APM466 Assignment

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February 2022

Fundamental Questions - 25 points

1.

   (a) When the government issue bonds, it will not cause inflation and control money supply; however, if government is choosing to print money, it will cause inflation and devalue the money of people who already have saved or invested.

   (b) Suppose that money lenders would worry that loaning their money will have a lower interest rate and less demand for borrowing in the future, so they decide to loan their money today or decrease the interest rate in the future which will cause a recession which could increase the short term yield rate and decrease the long term rate which let the the yield curve be flatten.

   (c) Quantitative easing is a monetary policy strategy that US Fed buy long-term securities from banks in order to decrease the interest rate, increase the money supply and stimulate market and economic during the COVID-19 pandemic.

2. I would like to construct a yield a "0-5 year" yield and spot curves, so the 10 bonds which I selected have the maturity date from Feburary 28th 2022 to Feb 28th 2027(in 5 years). Also, each of these bonds will be in the market for more than 5 years and the coupon rate on these bonds do not differ much. Most of the bonds have the maturity date on Feburary 28th or August 31th. Since these 10 bonds are paid semiannually, it is convenient for further calculation.

   CAN 0.50 Feb 28 2022, CAN 1.75 Feb 28 2023, CAN 2.25 Feb 29 2024, CAN 2.50 May 31 2024, CAN 1.50 Aug 31 2024, CAN 1.25 Feb 28 2025, CAN 0.50 Aug 31 2025, CAN 0.25 Feb 28 2026, CAN 1.00 Aug 31 2026, CAN 1.25 Feb 28 2027

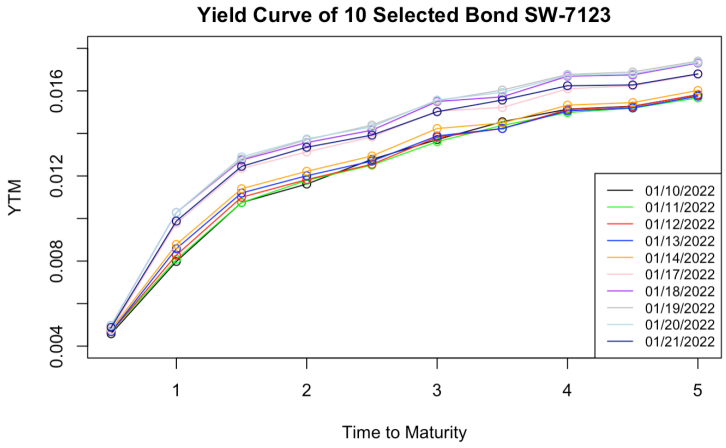
3. The Principal Component Analysis states that reducing the number of variables while preserving as much as possible for a data set. The variance of the process includes model building with the most important information. Since the eigenvectors of the covariance matrix are the directions of the axes where there is the most important information and that we call Principal Components. And eigenvalues are the amount of variance which are carried in each Principal Component. We could rank the eigenvectors from large number to small number based on the eigenvalues information. For example, students want to know how to get a higher gpa, after making the survey, study hard is the most important factor, second is keep health, and third is...etc.

Empirical Questions - 75 points

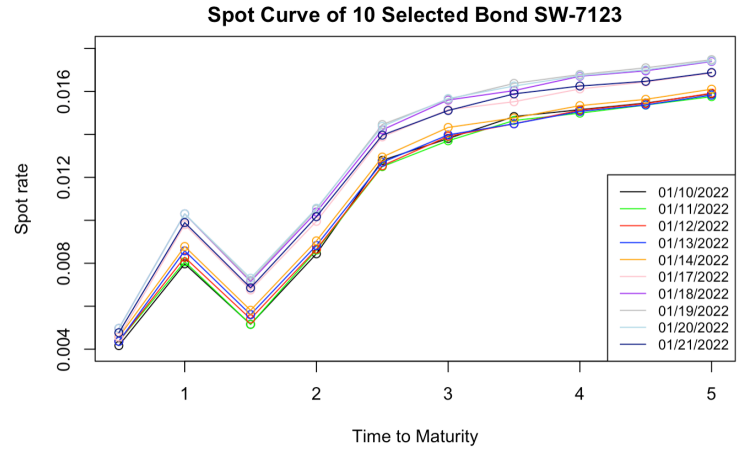
4.

