The New IS-LM Model: Language, Logic, and Limits

Robert G. King

R ecent years have witnessed the development of a New IS-LM model that is increasingly being used to discuss the determination of macroeconomic activity and the design of monetary policy rules. It is sometimes called an "optimizing IS-LM model" because it can be built up from microfoundations. It is alternatively called an "expectational IS-LM model" because the traditional model's behavioral equations are modified to include expectational terms suggested by these microfoundations and because the new framework is analyzed using rational expectations. The purpose of this article is to provide a simple exposition of the New IS-LM model and to discuss how it leads to strong conclusions about monetary policy in four important areas.

- *Desirability of price level or inflation targeting:* The new model suggests that a monetary policy that targets inflation at a low level will keep economic activity near capacity. If there are no exogenous "inflation shocks," then full stabilization of the price level will also maintain output at its capacity level. More generally, the new model indicates that time-varying inflation targets should not respond to many economic disturbances, including shocks to productivity, aggregate demand, and the demand for money.

- *Interest rate behavior under inflation targeting:* The new model incorporates the twin principles of interest rate determination, originally developed by Irving Fisher, which are an essential component of modern macroeconomics. The real interest rate is a key intertemporal relative

price, which increases when there is greater expected growth in real activity and falls when the economy slows. The nominal interest rate is the sum of the real interest rate and expected inflation. Accordingly, a central bank pursuing an inflation-targeting policy designed to keep output near capacity must raise the nominal rate when the economy's expected growth rate of capacity output increases and lower it when the expected growth rate declines.

- *Limits on monetary policy:* There are two limits on monetary policy emphasized by this model. First, the monetary authority cannot engineer a permanent departure of output from its capacity level. Second, monetary policy rules must be restricted if there is to be a unique rational expectations equilibrium. In particular, as is apparently the case in many countries, suppose that the central bank uses an interest rate instrument and that it raises the rate when inflation rises relative to target. Then the New IS-LM model implies that it must do so aggressively (raising the rate by more than one-for-one) if there is to be a unique, stable equilibrium. But if the central bank responds to both current and prospective inflation, then it is also important that it not respond too aggressively.

- *Effects of monetary policy:* Within the new model, monetary policy can induce temporary departures of output from its capacity level. However, in contrast to some earlier models, these departures generally will not be serially uncorrelated. If the central bank engineers a permanent increase in nominal income, for example, then there will be an increase in output that will persist for a number of periods before fully dissipating in price adjustment. Further, the model implies that the form of the monetary policy rule is important for how the economy responds to various real and monetary disturbances.

In summary, the New IS-LM model instructs the central bank to target inflation. It indicates that there are substantial limits on the long-run influence that the monetary authority can have on real economic activity and that there are also constraints on its choice of policy rule. But the New IS-LM also indicates that the monetary authority can affect macroeconomic fluctuations through its choice of the monetary policy rule, as well as via monetary policy shocks.

The plan of the article is as follows. Section 1 provides some historical background on the evolution of the IS-LM model since its origin in Hicks (1937). Section 2 then quickly lays out the equations of the closed economy version of the New IS-LM model. Section 3 uses the framework to show how a neutral monetary policy—a policy which keeps output close to its capacity level—implies a specific inflation targeting regime and, if certain exogenous shocks are small, rationalizes a full stabilization of the price level. Following

Goodfriend and King (1997), such a policy is called a "neutral monetary policy" and the new model is used to determine some rules for the setting of alternative monetary instruments that would yield the neutral level of output.

The article next turns to understanding the mechanics of the New IS-LM model. Proponents of IS-LM modeling typically stress that sticky prices are central to understanding macroeconomic activity (e.g., Mankiw [1990]) so that the discussion begins in Section 4 with this topic. Firms are assumed to set prices and adjust quantity in response to changes in demand. But in the New IS-LM model, firms are assumed to be forward-looking in their price-setting, in line with research that begins with Taylor (1980). Forward-looking price-setting has major effects on the linkage between nominal disturbances and economic activity, endowing the model with a mix of Keynesian and Classical implications. Section 5 considers the long-run limits on monetary policy given this "supply side" specification and several related topics.

Turning to the aggregate demand side, the new model's IS schedule is also forward looking. Section 6 starts by discussing why this is the inevitable attribute of optimizing consumption-investment decisions and then considers some macroeconomic implications of the new model's IS schedule.

The macroeconomic equilibrium of the New IS-LM model is employed to analyze three key issues that are relevant to monetary policy. Section 7 considers limits on interest rate rules. Section 8 highlights how monetary policy can produce short-run departures of output from its capacity level, either as a result of monetary shocks or as a result of a policy rule which differs from the neutral rules developed in Section 3. It also considers the origin and nature of the tradeoff between inflation and output variability that is present in this model. The article is completed by a brief concluding section.

1.   THE EVOLUTION OF THE IS-LM MODEL

Before detailing the model, it is useful to briefly review the historical process that has led to its development and influences its current uses. Since the 1930s, variants of the IS-LM model have been the standard framework for macroeconomic analysis. Initially, Hicks's (1937) version was used to explain how output and interest rates would be affected by various shocks and alternative policy responses. Subsequent developments broadened the range of issues that could be studied with the model, notably the introduction of an aggregate production function and a labor market by Modigliani (1944). With the rise of quantitative frameworks for monetary policy analysis—such as the Penn-FRB-MIT model, which was employed by the Federal Reserve System—the role of the IS-LM model changed in a subtle manner. After detailed explanations were worked out in these policy laboratories, the IS-LM model was used to give a simple account of the findings.

While the initial IS-LM model did not determine how the price level evolved through time, the addition of a price equation—or a wage/price block that featured a Phillips (1958) curve—made it possible to explore the implications for inflation.[1] The simultaneous occurrence of high inflation and high unemployment in the 1970s led macroeconomists to question this aspect of theoretical and quantitative macromodels. Further, during the rational expectations revolution spurred by Lucas (1976), fundamental questions were raised about the value of the IS-LM model and the related quantitative macroeconomic policy models. The IS-LM model was portrayed as being fatally inconsistent with optimizing behavior on the part of households and firms (Lucas 1980). The quantitative macropolicy models were criticized for not using microfoundations as a guide to the specification of estimable equations and also for avoiding central issues of identification (Sims 1980, Sargent 1981). The rational expectations revolution suggested that new macroeconomic frameworks were necessary—both small analytical frameworks like the IS-LM model and larger quantitative macropolicy models—and that these would lead to a substantial revision in thinking about the limits on monetary policy and the role of monetary policy.

