Dymanics of Nonlinear Robotic Systems
Homework 5

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1 Links love story

Hello, my dear friend! Today I will tell you a story about fat and lazy monk. And the name of him is Link2. All that he could do is just rotate around himself. He was too lazy to start moving. So, people called him Lazy Monk. No one was interested in his real name.

Once, a pretty girl was looking for a overnight stay. And she has come to Link2. Just when he saw her straight rigid form he was charmed:

-What is your name, the Beauty? - asked Link2.

- My name is Link1...

She was touched with the monk kindness. and they started to live together.

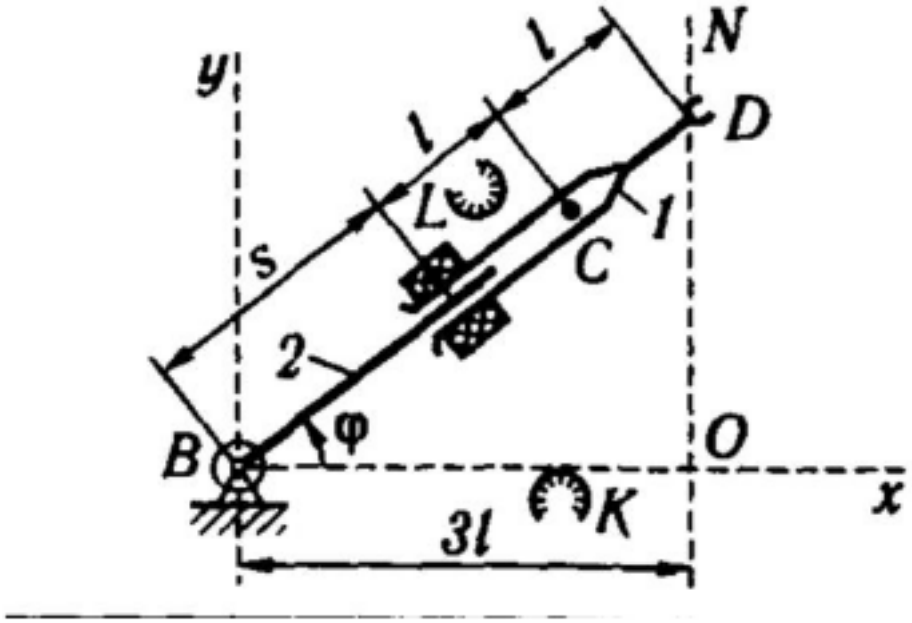

Figure 1: Link2 and Link1 hold hands

They lived happily together and nothing was supposed to tear them apart. But, once a powerful monk of higher rank noticed a beautiful Link2 and desired her for his own. He come to their home and pulled away from Link2. But link2 could nothing to do (Force and torque = 0), he was too fat and un-offsetable (only rotation).

(position growth because with ϕ = -$\pi/2$ link 2 moves in positive direction of s joint, with ϕ = $\pi/2$ - in negative, while applying only gravity force)

Link1 yelled:

- Please, do something, Link2! I do not want to leave you, neither go with him!

- But what can I do? - whispered Link 2 in despair...

And with magic a fairy has appeared.

- Hello, Link2! I heard your despair! I am Lagrange-Euler method. I will help you how to return your love! Think! what jacobian you have? Remember, because of your fat you are a really heavy!

