T - HCB = 0 \\ E - HCE = 0 \end{gathered} \tag{4}$$

where $K_1 + K_2 = K$ in formula(3).

Based on above theories, for a 2-Input-2-Output system like

$$\begin{aligned} x_{k+1} &= \begin{bmatrix} \frac{1}{2} & 1 \\ 0 & -\frac{1}{2} \end{bmatrix} x_k + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u_k + \begin{bmatrix} 1 \\ \frac{1}{2} \end{bmatrix} \omega_k \\ y_k &= \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} x_k \end{aligned} \tag{5}$$

where $x \in \mathbb{R}^2$ and u is given as below:

$$u = \begin{bmatrix} 2 + \sin(2\pi \cdot 0.01 \cdot k) \\ 2 + \cos(2\pi \cdot 0.01 \cdot k) \end{bmatrix} \tag{6}$$

and ω is white noise with an amplitude of 0.5 and standard derivation of 0.1 as shown in Fig.1.

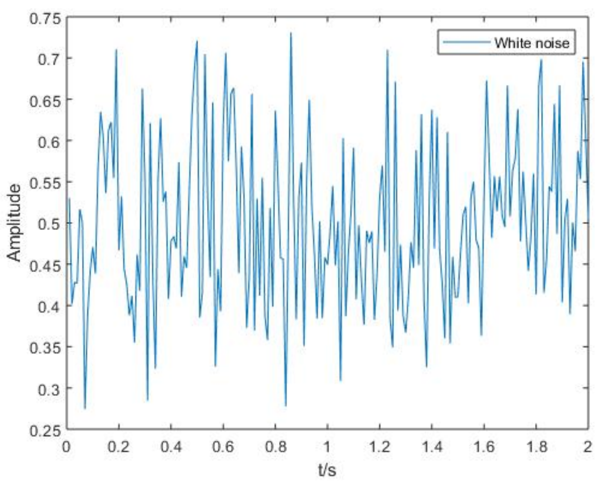

Fig.1 White noise

According to the parameters of the plant and convergence conditions of observers, we can design Luenberger observer as:

$$Luenberger\ \ Observer: \quad \begin{aligned} \hat{x}_{k+1} &= \begin{bmatrix} \frac{1}{2} & \frac{3}{2} \\ 0 & \frac{3}{4} \end{bmatrix} \hat{x}_k + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u_k + \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & \frac{1}{4} \end{bmatrix} y_k \\ \hat{y}_k &= \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} \hat{x} \end{aligned} \tag{7}$$

where $A - LC$ is a Schur matrix with eigenvalues of $\frac{1}{2}$ and $\frac{3}{4}$.

The Unknown Input Observer is designed as:

$$Unknown\ \ Input\ \ Observer: \quad \begin{aligned} z_{k+1} &= \begin{bmatrix} \frac{1}{2} & -1 \\ 0 & \frac{1}{2} \end{bmatrix} z_k + \begin{bmatrix} 0 & 0 \\ -1 & 2 \end{bmatrix} u_k + \begin{bmatrix} -1 & -2 \\ 0 & -3 \end{bmatrix} y_k \\ \hat{x}_k &= z_k + \begin{bmatrix} 2 & 0 \\ 1 & 0 \end{bmatrix} y_k \end{aligned} \tag{8}$$

where N, T, K, H all satisfy the conditions stated in (4).

In a healthy circumstance where no fault happens, the true output y and estimated output y_{LBG} from Luenberger observer, y_{UIO} from Unknown Input Observer are shown in Fig.2.

