# Final Project Report: Fault-tolerant switching control strategy based on multi-sensor fault diagnosis scheme

#### I. Introduction

Recently, multi-sensor fusion related topics draw more interests along with the advances in sensor technology. This also boosts up the research in multi-sensor fault diagnosis and fault-tolerant control strategies. Multi-sensor fault diagnosis and fault-tolerant control has a solid application background not only in the process control of industrial production but also in many other areas such as vehicle control, robot manipulator control, etc.

In this project, we implement a specific multi-sensor switching control strategy<sup>[1]</sup> in a second-order discrete system. We design the corresponding observers and then implement a simulation about the fault case for both sensors so as to analyze and evaluate the performance of designed control strategy.

### II. Methodology

Consider a second-order LTI discrete system as follows:

$$x(k+1) = Ax(k) + Bu(k) + Ew(k)$$
(1)

where  $A \in \mathbb{R}^{2\times 2}$ ,  $B \in \mathbb{R}^2$ ,  $E \in \mathbb{R}^2$ ,  $x \in \mathbb{R}^2$ ,  $u \in \mathbb{R}$ ,  $w \in \mathbb{R}$ .

We also have measurement equations:

$$s_{m1}(k+1) = A_{s_1} s_{m1}(k) + B_{s_1} Cx(k)$$

$$s_{m2}(k+1) = A_{s_2} s_{m2}(k) + B_{s_1} Cx(k)$$

$$y_1(k+1) = C_{s_1} s_{m_1}(k) + v_1(k)$$

$$y_2(k+1) = C_{s_2} s_{m_2}(k) + v_2(k)$$
(2-4)

where  $A_{s_i} \in \mathbb{R}$ ,  $B_{s_i} \in \mathbb{R}$ ,  $C \in \mathbb{R}^{1 \times 2}$ ,  $C_{s_i} \in \mathbb{R}$ ,  $s_{m_i} \in \mathbb{R}^2$ ,  $y_i \in \mathbb{R}$ ,  $v_i \in \mathbb{R}$ .

We also have following reference equations of the system and sensors:

$$x_{ref}(k+1) = Ax_{ref}(k) + Bu_{ref}(k)$$
  

$$s_{i,ref}(k+1) = A_{s_i}s_{i,ref}(k) + B_{s_i}Cx_{ref}(k)$$
(5,6)

and equations of state observers for corresponding sensors:

$$\hat{x}_{i}(k+1) = A\hat{x}_{i}(k) + Bu(k) + L_{i}(y_{i}(k) - C_{s_{i}}\hat{s}_{i}(k)) 
\hat{s}_{i}(k+1) = A_{s_{i}}\hat{s}_{i}(k) + B_{s_{i}}C\hat{x}_{i}(k) + L_{s_{i}}(y_{i}(k) - C_{s_{i}}\hat{s}_{i}(k))$$
(7,8)

Note that for each pair of sensor and observer, the following matrix pair need to be detectable:

$$\left(\begin{bmatrix} A & 0 \\ B_{s_i}C & A_{s_i} \end{bmatrix}, \begin{bmatrix} 0 & C_{s_i} \end{bmatrix}\right)$$

and the following matrix need to be a Schur matrix:

$$A_{L_i} = \begin{bmatrix} A & 0 \\ B_{s_i} C & A_{s_i} \end{bmatrix} - \begin{bmatrix} L_i \\ L_{s_i} \end{bmatrix} \begin{bmatrix} 0 & C_{s_i} \end{bmatrix}$$
 (9)

Beside these equations, we also defined many errors as follows:

$$e_{x_{i}} = x - \hat{x}_{i}$$

$$e_{s_{i}} = s_{i} - \hat{s}_{i}$$

$$e_{u} = u - u_{ref}$$

$$\hat{z}_{i} = \hat{x}_{i} - x_{ref}$$

$$\hat{e}_{s_{i}} = \hat{s}_{i} - s_{i,ref}$$

$$r_{i} = M_{i}(y_{i} - C_{s_{i}}\hat{s}_{i})$$

$$= M_{i}C_{s_{i}}e_{s_{i}} + M_{i}v_{i}$$
(10-15)

where errors of input, state of sensors, system, estimation of sensor state are included.

Then we come to the part of design of controller. The control law is expressed as

$$u^* = u_{ref} - K\hat{z}^* \tag{11}$$

where  $\hat{z}^* = \arg\min_{\hat{z}} \{\hat{z}^T P \hat{z}\}$  and P can be calculated from the following equation:

$$P = A^{T} P A + Q - K^{T} (R + B^{T} P B) K$$

$$(12)$$

Substituting the corresponding variables into formula (11), we have control law as

$$u^* = u_{ref} - K(\hat{x}_l - x_{ref} + r_l)$$
 (13)

Before moving to next section, we make a brief summary of this section. First, we design an LTI discrete system, then we design observers for the system and both sensors. After defining several error variables, we propound the switching control law based on the equations and variables.

#### **III. Simulation and Evaluation**

In this section, we choose appropriate parameters and calculate the specific variables that need to meet certain requirements then implement and analyze the performance of fault detection and switching control strategy in MATLAB. The detailed parameters can be found in the appendix.