One initial attempt at updating the IS-LM model was initiated in Sargent and Wallace (1975), who incorporated a version of the aggregate supply theory developed by Lucas (1972, 1973) in place of the Phillips curve or wage/price block. According to this rational expectations IS-LM model, systematic monetary policy could not influence real economic activity, although monetary shocks could cause temporary departures of output from its capacity level. This finding that systematic monetary policy was irrelevant led the related literature to be described, by some, as the New Classical macroeconomics. Sargent and Wallace also used their framework to argue against use of the nominal interest rate as the instrument of monetary policy—suggesting that this practice was inconsistent with a unique macroeconomic equilibrium. While this rational expectations IS-LM model was subsequently used to clarify issues of importance for monetary policy—for example, Parkin (1978) and McCallum (1981) showed that an appropriate nominal anchor could allow the interest rate to be used as the instrument of monetary policy—it did not gain widespread acceptance for three reasons. First, some economists—particularly macroeconomic theorists—saw the model as flawed, because its lack of microfoundations led it to lack the behavioral consistency conditions which are the inevitable result of optimization and the expectational considerations which are at the heart of dynamic economic theory. Second, other economists—particularly applied macroeconomists—were suspicious of the

---

[1] With this addition, the Hicksian setup was sometimes and more accurately called an IS-LM-PC model, but it has been more commonly referred to by its shorter title, as will be the practice in this article.

model because it suggested that departures of output from capacity should be serially uncorrelated. Third, many economists—including central bankers—remained convinced that the systematic choices of the monetary authority were important for the character of economic fluctuations and thus rejected the model due to the "policy irrelevance" implication.

In recent years, there has been the development of small, optimizing macro models that combine Classical and Keynesian features in a "New Neoclassical Synthesis."[2] The New IS-LM model is an outgrowth of this more general research program and is thus designed to incorporate the major accomplishments of the rational expectations revolution, including a more careful derivation from microfoundations, while retaining the stark simplicity that made the earlier IS-LM frameworks much employed tools. One important use of the New IS-LM model is to communicate results from other, more complicated macroeconomic models that are relevant to monetary policy. For example, Kerr and King (1996) first used the core equations of the New IS-LM model to exposit issues involving interest rate rules for monetary policy that had arisen in my research on small, fully articulated macroeconomic models with sticky prices and intertemporal optimization (King and Watson 1996; King and Wolman 1999).[3] The current article shows how the New IS-LM model is also useful in expositing many issues that arise in these sorts of small, fully articulated models and also in larger quantitative macroeconomic models that are currently employed for monetary policy analysis, including the new rational expectations framework of the Federal Reserve (the FRB-US model) and the various U.S. and international models developed by Taylor (1993). In fact, in using the model to discuss the implications of sticky prices, restrictions on interest rate policy rules, and the trade-off between the variability of inflation and output, the article will touch repeatedly on themes which have been central parts of Taylor's research program.

2. THE NEW IS-LM MODEL

Like its predecessors, the New IS-LM model is a small macroeconomic model designed to describe the behavior of economy-wide variables that enter in most discussions of monetary policy. There are five endogenous variables: the log level of real output/spending y, the log price level P, the real interest rate r, the inflation rate π, and the nominal interest rate R.[4]

---

[2] See Goodfriend and King (1997) for a detailed discussion of these developments.

[3] Bernanke and Woodford (1997) and Clarida, Gali, and Gertler (1999) have since made similar use of essentially the same framework to study various monetary policy issues. Related analyses using variations on the New IS-LM approach include McCallum and Nelson (1999) and Koenig (1993a,b); these authors use an alternative approach to aggregate supply.

[4] The New IS-LM model is most frequently presented in discrete time so as to keep the mathematical analysis as simple as possible (see Kimball [1995] for a continuous time analysis of

The Core Equations

Three specifications are present in all of the recent papers that employ the New IS-LM model. These are an IS equation, a Fisher equation, and a Phillips curve equation.

The **forward-looking IS equation** makes current real spending y_t depend on the expected future level of real spending $E_t y_{t+1}$ and the real interest rate r_t. There is also an aggregate demand shock x_{dt}: a positive x_{dt} raises aggregate spending at given levels of the endogenous determinants $E_t y_{t+1}$ and r_t.[5]

$$IS : y_t = E_t y_{t+1} - s[r_t - r] + x_{dt} \tag{1}$$

The parameter $s > 0$ determines the effect of the real interest rate on aggregate demand: If s is larger then a given rise in the real interest rate causes a larger decline in real demand. The parameter $r > 0$ represents the rate of interest which would prevail in the absence of output growth and aggregate demand shocks. The new IS equation is described as forward-looking because $E_t y_{t+1}$ enters on the right-hand side.

The **Fisher equation** makes the nominal interest rate R_t equal to the sum of the real interest rate r_t and the rate of inflation that is expected to prevail between t and t+1, $E_t \pi_{t+1}$.

$$F : R_t = r_t + E_t \pi_{t+1} \tag{2}$$

This conventional specification of the Fisher equation omits any inflation risk premium in the nominal interest rate.[6]

The **expectational Phillips curve** relates the current inflation rate π_t to expected future inflation $E_t \pi_{t+1}$, the gap between current output y_t and capacity output $\bar{y}_t$, and an inflation shock $x_{\pi t}$.

$$PC : \pi_t = \beta E_t \pi_{t+1} + \varphi(y_t - \bar{y}_t) + x_{\pi t} \tag{3}$$

The parameter β satisfies $0 \leq \beta \leq 1$. The parameter $\varphi > 0$ governs how inflation responds to deviations of output from the capacity level. If there is a larger value of φ then there is a greater effect of output on inflation; in this sense, prices may be described as adjusting faster—being more flexible—if φ is greater.

---

a related but more elaborate model). The discrete time approach also facilitates discussion of the relationship between the theoretical model's parameters and estimates obtained in empirical studies. Like many other macroeconomic theories developed since Sargent (1973), the model is constructed as a linear difference system, which makes it relatively straightforward to calculate the rational expectations equilibrium.

[5] The notation used in this case will carry over to the rest of the article: shocks are called x and their nature is identified with a subscript, such as d for demand in this case. The exact statistical properties of x_{dt} are not specified at present, but they are taken to be stationary random variables with a zero mean.

[6] See McCallum and Nelson (1999a) for additional discussion of this issue.

