Classical Growth Theories

Economic growth is one of the objectives of macroeconomic policy. It is measured as an increase in GDP over a period of time. In other words, we are interested in understanding what causes changes in total income or total output over time. In the previous section we learnt about the factors of productions: Capital, Labor, and Technology. It follows that changes in GDP, or output, must then come from changes to any of these three variables. The Solow growth model will set the structure to explore how changes in one of these could change the trajectory, or growth, of GDP over time. The Solow model shows how saving (affects capital), population growth (affects labor), and technological progress affect the level of an economy's output and its growth over time. But, importantly, it says *nothing* about how technological change comes about. In other words, technological progress is an exogenous variable in this literature.

Perspectives on Growth

Much of the research, and debate, around economic growth modeling lies in the essence of technological progress. Defined, loosely, as activities related to innovation (technological, cultural, or social), human capital, R&D investment, or anything that allows an economy to produce a higher quantity or variety of goods and services with available factors of production (i.e., capital and labor). Are these endeavors endogenous or exogenous? That is, is technological progress the result of intentional economic activities or not? Research strongly suggests positive effects of technological innovation, but it remains unclear what cultural, political, or market systems drive the rate of innovation.

Here is how I think about exogeneity vs endogeneity. It is about incentives and heterogeneity. Ask yourself: Does the effect of an innovation depend on the particular events, rules of the game, and institutions of a given economic system? If your answer is no, then such an effect must be fairly homogeneous across systems (e.g., countries, states, etc.). It is then akin to gravity across the globe, minimal differences in strength across locations but relatively the same effect everywhere. Following this analogy, gravity does not depend on the countries' laws, the incentives in place, nor the time period of analysis. In contrast, if your answer is yes, then the effect must be explained by processes of the system in which it takes place and therefore it calls for efforts to endogenize its impact on the variable of interest (e.g, growth rate). Endogenous variables are shaped by the interactions and feedback mechanisms within the system, much like how economic growth rates can be influenced by policy decisions, market conditions, and institutional frameworks.

One school, influenced by Solow's growth model, considers an exogenous[1] variable termed Total Factor Productivity (TFP) and focuses on the role of capital accumulation and labor to determine growth paths via their influence in aggregate production. TFP (the "A" variable in the Cobb-douglas we studied) is a generic technological change variable, assumed to grow at a constant rate regardless of any factors that might change in the model, that attempts to capture all contributors to growth aside from the measured inputs. Hence, it is measured as the residual of the model. That is, the growth that cannot be accounted for by the differences in measured inputs.

The main prediction of the Solow growth model is that, due to diminishing returns to capital, countries with smaller capital stock would grow at a faster rate than those with large capital stock - the so-called "catching-up growth". Empirical evidence shows that TFP explains 80% to 90% of labor productivity (output per man-hour) [2]. Modern growth accounting techniques have included the average level of human capital per worker and measures of social infrastructure (i.e., factors like institutions and government policies) in addition to the classical factors of capital and labor.

The "new" school, influenced by Paul Romer's growth model, seeks to explain the process of technological change that drives economic growth by including factors such as population size, R&D, and spillover effects as new knowledge diffuses over an economy. In contrast with Solow's model, endogenous growth theory poses a situation of increasing returns whereby the factors themselves may become more productive over time. For example, a worker might learn how to do their tasks more efficiently on the job ("learning by doing"), or human capital accumulation might contribute to new knowledge in the whole economy. Thus, spillover effects can have significant effects on productivity, innovation, and therefore growth. Measuring these effects must be an important component of this literature.[2]

Recent empirical analysis suggests that capital formation explains between 40% and 50% of differences in cross-country growth rates. The remaining unexplained growth can be attributed to breakthroughs in General Purpose Technologies (GPTs), a generic technology with many applications that triggers a series of innovations thereby affecting the entire economy. Some examples of GPTs include the steam engine, electricity, information technology, automation, railroads, computers, and AI. The conclusion from these models is that, in order to boost global growth, we must advance the "technological frontier" (aka state of the art) and that public

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[1] Solow himself [1] states that technological progress must at least be partially endogenous to the economy. To quote him "Valuable resources are used up in pursuit of innovation, presumably with some rational hope of financial success. The patent system intends to solidify that hope, [...]. It would be very odd if that activity had nothing to do with actual technological progress". But he is skeptical that there is anything sensible to be said about the process through which those decisions are incentivized in a form that "can be made part of an aggregative growth model".

[2] I must add that there are a set of additional assumptions built into these new growth models. Solow summarizes some of these [3]: A representative agent with infinite-horizon intertemporal optimization to determine investment, monopolistic (in contrast with perfect competition) as the underlying market form, and *only* asymptotic absence of diminishing returns to capital.

policies are vital for “nurturing the technological innovations that fuels the engine of global growth”.

