For office use only
T1 ________________
T2 ________________
T3 ________________
T4 ________________

Team Control Number
49274

Problem Chosen
A

For office use only
F1 ________________
F2 ________________
F3 ________________
F4 ________________

2016

MCM/ICM

Summary Sheet

Summary

Our goal is to establish a model which can simulate heat transfer in space-time under different temperatures and minimize the amount of water needed. We analyze dominant factors of 3 aspects, all of which can make great influence on heat transfer and the amount of water needed. Then, based on these factors, we determine multiple criteria to judge what temperature can save more water and make bathers feel more comfortable using a fuzzy synthetic evaluation (FSE).

In tackling question 1, we discuss the internal and external of the bathtub. As for the external, we see the bathtub as a whole, only considering the heat exchange between the top surface of bathtub and air and that between inflows and outflows. As for the internal, to express our heat transfer space-time model more directly, we separate time from the model to reduce the model dimension to 3. Then, we divide the three-dimension space into numerous vertical and horizontal planes to reduce the model dimension to 2. Besides, we do grid division on these planes. Further, we do analysis of heat transfer on these planes, take these heat transfer models as rules and apply the rules to our redesigned cellular automaton to do simulations. Finally, we can obtain heat transfer diagrams of each plane at different times and the minimum gross amounts of water needed under 3 different temperatures.

In tackling question 2, we extend the basic model in question 1 and take it as our optimized object. We consider dominant factors of 3 aspects: shape and volume of bathtub, shape and motions of body and heat dissipation, bubble agent. For each aspect, we adjust heat transfer model in question 1 and add new relations between temperature change and other factors. Moreover, we take these relations as new rules and apply them to our cellular automaton to do simulations. Finally, we obtain the change of the minimum gross amount of water needed under 3 different temperatures when influenced by various factors. Meanwhile, we get the reasonable tub size, body motions and the amount of bubble agent.

Temperature controlling is complicated, since it is impossible for us to adjust water inflows manually and real-timely to keep water temperature even and save water as much as possible. Therefore we need an automatic electronic temperature control system, but it remains to be seen whether it can be applied to the bathtub. In further discussion, we use a fuzzy synthetic evaluation to evaluate the optimal temperature for baths, and set this temperature as the setting temperature of temperature control sensor. Furthermore, we study and apply Fuzzy PID Model based on Grey prediction, and consequently prove the feasibility of an automatic control system. Finally, we conclude that it is not advisable for users to adjust temperature manually because of inaccuracy. Although it costs a certain amount of money to install automatic control system, we can gain economic profit and higher comfort level in the long run owing to the accurate control of the minimum gross amount of water.

Temperature Defender

Summary

Our goal is to establish a model which can simulate heat transfer in space-time under different temperatures and minimize the amount of water needed. We analyze dominant factors of 3 aspects, all of which can make great influence on heat transfer and the amount of water needed. Then, based on these factors, we determine multiple criteria to judge what temperature can save more water and make bathers feel more comfortable using a fuzzy synthetic evaluation (FSE).

In question 1, we discuss the internal and external of the bathtub. As for the external, we see the bathtub as a whole, only considering the heat exchange between the top surface of bathtub and air and that between inflows and outflows. As for the internal, to express our heat transfer space-time model more directly, we separate time from the model to reduce the model dimension to 3. Then, we divide the three-dimension space into numerous vertical and horizontal planes to reduce the model dimension to 2. Besides, we do grid division on these planes. Further, we do analysis of heat transfer on these planes, take these heat transfer models as rules and apply the rules to our redesigned cellular automaton to do simulations. Finally, we can obtain heat transfer diagrams of each plane at different times and the minimum gross amounts of water needed under 3 different temperatures.

In question 2, we extend the basic model in question 1 and take it as our optimized object. We consider dominant factors of 3 aspects: shape and volume of bathtub, shape and motions of body and heat dissipation, bubble agent. For each aspect, we adjust heat transfer model in question 1 and add new relations between temperature change and other factors. Moreover, we take these relations as new rules and apply them to our cellular automaton to do simulations. Finally, we obtain the change of the minimum gross amount of water needed under 3 different temperatures when influenced by various factors. Meanwhile, we get the reasonable tub size, body motions and the amount of bubble agent.

Temperature controlling is complicated, since it is impossible for us to adjust water inflows manually and real-timely to keep water temperature even and save water as much as possible. Therefore we need an automatic electronic temperature control system, but it remains to be seen whether it can be applied to the bathtub. In further discussion, we use a fuzzy synthetic evaluation to evaluate the optimal temperature for baths, and set this temperature as the setting temperature of temperature control sensor. Furthermore, we study and apply Fuzzy PID Model based on Grey prediction, and consequently prove the feasibility of an automatic control system. Finally, we conclude that it is not advisable for users to adjust temperature manually because of inaccuracy. Although it costs a certain amount of money to install automatic control system, we can gain economic profit and higher comfort level in the long run owing to the accurate control of the minimum gross amount of water.

Contents

1. Introduction

A bathtub is a kind of container for people's baths. People can take a bath in it.[1]Typically, healthy and appropriate baths do a lot of good including promoting blood circulation, removing body toxin, relieving pressure, etc. However, the hot water constantly flowing into the tub will be increasingly cooler in three modes---- heat conduction, heat convection and heat radiation, which will transfer heat into all directions and consequently influence the feelings in body and the health of people.[2]Of course, there are more influencing factors of water temperature than heat transfer ways. The shape, volume and temperature of the person in the bathtub, the motions made by the person, the shape and volume of the tub and other substances in water (bubbles) are also the influencing factors. Thus, it will be exhilarating if we can design an optimal water-saving model under the premise of ensuring comfort and a water-saving model that has maximum stability and can be popularized. We take different factors into consideration in the former model, so we obtain the optimal strategies under different circumstances. The latter model integrates all the influencing factors.

In this paper, we simulate the changing process of water temperature when influenced by various factors mentioned-above and compare them so as to decide the optimal strategy of adding hot water.

1.1 Restatement

We are required to build a mathematical model to analyze the changing process of water temperature when affected by different factors and get the optimal water-saving model. We decompose the first problem into two sub-problems:

- See the bathtub as a whole, analyze the temperature change caused by the water exchange with external environment.
    - The heat exchange between inflows and outflows
    - The heat exchange between surface water and air
- Analyze the temperature change inside water
    - The heat exchange caused by horizontal water flows
    - The heat exchange caused by vertical water flows
    - The heat exchange caused by water diffusion
    - The heat exchange caused by water convection

As for the second problem:

- After analyzing the influencing factors, adjust the basic model in problem 1, and add new relations of heat transfer
- Take adjusted basic model as new rules and apply them to our cellular automaton to do simulations. Finally we get the change of the minimum gross amount of water under 3 different temperatures when influenced by different factors.

In the whole process, we also propose a mathematical criterion to determine the minimum gross amounts of water when influenced by different factors. This is our main goal. It is impossible for users to adjust temperature manually because of inaccuracy. We need an automatic electronic temperature control system. Although it costs a certain amount of money to install automatic control system, we can gain economic profit and higher comfort level in the long run owing to the accurate control of the minimum gross amount of water.

In the first step, we seek to build a model to give a quantitative expression of heat transfer, then take the results as inputs and input them into our model. The inputs include the temperature of bathwater and other factors. Most importantly, the model should better satisfy the demand of saving water. Then we can change the inputs to do several simulations. Also, we compare and analyze different results by charts, make multiple appropriate rules and boundary conditions and apply them to our redesigned Cellular Automata simulation. Eventually, we can obtain the outputs of our Fuzzy PID model when it reaches dynamic balance state.

In the second step, we seek to use the influencing factors to propose a mathematical criterion to evaluate our results and assess our model. We will consider the shape and volume of the bathtub, the shape and volume of the person in the tub, the bubbles, etc. Besides, we will design other changing rules to determine which output is the best.

Then we attempt to adjust our model and provide people with the optimal strategies of adding hot water when they take a bath. We also consider the influence of intelligent system.

1.2 Literature Review

A model for the simulation of water temperature changing with time and space is unavoidable to the study of the fluid thermodynamics. Canadian thermodynamics Ron Weir, chemical thermodynamics Trevor Letcher, et al mentioned several aspects on the motion of heat in fluids: horizontal flows, vertical flows, heat radiation, heat convection, the effect of diffusion.[3]

To facilitate our study on these aspects, it is necessary for us to know the concrete definition of a fluid and the basic formulas of its motions. In physics, a fluid is a substance that continually deforms under an applied shear stress. Fluids are a subset of the phases of matter and include liquids, gases, plasmas and, to some extent, plastic solids. Fluids can be defined as substances that have zero shear modulus or in simpler terms a fluid is a substance which cannot resist any shear force applied to it.[4]

Having learnt about the relevant aspects of fluid thermodynamics, we must master the relevant basic knowledge. For example, heat equation is a vital partial differential equation in thermodynamics, which can describe how the water temperature changes with time in a region.[5] Certainly, boundary conditions, non-ideal conditions and relevant particle diffusion model should also be considered.

As a matter of fact, we still need to take account of many other factors. For instance, the irregular motions made by the person, in analogy with stir, can influence the contact area of water-air and the kinetic energy of bathwater, thus influencing a series of factors.[6]

2. Assumptions and Justifications

To simplify the problem, we make the following basic assumptions, each of which is properly justified.

- **A bathroom is a closed environment.** Usually, people close doors and windows of the bathroom to keep warm or for other reasons when they are going to take a bath.
- **The bathtub is initially full of water.** People will fill the tub with water before they settle in.
- **The bottom surface of bathtub is flat with no radian.** Bathtubs in the market have all kind of shapes. In our model we only consider bathtubs with a flat bottom to simplify our model.
- **Water flows are stable.** They will not change sharply.
- **The person in the bathtub is an adult with normal weight.** Typically, regular baths are not suitable for juveniles because of their delicate and sensitive skins. Besides, the majority of people have a normal weight.
- **The person in the tub will not stand when taking a bath.** Normally, the person only exposes shoulders, neck, head and arms in the air.
- **Heat transfer between bathwater and bathtub is left out in our model.** Nowadays most bathtubs are made of thermal insulation material so they are good at holding temperature.
- **The person in the tub is a normal person.** To make our model more popular, we set the parameters of the person as normal.

3. Notation

All the variables and constants used in this paper are listed in Table 1 and Table 2.