Using the definition of the inflation rate $\pi_t = P_t - P_{t-1}$, this specification might alternatively have been written as $P_t = P_{t-1} + \beta E_t \pi_{t+1} + \varphi(y_t - \overline{y}_t) + x_{\pi t}$. This alternative form highlights why (3) is sometimes called a "price equation" or an "aggregate supply schedule." It is a price equation in the sense that it is based on a theory of how firms adjust their prices, as discussed further in Section 4 below. It is an aggregate supply schedule because it indicates how the quantity supplied depends on the price level and other factors. But this article uses the Phillips curve terminology because this is the dominant practice in the new and old IS-LM literature.

The relationship between the output gap and the steady-state rate of inflation gap is given by $y - \overline{y} = \frac{1-\beta}{\varphi}\pi$ according to this specification. In fact, experiments with fully articulated models that contain the structural features which lead to (3)—including those of King and Wolman (1999)—suggest a negligible "long-run effect" at moderate inflation rates. Prominent studies of the monetary policy implications of the New IS-LM model—including that of Clarida, Gali, and Gertler (1999)—accordingly impose the $\beta = 1$ condition in specifying (3). In this article, β will be taken to be less than but arbitrarily close to one.

Money Demand and Monetary Policy

To close the model and determine the behavior of output, the price level and other variables, it is necessary to specify the monetary equilibrium condition. Researchers presently adopt two very different strategies within the literature on the New IS-LM model.

Specifying money demand and money supply. Under this conventional strategy, the **money demand function** is typically assumed to take the form

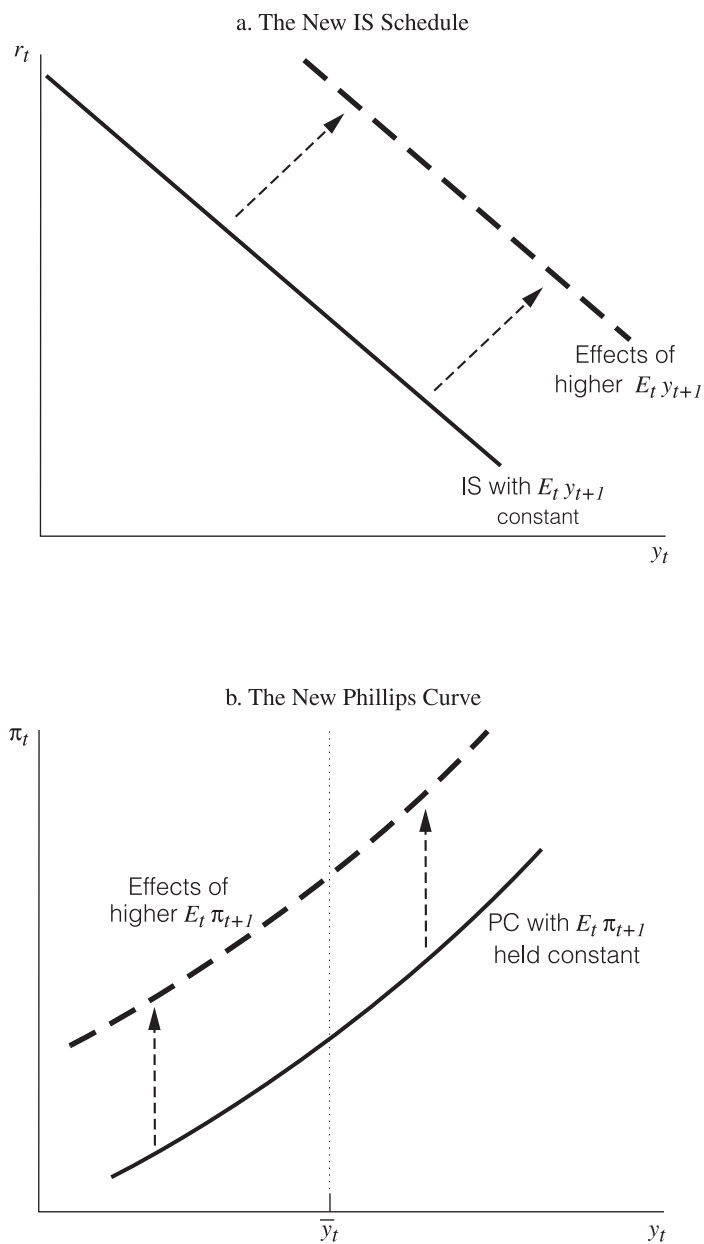$$MD : M_t - P_t = \delta y_t - \gamma R_t - x_{vt} \tag{4}$$

with $M_t - P_t$ being the demand for real balances. This demand for money has an income elasticity of $\delta > 0$ and an interest semielasticity of $-\gamma < 0$.[7] There is a shock which lowers the demand for money, x_{vt}: this is a shock to velocity when $\delta = 1$ and $\gamma = 0$.

The **money supply function** is assumed to contain a systematic monetary policy component, f_{Mt}, and a shock component x_{Mt} :

$$MS : M_t = f_{Mt} + x_{Mt.} \tag{5}$$

The monetary authority's systematic component may contain responses to the current state, lagged or expected future level of economic activity. Taken together, these equations determine the quantity of money and also provide

---

[7] Sargent (1973) showed that this semilogarithmic form is very convient for small rational expectations models.

Figure 1

a. The New IS Schedule

r_t

Effects of
higher $E_t\, y_{t+1}$

IS with $E_t\, y_{t+1}$
constant

y_t

b. The New Phillips Curve

π_t

Effects of
higher $E_t\, \pi_{t+1}$

PC with $E_t\, \pi_{t+1}$
held constant

$\overline{y}_t$ y_t

one additional restriction on the comovement of output, the price level and interest rates.

Specifying an interest rate rule for monetary policy. An alternative—and increasingly popular—strategy is to simply specify an **interest rate rule** for monetary policy,

$$IR : R_t = f_{Rt} + x_{Rt}, \tag{6}$$

which contains a systematic component, f_{Rt}, and a shock component x_{Rt}.

Under this rule, the quantity of money is demand-determined at the R_t which is set by the monetary authority. Thus, the behavior of the money stock can be deduced, from (4) and (6), as $M_t - P_t = \delta y_t - \gamma[f_{Rt} + x_{Rt}] - x_{vt}$. But since the stock of money is not otherwise relevant for the determination of macroeconomic activity, some analysts proceed without introducing money at all.[8]

What Is New about This Model?

The answer to this question depends on the chosen starting point in the history of macroeconomic thought.