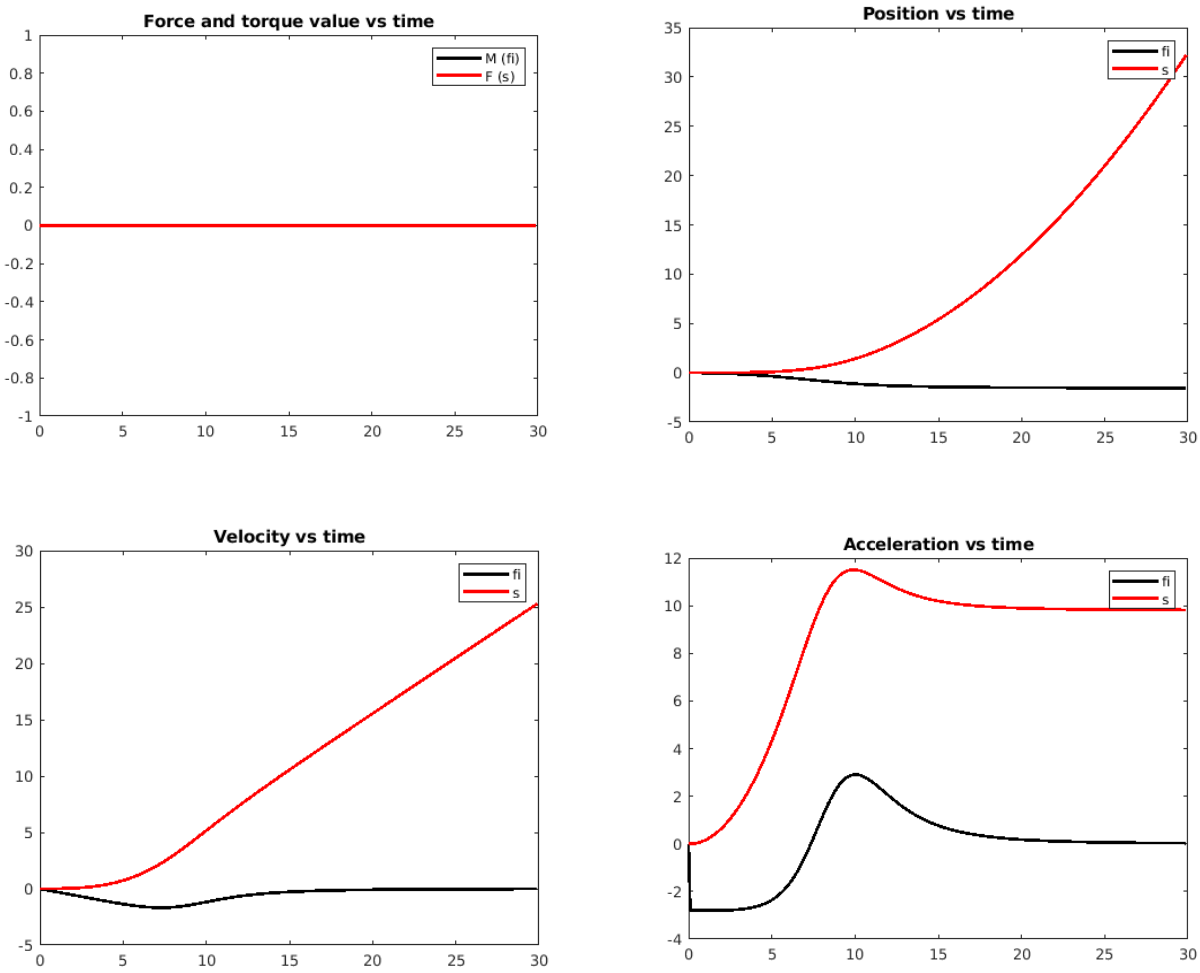

Figure 2: Gravity pulls Link1 from Link2

-Well, it is:

$$Jv1 = \begin{matrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{matrix} \quad Jw1 = \left|\begin{matrix} 0 \\ 0 \\ 1 \end{matrix}\right| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

- And what is Jacobian of Link2 when she was with you?

- It is:

$$Jv2 = \begin{matrix} \text{-(s+d2)*sin}(\phi) & \cos(\phi) \\ \text{(s+d2)*cos}(\phi) & \sin(\phi) \\ 0 & 0 \end{matrix} \quad Jw2 = \left|\begin{matrix} 0 \\ 0 \\ 1 \end{matrix}\right| \begin{matrix} 0 \\ 0 \\ 0 \end{matrix}$$

- Correct! Because your center of mass is in (y,x) = (0, 0). You are very heavy. And her is in the center of her body. Now we need to find a Kinetic energy. It is

$$D = \sum_i (m_i * Jv_i' * Jv_i + Jw_i' * R_i * I_i * R_i' * Jw_i);$$

$$D = \begin{matrix} \text{m2*L2}^2)/4 + \text{m2*L2*s} + \text{m2*s}^2 + \text{I1} + \text{I2} & 0 \\ 0 & \text{m2} \end{matrix}$$

- You both rotate around you, since you are very heavy, so

$$R = \begin{matrix} \cos(\phi) & \text{-}\sin(\phi) & 0 \\ \sin(\phi) & \cos(\phi) & 0 \\ 0 & 0 & 1 \end{matrix}$$

- Okay. now calculate your and Link1 Potential energy using:

$$P1 = m1 * g * h$$

$$P2 = m2 * g * (h + (s + d2) * sin(\phi))$$

$$P = P1 + P2$$

$$G1 = diff(P, \phi)$$

$$G2 = diff(P, s)$$

$$G = [G1; G2]$$

$$G = \begin{matrix} \text{g*m2*cos}(\phi)\text{*(L2/2 + s)} \\ \text{g*m2*sin}(\phi) \end{matrix}$$

- And also we need a Coriolis force:

$$C = \begin{matrix} \text{(ds*m2*(L2 + 2*s))/2} & \text{(dfi*m2*(L2 + 2*s))/2} \\ \text{-(dfi*m2*(L2 + 2*s))/2} & 0 \end{matrix}$$

- But how it will help me in my problem, Master? - asked Link2 wonderingly.