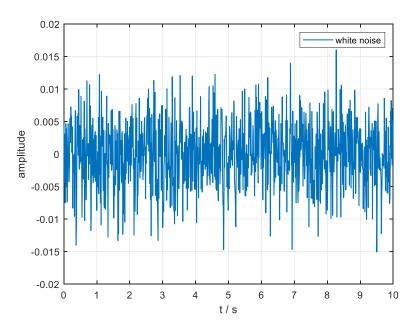


Figure.1 White noise

The noise w(t) is shown in Figure.1, it is a white noise with mean of 0 and variance of 0.0001. The noises  $v_1$ ,  $v_2$  in sensors are set as sinusoidal functions.

Then, we set a fault happening in sensor 1 at t = 2s and ending at t = 4s, and another fault happening in sensor 2 at t = 6s and ending at t = 8s.

Here we use two different input signals, sinusoidal function and sigmoid function so as to evaluate the performance of designed switching control strategy.

From Figure.2 and Figure.3 we can see that when a specific fault happens in certain sensor, the corresponding sensor show immediate deteriorated performance. But this fault does not affect the overall control performance since the control strategy immediately change to adopt information from the better (healthy) sensor.

Therefore, the control strategy can have satisfied control and signal tracking performance as long as there exists one healthy sensor at least.

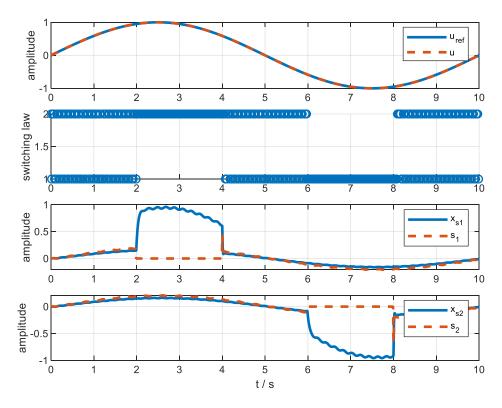


Figure.2 Evaluation of switching control strategy (sinusoidal input)

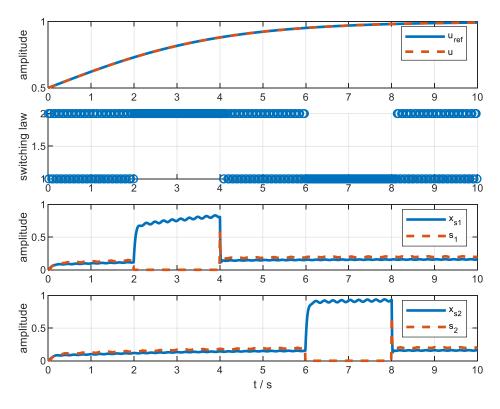


Figure.3 Evaluation of switching control strategy (sigmoid input)

#### **IV. Conclusions**

In this project, I learned about a fault-tolerant switching control strategy based on multi-sensor fault diagnosis. It shows good performance when certain fault happens in sensors with random noises due to the robustness brought up by the multi-sensor fault diagnosis scheme.

To implement this control strategy, I have to read the paper very carefully and deduce the formula in the paper over and over. The process of formula deduction is very tedious but it does help a lot for understanding the main idea of this paper. I also improved my coding skill during debugging the codes for iterations.

However, there still exists some flaws. For examples, I didn't calculate the invariant set of this fault diagnosis and fault-tolerant control scheme. The parameters are chosen and calculated by chance, which means it might show worse performance when encountered with unexpected inputs or larger noises or disturbances.

## References

- [1]. Seron M M, Zhuo X W, De Dona J A, et al. Multisensor switching control strategy with fault tolerance guarantees[J]. Automatica, 2008, 44(1): 88-97.
- [2]. Xu F, Puig V, Ocampomartinez C, et al. Set-theoretic methods in robust detection and isolation of sensor faults[J]. International Journal of Systems Science, 2015, 46(13): 2317-2334.
- [3]. Mi Y, Xu F, Tan J, et al. Fault-tolerant control of a 2-DOF robot manipulator using multi-sensor switching strategy[C]. chinese control conference, 2017: 7307-7314.

# **Appendix**

The detailed parameters of the system, sensors and corresponding observers are:

$$A = \begin{bmatrix} 0.75 & 0.1 \\ 0 & 0.75 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0.1 \end{bmatrix}^T$$

$$E = B$$

$$C = \begin{bmatrix} 1 & 0 \end{bmatrix}$$

$$A_{s_1} = 0.6065$$

$$B_{s_1} = 0.5$$

$$C_{s_1} = 0.7869$$

$$A_{s_2} = A_{s_1}$$

$$B_{s_2} = B_{s_1}$$

$$C_{s_2} = C_{s_1}$$

$$L_1 = \begin{bmatrix} 1.1064 & 0.3010 \end{bmatrix}^T$$

$$L_{s_1} = 1.3089$$

$$L_2 = L_1$$

$$L_{s_2} = L_{s_1}$$

$$P = \begin{bmatrix} 0.0236 & 0.8602 \\ 0.8602 & 3.0214 \end{bmatrix}$$