Relative to the original model of Hicks, the New IS-LM model is different in that it makes the price level an endogenous variable, which is influenced by exogenous shocks and the monetary policy rule. In the language of Friedman (1970) and other monetarists, the New IS-LM model views the price level as a monetary phenomenon rather than as an unexplained institutional phenomenon. In terms of formal modeling, the idea that the price level is a monetary phenomenon is represented in two ways. First, the model cannot be solved for all of the endogenous variables without the specification of a monetary policy rule. Second, under a money stock rule, even though some individual prices are sticky in the short run, the price level responds to exogenous, permanent changes in the level of the money stock in both the short run and the long run. But, since the 1970s, textbook presentations of the IS-LM model have added a pricing block or aggregate supply schedule, which makes the price level endogenous.

The New IS-LM model also incorporates expectations in ways that the traditional IS-LM model did not. But the rational expectations IS-LM model of Sargent and Wallace (1975) also incorporated the influence of expectations of inflation into both the Fisher equation and the aggregate supply schedule. Modern textbook treatments discuss these expectations mechanisms in detail.

---

[8] For example, Kerr and King (1996) discuss how one can manipulate an "IS model" to study limits on interest rate rules and Clarida, Gali, and Gertler (1999) conduct their discussion of the "science of monetary policy" within this model without specifying the supply and demand for money.

Figure 1 shows two of the New IS-LM model's key equations. As in modern textbooks, there is an IS curve which makes output depend negatively on the (real) interest rate and a Phillips curve or aggregate supply schedule which makes output depend positively on the inflation rate. Relative to these presentations, the New IS-LM model differs (i) in the stress that it places on expectations in both aggregate demand and aggregate supply and (ii) in the particular ways in which expectations are assumed to enter into the model. In particular, the new IS schedule (1) identifies expected future income/output as a key determinant of current output, while this is missing in the Sargent-Wallace model. The new aggregate supply schedule or Phillips curve (3) identifies expected future inflation as a key determinant of current inflation, while in the Sargent-Wallace model it is yesterday's expectation of the current inflation rate that is relevant for supply.

These channels of influence are highlighted in Figure 1. In panel a of the figure, an increase in expected future output shifts the IS curve to the right, requiring a higher real interest rate at any given level of output. In panel b of the figure, an increase in expected future inflation shifts the Phillips curve to the left, requiring a higher current inflation rate at any given level of output.

However, while it is possible to express these behavioral equations in familiar graphical ways, the reader should not be misled into thinking that macroeconomic analysis can be conducted by simple curve-shifting when expectations are rational in the sense of Muth (1961).[9] Instead, it is necessary to solve simultaneously for current and expected future variables, essentially by determining the complete path that the economy is expected to follow. Once this path is known, it is possible to return to the individual graphs of the IS curve or the Phillips curve to describe the effects of shocks or policy rules.[10] But this is not the same as deriving the result by shifting the curves.

3. NEUTRAL MONETARY POLICY

If the monetary authority's objective is to stabilize real economic activity at the capacity level, the New IS-LM model provides a direct case for an inflation-targeting monetary policy.

---

[9] Expectations are assumed to be rational in Muth's sense in this article and related literature. It is also worth noting that this article and much of the related literature also assumes that there is full current information and that monetary policy rules are credible.

[10] This point is related to the discussion in King (1993), where I argued that the traditional IS-LM model is flawed due to its treatment of expectations and could not be resurrected by the New Keynesian research program. In particular, while I noted that "every macroeconomic model contains some set of equations that can be labelled as its IS and LM components, since these are just conditions of equilibrium in the goods and money markets," I also stressed that "while some of us may choose to use the IS-LM framework to express results that have been discovered in richer models, it is not a vehicle for deriving those results. To simplify economic reality sufficiently to use the IS-LM model as an analytical tool, economists must essentially ignore expectations...."

Inflation Implications

In the New IS-LM model, there is a direct link between the objective of keeping output at a capacity level—which Goodfriend and King (1997) call a neutral monetary policy objective—and the dynamics of inflation. Setting $y_t = \overline{y}_t$ in (3) and solving this expression forward implies that

$$\pi_t = \beta E_t \pi_{t+1} + x_{\pi t} = \sum_{j=0}^{\infty} \beta^j E_t x_{\pi,t+j}. \tag{7}$$

This solution has three direct implications.

The case for price stability: If there are no inflation shocks ($x_{\pi t} = 0$ for all t) then the solution is that the inflation rate should always be zero. This is a striking, basic implication of the New IS-LM model. Reversing the direction of causation, it means that a central bank which keeps the price level constant also makes output always equal to the capacity level. Finally, it means that shocks to aggregate demand such as x_{dt} and to the determinants of capacity output $\overline{y}_t$ do not affect the price level under a neutral monetary policy regime.

The case for simple inflation targets: If there are inflation shocks, there continues to be an average inflation rate of zero under a neutral monetary policy.[11] However, as Clarida, Gali, and Gertler (1999) stress, the New IS-LM model suggests that there may be sustained departures from the zero long-run inflation target as a result of inflation shocks. For example, if the shock term is a first-order autoregression, $x_{\pi t} = \rho x_{\pi,t-1} + e_{\pi t}$, then the solution for the neutral inflation rate is

$$\overline{\pi}_t = \frac{1}{1 - \beta\rho} x_{\pi t} = \rho \overline{\pi}_{t-1} + \frac{1}{1 - \beta\rho} e_{\pi t},$$

so that the inflation target inherits the persistence properties of the inflation shock. If the persistence parameter ρ is positive, then a higher-than-average current inflation target implies that there will be, on average, a higher-than-average inflation target in the future.

In this setting, a central bank must more actively manage inflation in order to keep output at its capacity level. The New IS-LM model, however, implies that many shocks do not affect the inflation rate if it is managed to keep output at capacity, including aggregate demand shocks x_{dt}, shifts in determinants of capacity output $\overline{y}_t$, and shocks to the demand for money x_{vt}.

Appraising This Policy Implication

This strong policy conclusion raises a number of questions, which are considered in turn. In trying to answer these questions, we encounter a natural

---

[11] Recall that the inflation shocks are assumed to have a zero mean.

limitation of IS-LM models, new and old. Since these models are not built up from microfoundations, the answers frequently will require stepping outside the confines of the model to discuss other, related research.

