Team Control Number

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Problem Chosen

B

2015 Mathematical Contest in Modeling (MCM) Summary Sheet

Summary

Recently, a report on missing Malaysia plane MH370 is released in BBC. The progress of search and rescue work for Malaysia plane MH370 has gotten so many attractions. As we can see, a reasonable searching scheme plays a significant role in maritime search and rescue. In this paper, we propose a reasonable and effective scheme to assist the "searchers".

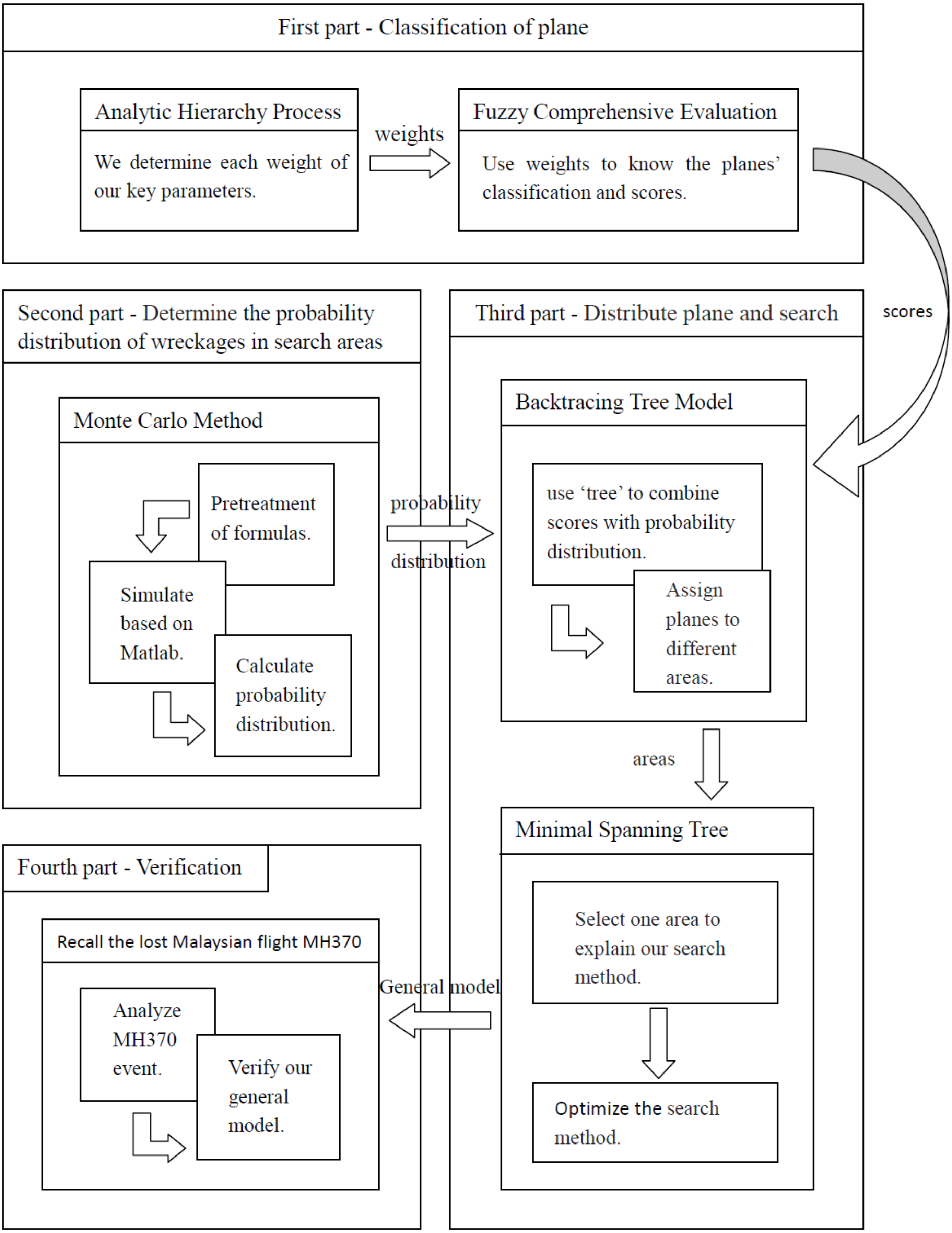
1 In the first part, to obtain a reasonable allocation scheme of search planes, we choose search time, searching distance and the security of the search planes as three key factors. and then determine the weights of the three key factors by analytic hierarchy process. Taking advantage of the three key factors above, we calculate the scores of search capability performance by Fuzzy Comprehensive Evaluation.

2 In the second part, to decide the possible search region, we use Monte Carlo method to simulate the general search region in the following two steps: (a)we use compound trapezoid formula, we integrate the distance of any point along x and y axes. (b) we simulate the probability distribution across every search region by division. Finally, we consider the influence coefficient of the wind and find it is sensitive for our model.

3 In the third part, combining the scores of search capability of each plane and probability allocation across in the general search region, we propose a reasonable and effective scheme for searching. Firstly, we divide the total search region into several unit regions with same shape and acreage. Secondly, to obtain the best search efficiency, we allocate each unit region to one search plane according to the score and probability using Backtracing model, and then we obtain the best scheme by Minimal Spanning Tree. What's more, taking the influence of current into consideration, we amend the basic model so that it can be adopted to dynamic search. At last, we also do the sensitivity analysis and find the sensitive results and others.

4 In the fourth part, we use MH370 to test our model. The result we calculate conforms to the real place MH370 crashed, moreover, we provide a general model to design the general search scheme for a crashed plane.

Keyword: Analytic Hierarchy Process; Fuzzy Comprehensive Evaluation; Monte Carlo Method; Backtracing Model; Minimal Spanning Tree; Different Integration