An important difference in these approaches lies in their assumptions. The neoclassical growth models (Solow’s camp) considers Constant Returns to Scale (CRS) but Diminishing Marginal Returns to Capital (DMR-K), holding labor constant (we will work out this in detail in the next class). That is, increasing all inputs (capital and labor) by some constant factor leads to an increase in output proportionally by the same factor. It implies that the size of the economy **does not** affect its growth rate. Moreover, DMR-K implies that as capital accumulates, its marginal product decreases, eventually reaching a point at which new investment just offsets the depreciation of the capital. Hence, why the model predicts a convergence in growth rates to a steady state *(The dynamic version of equilibrium, think of this as an equilibrium that is sustained over time. In math, it would be the case in which the partial derivative wrt to time is 0 and stays at 0)* where per-capita income growth is driven solely by exogenous technological progress. The only factors that may explain differences in per-capita income levels across countries are 1) savings rates, 2) population growth rates, and 3) initial capital stocks. In short, this model predicts conditional convergence: ***countries with similar structural characteristics (savings rates, population growth, etc.) should converge to the same steady state, with poorer countries growing faster along the transition path due to DMR-K.***

The new growth models assume Constant Returns to Capital (CR-K). That is, output increases proportionally with increases in capital, holding labor and technology constant. This is assumed to hold because capital might become more productive over time through some endogenous technological progress. So, DMR-K implies that capital accumulation alone cannot sustain long-run growth. In contrast, CR-K implies that it can. For example, in the simple AK model, $Y = AK$, where A represents the level of technology and K includes both physical and human capital. This formulation allows capital accumulation to drive long-run growth. But, as Solow points out [1], this leads to explosive behavior in the model. Hence why endogenous models often fail to converge.

Naturally, this leads to drastically different policy prescriptions. Exogenous models suggest that policies can affect the steady-state level of income and the speed of convergence, but have limited impact on long-run growth rates. They might prescribe management of the savings rate or allocation of investments. (I shall say more about this when we are done studying the model) but overall little policy (after all, exogenous technology means that there is little we can do to promote its creation). Endogenous models, on the other hand, emphasize that policies promoting capital accumulation, R&D, and knowledge creation can have lasting effects on long-run growth rates. Both types of models agree on the importance of some policies, such as promoting education and maintaining stable institutions, but differ on their predicted long-term effects.

This literature is growing, and there is much more left to be done. We won’t get into any details about endogenous models in this course, but understanding the key differences is crucial and thus why I wanted to start this new topic with such a long winded introduction.

Regardless of the modeling framework, exogenous TFP or endogenous technological change, a measurement error persists. How should we measure economic growth? Usually, economists rely on national production (GDP) as a “good-enough” proxy of living standards. Although it ignores household production, leisure time, effects on the environment, and other factors of well-being, GDP still represents an important aspect of living standards. I do worry that a blind emphasis on increases in production incentivizes overconsumption, reliance on material goods and creates a society with short-time preferences; at the cost of appreciation for family, the environment, and sustainability. Another issue with GDP, which calls for an innovation in data collection techniques, is that it does not capture the benefits of “free services” such as social media or the internet. It captures the benefit to advertisers, but not those enjoyed by the consumer. Effectively ignoring any consumer surplus from these transactions.

References

1. Solow, “Perspectives on Growth Theory”, 1994
2. Broughel and Thierer, “Technological Innovation and Economic Growth: A Brief Report on the Evidence”, 2019, Mercatus Research

Solow Growth Model

Concepts: Production with constant returns to scale, Dynamic model of capital accumulations, exogenous savings rate, exogenous labor supply determined by population growth, exogenous technological progress, capital-labor ratio (capital per worker), closed economy, steady-state, depreciation = dim. Marg. returns.

Background

The Solow-Swan model of economic growth is a classical model of long-run developed by Robert Solow and Trevor Swan in 1956. It was motivated by a critique of the Harrod-Domar model, one of the mainstream models of growth of that era, which assumed capital and labor were employed in **fixed proportions.** That is, firms could not substitute labor for capital, or vice versa. As Solow points out in his original paper, this assumption implies that very small changes to either of the parameters (i.e., factors of production) will lead to instability. This sensitive equilibrium was termed “knife-edge equilibrium”.

As you might intuit, the Solow-Swan model is an extension that relaxes this assumption. Or, in fact, replaces it with an assumption of **substitutability.** So firms can indeed substitute capital for labor. This makes sense. A firm might opt to automate for example, which is basically substituting human labor for machines. This is the first component of the model.

Now, remember Say’s law? An increase in output will be matched by an equivalent increase in income. So, in the long-run, total output produced (GDP) is the same as the total income (GDI) generated in the economy. From our studies of Consumption, we know that some of this income will be saved and some will be invested. Solow assumes, just like we did a few weeks ago, that the marginal propensity to save is exogenous. It is determined by people as they please, based on their preferences, so it is not explained by the model. This is the second component of the model.

Coupled together, this set of assumptions (long-run + exogenous saving) lay the groundwork for developing the mathematics of the Solow-Swan growth model.