Is this result a special one or does it hold in other related models? In fact, King and Wolman (1996) found that a constant inflation target causes real activity to remain at essentially the capacity level when there are changes in productivity or money demand within a fully articulated, quantitative model (a setting where sticky prices, imperfect competition and an explicit role for monetary services were added to a standard real business cycle model). The generality of this conclusion is suggested by the fact that Rotemberg (1996) was led to call it a "mom and apple pie" result in his discussion of King and Wolman (1996).[12]

What is capacity output? When explicit microfoundations are laid out, it is potentially possible to define a measure of capacity output more precisely. Goodfriend and King (1997) followed this approach—within a class of models with sticky prices, imperfect competition, and flexible factor reallocation—to identify capacity output as the level of output which would obtain if all nominal prices were perfectly flexible, but distortions from imperfect competition remained present in the economy.

Is stabilization at capacity output desirable? If output is inefficiently low due to monopoly or other distortions, then it may not be optimal to always keep output at its capacity level: optimal monetary policy may seek to produce deviations of output from capacity in response to underlying shocks. To study this issue carefully, though, it is again necessary to develop microeconomic foundations and to consider the design of monetary policies which maximize the welfare of agents in response to various shocks (as with the productivity shocks analyzed in Ireland [1996]). Studying a fully articulated economy with multiperiod price stickiness, King and Wolman (1999) show it is efficient—in the sense of maximizing welfare—to fully stabilize the price level and to keep output at its capacity level in response productivity shocks.[13]

Economic Activity under Neutral Policy

In the analysis above, the Phillips curve (3) was used to determine the behavior of inflation which is consistent with output being at its capacity level ($y_t = \overline{y}_t$). The other equations of the model economy then restrict the behavior of the remaining variables.

---

[12] He also verified that it held in other, related fully articulated models (Rotemberg and Woodford 1997, 1999).

[13] See also Goodfriend and King (1997) and Rotemberg and Woodford (1997).

Given that output is at its capacity level, the IS curve then implies that the real rate of interest is

$$\bar{r}_t = \frac{1}{s}[E_t\bar{y}_{t+1} - \bar{y}_t + x_{dt}].\qquad(8)$$

This is a neutral or "natural" real rate of interest, the idea of which is developed in more detail in Section 5.2 below. The real rate of interest is positively affected by growth in capacity output $E_t\bar{y}_{t+1} - \bar{y}_t$ and by aggregate demand shocks x_{dt}.

Taking this natural rate of interest $\bar{r}_t$ together with expected inflation, the Fisher equation (2) then implies that the nominal interest rate is

$$\bar{R}_t = \bar{r}_t + E_t\bar{\pi}_{t+1}.\qquad(9)$$

That is, a neutral interest rate policy must make the nominal interest rate vary with the natural rate of interest and the inflation target (7). For example, if the real economy is expected to display strong real growth in capacity output, then the nominal interest rate must be raised.[14]

Finally, the money demand function (4) implies that the stock of money evolves according to $M_t = (\bar{\pi}_t + P_{t-1}) + \delta\bar{y}_t - \gamma\bar{R}_t - x_{vt}$. That is, money growth obeys

$$\overline{M}_t - \overline{M}_{t-1} = \bar{\pi}_t$$
$$+ [\delta(\bar{y}_t - \bar{y}_{t-1}) - \gamma(\bar{R}_t - \bar{R}_{t-1}) - (x_{vt} - x_{v,t-1})],\quad(10)$$

which is the sum of the chosen inflation target and the change in the real private demand for money.

Implementation via a Money Stock Rule

One way to implement a neutral monetary policy is via a money stock rule. The solution (10) indicates that in order for the economy to stay at capacity output, the money stock must respond to the state of the economy. In particular, the growth of the neutral money stock is a complicated function of the exogenous variables of the model. Money growth must move one-for-one with the target rate of inflation $\bar{\pi}_t$, which in turn depends on the inflation shock $x_{\pi t}$. Money growth must also accommodate the changes in real demand for money brought about by growth in the capacity level of output $\bar{y}_t$, as stressed by Ireland (1996). It must also accommodate shocks to the demand for money and changes in the neutral nominal interest rate (which in turn depend on changes in the expected growth in capacity output and changes in the inflation target from (7)). This policy rule involves choices in the general money supply function (5), namely that there are no money supply shocks ($x_{Mt} = 0$) and that the systematic

---

[14] Unless there is simultaneously a negative price shock for some reason.

component of policy is given by $f_{Mt} = M_{t-1} + \overline{\pi}_t + [\delta(\overline{y}_t - \overline{y}_{t-1}) - \gamma(\overline{R}_t - \overline{R}_{t-1}) - (x_{vt} - x_{v,t-1})]$.

Under this rule, the central bank is not responding directly to output, inflation and so forth. Instead, it is responding to the fundamental determinants of economic activity.[15] Further, implicit in treating the solution (10) as a policy rule is the statement by the monetary authority, "if inflation deviates from the neutral level then no adjustment in the path of the money stock will occur." In the rational expectations equilibrium of the New IS-LM model, this statement turns out to be sufficient to assure that no departures of inflation from the neutral inflation rate ever occur.

Implementation via an Interest Rate Rule

There has been a great deal of research on interest rate rules in recent years for at least three reasons. First, as argued by Goodfriend (1991), this research focus matches well with the fact that the Federal Reserve actually implements monetary policy by choosing the setting of the federal funds rate, a very short-term nominal interest rate. Second, as shown by Taylor (1993), some simple interest rate rules appear to yield a quantitative match with the behavior of the FRS over various time periods. Third, there are interesting conceptual issues that arise regarding the determination of macroeconomic activity under an interest rate rule.

In looking for an interest rate rule that would yield the neutral level of output, a reasonable first idea would be to select the interest rate solution (9). In the New IS-LM model, as in other many frameworks considered by monetary economics dating back at least to Wicksell, this choice would not be enough to assure that the neutral level of real activity would occur. It might, but other levels of economic activity could also arise. One way of thinking about why multiple equilibria may occur is that money is demand-determined under an interest rate rule, so that the monetary authority is implicitly saying to the private sector, "any quantity of money which you desire at the specified nominal interest rate $\overline{R}_t$ will be supplied."

To eliminate the possibility of multiple equilibria, it is necessary for the monetary authority to specify how it would behave if the economy were to depart from the neutral level. For example, a specific interest rate rule—which responds to deviations of inflation from neutral inflation—is

$$
\begin{aligned}
R_t &= \overline{R}_t + \tau(\pi_t - \overline{\pi}_t) \\
&= [\overline{r}_t + E_t \overline{\pi}_{t+1}] + \tau(\pi_t - \overline{\pi}_t).
\end{aligned}
$$

---

[15] From this standpoint, it is clear that the assumption above—that the central bank and other actors have complete information about the state of the economy—is a strong one.