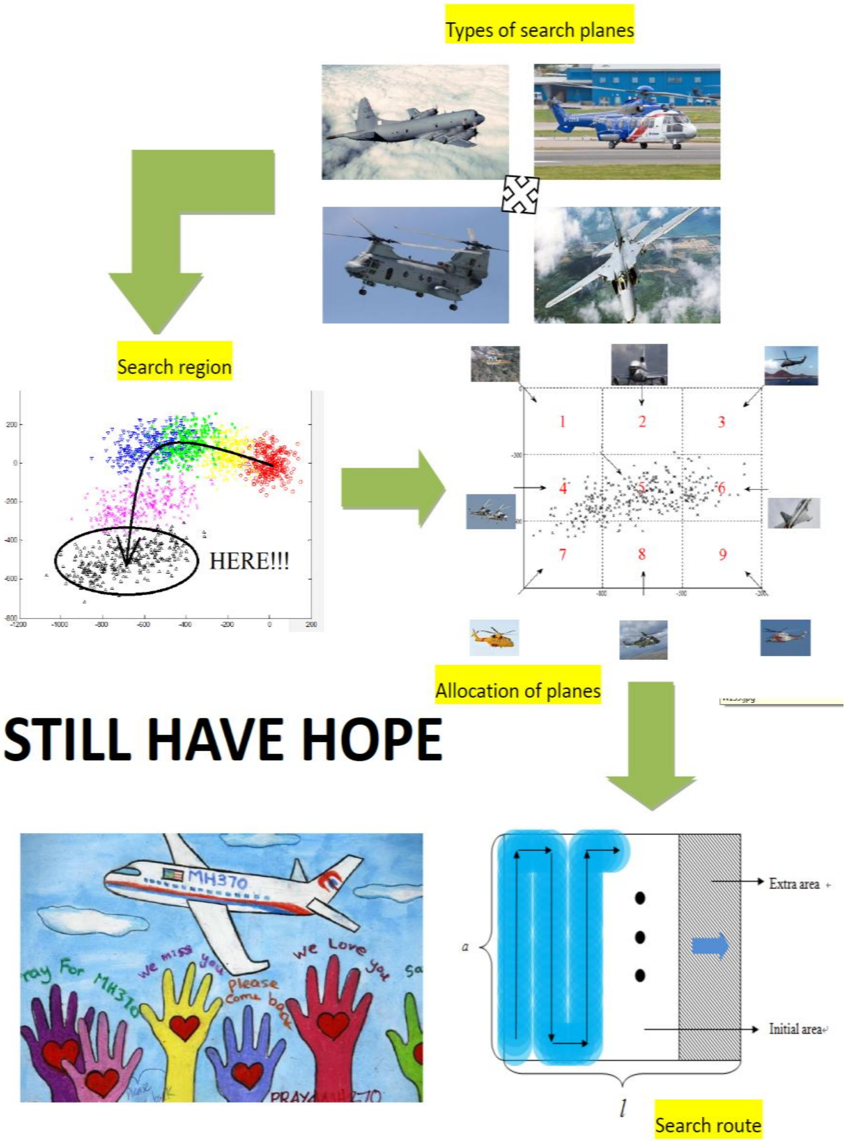

First part - Classification of plane
Analytic Hierarchy Process
We determine each weight of our key parameters.
weights
Fuzzy Comprehensive Evaluation
Use weights to know the planes' classification and scores.
scores
Second part - Determine the probability distribution of wreckages in search areas
Monte Carlo Method
Pretreatment of formulas.
Simulate based on Matlab.
Calculate probability distribution.
probability
distribution
Third part - Distribute plane and search
Backtracing Tree Model
use 'tree' to combine scores with probability distribution.
Assign planes to different areas.
areas
Minimal Spanning Tree
Select one area to explain our search method.
Optimize the search method.
General model
Fourth part - Verification
Recall the lost Malaysian flight MH370
Analyze MH370 event.
Verify our general model.

Where is the next Malaysian flight MH370 ?

According to a list underneath, we can realize that air crash is becoming more and more common. So our airlines provide a reasonable scheme for searching plays a significant role in maritime search and rescue.

Date	Downd Plane
1985	Air India Flight 182
1996	TWA Flight 800
2002	China Airlines Flight 611
2007	Adam Air Flight 574
2008	British Airways Flight 38
2009	Air France Flight 447
2011	Malaysia Airlines Flight 370
2013	Asiana Airlines Flight 214
2014	Malaysia Airlines Flight 17

List some of them.



In the future, every time we plan to do the search task, we will follow the three s

- step1: Classify our planes will help us search more efficiently. So, we should it to prepare for our search scheme.
- step2: Because of the influence wind and current, wreckages of downed aircrafts will move in some way. So, we should determine the search areas and allocate the planes to the right places quickly.
- step3: As soon as our planes arrive, we will search in one special method by the great efficient search scheme.

We will try our best to save lives, and we will never give up any life.

Contents:

I.Intruduction

1.1 Background:

BBC has reported:
The Malaysian government has officially declared the disappearance of Malaysian Airlines flight MH370, en route from Kuala Lumpur to Beijing on 8 March 2014, an accident.
Missing Malaysia plane MH370.

1.2 The description of the problem

Nowadays, aircraft crash is becoming more and more frequent. A reasonable and effective scheme for searching can assist "searchers" in a large extent in searching and rescuing. The progress of tackling can be divided into three phases. Aiming to each phase, we tackle corresponding problem：

- Classification of search planes.
- To determine the rough search region.
- The best Scheme for searching.

To tackle the first problem, we select sets of parameters to quantify the performance of each search plane and use the scores of searching capacity as the main standard for our classification process.
Aiming at the second problem, we introduce drifting trajectory to estimate the possibility in different regions. As a result, the rough search region can be simulated.
For the last question, firstly, we divide the whole search region into several unit search regions. Next, each unit region is distributed to one search plane. Finally, we provide the best trajectory for a plane searching one unit region.

II. Total assumptions

According to the question above, we make some assumptions in the following.

- When the accident happened to the plane, it has no sign immediately.
- If the plane falls down to the sea, it breaks up into many pieces. Because of the light weight of each piece, pieces are often floating on the surface of sea.
- Without current and wind, there are no other severe impact factors.

III. Plane classification and grade based on Analytic Hierarchy Process and Fuzzy Comprehensive Evaluation

3.1The key parameters

When searching for a crashed plane in open water, there is no doubt that the searching time, searching distance and the security of the search planes are the most important three elements in the search process.

Searching time can be represented by the planes' maximum speed, searching distance is related to the planes' maximum ranges, and the security of the searchplanes is about to the climb rate and maximum speed of the planes, which can help search planes escape from the dangerous situation which happens suddenly. Now, we describe them briefly as follows:

Maximum speed

Maximum speed refers to the permissible speed that the aircraft in the engine work achieving maximum power and maximum thrust. The higher this maximum speed is, the less time the plane use in arriving in the fixed searching area.

The Range of the plane

Maximum range is the farthest distance that a plane can arrive at once fly, it affects the search distance greatly.

Rate of climb

The height that airplane climbs per unit time can influence the time of changing vertical distance in searching progress, mainly related to search planes' security.

3.2The proportion decision of the three factors with AHP

Now, we want to do the classification with different planes. In the beginning, we collect so many important data about the three factors of different planes from Ref.[1], and list them in Table1**.**

"Table1 Each index of common plane used for search"[1]

num	Types of aircrafts	Maximum speed(km/h)	Range(km)	Rate of climb(km/h)
1	AgustaWestland AW109	285	932	35.28
2	AgustaWestland AW139	310	1,250	39.24
3	AgustaWestland CH-149 Cormorant	278	1389	36.72
4	Bell UH-1 Iroquois	217	507	32.112
5	Boeing Vertol CH-46 Sea Knight	267	1020	31.356
6	Eurocopter AS332 Super Puma	262	841	31.356
7	Eurocopter AS532 Cougar	249	573	25.92
8	Eurocopter EC225 Super Puma	275.5	857	31.32
9	Lockheed P-3 Orion	750	4400	57.6
10	Sikorsky SH-60 Seahawk	270	834	30.168
11	Sikorsky S-70 Blackhawk	361	461	41.148
12	Westland Sea King	208	1230	37.08
13	Westland Wessex HC2	213	499	30.24

To obtain the weights of the three key factors, AHP is a qualitative and quantitative method, and it use Random Math as a tool to find the weights of different factors if they are important, and describe briefly in the following:

(1) Scale

To obtain the relative weighs by doing pairwise comparison under a standard or various standards, we introduce scale of the relative importance in Table 2. The scales themselves are based on our own subjective decisions.

Table 2.The Meaning of the scale from 1 to 5.

Scale	Meaning
1	Requirements i and j have equal scale.
3	Requirement i has a slightly larger scale than j.
5	Requirement i has a strongly larger scale than j.
2,4	Intermediate scales between two adjacent judgments.
Reciprocals	Requirement i has a smaller scale than j.

(2) Pairwise comparison matrix

Based on the above scale, in pairwise comparison matrix which is the second layer to the first layer, the range has a strongly larger scale than the rate of climb, the range has a slightly larger scale than the maximum speed, the scale between the maximum speed and rate of climb is between equal and sligthtly larger , so pairwise comparison matrix which is the second layer to the first layer:

$$\begin{array}{l} \\ \text{Maximum speed} \\ \text{Range} \\ \text{Rate of climb} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Maximum speed} & \text{Range} & \text{Rate of} \end{array} \\ \begin{pmatrix} 1 & 1/3 & 2 \\ 3 & 1 & 5 \\ 1/2 & 1/5 & 1 \end{pmatrix} \end{array}$$

(3) Calculating the weight of each factor

We use Matlab to calculate each proportion of the three factors, as given in Table 3.

Table 3.Each proportion of the three factors

Factor	Maximum speed	Range	Rate of climb
Proportion	0.2297	0.6483	0.1220

From the Table 3 we can know the weights of maximum speed, the trange and the rate of climb are 0.2297, 0.6483 and 0.1220, respectively, they mean the importance of different factors in planes in our model.