Table 1 Symbol Table–Constants

Symbol	Definition	Units
	Constants	
C	Specific heat capacity of water	$\mathrm{J}/(\mathrm{kg}\cdot\mathrm{K})$
k_0	Coefficient of heat transfer	$W/m^2\cdot K$
Δk_1	The change of coefficient of heat transfer caused by body motions	$W/m^2\cdot \mathrm{K}$

Δk_2	The change of coefficient of heat transfer caused by bubble agent	$W/m^2 \cdot K$
k	Heat transfer coefficient	$W/m^2 \cdot K$
λ	Thermal conductivity	$W/m \cdot K$
D_m	Diffusion coefficient of water molecule	m^2/s
E	Turbulent diffusion coefficient	m^2/s
τ	Time period	
ρ_0	Density of water at 0℃	kg/m^3
$V_{capacity}$	Capacity of the bathtub	L
a	Coefficient of relationship between density and temperature	unitless
b	Coefficient of relationship between density and temperature	unitless
Δ_y	Changing rule of cross-sectional area	unitless
μ(k)	The control function of time k	unitless

Table 2 Symbol Table–Variables

Symbol	Definition	Units
	Variables	
ρ	Density of bathwater	kg/m^3
ρ'	density of inflows	kg/m^3
$\Delta\rho$	Change of density caused by bubble agent	kg/m^3
q	Outflow for unit tub height	m^3/s
q'	Inflow for unit tub height	m^3/s
Q_y	Net flow through plane y in unit period	m^3/s
Q_v	Heat transfer rate	W
Q_1/Q_1'	Difference of energy between inflws and outflows	W
Q_2/Q_2'	Heat loss caused by natural cooling	W
Q_3/Q_3'	Heat transfer caused by vertical flows	W
Q_4/Q_4'	Heat change caused by heat diffusion	W
Q_5/Q_5'	Temperature change caused by heat change	W
Q_6	Heat flow rate in the heat transfer process	W
Q_7	Heat loss caused by body heat radiation	W
Q_p	Heat exchange between body and water	W
Q_{total}	Total energy produced in period τ	W
T	Water temperature in tub	℃
T'	Temperature of inflows	℃
T_p	Normal human body temperature	℃
T_0	Environment temperature	℃
T_{zero}	*Temperature of top surface when t=0*	℃

S_p	Total surface area of human body	m^2
$S_{p'}$	Water-body contact area	m^2
H_p	Height of bather	m
W_p	Weight of bather	kg
A	Heat transfer area	m^2
A(y)	Surface area of top water surface at the height y	m^2
V_{min}	The minimum gross amount of water	L
$\frac{dT}{d\tau}$	Rate of heat dissipation	J/s
M	Energy metabolic rate	W/m^2
W	Mechanical work made by body	W/m^2
C	Heat loss by convection from body surface	W/m^2
R	Heat loss by radiation from body surface	W/m^2
E	Heat loss caused by sweat and breaths	W/m^2
S	Human body heat storage	W/m^2
N	Gross amount of times for adding water	unitless
V_{min}	The minimum amount of water added of each time	unitless
G	Degree of satisfaction of human	unitless
u	minimum value of impact factors	unitless
a_{ij}	Element in matrix of impact factors	unitless
R	Fuzzy evaluation matrix	unitless
r_{ij}	Element in fuzzy evaluation matrix	unitless
s_j	Standard deviation of the j column in R	unitless
$\bar{x}_j$	Average of the j column in R	unitless
v_j	Ratio of s_j to $\bar{x}_j$	unitless
w_j	Weight of v_j in column j	unitless
w	Weighted vector	unitless
F_i	Relative deviation of influencing factor	unitless
e(k)	Error	unitless
Δe(k)	Change rate of error	unitless
δ	Proportion band	unitless
T_i	Integral time	unitless
T_d	Derivative time	unitless

4. Model Overview

Most research for water heat transfer can be classified as microscopic and macroscopic. To study this problem in greater detail, we also adopt microscopic and macroscopic techniques. We seek to express the change of temperature with time and space in detail, and express the fluctuation when the model is influenced by multiple factors.

Our basic model for question 1 has a close look at the temperature change with time and space when influenced by multiple fixed factors. We adopt descending dimension method. First we separate time from the model to reduce model dimension to 3. Second we divide a 3D space into countless horizontal planes and vertical planes, and focus on the temperature change with time. Most importantly, we analyze and quantify the external and internal heat transfer. Then we design multiple appropriate rules and boundary conditions using the quantified results, and apply them to our redesigned cellular automaton. This model gives us some intuition about the situation of multiple influencing factors and serves as a stepping stone to question 2.

When tackling question 2 we extend the basic model in question 1 to view the problem from a broader perspective. We consider the influence of multiple factors on the basic model and the extent of the influence, and then we adjust them. Consequently, we apply the adjusted model to cellular automaton and do simulations. Finally, we get the minimum gross amounts of water needed throughout baths.

Our refined model attempts to tackle a more realistic while more challenging problem. We get the optimal temperature among the three initial setting temperature of cellular automaton by a fuzzy synthetic evaluation. we apply it to fuzzy PID control model based on Grey prediction. Finally, it proves to be true that an automatic electronic control system is suitable for our temperature control model of bathtubs. For the economic profit and relatively high comfort level in the long run, controlling temperature manually is not suggested.

From question 1 to question 2, our model is adjusted and refined gradually. Besides, our model combines with other mature models. Finally, we get the goals of saving water and enhancing the comfort level.

5. Basic Space-time Model of Temperature

We make a basic hypothesis in this model, that is, the heat change calculated by the temperature difference between the beginning and the end represents the gross heat change in the whole process. To simplify our model and provide convenience for subsequent studies, we take no account of secondary factors such as the bathtub, motions made by the person these factors. In addition, we make some assumptions according to this basic model.

- **The water temperature keeps even in the process of falling down from the faucet.** The faucet of a normal bathtub is usually 20-30 cm higher than the water surface when the tub if full. Considering the temperature of the bathroom is relatively high, we take no account of heat exchange with the air in the falling down process. This assumption can be justified by conservation of energy and heat equation.

- **The bathwater viscosity is infinite, namely the water surface is always calm.** The water viscosity is infinite, namely the water surface is always calm. In fact, when water flows into bathtub, it will make waves in the tub. The kinetic energy of the inflows mainly transfers to the waves but hardly transforms. The energy loss is less so we make this assumption.
- **The shape of bathtub is a cylinder whose upside and underside have the same area.** Actually, the shapes of bathtubs are diverse. However, we need to make this assumption for the purpose of designing a model.
- **Take the concept of micro-unit into our model, use a cylinder of low height when calculating. Figure1** shows a micro-unit taken out from a bathtub model according to the contour line of the bathwater.

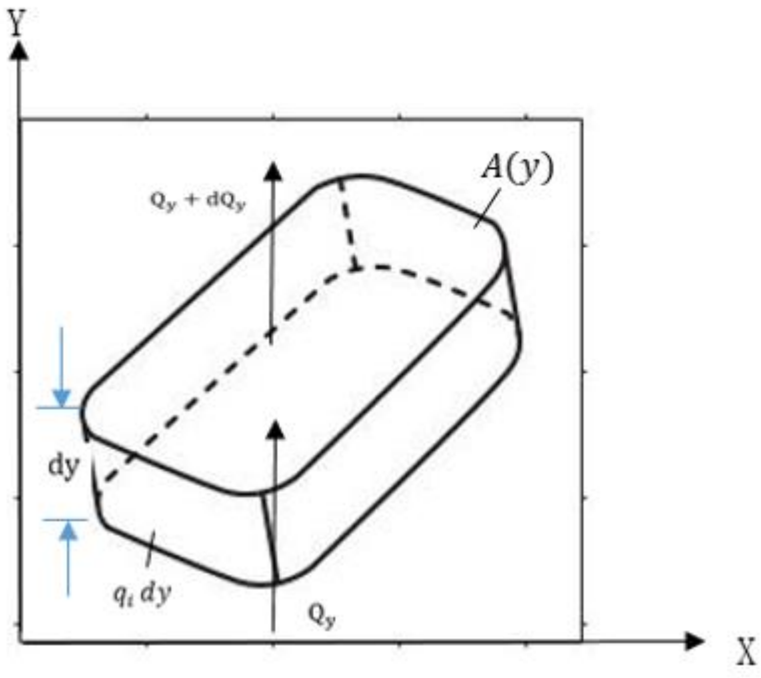


Figure 1 A micro-unit of the tub

- **We only consider the heat transferred from high temperature objects to low temperature objects.** Heat transfer is a two-way dynamic process. There is some heat transferred from low temperature objects to high temperature objects but it can be neglected because it is extremely little.
- **The size of bathtub is 1500*700*420, and the capacity of bathtub is 1000*450*340.**[1]

The bathwater temperature change is the result of internal and external heat exchange. Thus, the establishment of the model can be divided into two steps.

In the first step, we consider the heat exchange with the outside air:

- The heat brought by the inflows
- The heat loss in the form of radiation

In the second step, we take internal heat exchange into account:

- The heat exchange caused by horizontal water flows
- The heat exchange caused by vertical water flows
- The heat exchange caused by water convection
- The heat exchange caused by water diffusion

With regard to the requirement of never wasting too much water, we first study the way of adding water. This paper discuss further on two-dimension based on vertical temperature diffusion model by Dake and Harleman. [7]We make the time of the whole process τ. Then we study the heat change of every micro-unit per unit of time. Finally, we do integral of time and obtain a model of time.

5.1 Determine the water feeding mode

In order to keep the temperature even, we add a constant trickle of hot water from the faucet to reheat the bathwater. Meanwhile, there is an overflow drain from which water escapes when the tub is full. Undoubtedly, if the flow of the faucet is large, water will be wasted. Therefore we need to find a proper feeding mode of hot water to keep the water temperature even as far as possible and save water in the meantime.

According to the heat equation Q=CmΔT, we can get

$$Q_1 = (C\rho' q' T' - C\rho q T)dy \tag{1}$$

where C is the heat capacity of water, ρ' is the density of inflows, q' is the amount of inflow water that can fill unit height of the tub, T' is the temperature of the inflows, ρ is the density of the water in the tub, q is the amount of outflow water that can fill unit height of the tub and T is the temperature of the water in the tub.

Combining with our goal, we should keep q' as low as possible and keep Q_1 as high as possible. To achieve our goal, we enhance the temperature of the inflow water T', but infinitely enhancing the temperature will do great harm to body.

5.2 The external heat exchange

(1) In real terms, the external exchange is caused by the heat of inflows and outflows. We can get the heat difference caused by inflows and outflows using equation1.

(2) We can calculate the heat loss of exchange with air according to Newton's Law of Cooling. Newton's Law of Cooling states. The objects whose temperatures are higher than that of the environment will transfer heat to the environment and cool down. There is an equation according to the law.

$$\frac{dT}{d\tau} = k_0(T - T_0) \tag{2}$$

It indicates that the rate of heat loss is in direct proportion to the temperature difference between the environment and objects when the objects are in natural cooling state.