By specifying $\tau > 0$ then, the monetary authority would be saying, "if inflation deviates from the neutral level, then the nominal interest rate will be increased relative to the level which it would be at under a neutral monetary policy." If this statement is believed, then it may be enough to convince the private sector that the inflation and output will actually take on its neutral level.

Thus, a substantial amount of work on the New IS-LM model has concerned finding the conditions which assure a unique equilibrium. Section 7 below exemplifies this research. For the interest rate rule above, it shows that one way of assuring a unique equilibrium is to have a strong positive response, $\tau > 1$, as Kerr and King (1996) previously stressed. But, it also stresses that (i) a rule which specifies a strong negative responses to current inflation may also lead to a unique equilibrium, and (ii) that strong positive responses may lead to multiple equilibria if policy is forward looking.

4.   PRICE STICKINESS AND ECONOMIC ACTIVITY

Milton Friedman (1970, p. 49) focused attention on the importance of determining how a change in nominal income is divided between responses of real output and the price level at various horizons. In the New IS-LM model, changes in monetary policy can affect real output because there is price stickiness of a sort long stressed in Keynesian macroeconomics. But since stickiness of prices is modeled in a New Keynesian manner—with pricing rules based on firms' optimizing behavior—there are some novel implications for the dynamics of real output and the price level.

The Structure of the New Phillips Curve

The New Keynesian research on aggregate supply was designed to produce an "an old wine in a new and more secure bottle" by providing a better link between inflation and real activity, with microfoundations that earlier Keynesian theories lacked.[16] Four key ideas are stressed in the twin volumes edited by Mankiw and Romer (1991) on this topic: costly price adjustment, asynchronous price adjustment, forward-looking price setting, and monopolistic competition.

These ideas have been implemented in a variety of applied macroeconomic models beginning with Taylor's (1980). All of these sticky price models contain two central ingredients. First, since price adjustment does not take place simultaneously for all firms, the price level is a weighted average of current and past prices. Second, since firms have market power and recognize that their nominal prices may be fixed for some time, the models display a

---

[16] See Phelps and Taylor (1977), p. 166.

richer, forward-looking pattern of price-setting than that which arises in the standard, static monopoly pricing model.

These general ideas have been implemented in a variety of different approaches to pricing. Models in the style of Taylor (1980) assume that firms adjust their prices every J periods, where J is assumed to be fixed. Calvo (1983) proposed an alternative stochastic adjustment model, in which each firm has a constant probability of being able to adjust its price every period. The Calvo model has been incorporated into the New IS-LM model for four reasons. First, it seems to capture a key aspect of price dynamics at the level of individual firms, which is that these involve discrete adjustments which occur at irregularly spaced intervals of time. Second, it leads to price level and price-setting expressions which can be readily manipulated analytically. Third, this approach has provided a tractable base for recent studies which have provided empirical support for the New Keynesian approach to pricing.[17] Fourth, it also turns out to be observationally equivalent at the aggregate level to a popular alternative model of price adjustment—the quadratic cost of adjustment model for prices—as shown by Rotemberg (1987).[18] At the same time, the Calvo and Taylor models are similar in the broad predictions developed in this section, so that the increased tractability comes at a small apparent cost.[19]

In the Calvo model, the microeconomic extent of price stickiness is determined by a single parameter, the probability that a firm will be unable to adjust its price in a given period, which will be called η.[20] Since a firm's adjustment probabilities do not depend on the duration of its interval of price fixity, there is a probability η^j of being stuck in period $t + j$ with the price that is set at t and the probability of first adjusting in j periods is $(1 - \eta)\eta^{j-1}$. Accordingly, the expected duration of price stickiness is $1(1 - \eta) + 2(1 - \eta)\eta + \ldots (j + 1)(1 - \eta)\eta^j + \ldots = \frac{1}{\eta}$, which depends on η in a convenient manner.

This degree of microeconomic stickiness plays a role in both the nature of the price level and the nature of the pricing decision. In the model economy, there are many, essentially identical firms which face stochastic individual opportunities to adjust prices. With a large number of firms in the economy,

[17] Recent interesting empirical studies of this approach include Roberts (1995), Gali and Gertler (1999), and Sbordonne (1998).

[18] Rotemberg (1982) used the quadratic cost of adjustment model to study U.S. price dynamics. Generalizations of this approach, developed in Tinsley (1993) are employed in the Federal Reserve System's new rational macroeconometric model.

[19] However, Wolman (2000) stresses that they can be quite different in some detailed implications for price dynamics.

[20] This model is sometimes criticized on a number of grounds. First, the probability of being able to adjust price is independent of the time since the last price adjustment, so that firms face some chance of being trapped with a fixed price for a very long time. Second, the probability of price adjustment is exogenous. Dotsey, King, and Wolman (1999) study time-dependent and state-dependent pricing that overcomes each of these objections.

the fraction of firms adjusting price in a period is equal to the probability of price adjustment $(1 - \eta)$ and the fraction of firms stuck with a price that is j periods old is $(1 - \eta)\eta^j$.

A backward-looking price level: In general, the price level is an average of prices. In any model with staggered price-setting, some of these prices will be newly set by firms which are adjusting prices and some will have been set in prior periods. Taking P_t^* to be the price chosen by all adjusting firms in period t and P_t to be the price level as above, the following simple loglinear specification captures the idea that the price level is an average of prices:

$$P_t = (1 - \eta) \sum_{j=0}^{\infty} \eta^j P_{t-j}^* = \eta P_{t-1} + (1 - \eta) P_t^*. \tag{11}$$

The second equality derives from the definition of the lagged price level: it is a convenient expression for many analytical purposes. Notably, (11) can be rewritten as a partial adjustment mechanism, $P_t - P_{t-1} = (1-\eta)[P_t^* - P_{t-1}]$, so that the price level responds only gradually when P_t^* is raised above P_{t-1} with the extent of price level adjustment just being the microeconomic probability of price adjustment.

