A Forward-Looking Example for Identification
Comment on Backus, Chernov, and Zin (2013)

Tao Zha

15 October 2013

Forward-looking model: a simultaneous-equation approach

- Backus, Chernov, and Zin (BCZ, 2013) is an inspiring and thought-proving paper that repays careful study.
- Consider the following simple forward-looking Phillips new-Keynesian model:

$$\pi_t = E_t \pi_{t+1} + \kappa x_t + u_{st}, \tag{1}$$

$$x_t = E_t x_{t+1} - \tau (R_t - E_t \pi_{t+1}) + u_{dt}, \tag{2}$$

$$R_t = \phi_\pi \pi_t + u_{Rt}. \tag{3}$$

- For analytical tractability and illustration, assume that u_{st}, u_{dt}, and u_{Rt} are uncorrelated and i.i.d. that their distribution is Gaussian with mean zero and the variances σ_s^2, σ_d^2, and σ_R^2.
- Cochrane (2006) shows that when the unique equilibrium exists, if one regresses R_t on π_t, the estimate of ϕ_π will not depend on the true value of ϕ_π, which generates the data.
- In other words, ϕ_π is not identified by the OLS method.

Forward-looking model: a simultaneous-equation approach

- In this model, since all three variables are simultaneously determined, there is no "policy instrument" for an instrument-variable (IV) estimation of the policy reaction equation (3), just as there is no "right-hand-side variable" shifter (see Sections 2.3 and 5.3 in BCZ).
- As in the simultaneous-equation VAR literature, however, the policy parameter can still be locally identified using identifiable orthogonal shocks as "instrumental" shifters.
- For a unique equilibrium or an MSV equilibrium, we shall now show that all the other parameters in the model (including ϕ_π in the Taylor rule) are locally identified.

Compact form

- Denoting
$$y_t = \begin{bmatrix} \pi_t & x_t & R_t \end{bmatrix}'.$$
- We can write the equations (1)-(3) in the matrix form
$$y_t' \begin{bmatrix} \sigma_s^{-1} & 0 & -\phi_\pi \sigma_R^{-1} \\ -\kappa \sigma_s^{-1} & \sigma_d^{-1} & 0 \\ 0 & \tau \sigma_d^{-1} & \sigma_R^{-1} \end{bmatrix} = E_t y_{t+1}' \begin{bmatrix} \sigma_s^{-1} & \tau \sigma_d^{-1} & 0 \\ 0 & \sigma_d^{-1} & 0 \\ 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} \epsilon_{st} \\ \epsilon_{dt} \\ \epsilon_{Rt} \end{bmatrix},$$

MSV equilibrium

- The MSV equilibrium is

$$y_t' = \frac{1}{1+\kappa\tau\phi_\pi}\begin{bmatrix} \sigma_s & -\tau\phi_\pi\sigma_d & \phi_\pi\sigma_R \\ \kappa\sigma_s & \sigma_d & \kappa\phi_\pi\sigma_R \\ -\kappa\tau\sigma_s & -\tau\sigma_d & \sigma_R \end{bmatrix}\epsilon_t,$$

where

$$\epsilon_t = \begin{bmatrix} \epsilon_{st} \\ \epsilon_{dt} \\ \epsilon_{Rt} \end{bmatrix}.$$

SVAR form

- To see how the rest of the parameters are locally identified, we rewrite the equilibrium in the following simultaneous-equation SVAR form:

$$y_t' A_0 = \epsilon_t,$$

where

$$A_0 = \begin{bmatrix} a_{0,11} & 0 & a_{0,13} \\ a_{0,21} & a_{0,22} & 0 \\ 0 & a_{0,32} & a_{0,33} \end{bmatrix}, \tag{4}$$

with

$$a_{0,11} = \sigma_s^{-1},\ a_{0,13} = -\phi_\pi \sigma_R^{-1},\ a_{0,21} = -\kappa \sigma_s^{-1},$$
$$a_{0,22} = \sigma_d^{-1},\ a_{0,32} = \tau \sigma_d^{-1},\ a_{0,33} = \sigma_R^{-1}.$$

Necessary condition

Any two different points of the vector of DSGE parameters (σ_s, σ_d, σ_R, ϕ_π, κ, and τ) imply two different points of the vector of SVAR parameters ($a_{0,11}$, $a_{0,13}$, $a_{0,21}$, $a_{0,22}$, $a_{0,32}$, and $a_{0,33}$). Thus, a necessary condition for identification is satisfied.

Local identification

- The structural parameters are **locally identified** (although not globally identified) (see Rubio-Ramirez, Waggoner, and Zha 2010, ReStud).
- To see how it is is true, consider the following parameter point consistent with Lubik and Schorfheide (2004, AER):

$$\phi_\pi = 2.0, \kappa = 0.58, \tau = 0.54, \sigma_d = 1.0, \sigma_s = 2.0, \sigma_R = 0.2.$$

- We can obtain another distinct point in the parameter space that generates the same data dynamics:

$$\phi_\pi = 2.50, \kappa = 0.90, \tau = 0.56, \sigma_d = 1.02, \sigma_s = 2.49, \sigma_R = 0.2.$$

- Because the values of the Phillips slope parameter κ are different for these two parameter points, the inference about the frequency firms change their prices is very different.
- For each point, it can be verified that all the parameters are **locally identified**.