(4)Consistency check

Finally, we need to do the consistency check. We get the RI(Random Consistency Index) from the Table 4 and define the CI(Consistency Index):

$$CI = \frac{\lambda - n}{n-1} \tag{1}$$

Among them, λ is the highest eigenvalue, n is 3 × 3 matrix's the sum of the diagonal elements, and it is equal to the sum of eigenvalues in the matrix.

Table 4 Values of RI in different order.

Order	1	2	3	4	5	6	7	8	9
RI	0	0	0.52	0.89	1.12	1.26	1.36	1.41	1.46

According to the CR'(Consistency Ratio) equation

$$CR = \frac{a_1 CI_1 + a_2 CI_2 + ... + a_m CI_m}{a_1 RI_1 + a_2 RI_2 + ... + a_m RI_m}. \tag{2}$$

Among them, a_i is the element in matrix.

We can calculate the CI= - 0. 0018, CR= - 0. 0036, because of CR=0. 0036<0.10, so the consistency of our matrix can be accepted, and we can use each proportion of the three factors at ease.

3.3Plane classification and scores with Fuzzy Comprehensive Evaluation

Fuzzy Comprehensive Evaluation Method is based on the Fuzzy Math, it can change qualitative evaluation into quantitative evaluation, and help us know the comprehensive evaluation of each plane that is affected by variety of factors. Hence, we use it to get the categories totally and comprehensive score of each plane which will be used in Model 3.

Determine the different factors sets and evaluations sets

As mentioned above, Maximum speed, Range, Rate of climb and types of aircrafts are the crucial factors that affect searching process, and we use *A* represents Types of aircrafts, ***B*** represents Maximum speed, *C* represents Range, and *D* represents Rate of climb. so we can define different factors sets:

$$F = \{A, B, C, D\}$$

Because the goal of search is less search time, and it needs search security, So we can define the group of evaluations as E:

$$E = \{G, H\}$$

Among them, G is Search time, H is Search security.

Build the evaluation matrix in the beginning of each factor

Firstly, we evaluate F and E. Secondly, combine them with the Table 1, we can get the matrix R_i:

$$R_1 = (285, 932, 35.28) \quad R_8 = (275.5, 857, 31.32)$$
$$R_2 = (310, 1250, 39.24) \quad R_9 = (750, 4400, 57.6)$$
$$R_3 = (278, 1389, 36.72) \quad R_{10} = (270, 834, 30.168)$$
$$R_4 = (217, 507, 32.112) \quad R_{11} = (361, 461, 41.148)$$
$$R_5 = (267, 1020, 31.356) \quad R_{12} = (208, 1230, 37.08)$$
$$R_6 = (262, 841, 31.356) \quad R_{13} = (213, 499, 30.24)$$
$$R_7 = (249, 573, 25.92)$$

So, if we put all R_i into one evaluation matrix R, it can be described as follows

$$R = \begin{pmatrix} 285 & 932 & 35.28 \\ 310 & 1250 & 39.24 \\ 278 & 1389 & 36.72 \\ 217 & 507 & 32.112 \\ 267 & 1020 & 31.356 \\ 262 & 841 & 31.356 \\ 249 & 573 & 25.92 \\ 275.5 & 857 & 31.32 \\ 750 & 4400 & 57.6 \\ 270 & 834 & 30.168 \\ 361 & 461 & 41.148 \\ 208 & 1230 & 37.08 \\ 213 & 499 & 30.24 \end{pmatrix}$$

Build the Fuzzy Comprehensive Evaluation model

AS for building the Fuzzy Comprehensive Evaluation model easier, we have make the AHP, and get the each proportion of the three factors that Fuzzy
Comprehensive Evaluation method needs.

So, from the Table 2, we can know each proportion of the three factors. Firstly, we put them into matrix $\boldsymbol{A}$. Then, we define matrix $\boldsymbol{B}=\boldsymbol{R}\times\boldsymbol{A}$:

$$B = R \times A = \begin{pmatrix} 285 & 932 & 35.28 \\ 310 & 1250 & 39.24 \\ 278 & 1389 & 36.72 \\ 217 & 507 & 32.112 \\ 267 & 1020 & 31.356 \\ 262 & 841 & 31.356 \\ 249 & 573 & 25.92 \\ 275.5 & 857 & 31.32 \\ 750 & 4400 & 57.6 \\ 270 & 834 & 30.168 \\ 361 & 461 & 41.148 \\ 208 & 1230 & 37.08 \\ 213 & 499 & 30.24 \end{pmatrix} \times \begin{pmatrix} 0.2297 \\ 0.6483 \\ 0.1220 \end{pmatrix} = \begin{pmatrix} 673.984 \\ 886.369 \\ 968.823 \\ 382.451 \\ 726.421 \\ 609.227 \\ 431.833 \\ 622.697 \\ 3031.822 \\ 606.382 \\ 386.808 \\ 849.710 \\ 376.117 \end{pmatrix}$$

we change ***B*** into normalized ***B'*** and sort the X_{Num} of ***B'*** from low to high to get matrix **C**:

$$
\begin{array}{cc}
 & X_{Num} \quad \text{Num} \\
B' = & \begin{pmatrix}
0.0639--1 \\
0.0840--2 \\
0.0918--3 \\
0.0362--4 \\
0.0690--5 \\
0.0578--6 \\
0.0402--7 \\
0.0592--8 \\
0.2875--9 \\
0.0576--10 \\
0.0367--11 \\
0.0805--12 \\
0.0356--13
\end{pmatrix}
\end{array}
\qquad
\begin{array}{cc}
 & X_{Num} \quad \text{Num} \\
C = & \begin{pmatrix}
0.0356-13 \\
0.0362-4 \\
0.0367-11 \\
0.0402-7 \\
0.0576-10 \\
0.0578-6 \\
0.0592-8 \\
0.0639-1 \\
0.0690-5 \\
0.0805-12 \\
0.0840-2 \\
0.0918-3 \\
0.2875-9
\end{pmatrix}
\end{array}
$$

From matrix **C**, we can calculate the evaluation index of any two adjacent numbers. Firstly we define $X_{i-j} = \left|X_i - X_j\right| \times 100$ according to our goal, and we get the every X_{i-j} in Table 5

Table 5 The value of X_{i-j}

X_{i-j}	X_{13-4}	X_{4-11}	X_{11-7}	X_{7-10}	X_{10-6}	X_{6-8}
Value	0.06	0.05	0.35	1.74	0.02	0.14
X_{i-j}	X_{8-1}	X_{1-5}	X_{5-12}	X_{12-2}	X_{2-3}	X_{3-9}
Value	0.47	0.51	1.15	0.35	0.78	19.57

From Table 5, if we let the threshold value T=1, $X_{i-j} > 1$ is the threshold value which means , we can get four Categories totally:

Table 6 A list of Categories

Category 1:	●Category 2:	Category 3:	●Category 4:
•13(Westland Wessex HC2) •4(Bell UH-1 Iroquois) •11(Sikorsky S-70 Blackhawk) •7(Eurocopter AS532 Cougar)	•10(Sikorsky SH-60 Seahawk) •6(Eurocopter AS332 Super Puma) •8(Eurocopter EC225 Super Puma) •1(AgustaWestland AW109) •5(Boeing Vertol CH-46 Sea Knight)	•12(Westland Sea King) •2(AgustaWestland AW139) •3(AgustaWestland CH-149 Cormorant)	•9(Lockheed P-3 Orion)

If we set the highest score is 100, we can know every plane's score thtotal matrix **B'** ,and then we put them into Table 7.

Table 7 The scores of search planes

num	Types of aircrafts	Scores(100)
1	AgustaWestland AW109	6.39
2	AgustaWestland AW139	8.40
3	AgustaWestland CH-149 Cormorant	9.18
4	Bell UH-1 Iroquois	3.62
5	Boeing Vertol CH-46 Sea Knight	6.90
6	Eurocopter AS332 Super Puma	5.78
7	Eurocopter AS532 Cougar	4.02
8	Eurocopter EC225 Super Puma	5.92
9	Lockheed P-3 Orion	28.75
10	Sikorsky SH-60 Seahawk	5.76
11	Sikorsky S-70 Blackhawk	3.67
12	Westland Sea King	8.05
13	Westland Wessex HC2	3.56

IV. The determination of search region using Monte Carlo method

When an object floats at sea, it must be influenced by variety natural factors, such as speed of itself, wind power, current-force and wave-force. In the following,we will discuss them one by one.