In the equation, $\frac{dT}{dt}$ is the rate of heat loss, T is the temperature of surface water in the tub, T_0 is the room temperature, k_0 is the coefficient of heat transfer which is influenced by heat-transfer surface finish, surface temperature and environment temperature. Do some calculation with equation 2 we get:

$$k_0 = \frac{ln\left(\frac{T-T_0}{T_{zero}-T_0}\right)}{t} \tag{3}$$

T_{zero} is the water temperature of upside surface when time is t = 0.According to the measurement we obtain $k_0 = -0.0328$

Finally, we get the equation of water temperature and time in the changing process under natural condition (room temperature is 20℃):

$$T = 20 + 20e^{-0.0328*\tau} \tag{4}$$

And the curve is showed in **Figure 2**.

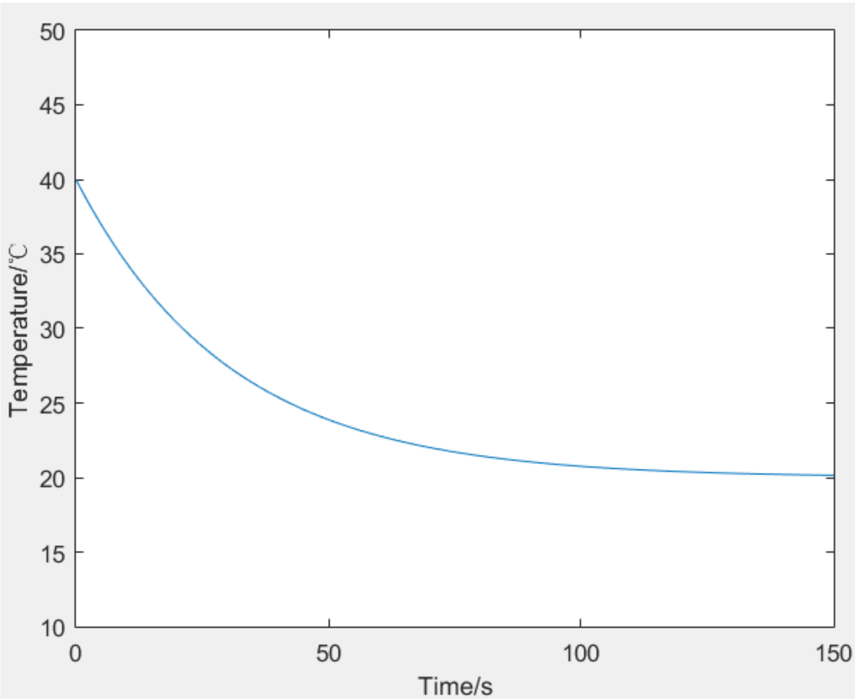


Figure 2 The curve of cooling process

We can find that the temperature of high-temperature objects tend to be environment temperature.

The heat loss of micro-unit caused by natural cooling per unit time is:

$$Q_2 = -C\rho AT\,dy \tag{5}$$

A is the area of horizontal plane at the height of y.

5.3 The internal heat exchange

First, we separate time from our model and consequently get a 3-D space. Then we divide the space into numerous vertical and horizontal 2-D planes. In that way, we can simplify the model and do some analysis.

According to the Fourier Law, the temperature distribution of objects satisfies an function related to space and time, that is, $T = f(x, y, z, \tau)$. On a horizontal plane, we only consider two-dimension temperature field, so we can simplify the function like this T= $f(x, \tau)$. **Figure 3** shows the temperature gradient.

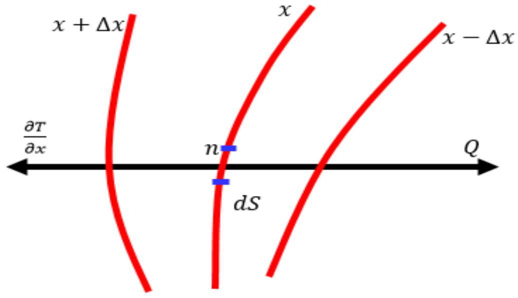


Figure 3 Temperature gradient

In the two-dimension temperature field, the temperature of isothermal surface x is $t(x, \tau)$, the temperature of isothermal surface $x + \Delta x$ is $t(x + \Delta x, \tau)$. See **Figure 4**.

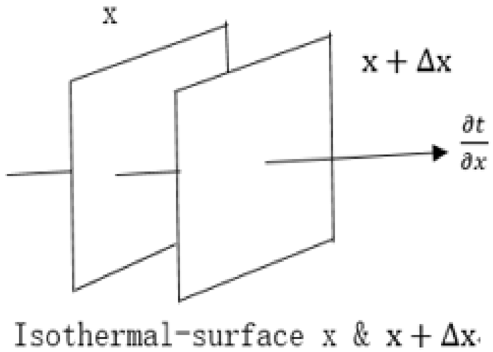


Figure 4 Isothermal surface x and x+Δx

The average temperature change between the two isothermal surfaces can be expressed as:

$$\frac{T(x+\Delta x,\tau)-T(x,\tau)}{\Delta x} \tag{6}$$

The gradient:

$$gradt = \lim_{\Delta x \to 0} \frac{T(x+\Delta x,\tau)-T(x,\tau)}{\Delta x} = \frac{\partial T}{\partial x} \tag{7}$$

We can use the equation of the Fourier Law as for the bathwater in the tub.

$$dQ_v = -\lambda dA \frac{\partial T}{\partial x} \tag{8}$$

Q_v is the rate of heat transfer, λ(W/m k) is the thermal conductivity, and A is the area of heat transfer surface.

If there are two points on the same horizontal plane but on different isothermal surfaces, heat transfer will happen. The thermal conductivity of water is 0.62, then the quantity of transferred heat per unit time can be calculated by:

$$Q' = -0.62 * A \frac{\partial T}{\partial x}$$

(9)

It is precisely because the heat transfer law that the water temperature can become uniform gradually under static condition.

Moerover, the inflows have a vertical speed which will cause vertical flows in bathwater, and consequently influence the heat of water. To simplify the model, we only consider up-down water flows. Assume in a unit period of time the net flow through the upside surface of micro-unit is Q, the heat change in micro-unit caused by vertical flows is

$$Q_3 = -C\rho QT\,dy \tag{10}$$

Because of heat diffusion, the heat loss of micro-unit is

$$-C\rho A(D_m + E)\frac{\partial T}{\partial y} \tag{11}$$

So, in a unit period, the heat change in bathwater is

$$Q_4 = \frac{\partial}{\partial y}\left[C\rho A(D_m + E)\frac{\partial T}{\partial y}\right]dy \tag{12}$$

where $D_m$is the diffusion coefficient of water molecules, E is the turbulent diffusion coefficient.

The heat change will cause the temperature change

$$Q_5 = -C\rho\frac{\partial T}{\partial \tau}A\,dy \tag{13}$$

According to the thermal balance, we know

$$Q_1 = Q_1 + Q_2 + Q_3 + Q_4 + Q_5 \tag{14}$$

So in the time period τ, the heat change is

$$Q_{total} = \int_0^{\tau} Q dt \tag{15}$$

As for the water in normal state, we believe that the water cannot be condensed. The relationship between density ρ （kg/m^3） and temperature T can be expressed as

$$\rho = \rho_0 + aT^2 + bT \quad (T \geq 0) \tag{16}$$

where ρ_0 is the density when the temperature is 0 ℃ , $\rho_0 = 999.87$, a and b are constant, a=-0.0067,b=0.07.

Heat exchange will happen between water with different temperatures. By equation 10 we can get

$$\frac{\partial \mathrm{T}}{\partial \tau} = \frac{q'}{A}(T' - T) - T\left(\frac{Q}{A} + 1\right) + (D_m + E)\frac{\partial^2 T}{\partial y^2} \tag{17}$$

This equation is exactly the control equation of water temperature control on two-dimension temperature field.

5.4 Application of cellular automaton

We apply this model to our redesigned cellular automaton. First，we explain the algorithm. Then, we introduce our additional work in the refined model. Finally, we can get the diagrams of the change of the temperature with space-time.

5.4.1 Initialize the input values

The input values include the temperature of the inflows, the amount of water added into the tub each time, etc. The temperature of inflows is related with the amount of water used throughout the bath. The amount of water added each time is related with the time taken for uniform temperature in the tub. Moreover, the time is related with the body comfort level. The optimal body comfort level is that the person in the tub never feels ”cold” throughout the bath. Therefore, we need to obtain the optimal temperature of inflows, the optimal amount of water added each time, the interval of adding water, etc.

5.4.2 Determine the standard temperature for judgment in the program

We set the input standard temperature between38℃ and40℃, then we set different standard temperatures in the iterative operation, and finally we determine the optimal standard temperature. [8]

There are two main considerations:

- The influence of standard temperature on body comfort level
- Reliable temperature of bathwater

5.4.3 Calculate the temperature change at different time points in continuous iterative

The time and space model of water temperature can reach the complicated stage of four-dimension, so we must reduce the dimension of the model twice. We divide it into countless two-dimension horizontal and vertical planes to do grid partition. We think space can be diveded into countless extremely small cubes with a size of 1x1. We believe the water temperature in the extremely small cube is uniform. Heat horizontally diffuses towards the 4 vertical planes and vertically diffuses towards upside and downside horizontal planes. We use the conclusions obtained from the model mentioned-above to do calculations of these cubes. Additionally, we call these cubes “point” in the following pages.

- calculate the water temperature change of everywhere on the inside horizontal plane
- calculate the water temperature change of everywhere on the inside vertical plane
- calculate the water temperature change of everywhere on the vertical water-air contact area

5.4.4 Modeling Using Periodic Boundary Conditions

To run a cellular automaton, we have to decide the boundary conditions and the initial state.

First, on the boarder of points we need elements like the heat transfer model above and the standard temperature, etc. Of course, because we have massive differential equations in our conclusions, we still need to decide how to process them when every point transfers heat towards five directions. For this purpose, we should more deeply understand Neumann boundary condition [9] and Dirichlet boundary condition.[10]

Second, we assume that when the water is added into the tub, the excess water escapes through the overflow drain immediately. So the amount of bathwater is in dynamic balanced condition all the time. Besides, when the water molecules diffuse on the boarder of point, there will be some molecules transfer from lower temperature to lower temperature, but we neglect this. Consequently, we can accurately define a system with constant amount of water, constant water temperature and unidirectional heat transfer towards five directions. In reality, heat transfer is a bidirectional process, but this still verifies the system. As we can imagine, the essence of heat transfer in this problem is the motion of water molecules, and the molecules take heat on the move. Most molecules transfer from higher temperature to lower temperature so only very few molecules transfer from lower temperature to higher temperature. Therefore, our assumption satisfies the periodic boundary conditions. Henceforth, we will study further from unidirectional heat conduction after simplifying the way of heat transfer.