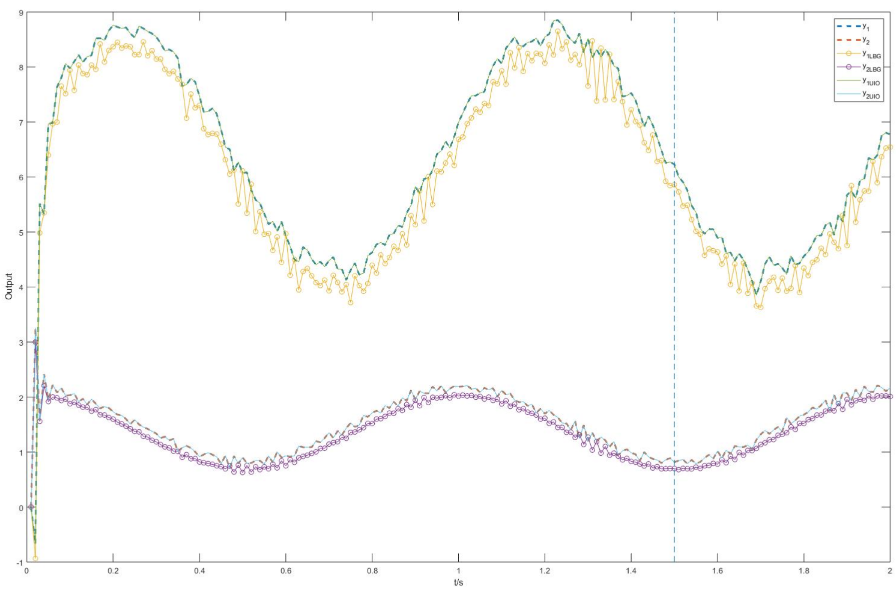

Fig.2 Comparison of output y and estimation

It can be seen that the Unknown Input Observer has a nearly no-error estimation of y while Luenberger observer does not. Because Luenberger observer cannot eliminate the influence of noise ω.

Likewise, the estimation of state variable x shows the same trend as shown in Fig.3.

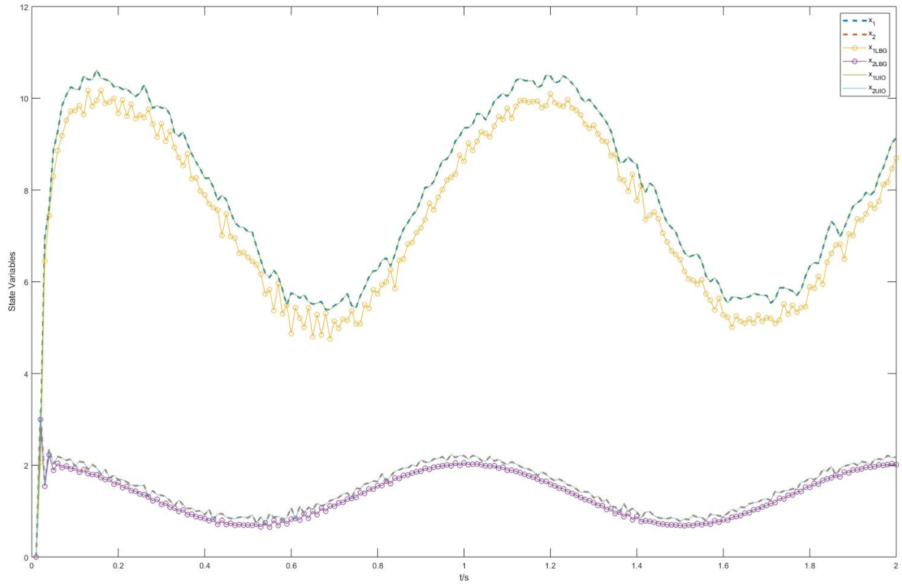

Fig.3 Comparison of state variables x and estimation

In order to have a more intuitive recognition of the difference between these two observers, we can see the comparison of residual errors in following picture.