- Look. I am Lagrange-Euler method. And i present you my formula. F is your force, that is Moment for rotational. q is your and Link1 positions ϕ and s, since she is a prismatic joint link.

$$F = D * \ddot{q} + C * \dot{q} + G;$$

- Now, use the Force... - told fairy and disappeared. Link2 gather all his straight. He can't to move from zero point, but he can use the Force to pull Link2, his love to him.

$$F = [0, -9.81 * m2](exact force to compensate gravity in \phi = -pi/2)$$

(Force direction (minus shows) is opposite to s positive direction)

Link2 pulled Link2 to him. Hugged her tightly and menacingly looked into Gravity's eyes. He understood this fat monk is not such week as earlier and gone away. Link1 pressed against hero's chest and said:

- Never let me go, I love you, Link2!

- I love you too, my dear! I can now not only pull you with the Force, but also I can fix a zero ϕ, so there is no force needed for you to stay with me. (But I don't want to calculate exactly value of this Moment needed, tried several times without calculations but did not guess, so example is only for linear force).

...T̋HE END...

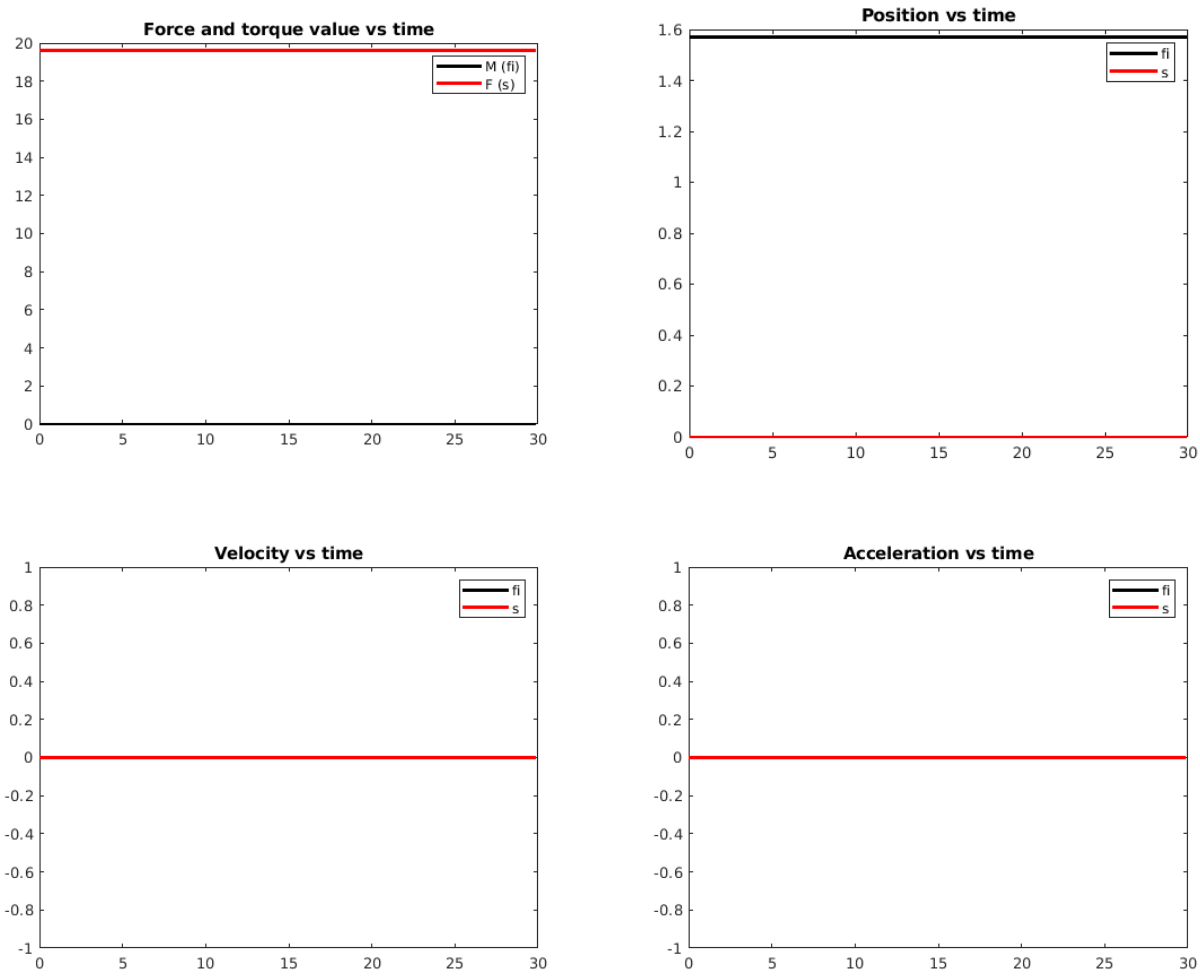

Figure 3: Gravity pulls Link1 from Link2

2 Git

`https://github.com/Bonilapa/DLNRS5.git`