Forward-looking price-setting: A key aspect of New Keynesian models is that firms know that their prices may be sticky in future periods. For this reason, they rationally consider future market conditions when they set prices. The idea of forward-looking price-setting by firms may be captured with the specification

$$P_t^* = (1 - \beta\eta) \sum_{j=0}^{\infty} (\beta\eta)^j E_t[\psi_{t+j} + P_t] + x_{Pt} \tag{12}$$

$$= \eta\beta E_t P_{t+1}^* + (1 - \beta\eta)[\psi_t + P_t] + x_{Pt} - \beta\eta E_t x_{P,t+1}, \tag{13}$$

which can be developed from the Calvo model as in Rotemberg's survey of New Keynesian macroeconomics (1987). The price chosen by firms adjusting at date t, P_t^*, is a distributed lead of nominal marginal cost (real marginal cost is ψ_t so that nominal marginal cost is $\psi_t + P_t$ in this loglinear world). There are two parts to the discounting: β, which represents a conventional market discount factor (so that β is very close to, but less than one) and η, which reflects the fact that firms know that there is a lower probability of being stuck with today's price as they look further ahead. The shock x_{Pt} is a structural shock to the level of prices set by firms in period t and its relationship to the inflation shock introduced earlier in (3) will be determined later. The second line of (12) involves using the definition of P_{t+1}^* to eliminate the distributed lead of future nominal marginal cost.

The forward-looking pricing rule (12) implies that a current change in nominal marginal cost affects P_t^* very differently if it is expected to be permanent than if it is expected to be temporary. If nominal marginal cost is

expected to be the same in all future periods, then there is a one-for-one effect of its level on P_t^* since $(1 - \beta\eta)\sum_{j=0}^{\infty}(\beta\eta)^j = 1$: a firm will raise its price proportionately if changes in marginal cost are expected to be permanent. By contrast, P_t^* will respond by a smaller amount, $(1 - \beta\eta)$, if the change in marginal cost is expected to be temporary, affecting only date t marginal cost.

Output and demand: New Keynesian macroeconomists stress that an optimizing, monopolistically competitive firm will rationally supply additional output in response to an expansion of demand, rather than rationing customers, when its price is sticky (see, for example, Romer [1993]). This output response is profitable so long as the firm's sticky nominal price is greater than its nominal marginal costs. The specification (3) assumes that this is true over the range of disturbances considered in the New IS-LM model.

A heroic assumption: To generate (3), a final—heroic—assumption is needed. In particular, assume that real marginal cost is positively related to the output gap, with the parameter h being the elasticity of this response. That is,

$$\psi_t = h(y_t - \overline{y}_t). \tag{14}$$

The parameter h is positive under conventional assumptions about the aggregate production function and factor supply elasticities. Real marginal cost would necessarily rise with the level of economic activity if the economy had some fixed factors (such as a predetermined capital stock) or if higher real wage rates were necessary to induce workers to supply additional hours.

The specification involves a shortcut that avoids modeling of the labor market, which is complicated, difficult, and controversial. Some fully articulated models suggest that (14) is a useful approximation and also suggest particular values of h. Others may suggest that this assumption is a weakness of the New IS-LM model.

Putting the elements together: Combining (11), (12), and (14), as is done in Appendix A, leads to

$$
\begin{aligned}
P_t - P_{t-1} \;=\;\; & \beta(E_t P_{t+1} - P_t) \\
& + [h\frac{(1 - \eta)(1 - \beta\eta)}{\eta}](y_t - \overline{y}_t) \\
& + (\frac{1 - \eta}{\eta})[x_{Pt} - \beta\eta E_t x_{P,t+1}].
\end{aligned} \tag{15}
$$

This is identical to (3), but there is an explicit linking of the parameter $\varphi = h\frac{(1-\eta)(1-\beta\eta)}{\eta}$ to deeper parameters of the price adjustment process and the elasticity of marginal cost with respect to the output gap.[21]

---

[21] There is also a linking of the inflation shock $x_{\pi t}$ to underlying shocks to the price setting equation x_{Pt} above, which is $x_{\pi t} = (\frac{1-\eta}{\eta})[x_{Pt} - \beta\eta E_t x_{P,t+1}]$. This latter linkage is important

Long-run neutrality: The form of the equation (15) highlights the fact that a purely nominal disturbance, which permanently affects the level of prices at all dates by the same amount, will have no effect on the level of real economic activity within the New IS-LM model. Specifically, if the price level is constant at all dates ($E_t P_{t+1} = P_t = P_{t-1} = P$) and there are no inflation shocks ($x_{\pi t} = 0$), then output is equal to capacity ($y_t = \bar{y}_t$).

The Nonneutrality of Nominal Shocks

Many New Keynesian authors, including Taylor (1980) and Mankiw (1990), have stressed that the new Phillips curve implies that nominal disturbances can have effects on real economic activity because prices are sticky and output is demand-determined. In this subsection, the implications of price stickiness for the division of nominal income changes into prices and output are explored.

Implications from analytical solutions for output and prices: Suppose that nominal income is exogenous and governed by the simple rule $Y_t - Y_{t-1} = \rho(Y_{t-1} - Y_{t-2}) + x_{Yt}$ with x_{Yt} being a series of "white noise" shocks.[22] For simplicity, assume that capacity is expected to be constant through time at $\bar{y}$ and that there are no price shocks.

Since (15) is a much-studied second order expectational difference equation, whose solution is reported in Appendix B of this article, it is easy to compute the solution for the price level. The solution takes the form

$$P_t = \theta P_{t-1} + (1 - \theta)(1 - \beta\theta) \sum_{j=0}^{\infty} (\beta\theta)^j E_t(Y_{t+j} - \bar{y}) \qquad (16)$$

$$= \theta P_{t-1} + (1 - \theta)(Y_{t-1} - \bar{y}) + \frac{1 - \theta}{1 - \theta\beta\rho}(Y_t - Y_{t-1})$$

where θ is the smaller root of the equation $\beta z^2 - [1 + \beta + \varphi]z + 1 = 0$, which may be shown to be between zero and one (see Appendix B). Further, since $y_t = Y_t - P_t$, the model's implications for output are readily calculated

$$y_t - \bar{y} = (\theta \frac{1 - \beta\rho}{1 - \theta\beta\rho})[Y_t - Y_{t-1}] + \theta[y_{t-1} - \bar{y}] \qquad (17)$$

There are several aspects of these solutions that warrant discussion. First, the coefficient θ provides one measure of the degree of gradual price level

---

in terms of assessing the magnitude of inflation shocks. If price shocks are independent through time, as some theories of mistakes suggest, then $x_{\pi t} = (\frac{1-\eta}{\eta})x_{Pt}$ and with one-quarter of firms adjusting prices each period ($\eta = .75$), then inflation shocks will be only one-third as large as price-setting errors.