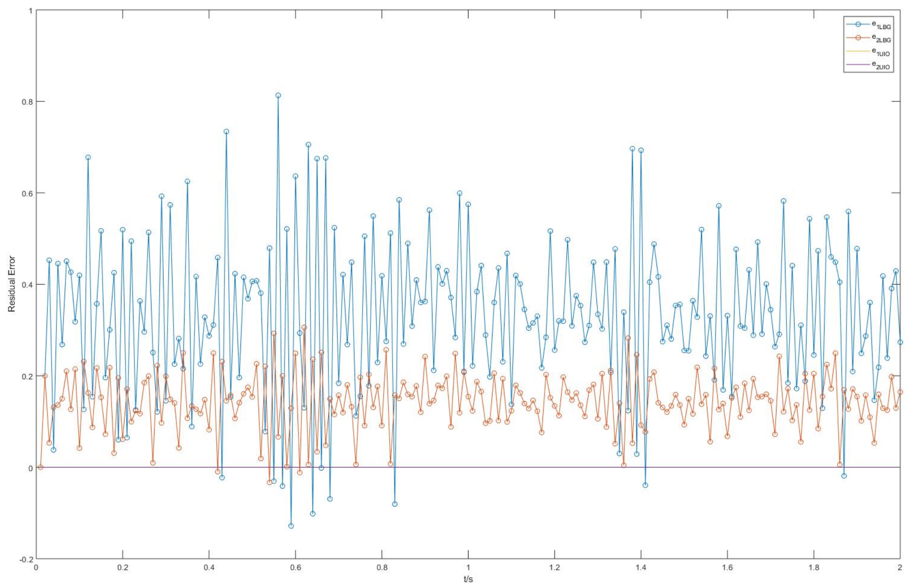

Fig.4 Comparison of residual errors e

From Fig.4 we can see that Unknown Input Observer does have a no-error estimation of x and y while Luenberger Observer has quite large errors.

As for the fault circumstance where a fault of actuator 1(mathematically shown in T and has effects on u_1) shuts down at $t = 1.5$s, the performances of Luenberger Observer and Unknown Input Observer are shown in Fig.5~7.

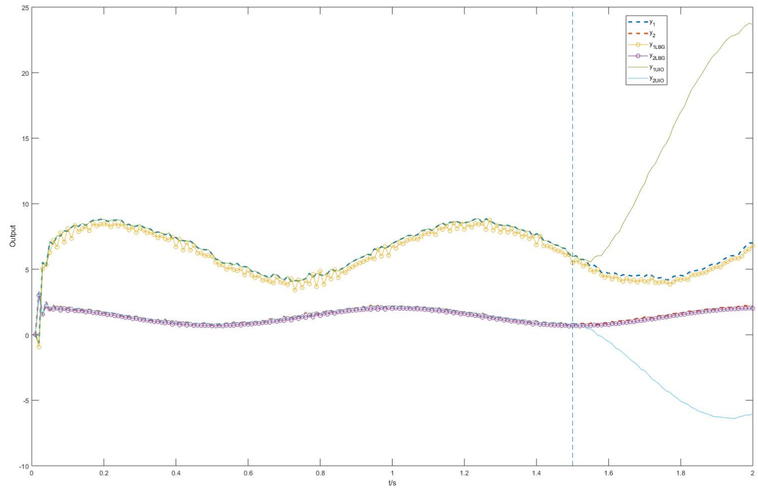

Fig.5 Comparison of output y and estimation when fault happens at t = 1.5s

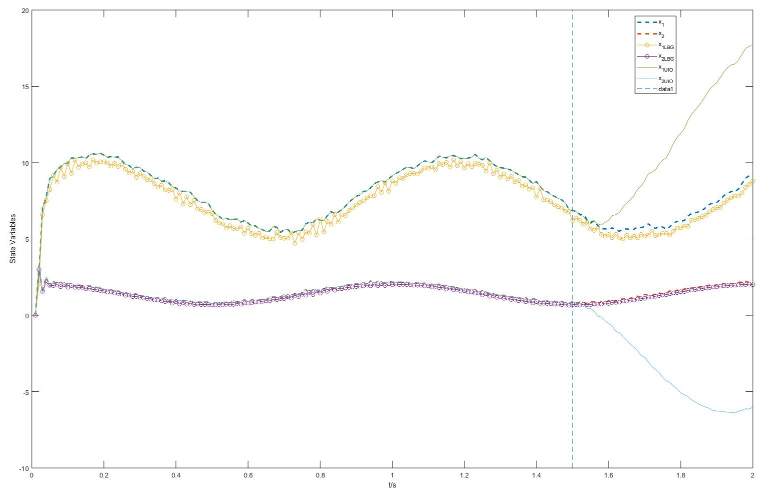

Fig.6 Comparison of state variables x and estimation when fault happens at t = 1.5s