4.1 Notations and definitions

All the variables and constants used in this paper are listed in Table7:

Table8 The variables and constants

Symbol	Definition
Q_0	Initial amount of fuel when plane took off
T_0	Starting time
T	Final time of the plane being kept in touch with
R	The radius of crash region
P	The position where plane is out of contact
P	The remaining amount fuel as plane is in lost
q	The plane consumption of fuel per hundred kilometers per passenger
n	The amount of passengers whom plane carrys
V	Speed of plane

4.2 Determination of the range for the situation of plane crash

Plane flying from A to B happens to be out of contact, however we can not make sure if the plane is in an accident. And then we make the following assumptions about parameters in this paper

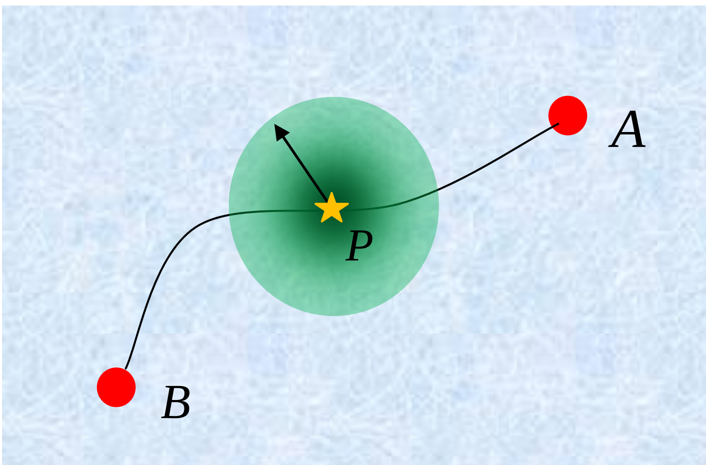


Figure 1 the the range for the situation of plane crash

$$Q_1 = Q_o - nq\frac{\int_{T_0}^{T} vdt}{100} \tag{3}$$

$$R = 100\frac{Q_1}{nq} \tag{4}$$

Hence, the location of the plane crash is with in the radius of circle center by P R . All the variables and constants used in this paper are listed in Table 8.

4.3 Dynamic drift trajectory simulation using Monte Carlo method

To get the trajectory of drift, we introduce speed function. Due to the speed of current and the speed of water are varying as time goes on, so it is difficult to determine the speed of a certain wreckage. The speed of an object floating in the sea is determined by $V_{current}$ and V_{wind}. Using composition of speed vectors, the velocity of object is as follows [2]

$$“\vec{V}_{object} = \vec{V}_{current} + \alpha\vec{V}_{wind}” \tag{5}$$

Where α is the factor of wind, With 0.01< α <0.05, this speed function is reasonable. Monte Carlo method is a broad class of computational algorithms that rely on repeated random sampling to obtain numerical results. Because each wreckage has different shape, mass , and the volume on the water, the impact of wind on the wreckage is different and changing with time going on. As a result, in the progress of drifting on the sea, the speed and the direction of wreckage are uncertain at any time. Above all, we adapt Monte Carlo method to simulate the drifting trajectory.

4.3.1Solution of drift trajectory using difference integration

To simulate drift trajectory, some experimental data are introduced in Table 8. From Table 8, the relationship between time and the velocity of current and wind in two vertical directions is shown in Table. in the experiment on actual drifting. Combining data in Table and equation(). it is not difficult to find that the impact of current-power is great and meanwhile the effect of wind-power is slightly inferior. So in practice we just use five data among the whole twelve data in order to avoid effect of current-power.

"Table.9 The relationship between current speed and wind speed on x-y plane and time"[3]

T min	Current speed of x m/s	Current speed of y m/s	Wind speed of x m/s	Wind speed of y m/s
T_0 =0	-0.1827	0.1398	-4.8300	-2.0768
T_1=10	-0.1827	0.1398	-4.9308	-1.6914
T_2 =54	-0.1180	0.0100	-4.9762	-0.9965
T_3 =117	0.0455	-0.4711	-4.2475	-0.9858
T_4 =139	-0.0218	-0.4437	-3.7766	-0.9790

To demonstrate the trajectory of wreckage drifting as the time goes on, we must be given displacement along X axis and Y axis at different time. Referring to known data, we tackle trajectory equation using difference method by the following steps

Step 1: we divide the domain of T [T_0 =0, T_4 =139] into five asymmetrical intervals.

Step2: aiming at the subdomain, we conclude displacement equation

$$S = \int_{T_k}^{T_{k+1}} V(T)dT \approx \frac{T_{k+1}+T_k}{2}\left[V(T_{k+1})+V(T_k)\right] \tag{6}$$

Due to intervals of definite integral is additive, hence the equation of difference integration is

$$\int_{T_0}^{T_n} V(T)dT = \sum_{k=0}^{n}\int_{T_k}^{T_{k+1}} V(T)dT \tag{7}$$

Then we introduce composite trapezoidal rule to the equation above, thus

$$\int_{T_0}^{T_n} V(T)dT = \frac{\Delta T}{2}\left[V(T_0)+2\sum_{k=1}^{n-1}V(T_k)+V(T_n)\right] \tag{8}$$

Step 3:Based on calculating above, we get the displacement equation along X axis and Y axis at different time

$$\begin{cases} X_n = X_0 + \dfrac{\Delta T}{2}\left[V_x(T_0) + 2\sum_{k=1}^{n-1} V_x(T_k) + V_x(T_n) \right] \\ Y_n = Y_0 + \dfrac{\Delta T}{2}\left[V_y(T_0) + 2\sum_{k=1}^{n-1} V_y(T_k) + V_y(T_n) \right] \end{cases}. \tag{9}$$

4.3.2Simulation results

We define a as the influence coefficient of wind In equation (3) , the range of the value of a is totally from 0.01 to 0.05. Therefore, we generate some data in a normal allocation with μ =0.04 and σ =0.0067. As we can known, the speed of each wreckage is fluctuating all the time.

We take T_0 as the starting point, when total initial points generated randomly are getting together near the coordinate (0,0). We simulate wreckage drifting trajectory under different cases:

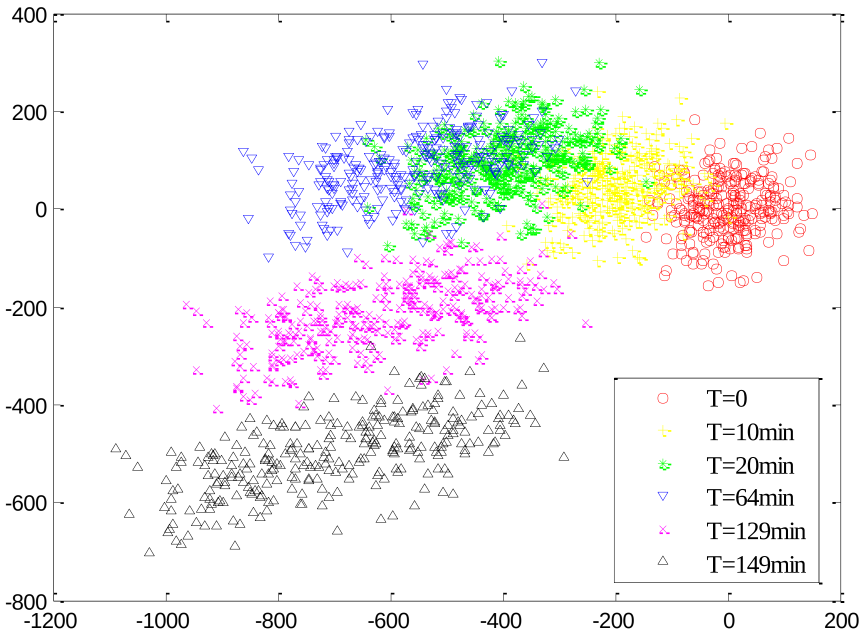


Figure 2 The superposition of wreckage drifting trajectory at different T

At each time, the wreckage spot is shown in Figure 3.