Figure 5 is the flow chart of cellular automaton.

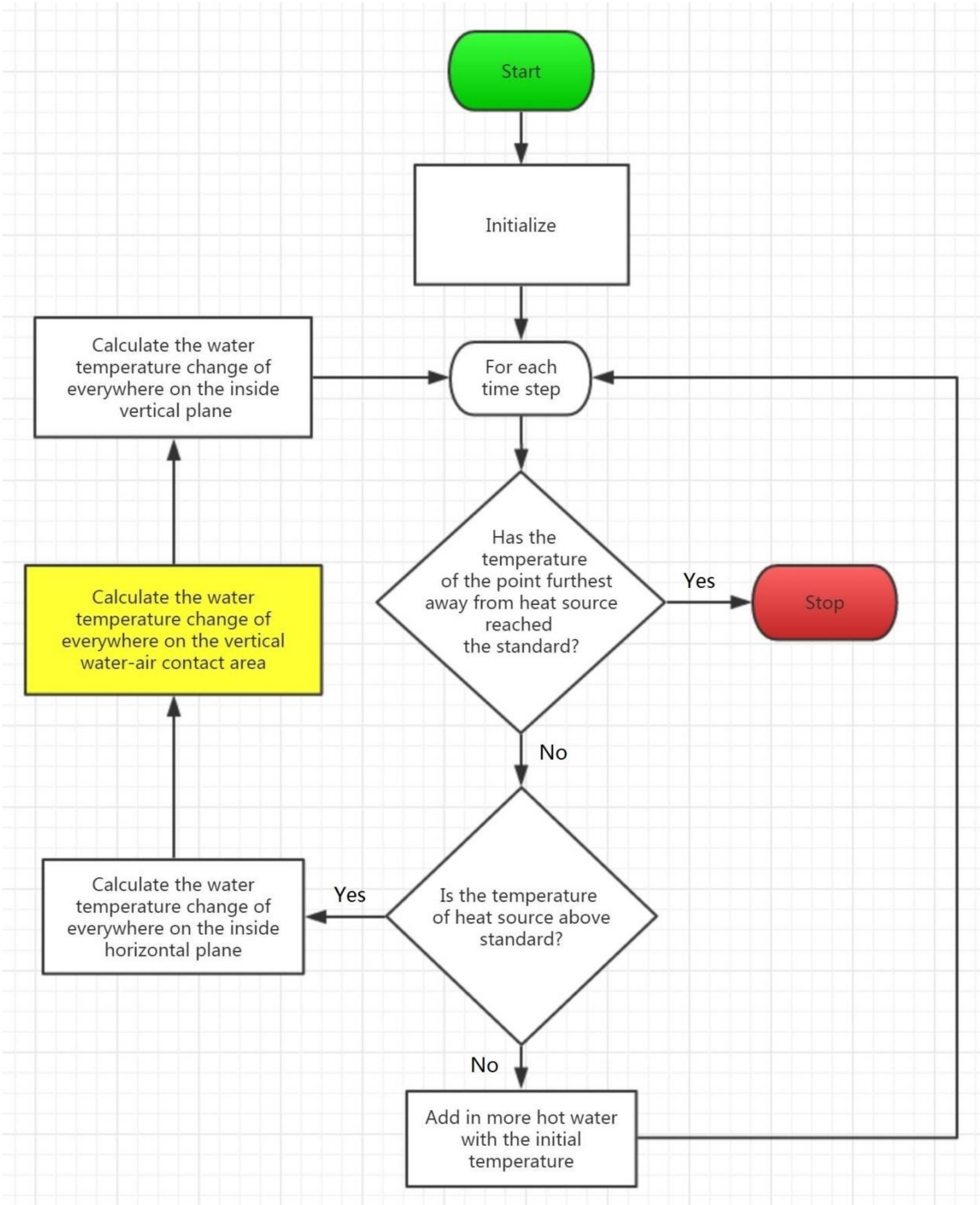


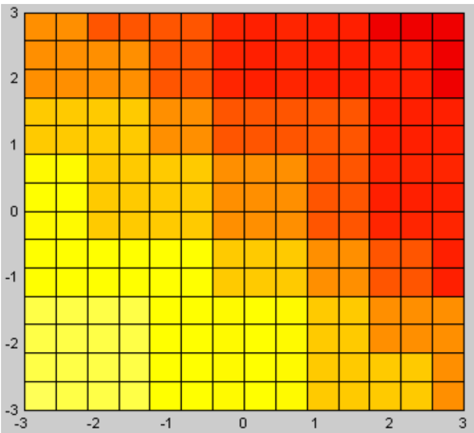
Figure 5 Flow chart of cellular automaton

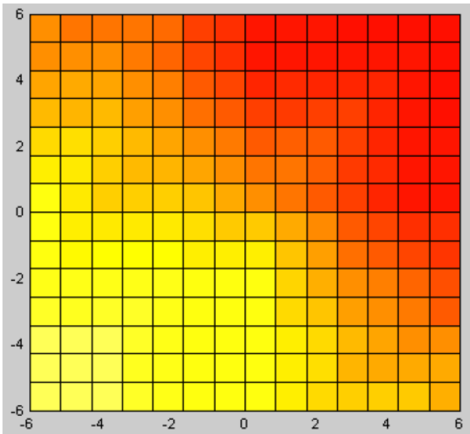
5.5 Results

In question 1, our primary goal is to obtain space-time model of temperature and save water as much as possible under the condition of making bather feel comfortable.

5.5.1 Two-dimension representation of the temperature change with space and time

In this section, we must get space-time model of temperature. To simplify and visualize the model, we reduce the dimension of the model to two-dimension horizontal planes and vertical planes. We set the standard temperature of cellular automaton as 39 ℃ and set the temperature of inflows as 46℃ **Figure 6** is cellular automaton sketch of heat conduction after the first iteration. Of course, iteration at different time will cause different sketches. So the number of figures is infinite and in the end the temperature tends to be uniform (the abscissa and ordinate are only used to determine positions).



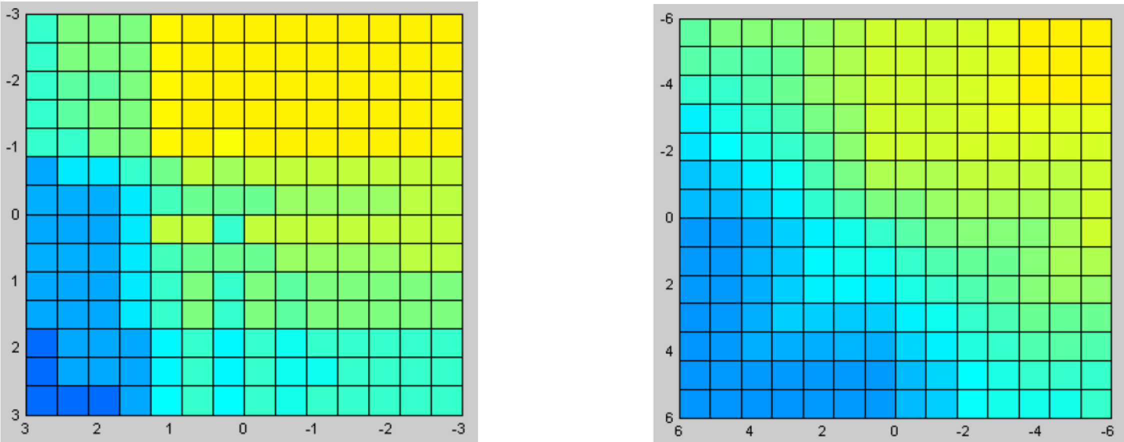


(a) Time space diagram (t=1, horizontal plane) (b) Time space diagram (t=1, vertical plane)

Figure 6 Two planes of the point at the heat source

From the two diagrams, we can see the heat source is located in the top right corner. In horizontal plane, heat transfers to 4 horizontal directions. In vertical plane, heat only transfers downward and with air. Obviously, the rate of heat conduction on vertical plane is bigger than that on horizontal plane. When heat is transferred to the sixth square in the right diagram, heat in the left diagram is transferred to the third square.

After the fifteenth iteration, namely the fifteenth unit interval comes to an end, the temperature change of the farthest point from heat source can be seen in **Figure 7.(** the abscissa and ordinate are only used to determine positions.**)**



(a) Time space diagram (t=15, horizontal plane) (b) Time space diagram (t=15, vertical plane)

Figure 7 Two planes of the point farthest from the heat source

From the two diagrams, dark blue represents the standard temperature (the most appropriate water temperature for baths). The in the right diagram is brighter than that in the left diagram, which indicates that more heat is transferred to this position. When the colors of left bottom in both of the two diagrams get brighter, the amount of hot water added in the whole process can reach the goal of keeping the temperature even, and the program terminates automatically. After a while, the color of the top right corner in either diagram gets darker. This means the bathtub needs more hot water and the program starts again.

5.5.2 Results: The minimum gross amount of water

We choose the minimum gross amount of water as the goal of our model. To judge the effectiveness of different influencing factors, we propose the criteria.

The minimum gross amount of water: our goal is to save water as much as possible. In this section, the volume and the shape of the bathtub are fixed so we only consider the amount of used water under different temperature. N is the number of times of adding hot water, $V_{capacity}$ is the capacity of the tub whose value is 153 L here, and $V_{interation}$ is the minimum of water inflow each time calculated by iteration. The minimum gross amount of water is V_{min}

$$V_{min} = N * V_{iteration} + V_{capacity} \tag{18}$$

(1) The temperature of water inflows: the temperature of inflows can influence the amount of water needed, and the temperature cannot be too high because excessively high temperature will do great harm to body and consume much electrical power. After searching online, we know that the setting temperature of household water heaters is usually about45℃. Thus we should determine an appropriate temperature within this extent.[11]

The determined input values are listed in **Table 2**

Table 2 Determined input values

The setting standard temperature of cellular automaton	The temperature of water inflows				
38	44	45	46	47	48
39	44	45	46	47	48
40	44	45	46	47	48

(2) Use iterative calculation method to get the minimum amount of water of each time. To guarantee the comfort level of body, we set that we must add hot water in advance and heat must have been transferred to the farthest point away from the heat source before the temperature of the farthest point is lower than the standard temperature. Under the condition of fixed temperature of inflows and fixed standard bath temperature, cellular automaton doesn't know at initialization what amount of water of each time can meet the condition we set. Therefore, we must run the iterative operation program repeatedly. Since the relatively big amount of water certainly can keep temperature even, we choose the amount from large to small to find the critical point. Finally, the minimum result is precisely the minimum amount of water needed of each time.

$$V_{minimum\ of\ each\ time} = V_{minimum\ iterative\ result\ of\ each\ time} \tag{19}$$