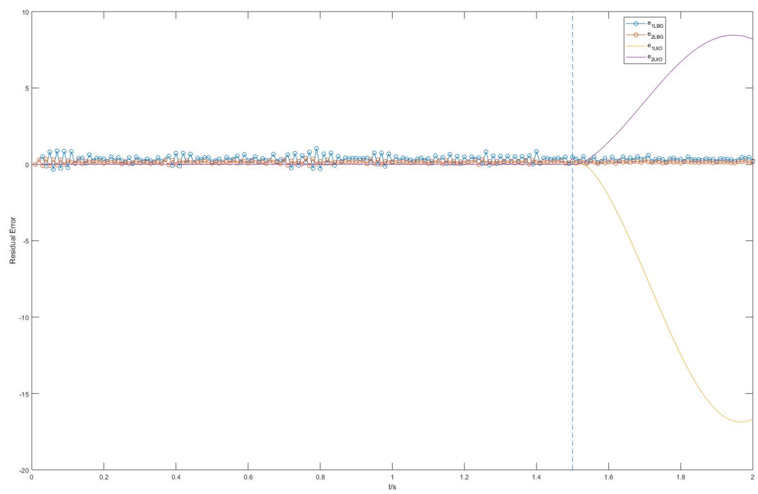

Fig.7 Comparison of residual errors e when fault happens at t = 1.5s

From Fig.5~7 we can find that when a fault happens, only Unknown Input Observer can detect the fault while Luenberger observer shows just the same behavior as in healthy circumstance.

More specifically, we can detect the errors just by checking the residual errors of Unknown Input Observer. This is shown in Fig.8.

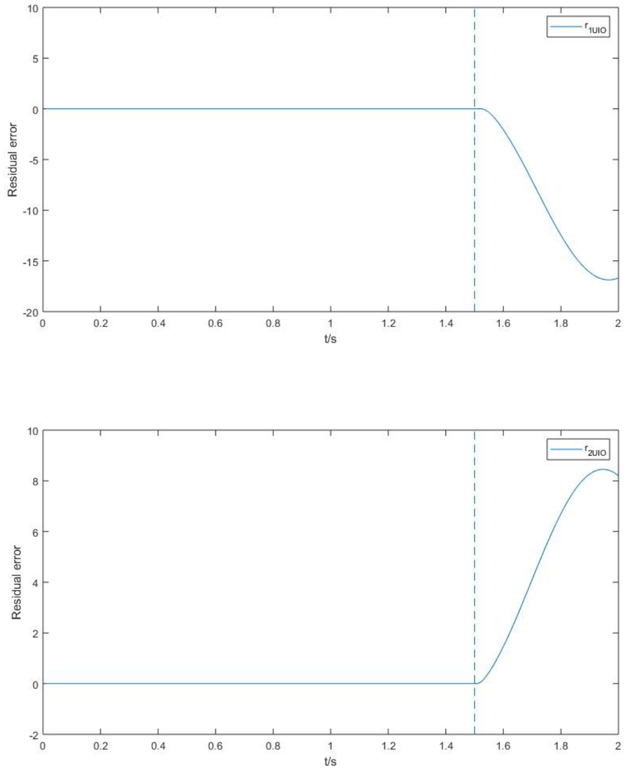

Fig.8 Residual errors of Unknown Input Observer

To conclude, Luenberger observer is just a basic state observer and cannot eliminate the effects of unknown inputs such as exterior disturbances or noise while Unknown Input Observer shows better performance on this estimation task and therefore can be used in fault detection.