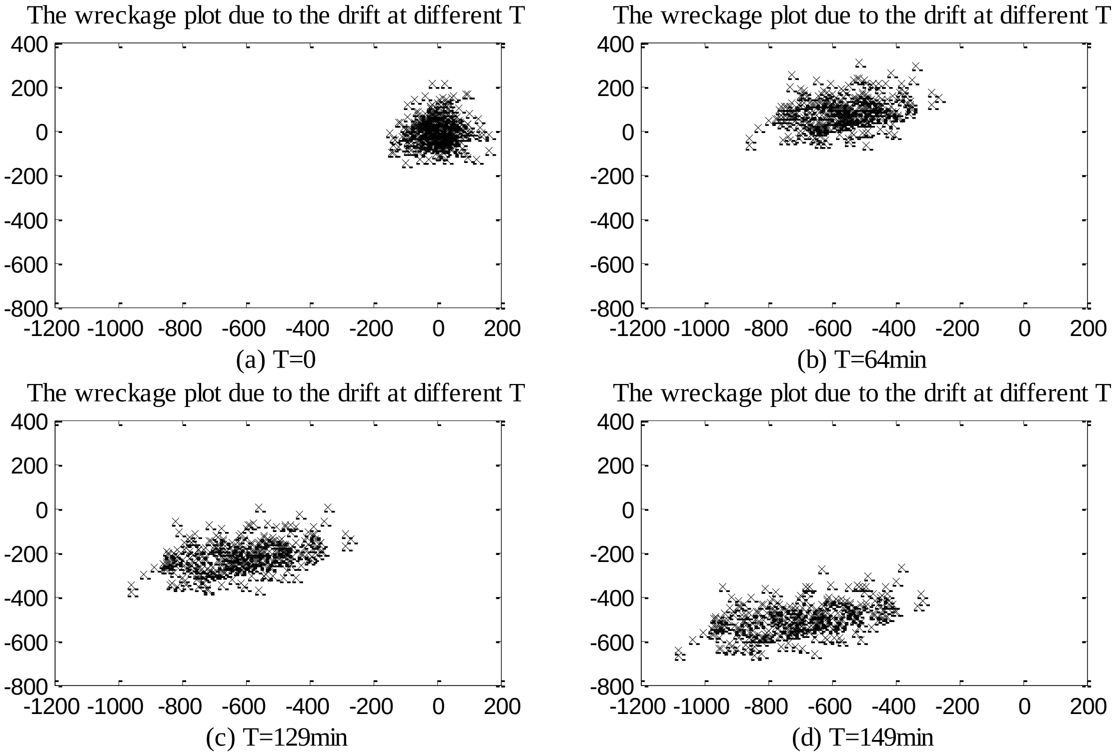


Figure3 Wreckage drifting trajectory at different T

Figure 3 shows that as the influence coefficient of wind grows, drift path deviation of the remains is getting bigger and bigger, so the influence coefficient of wind has a certain influence on the actual drift trajectory.

4.3.3 Sensitivity analysis

According to our analysis of the problem, the influence coefficient of wind can affect the deviation angle of wreckage and affect search region and rescue ultimately. In the actual search and rescue, it is of great necessity to realize that searching region is varying caused by the varied influence coefficient of the wind. To determine and make out the final rescue region, it is necessary to analyze the sensitivity of wind influence coefficient in advance.

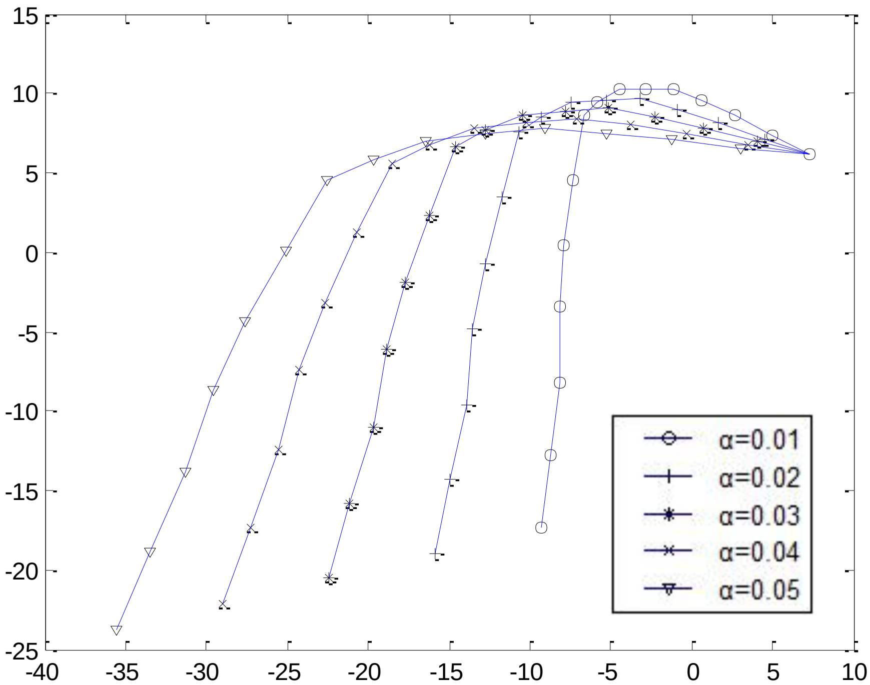


Figure 4 Trajectory of wreckage drifting when α is varied

In the above models, the influence coefficient of the wind falls within the scope of 0.01 to 0.05 according to normal allocation. To check the effect of the wind drift coefficient of debris, we, in turn, make the influence coefficient of wind by Monte Carlo simulation to fit different curve as is shown in Figure

V. The best scheme for searching

5.1Backtracing tree model for the best allocation of search planes

5.1.1 The best scheme for Assigning search planes using backtracing

Step 1: we label three planes with A, B and C alphabetically. The three planes are represented by each vertex in Tree. In the same way, the three searching regions are labeled with 1, 2 and 3. And similarly, each edge represents a region.

Step 2: we create a tree to illustrate the distribute of planes and search regions. As one vertex is adjacent to its outgoing edge , we define that this vertex representing plane is distributed to the edge representing region. However, it is noteworthy that when a vertex is adjacent to its incoming edge, there was no relationship between them.

Step 3: we define each vertex as a layer, so the depth of tree given in Fig is 3. Significantly, we compare allocation to one search path in the tree. One search path is matched with the only allocation.

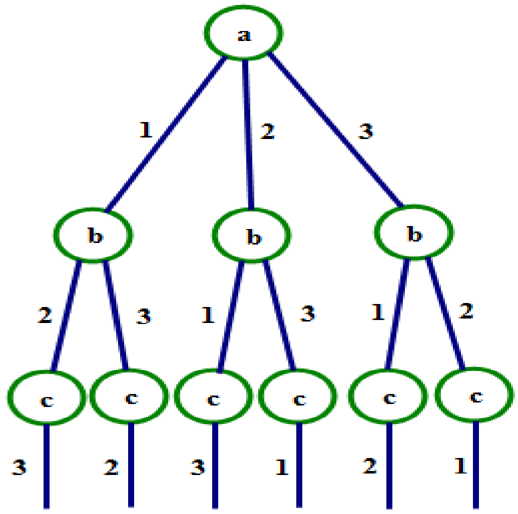


Figure 5 The tree used for illustrating

Step 4:to avoid failure of searching, if a search plane exists, it should be distributed to search region with greatest chance. So we introduce an index to measure the stand or fall of allocation. The value of this index is the product of performance parameter for per plane and possibility of region. To achieve maximum efficiency, we should get summation of each matched product to be the greatest.