   (a) To calculate the yield to maturity on my 10 selected bond, first I collected Janurary 10th to Janurary 21st(10 weekdays) bond clean price, and used Dirty Price = accrued interest + clean

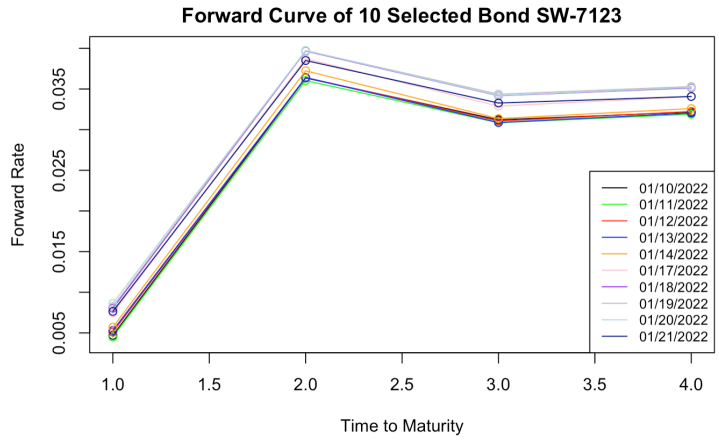
price to get the dirty price for each day. Then I used the formula $P_n = \sum_{i=1}^{n} P_i(1+\frac{r}{n})^{-\frac{t_i}{n}}$ to calculate yield rate in the excel. The left side of the formula is dirty price, n means pay n times coupon in a year. Then, I plot the graph with time on maturity on x-axis and yield to maturity on y-axis.



(b) To calculate the spot rate on my 10 selected bond, I used the bootstrapping technique is $P_n = \frac{C_1}{(1+r_1)^1} + \frac{C_2}{(1+r_2)^2} + \frac{C_3}{(1+r_3)^3} + ... + \frac{C_n}{(1+r_n)^n} + \frac{P}{(1+r_n)^n}$ P_n denotes the dirty price, C_n denotes cash flow for each period, r_n denotes the spot rate which is we want to gain. Then we put the equation into R and use uniroot to get the 5-year spot curve with time to maturity on x-axis and spot rate on y-axis.



(c) To calculate the forward rate, first I chose 5 bonds that are one year, two years, three years, four years and five years away from maturity. Then I used the formula $(1+f_{A,B-A})^{B-A} * (1+Z_A)^A = (1+Z_B)^B$ based on the spot rates. Then we could get the 1y1y, 1y2y, 1y3y and 1y4y forward rate. Then I plot the forward curve graph with time to maturity on x-axis and Forward rate on y-axis.



5. The first graph is the covariance matrices(5x5) for the time series of daily log-returns of yield. The second graph is the covariance matrices(4x4) for the time series of daily log-returns of forward rate.

```
             [,1]         [,2]         [,3]         [,4]         [,5]
[1,] 0.0003197312 0.0002624205 0.0001707518 0.0002046212 0.0001000925
[2,] 0.0002624205 0.0016472579 0.0012199286 0.0010550880 0.0010115701
[3,] 0.0001707518 0.0012199286 0.0009413716 0.0007843690 0.0007895851
[4,] 0.0002046212 0.0010550880 0.0007843690 0.0007230808 0.0006733042
[5,] 0.0001000925 0.0010115701 0.0007895851 0.0006733042 0.0007794923
```

```
             [,1]         [,2]         [,3]         [,4]
[1,] 0.011687451 0.0020306444 0.0022029023 0.0022041488
[2,] 0.002030644 0.0004255877 0.0004680772 0.0004599655
[3,] 0.002202902 0.0004680772 0.0005634264 0.0005320385
[4,] 0.002204149 0.0004599655 0.0005320385 0.0005146637
```

6. The first graph is the eigenvalues and eigenvectors for yield covariance. The second graph is the eigenvalues and eigenvectors for forward covariance.

```
[1] 3.957231e-03 3.089708e-04 9.540285e-05 3.304040e-05 1.628902e-05
           [,1]       [,2]        [,3]        [,4]        [,5]
[1,] -0.1037489  0.93118731 -0.28922116  0.19199075  0.04021241
[2,] -0.6395217  0.04355469  0.48455821  0.04886555  0.59323725
[3,] -0.4821797 -0.09133018  0.14107861  0.53577605 -0.67245997
[4,] -0.4186095  0.13038449 -0.02436844 -0.81704818 -0.37363679
[5,] -0.4153517 -0.32504134 -0.81305441  0.07828154  0.23376410
```

```
[1] 1.289891e-02 2.729008e-04 1.719850e-05 2.123281e-06
           [,1]       [,2]        [,3]         [,4]
[1,] 0.9508364  0.3065730 -0.04292340 -0.008984149
[2,] 0.1684670 -0.4178804  0.82074674 -0.351240125
[3,] 0.1841055 -0.6637635 -0.56833155 -0.450025002
[4,] 0.1833964 -0.5392652  0.03913578  0.820991570
```

We could observe that for those eigenvalues, they all show from the biggest number to the smallest which indicates that the first eigenvalue for two graphs include the most important information which is the principle component and based on the eigenvalues, we could also rank eigenvectors and pick them from large to small. From above data, we could conclude that the more recent the bond price is, the more valuable it is.

References and GitHub Link to Code

1. Bond data are collected from http://www.businessinsider.com and github for full code https://github.com/Bonnierbb/Mathematical-Finance-as1.git