Referee Report on

"Identifying Taylor rules in macro-finance models"

1. Summary

This paper revisits the identification problem of Taylor rules discussed by Cochrane (2011) and it links it with macro-finance models.

2. A State-space representation

I never liked Cochrane's 2011 paper. I always thought that the presentation was unnecessarily complicated and that the lack of clarity stayed in the way of insight. In particular, neither Cochrane nor this paper use state-space representations, which a powerful yet simple way to deal with the issues at hand.

To write such state-space representation and explain the problem, we start with a transition equation for the states

$$x_{t+1} = Ax_t + Cw_{t+1} \tag{1}$$

and a measurement equation:

$$\begin{pmatrix} i_t \\ \pi_t \end{pmatrix} = Dx_t \tag{2}$$

where A, C, and D are potentially non-linear functions of a set of structural parameters γ that describe the preferences and technology in the model. These non-linear functions are the cross-equation restrictions imposed by equilibrium.

The Kalman filter let us to use data on inflation, i_t, to estimate A, C, and D. Note that I do not include the state x_t in the observables. I only do so to save on notation, but my argument can be easily re-arranged to accommodate the case (the paper, instead, presents first the case where x_t is observable and then moves to the general case; this seems to me the wrong strategy, since it obscures the core of the problem by adding extra unneeded structure on the measurement equation).

The question is: Can we recover γ from knowledge of A, C, and D? In general, no. As we argued before, A, C, and D are non-linear functions of a set of structural parameters γ and such functions will not be, in general, invertible.

To show this lack of invertibility, we can go to example 1, where we derive the measurement equation from the model

$$i_t = \mathbb{E}_t \pi_{t+1} \tag{3}$$
$$i_t = \tau \pi_t + s_t \tag{4}$$
$$s_t = d' x_t \tag{5}$$

This model can be solved by noticing that:

1. Combining equations (3) to (4), we get:

$$\mathbb{E}_t \pi_{t+1} = \tau \pi_t + d' x_t$$

2. The linear structure of our problem implies that the equilibrium function for π_t must be a linear function of the states x_t:

$$\pi_t = b' x_t$$

   with coefficients b to be determined.

Therefore, and using (1),

$$b' A x_t = \tau b' x_t + d' x_t$$

from which we get:

$$b' = -d' \left(\tau I - A\right)^{-1}$$

Then:

$$D = \begin{pmatrix} -d' \left(\tau I - A\right)^{-1} A \\ -d' \left(\tau I - A\right)^{-1} \end{pmatrix}$$

Note that the first and the second entry of D give us the same information (A is known) and that we cannot separate d from τ. More precisely, d and τ are not point-identified, but they are set-identified.[1]

Example 2 illustrates that knowledge of the pricing kernel

$$i_t = -\log \mathbb{E}_t e^{m^{\$}_{t+1}} = \delta' x_t$$

does not solve the problem, as

$$\delta' = -d' \left(\tau I - A\right)^{-1} A$$

[1] There is a large and growing literature on set identification that I think the paper could use or, at least, cite.

still does not let us distinguish d from τ.

How can we get around the identification problem? A simple solution (as the paper explains in section 4) is to impose further structure on d. The baseline "plain vanilla" DSGE model assumes that a component of x_t is a serially uncorrelated normal random variable and that d' is a vector of zeros except in the row corresponding to such normal random variable, where the entry is a one. Thus, of all the members of the set

$$\widehat{D}_{21} = -d' \left(\tau I - A\right)^{-1}$$

where $\widehat{D}_{21}$ is the entry 2,1 of the estimated matrix D, we just pick the τ associated with the d we just specified. An alternative interpretation of this approach is that we have a dirac prior on the parameters of d and such prior collapses our set identification into a point identification.[2]

But, of course, as the paper explains in section 3 and *en passant* along the text, there are many other possibilities, some borrowed from macro-finance, some from narrative approaches, etc.

Unfortunately, the paper never tries to explore the issue of characterizing these possibilities systematically (or even partially!). Instead of digging deeper and searching for a more thorough understanding, it only presents some examples and covers material that is (or should be) already well understood by macroeconomists.

3. Assessment

I can see the case for a pedagogical paper where the identification of Taylor rules is exposed in a clear way (I'd strongly suggest with the help of state-space representations: with them you can rewrite the whole paper in less than 10 pages) and related to asset pricing. But such paper is for the *JME* or *AEJ Macro*, but definitively not for *AER*. In the current draft, there is just not enough new material with respect to Cochrane's 2011 paper.

[2] Personally, I find this solution not only to be the cleanest, but also the one that is closest aggrement with John Taylor's own idea of what a monetary policy rule is.