Combinatorial aspects of surface Cluster algebras and applications to Frobenius' conjecture

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Overview

1. Introduction to Cluster algebras
2. Frobenius' Conjecture
3. Connection to Cluster algebras
4. Continued fractions
5. Snake graphs
6. Palindromification
7. Markov numbers & reformulation of the conjecture

Introduction

Figure: Cluster algebra was first introduced, in 2002, by Sergey Fomin and Andrei Zelevinsky.

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- Triangulated surfaces,
- Coxeter groups,
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- Hyperbolic geometry,
- Combinatorics,
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e.g. $y_1^3 y_2^{-4} y_3 \oplus y_1^{-1} = y_1^{-1} y_2^{-4}$.

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Consider $(\mathcal{G}, \oplus, \cdot)$; then take its group ring $\mathbb{Z}\mathcal{G}$ (also known as a *tropical semifield*). This will be our ambient field[1].

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Cluster algebras

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- $\mathcal{Q}$ a cluster quiver.

Cluster algebras

A *quiver* $\mathcal{Q}$ is a directed graph; i.e. a 4-tuple $(\mathcal{Q}_0, \mathcal{Q}_1, h, t)$.

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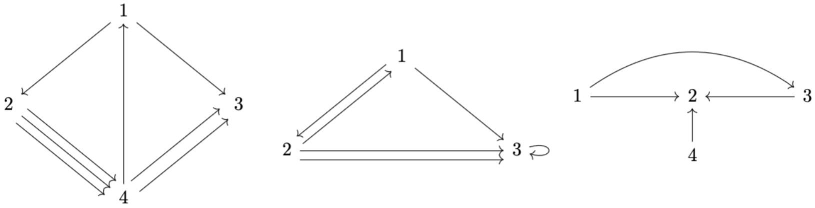

Figure: Example of 2 *cluster quivers* (left and rightmost) and a non-cluster quiver (middle)

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The Cluster algebra is generated by applying a recursive method to the initial seed, called a *cluster mutation*. A *cluster mutation* μ_k acts on $\mathbf{x}$ as follows; $\mu_k((x_1, \ldots, x_k, \ldots, x_n)) = (x_1, \ldots, x'_k, \ldots, x_n)$, where;

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$$x_k' = \frac{1}{y_k \oplus 1} \frac{y_k \prod_{i \to k} x_i + \prod_{k \to j} x_j}{x_k}.$$

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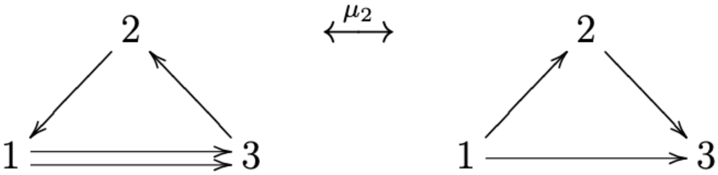

Figure: Example of mutation.

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e.g. $(1, 5, 13), (5, 13, 194), (1, 89, 233), (5, 29, 433)$.

Frobenius' Conjecture

Conjecture

Given any two ordered[a] Markov triples (a_1, a_2, τ), (b_1, b_2, τ), then $a_1 = b_1$, and $a_2 = b_2$.

[a] An *ordered* Markov triple is a solution (a, b, c) to Markov's equation, such that $a \leq b \leq c$.

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Connection?

How does Cluster algebra factor in when thinking about Frobenius' conjecture?

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Consider the triangulated punctured torus $\mathbb{T}^2$, as the quotient space

$$\mathbb{T}^2 = I \times I / \sim_{vert} / \sim_{hor},$$

where $I = [0, 1] \subset \mathbb{R}$ and $\sim_{vert}$ and $\sim_{hor}$ is the equivalence relation $(x, 1) \sim_{vert} (x, 0))$, and $(0, y) \sim_{hor} (1, y)$...