Step 5:let's suppose that we match plane at first. Plane A has three outgoing edges, so there are three regions it can be assigned to. Similarly, there exist two for plane B. when travelling at the last plane, the constrained condition is reached. Then we track back to the prior plane to continue the next search, and search this time is finished. However, the traditional travelling method has some shortcomings, such as long time consuming and large work. Thus if the method of backtracking comes into use, we will avoid some needless work.

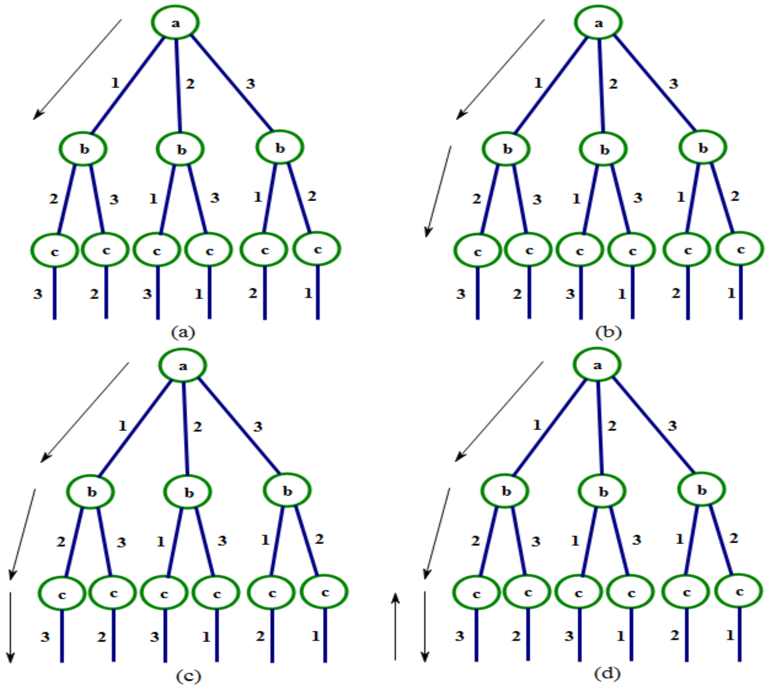


Figure 6 The progress of tracing back

By backtracing , we can obtain all the bathes of travelling. Comparing each possible bathes, we select the best bath.

5.2.1 Inspection for the model

According the data in Table, we select three hundred points randomly to simulate actual values of probabilities on x-y plane. And then we count values of possibilities of the three hundred points. According to the data above, we determine the search scope. Assuming that all the plane starting place is at the top left corner , we can conclude two matrixes.

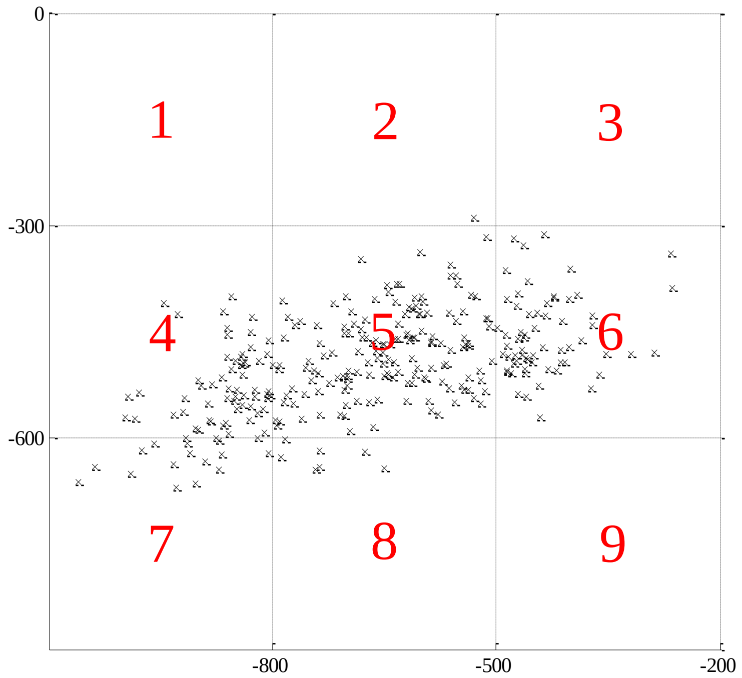


Figure 7 Wrechage drifting trajectory at different T

We randomly select nine planes. Then nine regions given in Figure above is distributed to the nine planes. Using backtracing method introduced above, we tackle the result of this simulative experiments showed in Table.10.

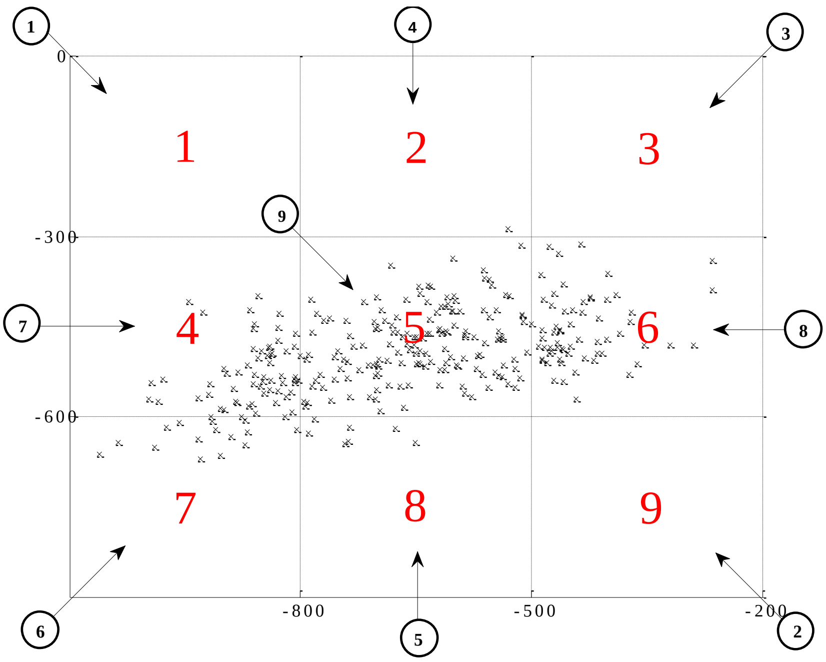


Figure 8 The best allocation in simulating

5.2 Trajectory of unit square searching region determined by Minimum Spanning Tree

After making sure the whole total search region, we divide it into several square regions for further searching.

5.2.1Assumptions

- We assume that there exists a square area preparing for being searched.
- The square area is divided into nine unit squares.
- When center of each unit square region is passed by searching route, searching for this region is finished.
- The length between two adjacent horizontal points or vertical points is given as unit 1.

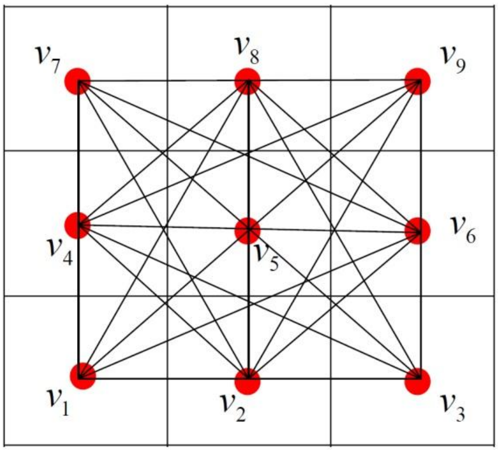


Figure 9 Complete gragh of total searching trajectories

With the unit length =1 in Figure 9, we calculate values of other adjacent edges in the following matrix.