(3) The length of bath time: the length of bath time will undoubtedly influence the times of adding water throughout the bath. After searching online, we know that the bath time is determined by the water temperature. If the temperature is 40℃, the bath time can be 10 minutes; if the temperature is $37-38$℃, the bath time can be 20-30 minutes.[12] What we set is listed in **Table 3.**

Table 3 The input values

The setting standard temperature of cellular automaton	37	38	39	40
The length of bath time	25	20	15	10

Combine the criteria (1) (2) (3) with the mentioned-above time and space model of temperature, then we can obtain the optimal gross amount of water needed presented in **Figure 8.**

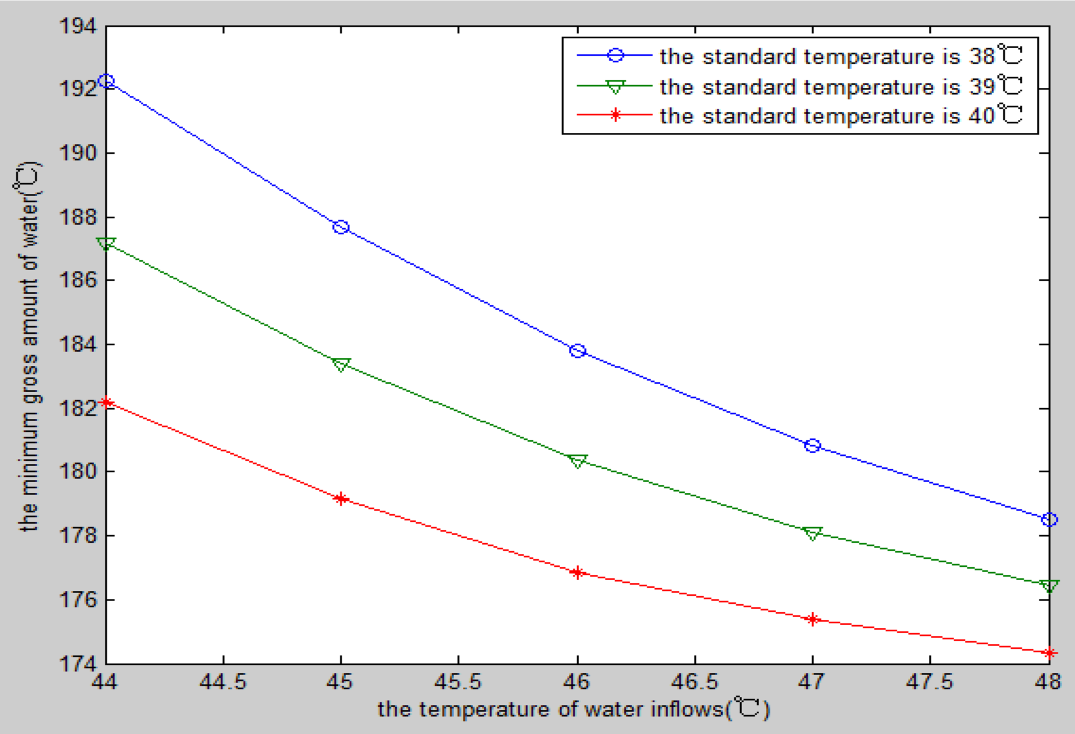


Figure 8 The optimal gross amount of water

From the diagram, we can see that there is negative correlation between the standard temperature and the gross amount of water. This is closely related with the length of bath time. But when the temperature of inflows is higher, the amount of water will not decrease in the same proportion. One of the factors of this phenomenon is water-air heat transfer model led into the cellular automaton. The general trend is that, the higher the water temperature is, the more severe the heat transfer with air is.

6. The Refined Model

Through the basic study on heat change above, we have a preliminary method of water temperature control to keep the water temperature even. However, since water temperature is affected by many factors, such as the shape and volume of the tub, the shape /volume/temperature of the person in the bathtub, the motions made by the person in the bathtub and so on, we should take these factors into account to make our model more reliable.

6.1 The influence of the shape and volume of the bathtub

The majority of bathtubs in the market are square, round or oval. A circle can be considered as an oval with the same minor axis and long axis so we need to consider only two kinds of tubs with different shapes. Now that we assume the initial state of tub is full, a higher capacity influences $V_{capacity}$ and the area of the top surface.

The shape of bathtub has great influence on heat loss. When the volume of the tubs is the same, the volume can be expressed as $\int_0^{y_0} yA(y)dy$ where y_0 is the height of the overflow drain, which determines the maximum amount of water in the bathtub. We assume that the tub is a truncated cone no matter the shape is square or round, but the cross sectional area obeys the change of Δ_y

(1) Square

$$A(y) = (a + \Delta_y a)(b + \Delta_y b) \tag{20}$$

a is the length of the square, and b is the width.

(2) Oval

$$A(y) = \pi(\frac{a}{2} + \Delta_y a)(\frac{b}{2} + \Delta_y b) \tag{21}$$

a is the minor axis of the oval, and b is the long axis.
A(y) is the area of water surface at the height of ***y***.

Figure 9 illustrates the relationship between the area of water surface and the height.

As we mentioned in our basic model, the heat loss caused by natural cooling is

$$Q_2' = -\frac{\partial}{\partial y}[C\rho y A(y)T]\,dy \tag{22}$$

Moreover, the bigger the air-water contact area is, the more heat loss is.

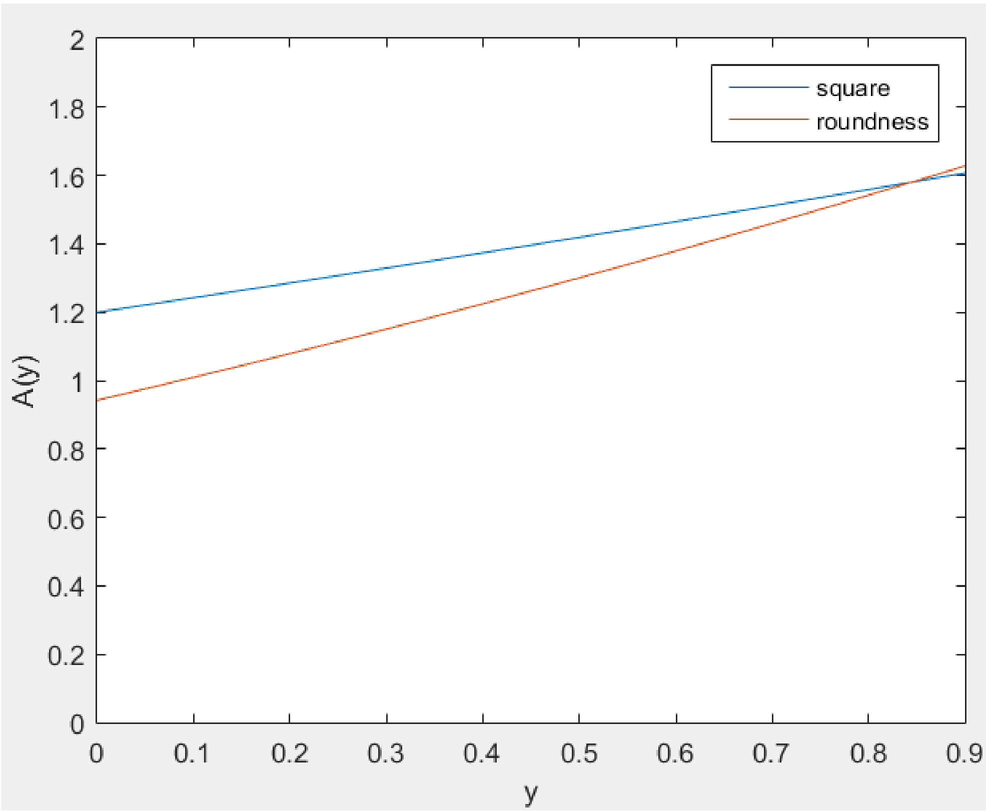


Figure 9 Relation between the area of water surface and the height

From figure 9 we can find that the surface area of oval is smaller when the height range is 0~0.85, so heat loss will be less. By this token, oval is more appropriate choice for the shape of tub.

Considering the surface area is different when y is different, we need to adjust our basic model.

If the net flow through plane ***y*** in a unit interval is Q_y, causing the heat change of $C\rho Q_y T + \frac{\partial}{\partial y}(C\rho Q_y T)\,dy$ in the micro-unit.

$$Q_3{}' = -\frac{\partial}{\partial y}(C\rho Q_y T)\,dy \tag{23}$$

Because of heat diffusion, the heat transferred in a unit interval through plane y is

$$-C\rho A(y)(D_m + E)\frac{\partial T}{\partial y} \tag{24}$$

That through plane $y + \Delta y$ is

$$-C\rho A(y)(D_m + E)\frac{\partial T}{\partial y} + \frac{\partial}{\partial y}\left[-C\rho A(D_m + E)\frac{\partial T}{\partial y}\right] \tag{25}$$

So the heat change in an interval caused by heat diffusion is

$$Q_4{}' = \frac{\partial}{\partial y}\left[C\rho A(y)(D_m + E)\frac{\partial T}{\partial y}\right]dy \tag{26}$$

The heat change in micro-units will cause the heat change in the bathwater.

$$Q_5{}' = -C\rho\frac{\partial T}{\partial \tau}A(y)\,dy \tag{27}$$

Assume the temperature of water inflows is 48℃ , and the fixed volume of bathtub is 0.153m^3. We add the oval shape into the rules of cellular automaton simulation. Then by iterative computing we get the curves that denote the change of the minimum gross amount of added water under 3 different standard temperatures when the top surface area changes. See **Figure 10**.