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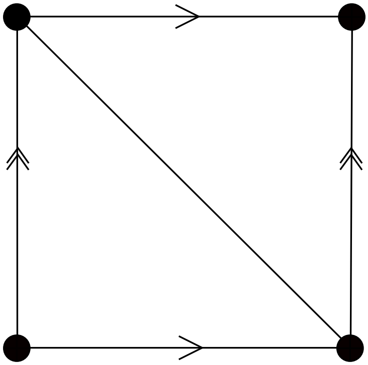

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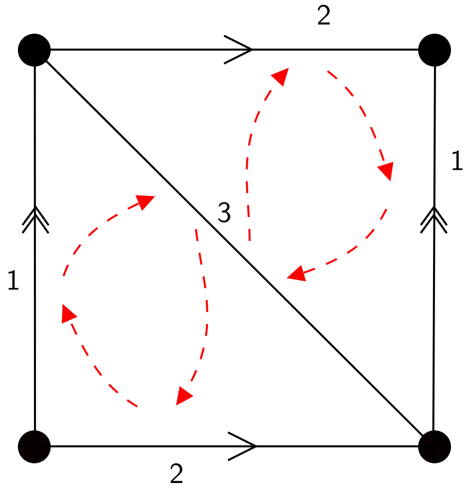

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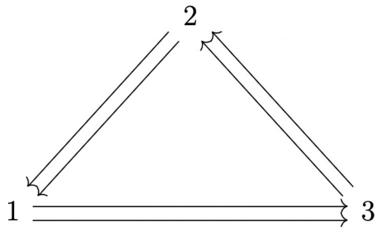

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Observe that by starting with $(x_1, x_2, x_3) = (1, 1, 1)$ (the smallest Markov triple), we obtain

$$\begin{aligned}
\mu_1(1,1,1) &= (2,1,1) \sim (1,1,2), \\
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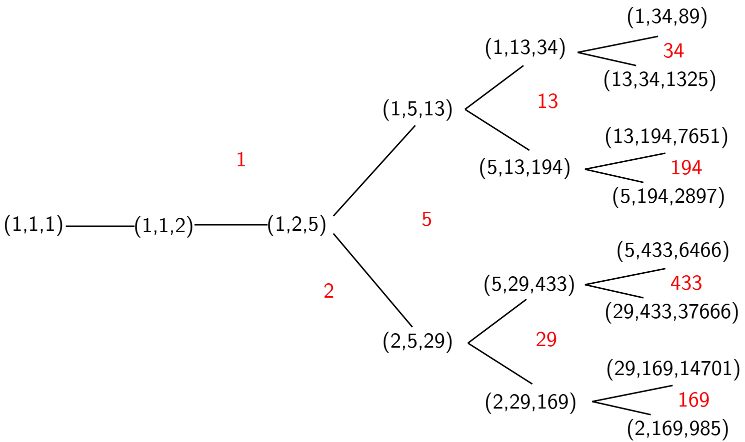
Markov Tree


(1,1,1)
(1,1,2)
(1,2,5)
1
2
5
(1,5,13)
(2,5,29)
13
29
(1,13,34)
(5,13,194)
(5,29,433)
(2,29,169)
34
194
433
169
(1,34,89)
(13,34,1325)
(13,194,7651)
(5,194,2897)
(5,433,6466)
(29,433,37666)
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Continued Fractions

A continued fraction is a way to represent a real number; as follows,

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$$\sqrt{2} = 1 + \cfrac{1}{2 + \cfrac{1}{2 + \ddots}} = [1; 2, 2, 2, \dots].$$

Continued Fractions

A continued fraction $[a_1, \ldots, a_n]$ is called *palindromic of even length* if $(a_1, \ldots, a_n) = (a_n, \ldots, a_1)$, as sequences and n is even.

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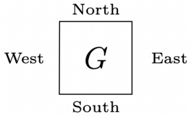

Figure: A tile G with sides labeled to denote the orientation

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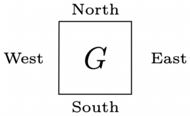

Figure: A tile G with sides labeled to denote the orientation

Any tile can be attached on either the north or east edge of the previous tile.