Table 10 The matrix for values of edges

$$
\begin{vmatrix}
0 & & & 1 & \sqrt{2} & \sqrt{5} & & \sqrt{5} & \\
 & 0 & 1 & \sqrt{2} & 1 & \sqrt{2} & \sqrt{5} & & \sqrt{5} \\
 & 1 & 0 & \sqrt{5} & \sqrt{2} & 1 & & \sqrt{5} & \\
1 & \sqrt{2} & \sqrt{5} & 0 & 1 & & 1 & \sqrt{2} & \sqrt{5} \\
\sqrt{2} & 1 & \sqrt{2} & 1 & 0 & 1 & \sqrt{2} & 1 & \sqrt{2} \\
\sqrt{5} & \sqrt{2} & 1 & & 1 & 0 & \sqrt{5} & \sqrt{2} & 1 \\
 & \sqrt{5} & & 1 & \sqrt{2} & \sqrt{5} & 0 & 1 & \\
\sqrt{5} & & \sqrt{5} & \sqrt{2} & 1 & \sqrt{2} & 1 & 0 & 1 \\
 & \sqrt{5} & & \sqrt{5} & \sqrt{2} & 1 & & 1 & 0
\end{vmatrix}
$$

5.3.2 Prim algorithm:

- Get S= $\left(V_1, V_2 \ldots, V_9\right)$.
- Get T= $\left(e_{12}, \mathrm{e}_{23} \ldots, \mathrm{e}_{89}\right)$.
- Starting states of S and T are empty .
- Select a vertex as the primitive point randomly.
- Repeated progress using Prim algorithm.
- We get Minimum Spanning Tree T= (S, T) from the process above.

5.3.3Results

By calculating the Prim algorithm using Matlab, we get the best searching route

$$V_1 \text{->} V_2 \text{->} V_3 \text{->} V_6 \text{->} V_5 \text{->} V_4 \text{->} V_7 \text{->} V_8 \text{->} V_9$$

Which is illustrated in Figure 10

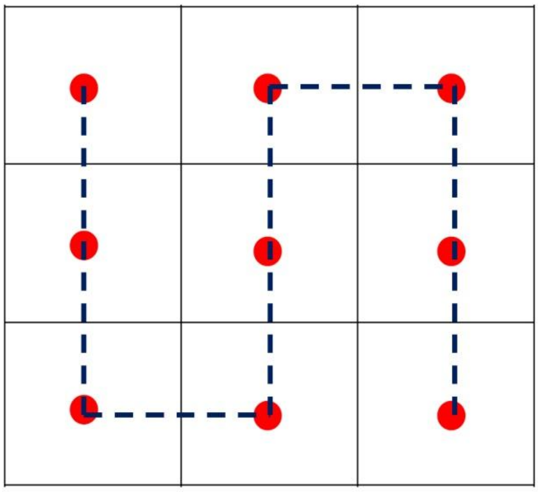


Figure 10 the best searching route

5.4 Dynamical trajectory of unit square search region

5.4.1 Notations, definitions and Assumptions

All the variables and constants used in this paper are listed in Table 11

Table 11 The variables and constants

Symbol	Definition
r	Search radius of plane
n	Times of search along the direction vertical to current
$v_{airplane}$	Search speed of plane
t	Total time that plane spends on searching
v_{water}	Speed of water
a	Half length of the edge of the initial square region

In the searching progress, search regions are moving as time goes on. Consequently, each search region is not as big as the region that is distributed initially . We should also consider extra area caused by current. Hence, to make the search more efficiently, we add extra area to initial resting region. Assuming that each search plane has a circular searching range, the distance between any two parallel tracks is twice as much as the search radius of plane.

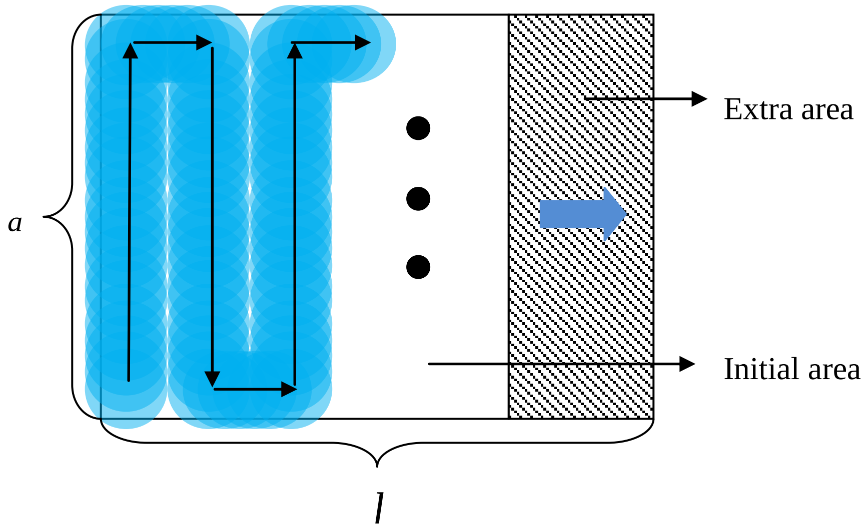


Figure 11 The search route considering the impact of current

The equation of search length l along direction of current is given as

$$l = a + \int v_{water} dt \tag{10}$$

According to the correlation of several factors in Figure , we conclude the equation

$$n(a - \sqrt{2}r) + l - \sqrt{2}r = \int v_{airplane} t \tag{11}$$

$$(n-1) \cdot 2r = a - \sqrt{2}r \tag{12}$$

Here, it is assumed that both water speed and the speed of plane are constant quantities. Then

$$l = a + v_{water} t$$

$$n(a - \sqrt{2}r) + l - \sqrt{2}r = v_{airplane} t \ .$$

Thus

$$n = \frac{a - \sqrt{2}r}{2r} + 1 \tag{13}$$

$$t = \frac{\left(a - \sqrt{2}r\right)^2}{2\left(v_{airplane} - v_{water}\right)r} + \frac{a - \sqrt{2}r}{v_{airplane} - v_{water}} \tag{14}$$

$$l = a + \frac{v_{water}\left(a-\sqrt{2}r\right)^2}{2\left(v_{airplane}-v_{water}\right)} + \frac{v_{water}\left(a-\sqrt{2}r\right)}{v_{airplane}-v_{water}} \tag{15}$$

5.4.2 Sensitivity analysis

In this model, the speed of water, the speed of plane, half length of the edge of the initial square region or search radius of plane could have an effect on the total time that plane spends on searching. To explore the influence that each parameter has on the model, it is of necessity to analyze the sensitivity of each parameter in advance.

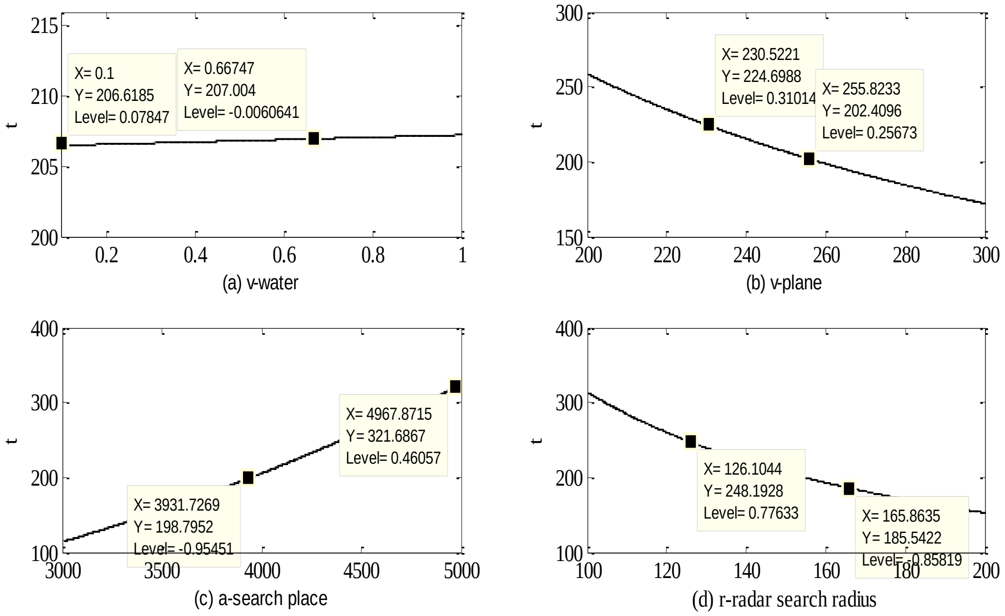


Figure 12 Values of the T when different parameters vary

In the Figure 12 ,we define these parameters as some reasonable values. We make $v_{airplane}$=250km / h , $a = 40km$ and $r = 150km$.

