96. Unique Binary Search Trees

题目描述：https://leetcode.com/problems/unique-binary-search-trees/

> 给出节点个数n，求从1-n共n个节点共能组成多少种二叉平衡树。
> 例如：给定 n = 3
>
> ```
> Given n = 3, there are a total of 5 unique BST's.
>  1         3     3      2      1
>   \       /     /      / \      \
>    3     2     1      1   3      2
>   /     /       \                 \
>  2     1         2                 3
> ```

解题思路：

动态规划问题。
f[i][j]的意思是从i节点到j节点有多少种构造BST的方式
f[i][j] = sum(f[i][k-1] * f[k+1][j]) + f[i][j-1] + f[i+1][j];

代码：

```
class Solution {
public:
    int numTrees(int n) {
        vector<vector<int> > f(n+1, vector<int>(n+1, 0));
        for(int i = 1; i <= n; i++) {
            f[i][i] = 1;
        }
        for(int l = 2; l <= n; l++) {
            for(int i = 1; i + l - 1 <= n; i++) {
                int j = i + l - 1;
                f[i][j] += f[i+1][j];
                f[i][j] += f[i][j-1];
                for(int k = i+1; k < j; k++) {
                    f[i][j] += f[i][k-1]*f[k+1][j];
                }
            }
        }
        return f[1][n];
    }
};
```