Theory and mathematics

We consider a **composite commodity**, a useful economic artifact that allows us to study an entire market by abstracting the different commodities that may exist within it. Suppose we are interested in the market for Beverages. There exist various kinds: coca cola, iced tea, water, sports drinks, etc. By considering a composite commodity we bundle all of these together into a single theoretical commodity called Beverages. Thus, the production of this composite commodity will in fact be the total output/income **Y** of that market and total capital **K** is the accumulation of the composite commodity.

Also, because growth happens over time we are dealing with flow variables. The purpose of growth models is to give a mathematical rationale to explain how this gets accumulated over time. The Solow-Swan model focuses specifically on how capital is accumulated. The mathematics that deals with changes over time is differential equations. Don't worry if you never took a class on this, we will develop the simple mathematics underlying this model from scratch and I am confident everything will make sense.

Next, since the MPS is exogenous we define it as a fraction **s** of total income.

$$\frac{dK}{dt} = s \cdot Y = s \cdot F(K, L)$$

The left-hand side defines the accumulation of capital over time, which is aggregated as the fraction of total output/income saved. In the current form, we have an equation with two unknowns. To close the system, we use the assumption of full employment. *Remember what this assumption means?* Following Harrod-Domar, we assume that the availability of labor to be employed is also exogenously determined by an *exponentially growing population*. There are variations that consider different mechanisms via which the total labor available is determined, but the concept is the same. Since firms fully employ all resources, the entire population is available to be employed. In math…

$$L(t) = L_0 \cdot e^{nt} \Rightarrow \frac{dK}{dt} = s \cdot F(K(t),\ L_0 \cdot e^{nt})$$

n is a positive constant value that determines the rate at which the population grows over time **t.**

We now have a differential equation with a single unknown variable, K(t), which determines the time path of capital accumulation under full employment.

I mentioned that differential equations are all about accumulation. In this case, we have two variables to accumulate: Capital and Labor.

We defined the rate of capital accumulation as the fraction of income saved

$\frac{dK}{dt} = s \cdot Y$, where s is the marginal propensity to save

And, implicitly, we defined the rate of labor accumulation as the rate in which available labor grows.

$\frac{dL}{dt} = n \cdot L$, where n>0 is the per-capita growth rate.

Since we cannot solve this single equation with two unknowns, we borrowed Harrod-Domar assumption of the relationship between population and labor growth. This assumption is comes from solving the first-order differential equation dL/dt

$$\frac{dL}{dt} = n \cdot L \Rightarrow \int \frac{1}{L} dL = \int n\, dt \Rightarrow ln(L) = n \cdot t + C \Rightarrow L = e^{nt+C}$$

A derivative gives us the rate of change, and the integral gives us how much we have in the area under the curve. The integral is nothing more than a method to sum, or accumulate, the values of a variable. So, solving a differential equation (which is stated as a derivative function) means integrating it. If you recall from your calculus classes, the derivative of ln(x) is 1/x, which is why the integral of 1/L is ln(L). To get the log out of the LHS, we exponentiate both sides. Yielding us the final solution.

The next, and final step, in solving first-order differential equations is finding the "coefficient of integration". That is, the value of the constant C. This is the initial value of the variable interest, so to find it we set t=0.

$$L_0 = e^C \Rightarrow ln(L_0) = C$$

Finally, plug this back into our equation for L

$$L = e^{nt+C} = e^{nt+ln(L_0)} = L_0 \cdot e^{nt}$$

We can now rewrite the rate of capital accumulation as

$$\frac{dK}{dt} = s \cdot F(K, L_0 e^{nt})$$

The dynamic process of aggregation follows the following steps:

1. Available supply of labor is given by $L(t) = L_0 \cdot e^{nt}$ and the available stock of capital is a datum.
2. Because of flexible prices we justify the full employment of K and L. So, Y = F(K, L) yields the current rate of output/income. The only constraint on the production function is that it should have Constant Returns to Scale (CRS)
3. The propensity to save, s, gives us the fraction of Y(t) that is saved by economic agents. In turn, this reveals the net accumulation of capital in the current period.
4. Add this to the previously accumulated capital (which is assumed to be inelastically supplied because at each period all resources are fully employed) to obtain the capital available for the next period.
5. Repeat the whole process to continue aggregating.

We now have a **dynamic model** that aggregates the use of **capital and labor per-capita** *(can you see why it is per-capita? Think about the mathematical definition of CRS and how you could rewrite the equation for the rate of capital accumulation.)* over time. With it, we aim to provide answers to questions such as:

- What determines long-run output level per-capita?
- How do savings, population growth, and technological progress affect income Y and economic growth?
- What is the role of accumulating capital K in the growth process?
- How does the economy converge to its steady-state growth?

To find solutions of this model we need one more thing: The production function F(K,L). The only constraint we impose is Constant Returns to Scale (CRS). This simply means that a change in the factors of production will lead to the same proportional change in output.