As the Figure 12 shown, from $v_{water} = 0.1km/h$ to $v_{water} = 0.66747km/h$, Δt is slight. So, in this case, the speed of water has little influence on search time. On the same way, we do sensitivity analysis on other parameters and get Figure(b)(c)(d).

VI. The inspection and Analysis for the model

6.1 Strenghs and weakness

Strengths

- The model describes the motion in detail, with coordination among the many physical quantities.
- The whole process is rational and novel.
- The simulation and numerical computations are precise.

Weakness

- 1There are some errors in the estimated data because of the shortage of data. In the progress of searching, though we cannot take all the situation into account, but the key factors and the normal situation are taken into consideration.

6.2 The inspection for the model

To test the feasibility of the model, we select the data from MH370 aircraft crash in last year to determine a scheme for reaching.

Step 1: we inquire for the types and related parameters of planes that are involved into search work given in Table 12

Table 12 Each index of common plane used for search

Types of aircrafts	Maximum speed(km/h)	Range(km)	Rate of climb(km/h)
Lockheed C-130 Hercules[4]	620	3800	33.48
P-3 Orion[5]	750	9000	57.6
Ilyushin Il-76[6][7]	860	6700	54.864

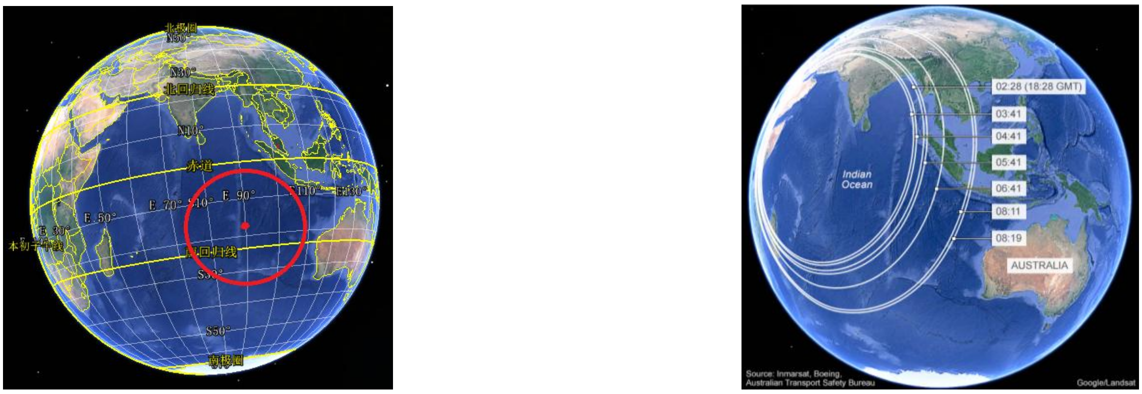
we conclude the scores of search capacity.

Table 13 The scores of each plane is shown

Type	LockheedC-130 Hercules	P-3 Orion	Ilyushin Il-76
score	19.81	45.66	34.53

Step 2:

According to the remaining fuel capacity , we calculate the farthest flight distance. When satellite reconnaissance is out of contact to the plane, we can conclude the coordinate of this plane at that time. Using the method mentioned above, we can determine the location of the crash within the range of the circle shown in Figure (A). According to the aircraft flight direction, we can determine that the total location is located in the north-south corridor. The search region that we determine is consistent to that experts predict.



(a) **(b)**

Figure 13 The search region(a) that we draw and that reperts conclude(b)

Based on the local current and wind, we obtain the possible speed and direction of wreckage. On the basis of result by Monte Carlo simulation, we can draw a wreckage plot given in Figure(A). The region that we draw above and the official reach region are basically the same.

(c) （d）

Figure 14 The search region(c) that we draw and that reperts conclude(d)

Step 3:

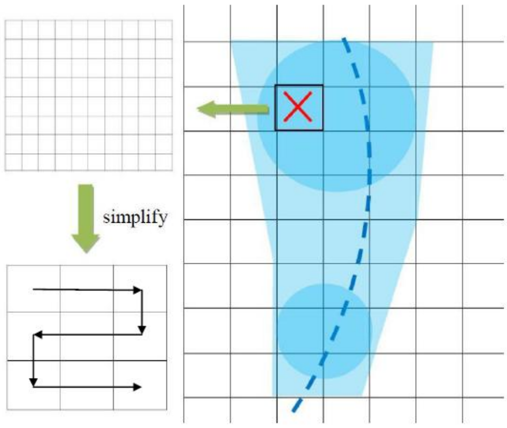


Figure 15 The best scheme for searching

We divide the whole region that we will search into many parts. According to the scores of search capability, we distribute each plane to each search region reasonably .We make plane that has higher score search in a region where we can find the wreckage more easily .In each region, each plane follow the best route that our model make.

6.3

Our model is not only for the MH370 flight crash.Our model can be applied to other flight crash in open water, such as Air France Flight 447 crash.According to the data we have,we can make a effective plan.

VII .References

[1] http://en.wikipedia.org/wiki/Search_and_rescue (2/8/2015)

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[3]SU T sung-chow. On predicting the person in water for search and rescue [C].Defend Technical Information Center, 1993.

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[5]http://zh.wikipedia.org/wiki/%E4%BC%8A%E7%95%99%E7%94%B376 (2/8/2015)

[6]https://translate.google.com.hk/translate?hl=zh-CN&sl=en&tl=zh-CN&u=https%3A%2F%2Fdoc8643.com%2Faircraft%2FIL76&anno=2(2/8/2015)

[7]http://zh.wikipedia.org/wiki/P-3%E7%8D%B5%E6%88%B6%E5%BA%A7%E6%B5%B7%E4%B8%8A%E5%B7%A1%E9%82%8F%E6%A9%9F(2/8/2015)

[8]http://zh.wikipedia.org/wiki/C-130%E9%81%8B%E8%BC%B8%E6%A9%9F(2/8/2015)

[9]http://news.china.com/focus/mhkjsl/11152539/20140315/18395685.html(2/9/2015)

X. Appendix

Table.a relationship of planes and scores

Scores / Planes								
Sikorsky S-70 Blackhawk	3.67	3.67	3.67	3.67	3.67	3.67	3.67	3.67
Bell UH-1 Iroquois	3.62	3.62	3.62	3.62	3.62	3.62	3.62	3.62
Sikorsky SH-60 Seahawk	5.76	5.76	5.76	5.76	5.76	5.76	5.76	5.76
Eurocopter EC225 Super Puma	5.92	5.92	5.92	5.92	5.92	5.92	5.92	5.92
Boeing Vertol CH-46 Sea Knight	6.90	6.90	6.90	6.90	6.90	6.90	6.90	6.90
Westland Sea King	8.05	8.05	8.05	8.05	8.05	8.05	8.05	8.05
AgustaWestland AW139	8.40	8.40	8.40	8.40	8.40	8.40	8.40	8.40
AgustaWestland CH-149 Cormorant	9.18	9.18	9.18	9.18	9.18	9.18	9.18	9.18
Lockheed P-3 Orion	28.75	28.75	28.75	28.75	28.75	28.75	28.75	28.75

Table.b relationship of planes and scores

Probability(*300) / Place								
1	0	0	0	0	0	0	0	0
1	1	1	1	1	1	1	1	1
3	0	0	0	0	0	0	0	0
4	62	62	62	62	62	62	62	62
5	153	153	153	153	153	153	153	153
6	62	62	62	62	62	62	62	62

7	15	15	15	15	15	15	15	15
8	7	7	7	7	7	7	7	7
9	0	0	0	0	0	0	0	0