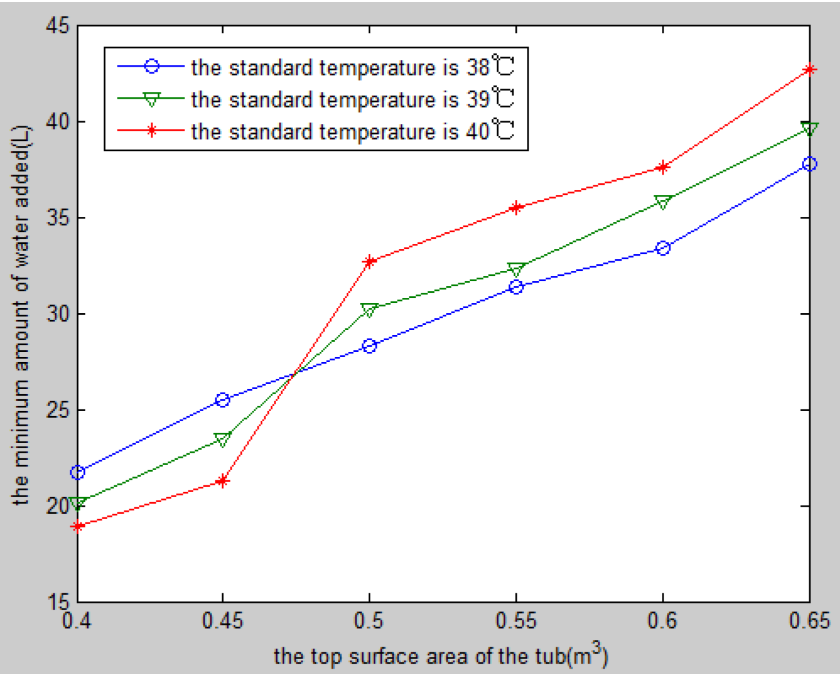


Figure 10 The curves of the change of water amount with top surface area

From the diagram, we know that under different standard temperatures the minimum amount of added water is roughly in direct proportion to the top surface area. At the intersection of the three curves, the top surface area is about $0.475m^3$. It is the most appropriate bathtub top surface area applied to popular bathtubs. So we select three optimal points:38℃: 25.5L, 39℃: 23.44L, 40℃: 21.24L.

6.2 The influence of shape and motions of bather and heat dissipation on water temperature

6.2.1 The influence of shape of bather on water temperature

The body shape mainly influences the total surface area. According to the equation for estimating the total surface area:

$$S_p = {H_p}^{0.725} \times {W_p}^{0.425} \times 0.007246 \tag{28}$$

S_p is the total surface area of human body. H_p is the height. W_p is the weight. According to statistics, the total surface area of a healthy person is about 1.5~$2m^2$

When bather is taking a bath, he only exposes the shoulders and above. Then the surface area in water is 1.35~1.86 m^2 . Considering heat exchange between water and skin,

$$Q_p = C\rho S_{p'}(\mathrm{T} - T_p) \tag{29}$$

where Q_p is heat exchange between water and body, and T_p is body temperature, $S_{p'}$ is the contact area..

Because human-beings are warm-blooded animals, heat exchange in human body is a dynamic balanced process. So we can consider that heat is released from human body. We can see it as a part of heat loss of the whole system.

$$Q_1' = (C\rho' q' T' - C\rho q T)dy - Q_p \tag{30}$$

6.2.2 The influence of the motions made by the person

When a person is taking a bath, he is not motionless. The body motions make waves on the water surface, consequently enlarge the water-air contact area and influence the cooling rate. In the meantime, they speed up the water flow and consequently speed up the convection velocity. Certainly, there are many other factors, such as the water loss and the change of air flow caused by the bather's motions. However, we only consider the two factors we mentioned first.

- **Assume the weight and volume and other physical properties of the bather are in line with standards.** To make our model more popular, we choose a standard body in our model.
- **Assume the amount of bathwater is invariable.** Because the bather will not have big range movement, compared with the amount of bathwater in the tub, the water loss is so little that we can neglect it.
- **Assume the bather's motions cannot cause any air flows.** Actually, every motion can affect the airflow. To simplify our model, we think the bather's motions cannot cause any air flows. In that way, they cannot cause any heat exchange between the bathwater and air.

- **Assume the bathwater doesn't generate any heat because of the motions.** The motions can transfer the mechanical energy to the bathwater, but the majority of the mechanical energy transforms into kinetic energy, so heat can be neglected.

The coefficient of heat transfer obtained from equation (3) is $\mathrm{k}_0 = -0.0328$, but the motions cause waves on the water surface, and consequently change the surface smoothness. k_0 is changed. Set the change as Δk_1. In the meantime, the water-air contact area is also changed. In the case of multiple changed factors, we recalculate the heat dissipation Q_2'.

$$\begin{cases} Q_2' = -\frac{\partial}{\partial y}[C\rho y A(y) T]\,dy \\ T = T_0 + \alpha e^{(\mathrm{k}_0 + \Delta k_1)\times\tau} \\ \mathrm{A(y)} = \pi[\left(\frac{a}{2} + \Delta_y \mathrm{a}\right)\left(\frac{b}{2} + \Delta_y \mathrm{b}\right) + \Delta A \end{cases} \tag{31}$$

Besides, the motions made by the person in the bath are also helpful to the bathwater. They can speed up the velocity of water flows, namely enlarge the heat transfer coefficient. In that way, the bathwater temperature will become uniform more quickly. The heat flow rate in the heat transfer process is

$$Q_6 = A(y)k(T' - T) \tag{32}$$

$\mathrm{A(y)}$ is heat transfer area and k is heat transfer coefficient.

6.2.3 The influence of body temperature on water temperature

As for normal people, when the environment temperature is close to the body temperature, the heat release of human body will decrease with the environment temperature. When environment temperature is considerably higher than body temperature, the higher the environment temperature, the slower the heat release is; otherwise, the lower the environment temperature is, the quicker the heat release is. In a word, the speed of heat dissipation depends on the temperature difference between the body temperature and the environment temperature. Besides, if a person moves more quickly, the heat dissipation will accelerate.

Temperature constancy and heat balance of the human body satisfies:[13]

$$\mathrm{M - W - C - R - E - S = 0} \tag{33}$$

M (W/m^2) is energy metabolic rate of human body, which depends on the degree of activity. W (W/m^2) is mechanical work done by body. C (W/m^2) is the heat convection from body surface to the environment. R (W/m^2) is the heat radiation from body surface to the environment. E (W/m^2) is the heat taken by sweat and breaths. S (W/m^2) is the body heat storage capacity, which is zero on stable and normal physical condition. In this section, we simplify this equation to:

$$\mathrm{M - C - R - E = 0} \tag{34}$$

To sum up, we set the water temperature as 48℃ and set the capacity of the tub as $0.153m^3$. We take body motions and shape and heat dissipation model as the new rules and apply them to cellular automaton. Since the shape of body is in direct proportion to water-body contact area and the speed of body heat dissipation is in direct proportion to the degree of activity, and the degree of activity is in direct proportion to Δk_1, we make an analogy among them. Finally, we determine the correlation index of them and make an analogy. Then we get the relationship between the minimum amount of water and Δk_1. **See Figure 11**.

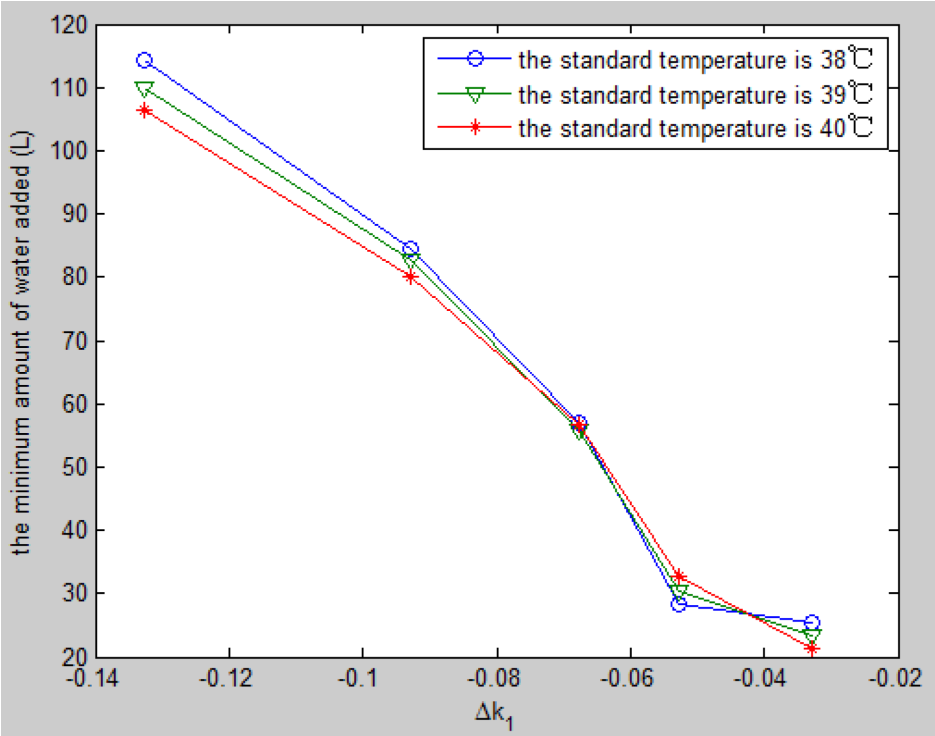


Figure 11 The relationship between the minimum amount of water and Δk_1

From the diagram, we find that the body motions and the heat dissipation have great influence on the water temperature. When the body motions are severe, the heat dissipation is quicker. That is, when Δk_1 is increasingly small, the heat dissipation will be increasingly quick. What's more, the heat dissipation replaces the standard temperature as the most important influencing factor of the minimum amount of added water. Therefore, for water saving concerns, it is not advisable to play in the tub. So we select 3 optimal points when Δk_1 ranges from -0.06to -0.04:
38℃: 28.305L, 39℃: 30.2376L, 40℃: 32.7096L

6.3 The influence of bubbles on the temperature

First, it is obvious that the bubble agent change the density of water. We set the change as Δρ, and then the density of water becomes ρ+Δρ. The heat exchange between water surface and air is

$$Q_1{}' = [C\rho' q' T' - C(\rho + \Delta\rho) q T] dy \tag{35}$$

Meanwhile, the bubble layer is equivalent to a layer of membranes. Detergent is a kind of interfacial agent. From molecule level, water molecules are surrounded by bubble agent. In that way, bubbles decrease the rate of evaporation and lower the room temperature because of their characteristics. So they break off the heat change to some extent. To see from a macroscopic perspective, the bubbles isolate the water surface from the air, so they change the coefficient of heat transfer. Set the change as Δk_2 and then Q_2' satisfies:

$$\begin{cases} Q_2' = -\frac{\partial}{\partial y}[C\rho y A(y) T]\, dy - Q_e \\ T = T_0 + \alpha e^{(\mathrm{k}_0 + \Delta k_2)\times\tau} \\ \mathrm{A(y)} = \pi[\left(\frac{a}{2} + \Delta_y \mathrm{a}\right)\left(\frac{b}{2} + \Delta_y \mathrm{b}\right) \end{cases} \tag{36}$$

Our overall goals will not change. That means the water temperature of the tub is constant throughout the bath. According to the energy conservation law

$$Q_{in} = Q_{out} \tag{37}$$

Therefore, finally we can get

$$\frac{\partial T}{\partial \tau} = \frac{q'}{A}(T' - T) + (D_m + E)\frac{\partial^2 T}{\partial y^2} - T\left(\frac{\partial}{\partial y}\left(\frac{Q_y}{A(y)}\right) + 1\right) \tag{38}$$