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Furthermore, on a snake graph, we assign a *sign function*

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- The sign on the south edge is different than the sign on the north edge.

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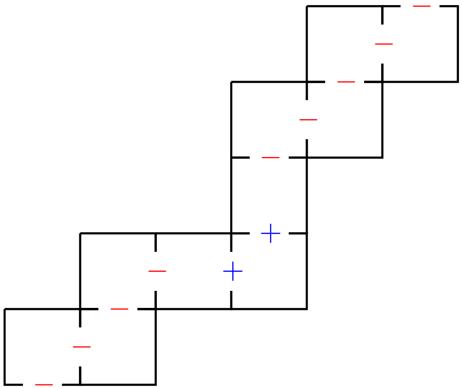

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$$(\underbrace{-, \dots, -}_{a_1}, \quad \underbrace{+, \dots, +}_{a_2}, \quad \underbrace{-, \dots, -}_{a_3}, \quad \underbrace{\dots,}_{\dots} \quad \underbrace{\epsilon, \dots, \epsilon}_{a_n}), \tag{1}$$

where $\epsilon = \begin{cases} + & \text{if } n \text{ is even;} \\ - & \text{if } n \text{ is odd} \end{cases}$.

Snake graph of a continued fraction

Then the snake graph $\mathcal{S}[a_1, \ldots, a_n]$ is the graph with precisely $a_1 + \cdots + a_n - 1$ tiles determined by its sign sequence.

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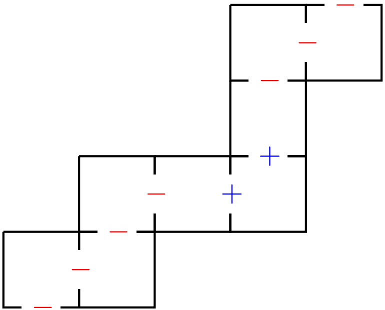

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$$[a_1, a_2, \ldots, a_n] = \frac{m(\mathcal{S}[a_1, a_2, \ldots, a_n])}{m(\mathcal{S}[a_2, a_3, \ldots, a_n])}$$

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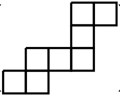

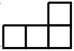

$$= \frac{31}{7}$$

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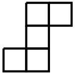
- The snake graph $\mathcal{S}[a_n, \ldots, a_1, a_1, \ldots, a_n]$ has a rotational symmetry at its center tile; e.g. 

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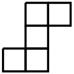
- The snake graph $\mathcal{S}[a_n, \dots, a_1, a_1, \dots, a_n]$ has a rotational symmetry at its center tile; e.g. 
- If $[a_1, \dots, a_n] = p_n/q_n$, then

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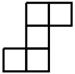
- The snake graph $\mathcal{S}[a_n, \dots, a_1, a_1, \dots, a_n]$ has a rotational symmetry at its center tile; e.g. 
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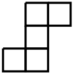
- The snake graph $\mathcal{S}[a_n, \ldots, a_1, a_1, \ldots, a_n]$ has a rotational symmetry at its center tile; e.g. 
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  e.g. for $[3, 1, 5] = 23/6$, we have that $[3, 1] = 4$; and $[5, 1, 3, 3, 1, 5] = \frac{565}{98} = \frac{23^2 + 6^2}{4 \cdot 23 + 1 \cdot 6}$.

Overview

1. Introduction to Cluster algebras
2. Frobenius' Conjecture
3. Connection to Cluster algebras
4. Continued fractions
5. Snake graphs
6. Palindromification
7. Markov numbers & reformulation of the conjecture

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Conjecture

Let $m > 2$ be a Markov number. Then there exist **unique** positive integers $a < b$ with $\gcd(a, b) = 1$, such that $m = a^2 + b^2$, $2a \leq b < 3a$; and the continued fraction corresponding to b/a consists entirely of 1's and 2's.

The End

Thank you for listening!