We set the water temperature as 48℃, set the capacity of the bathtub as $0.153m^3$. Then we take the bubble bath model as new rule and apply it into cellular automaton. Finally, we get the relationship between the minimum water amount and Δk_2. See **Figure 12.**

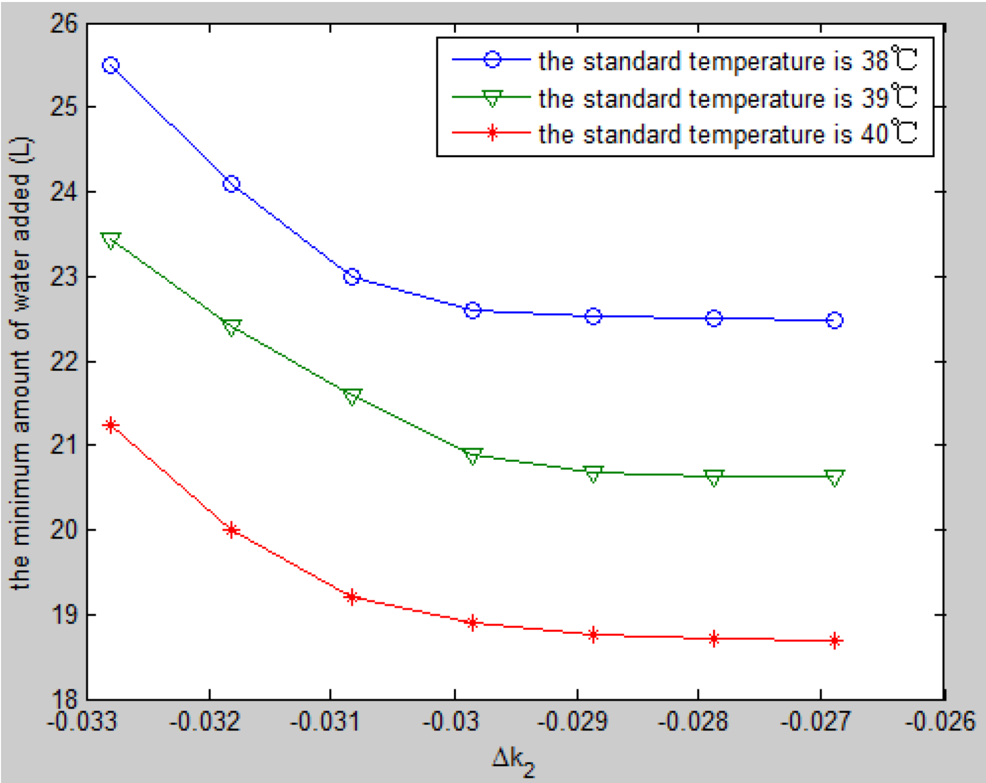


Figure 12 The relationship between the minimum water amount and Δk_2.

From the diagram, we find that when the bubbles reach a certain quantity, their influence on coefficient of heat insulation tends to be gentle and the insulating effect is restricted to the range of 8%~13%. So we recommend the amount of bubbles within the range of $-0.03 < \Delta k_1 < -0.029$. Therefore, we choose three optimal points:

38℃: 23.0L , 39℃: 21.6L , 40℃: 19.2L

7. Sensitivity Analysis

Some inputs of our model may be difficult to get or there might be some indeterminacy in our inputs. Both these kinds of deviation might affect the result of our model. For the purpose of testing the robustness of our model, we implement a sensitivity analysis. We test our model under different temperatures. Fortunately, the analysis proves that our model does not demonstrate a chaotic behavior, showing a good sensitivity.

In our model we set 3standard temperatures according to literature[8]. They are 38℃, 39℃ and 40℃. However, we did not discuss the deviations caused by temperature change near the range. Meanwhile, the setting temperature cannot be too high or too low because human body cannot bear. Therefore, we set 39℃ as the center, enhance temperature by 5%, 10% and lower temperature by 5%, 10% to obtain the changes in our criteria. We find that the bather factor increases by 17.7% when temperature is lowered by 15% and decreases by 7.5% when temperature is enhanced by 10%. We also find that the bubble agent factor increases by 13.8% when temperature is lowered by 5% and decreases by 8.5% when temperature is enhanced by 10%. The tub size has little change at relatively low temperature while bather factor and bubble agent have much change at both relatively high and low temperatures. **Table 8** lists the sensitivity analysis.

Table 8 The sensitivity analysis

Change of standard temperature change	Tub size	Bather factor	Bubble agent
-10%	5.0%	17.7%	7.2%
-5%	0.8%	3.9%	13.8%
0%	0.0%	0.0%	0.0%
5%	2.8%	2.9%	-3.2%
10%	-3.9%	-7.5%	-8.5%

8. Further Discussions

We hope that there can be an accurate automatic control system for the regular water inflows, and hence we need to prove that the automatic control system can be transplanted to our bathtub. Before that, we also need to determine the setting temperature of adding-water sensor and simulate the process of reheating to see whether the bath is satisfactory.

8.1 Use a fuzzy synthetic evaluation (FSE) to determine the optimal standard bath temperature

In most practical problems, according to different criteria we can get different results, but with so many criteria and results we can hardly determine the importance of each criterion. Meanwhile, we do not have more information, so we implement a fuzzy synthetic evaluation.[14]

FSE is a kind of comprehensive bid evaluation method based on fuzzy mathematics. This method transforms the qualitative evaluation into quantitative evaluation based on Membership degree theory of fuzzy mathematics, that is, use fuzzy mathematics to give an overall evaluation to the objects restricted by multiple factors. Meanwhile, we need determine the exclusive weight of every criterion based on the given data. Coefficient of Variation can help do this, which is the ratio of standard deviation to average number.

1. Identify Alternatives and Attributes

After coordinating the conclusions we get, we can obtain the optimal minimum amount of added water (exclude the capacity of tub) corresponding to each influencing factor in certain cases. **Table 4** lists the Criteria of Basic Rules in temperature controlling .See **Table 4**.

Table 4 Criteria of Basic Rules in temperature controlling

Temperature	Size of bathtub	Bather factor	Bubble agent
38	25.5	28.305	23.0
39	23.44	30.2376	21.6
40	21.24	32.7096	19.2

Then we derive the ideal values from Table 4.

$$\mathrm{u} = (u_1, u_2, u_3) = (21.24{,}28.305{,}19.2) \tag{39}$$

2. Determine Fuzzy Evaluation Matrix

The membership function is defined as

$$r_{ij} = \frac{|a_{ij} - u_i|}{\max\{a_{ij}\} - \min\{a_{ij}\}} \tag{40}$$

Then we have the fuzzy evaluation matrix

$$\mathrm{R} = \begin{bmatrix} 1 & 0 & 1 \\ 0.516 & 0.439 & 0.632 \\ 0 & 1 & 0 \end{bmatrix} \tag{41}$$

Using coefficient of variation method, we define v_j and w_j as:

$$v_j = \frac{s_j}{\bar{x}_j}, \qquad w_j = \frac{v_j}{\sum_{i=j}^{3} v_j}$$

s_j is the standard deviation of data in column j in table 4, and $\bar{x}_j$ is the average of data in column j in table 4. Then we calculate the weighted vector

$$\mathbf{w} = (\mathbf{0.1846}, \mathbf{0.1949}, \mathbf{0.1734}) \tag{42}$$

3. Aggregate using a fuzzy operator

Then we use a fuzzy operator to aggregate and obtain the relative deviation.

$$F_i = \sum_{j=1}^{3} w_j r_{ij} \tag{43}$$

The relative deviation measures the distance between a specific alternative to the ideal alternative. The lower the value is, the better the alternative is.

4. The Results

The relative deviations in both cases are listed below. See **Table 5**.

Table 5 Relative deviations of different rules in temperature controlling

The standard temperature (℃)	38	39	40
F_i	0.2798	0.2805	0.2830

Because the synthesis score of 40℃ is the highest, we choose 40℃ as the most appropriate standard temperature.

8.2 Fuzzy PID model based on Grey prediction

After reading the Design of the Temperature Controller based on Fuzzy-PID technology by YANG Sheng-quan, LIU Ping-ping, et al [15]and Grey Prediction Model by WANG Jian-zhong et al,[16] we are inspired. We hope to build a fuzzy PID model based on Grey prediction, and then we can apply it to our temperature control system of bathtub. We simulate the response time of the temperature control balance to judge whether the bath is satisfying. This is preparation for manufacturing our automatic temperature-controlled bathtub.

1. Grey Prediction Control

A Grey system consists of partly clear messages. All the systems difficult to construct an accurate model fall under the category of Grey systems. In Grey system theory, GM(1,1) [17]Model is built according to relevancy, Grey derivate of generation number and Grey differential equation. Grey prediction is built based on GM(1,1) Model. This paper is to predict the outputs of the system when time is k+M using GM(1,1) Model.

We can obtain the predicted result when time is k+M:

$$y_p^{(0)}(k+M) = \left[y_k^{(0)} - \frac{b_g}{a_g}\right] e^{-a_g M}(1 - e^{-a_g}) \tag{44}$$

When Grey prediction controlling, only by choosing an appropriate modeling dimension n and an appropriate amount of prediction step M can we predict the changing process of system behaviors accurately. In that way, we can make Grey prediction play a leading role and enhance the accuracy and real-time of control. Normally, the more lagging a system is, the more prediction step is. This paper makes modeling dimension n=5 and prediction step M=10.

2. Fuzzy PID Control System

Fuzzy PID Control System is mainly made up of two parts which are parameter adjustable PID and fuzzy control system. Parameter adjustable PID is to control the system and fuzzy control system is to automatically adjust the three parameters of PID. Usually, digital controller can be expressed as

$$\mu(k) = \frac{1}{\delta}\left\{e(k) + \frac{T}{T_i}\sum_{i=1}^{k} e(i) + \frac{T_d}{T}\Delta e(k)\right\} \tag{45}$$

Among this, μ (k) is the control function of time k, e(k) is error and Δe(k) is the change rate of error. They can be taken as the input lingual variables. T is the sampling period of the controller, δ is the proportion band, T_i is the integral time, T_d is the derivative time.

3. PID Control Laws Based on Fuzzy Tuning Rules

Grey prediction fuzzy PID control adds a Grey predictor to the feedback circuit. The predictor is based on Grey theory. It is a mathematical model in which the behaviors of system represent the system. Namely, using the five sequential sampling data before time k to obtain the predicted values when time is k+M based on Grey prediction algorithm, using the forecast error $\mathrm{e}(k) = r(k) - y_p(k+M)$ to replace the present measuring error and the corresponding change of error $\mathrm{e}(k) = r(k) - y(k+M)$, and taking part in the online adjustment to the parameters of fuzzy PID controller. Since we use the predicted values of error to control, this kind of prediction control can be regarded as "beforehand adjustment". **Figure 13** is the flow chart of the system.

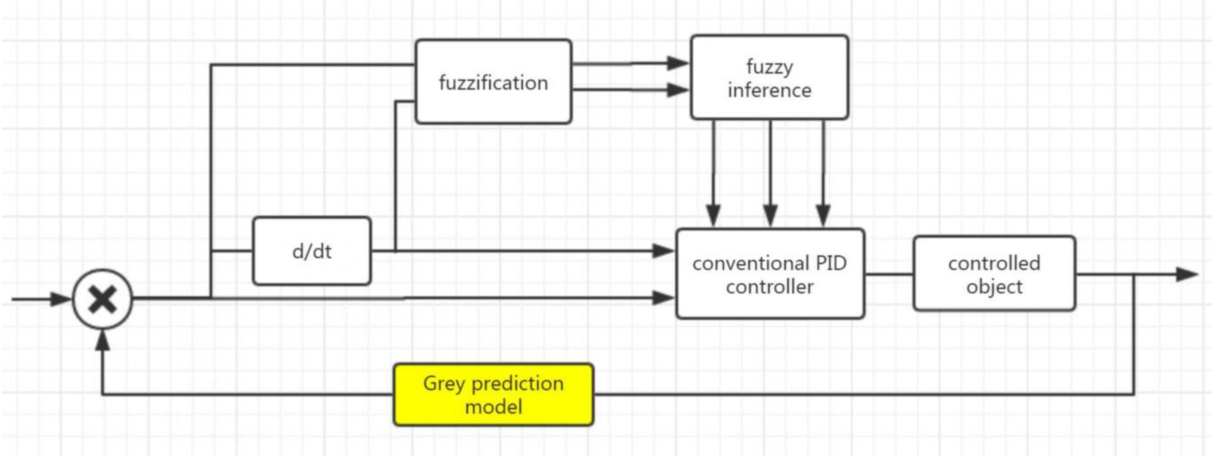
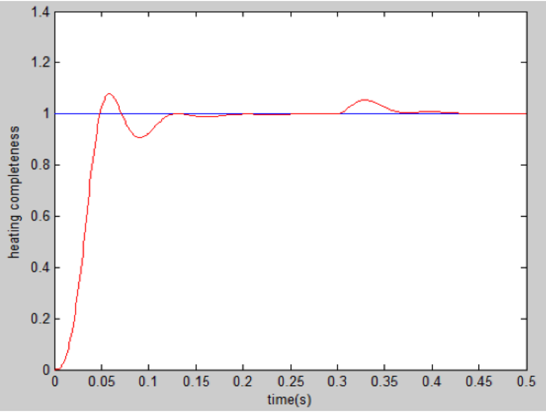


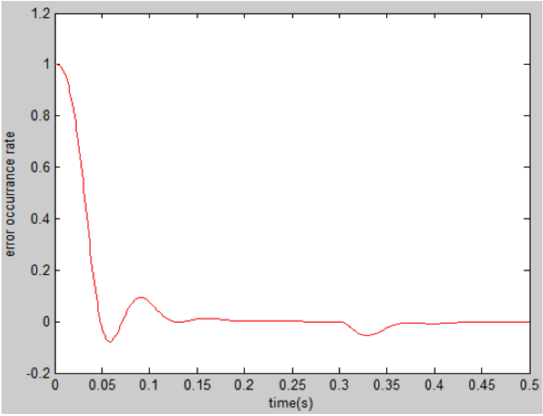
Figure 13 The flow chart of Fuzzy PID temperature model based on Grey prediction

4. The response time of the temperature control balance and error occurrence rate

Use the model above and the data to do simulation, and then we can get the figures of the response time of the temperature control balance and error occurrence rate. See **Figure 14.**



(a) Response time of the temperature control balance



(b) Error occurrence rate

Figure 14 Fuzzy PID temperature control model based on Grey Prediction

From the left diagram, we can see the response time of the temperature control balance is about 0.1s~0.5s. Before this time, heating completeness increases sharply. Although there appeared a short time of excessive heating, it hardly influences the accuracy of temperature control. The extremely short response time indicates the high sensitivity. The bather can hardly feel the change of temperature and totally enjoy the bath.

From the right diagram, we can see the same range of the response time. Before this time, error occurrence rate decreases sharply. Although entirely wrong control occurred(negative), the whole tends to be error-free.

To sum up, we can prove that automatic control system can be transplanted to our bathtub, and it does well. So we consider that, because of the high difficulty, we recommend an automatic electronic temperature control system to families for the purpose of enhancing the comfort level and saving water. Although it costs a certain amount of money to install automatic control system, we can gain economic profit and higher comfort level in the long run owing to the accurate control of the minimum gross amount of water.

9. Strengths and Weaknesses

Strengths:

- Our models are fairly robust to the changes in parameters based on sensitivity analysis. It means a slight change in parameters will not cause a significant change in the result.
- We use the theory of dimension reduction, the four dimensional space-time model is decomposed into countless two-dimensional models Through two decomposition. Ensure the model is correct and practical, it also can greatly simplify the modeling process.
- We simulate the process of temperature transfer using cellular automaton, deduct the reasonable size of bathtub, body motions and the amount of bubbles. Finally, we calculated the minimum amount of water used which make temperature reach a dynamic equilibrium throughout the bath.
- We use a fuzzy synthetic evaluation (FSE) to evaluate an optimal temperature among the three standard temperatures, and combine with fuzzy PID model based on Grey prediction, prove that the automatic temperature control system can be well transplanted into the temperature control system of bath, and achieve adjusting temperature precisely, as well as reaching the purpose of the minimum water consumption.

Weaknesses:

- Different people prefer different length of time for a bath, which is neglected in our model.
- We haven't considered the positions of the water inlet and water outlet. In our model, we modeled according to the bathtub sales in the markets. The location of the inlet determines the location of the bathtub heat source. While heat source position is different, heat distribution in the bathtub will also be different, that will influence of heat transfer in the water.
- Because of people's subjectivity, the human behavior is difficult to grasp. Human behavior will affect the water heat convection and heat dissipation in some extent. Our model will appear deviation when the body moved acutely.

10. Non-technical Explanation

When it comes to baths, the vast majority of us will admit that taking a bath is the best thing to do when we are tired and dirty from work. Most of the time, we take baths at home because of our budget and limited time. However, the bathtubs usually do not have a secondary heating system or circulating jets, which causes that we will feel increasingly cooler as the water temperature drops.

So why is it so difficult to keep the water temperature even throughout the bath? That is because there are a variety of influencing factors of bathwater temperature. It is difficult for us to control these factors accurately and simultaneously. The influencing factors and corresponding strategies are as follows.

The response time of reheating bathwater is influenced by the position of faucet. If the faucet is located in the middle of the bathtub edge, it will help the water temperature become uniform. Therefore, when you install the faucet for your bathtub, you are advised to choose the center of tub edge.

The shape and volume of tub have effect on the bathwater temperature. According to our study, the bathtubs with a top surface area of approximately 0.475m^3 are the best for keeping bathwater temperature even. Besides, cuboid bathtubs which fit the shape of human body most are the best for holding heat. So it is sensible for you to choose a cuboid bathtub with a top surface area of approximately 0.475m^3 when you purchase. Remember, the precondition is that the bathtub is big enough for you.

When you take a bubble bath, the bubbles also have influence on the bathwater temperature. A certain number of bubbles can help hold heat, but the effect is limited. Based on our study, it is the best state when bubbles can cover the water surface just right. Excessive bubble agent will cause waste.

These are dominant factors of bathwater temperature. Of course, there are many other influencing factors which are the same difficult to control. It is not advisable for users to adjust temperature manually because of inaccuracy. So we need an automatic electronic temperature control system for our bathtub. Although it costs a certain amount of money to install an automatic control system, you can gain economic profit and higher comfort level in the long run owing to the accurate control of the minimum gross amount of water.

Reference.

[1] Wikipedia, Bathtub, https://en.wikipedia.org/wiki/Bathtub, 2016/1/29.
[2] Wikipedia, Heat transfer, https://en.wikipedia.org/wiki/Heat transfer, 2016/1/29.
[3] Trevor Letcher, Ron Weir, et al, Applied Thermodynamics of Fluids, London: Royal Society of Chemistry, 2010/10/31.
[4] Wikipedia, Fluid, https://en.wikipedia.org/wiki/Fluid, 2016/1/29.
[5] Wikipedia, Heat equation, https://en.wikipedia.org/wiki/Heat_equation, 2016/1/29 .
[6] V.V.Zakharenko, T.N.Azyasskaya, Diffusion model of longitudinal agitation in heat and mass transfer processes, Journal of Engineering Physic and Thermophysics, Vol 70, No.4:718-728, 19 97.
[7]Dake J.M.K., Harleman D.R.F.. Thermal Stratification in Lake: Analytical. Water Resources Research,1969,5(2):484~495.
[8] Wikipedia, Take a bath, https://zh.wikipedia.org/wiki/%E6%B3%A1%E6%BE%A1, 2016/1/29
[9] Wikipedia, Neumann boundary condition https://en.wikipedia.org/wiki/Neumann_boundary_condition 2016/1/30
[10] Wikipedia, Dirichlet boundary condition https://en.wikipedia.org/wiki/Dirichlet_boundary_condition 2016/1/30
[11]Tubatu, the setting temperature of heater, http://www.to8to.com/ask/k1207019.html 2016/1/31
[12] Baidu, http://zhidao.baidu.com/link?url=kpKb7N_aPr1Qvd_JUIXos3qxXCHJ-uwDp0MTKm6p39pzQ-de3TAcpJE3wJlbcxjRbq0mpGdYaS3TTpdYPnj0b_ 2016/1/31
[13] Zhanfen Jin,Yingxin Zhu, Architectural Environment, Beijing: Chinese Architectural press , 2001.
[14] Sadiq, R., Husain, T., Veitch, B., & Bose, N. (2004). Risk-based decision–making for drilling waste discharges using a fuzzy synthetic evaluation technique. Ocean Engineering, 31 31(16), 1929~1953.
[15] Shengquan Yang, Pingping Liu, Baomin Li, The Design of a Temperature Controller Based on Fuzzy-PID technology, Automatization Appearance and Control Equipment , 2008, Vol.5: 14~16, 2008.
[16] Jianzhong Wang,J iangtao Ji, Hongru Wang, Grey Prediction Fuzzy Control of the Target Tracking System in a Robot Weapon, Journal of Beijing Institute of Technology, Vol.16, No.4: 424~429, 2007.
[17] Jun’an Wang, An Wang, Zhongliang Jin, Fuzzy-Gray, Predictive control algorithm and application of system engineering theory and practice, 2002, Vol.8: 132~135.