[22] There are two alternative ways to rationalize this. One is that there is a strong form of the quantity equation, with the money demand function (4) satisfying $\delta = 1$ and $\gamma = 0$ and the money supply equation (5) taking the form $M_t = x_{Mt}$ with x_{Mt} being a random walk. Another is that the monetary authority follows a monetary policy rule which makes nominal income equal to an exogenous random walk.

adjustment at the macroeconomic level, since it indicates the extent to which the past price level influences the current price level. This is different from the extent of price stickiness η at the microeconomic level, although increases in η lead to larger values of θ. In this example, θ is influenced by the elasticity of marginal cost h as well as η. If inflation is more responsive to departures of output from the capacity level, then the current price level becomes less sticky, in the sense that it is less dependent on the past price level. (More specifically, lower values of η or higher values of h lead to higher values of φ, which in turn make for smaller solutions for θ.) More generally, the importance of predetermined prices to the current price level depends on the structure of the entire macroeconomic model, i.e., it is a system property rather than a property of just the equations of the "price block", such as (11) and (12).[23]

Second, the degree of gradual price level adjustment is important for the persistence of output fluctuations: θ enters (16) as the coefficient on the lagged price level and enters (17) as the coefficient on the lagged output level. The simplicity of this linkage reflects the fact that nominal income is evolving exogenously in this model, but the general relationship between the extent of gradual price level adjustment and the degree of output persistence also carries over to richer setups.

Third, when the growth rate of nominal income is white noise (so that the level of nominal income is a random walk), then θ also controls the split of a change in nominal income between output and the price level. If prices are more sticky, then nominal income changes have a greater effect on real output.

Fourth, when the growth rate of nominal income becomes more persistent, then there is a larger effect of a surprise nominal income change on the price level and a correspondingly smaller one on output. In fact, if the changes in nominal income growth are permanent ($\rho = 1$) and market discounting is small ($\beta = 1$) then the coefficient on $Y_t - Y_{t-1}$ in the price level equation (16) becomes one and the coefficient in the output equation (17) becomes zero. In this limiting situation, there is neutrality independent of the degree of underlying price stickiness or the value of θ which is the indicator of the gradual adjustment of the price level.

Implications from simulated responses to an increase in nominal income: Figure 2 highlights some implications of (3) and a similar figure will be used later to highlight some implications of the full New IS-LM model. In constructing these figures, the time unit is taken to be one quarter of a year, which is a conventional macroeconomic modeling interval. The response of the price

---

[23] For this reason, it is affected by other parameters of the New IS-LM model when the full model is solved, as in Section 8 below.

Figure 2

a. Price Level

b. Output
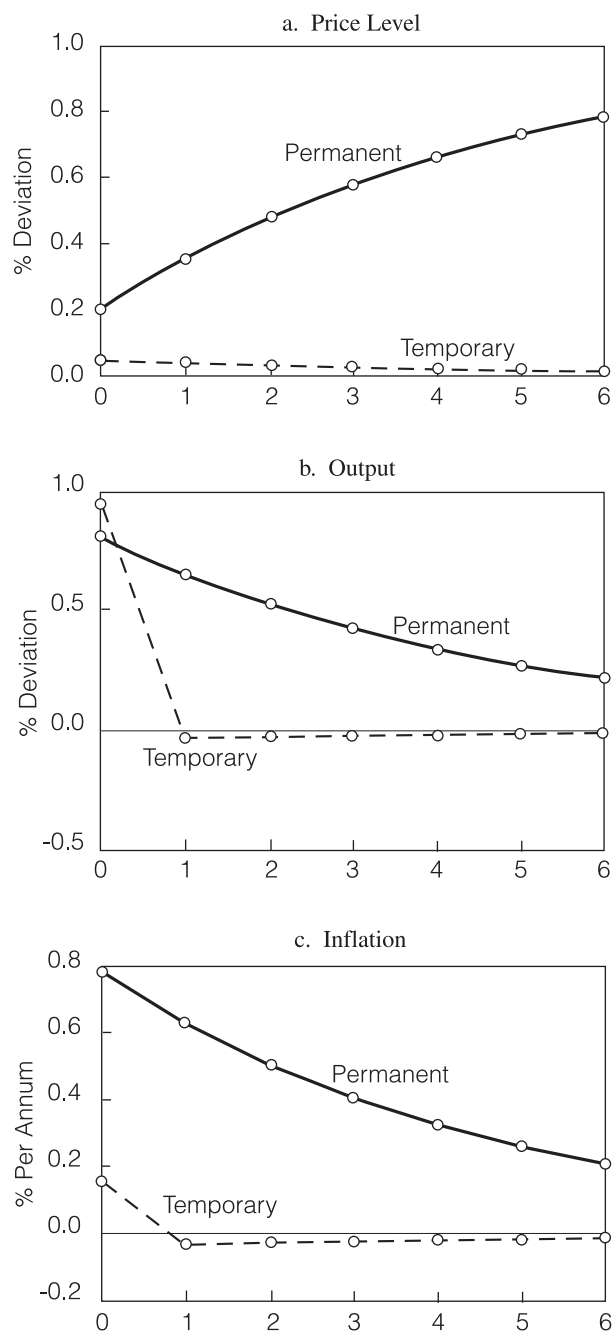
c. Inflation

level and output will be measured in percentage points and the responses of inflation rates and interest rates will be measured in percent per annum.[24]

The solid line in panel b of Figure 2 shows the (impulse) response of output to an unexpected and permanent one percent increase in nominal income which takes place at date 0. Given that nominal income is exogenously one percent higher and since $y = Y - P$, the path for output is a mirror image of the path for prices: output is high when prices are low relative to the level of nominal income. On impact, output rises by $0 < \theta < 1$ percent, with the figure constructed under the assumption that $\theta = .20$.[25] The price level rises by $0 < (1 - \theta) < 1$ percent, with the figure constructed under the assumption that $1 - \theta = .8$.

In subsequent periods, the price level gradually adjusts up to its new higher long-run level, while output falls back toward the capacity level. The speed of adjustment is again given by the value of θ. There is an output effect of θ percent in the first period, θ^2 in the second period, and so forth.

The inflation rate is shown by the solid line in panel c of Figure 2 and is given mathematically by differencing (16) under the assumption that $\rho = 0$, which results in $\pi_t = \theta \pi_{t-1} + (1 - \theta)(\Delta Y_t - \Delta \overline{y})$.[26] This is exactly the same solution as for the level of the output gap, so that a crude Phillips curve relationship of the form $\pi_t = (\frac{1-\theta}{\theta})(y_t - \overline{y})$ would work perfectly in this economy, given the assumed driving process. More generally, under a variety of driving processes, the model predicts that a rising price level (inflation) is positively associated with high output (relative to capacity).[27] In this sense, the model can generate a traditional empirical Phillips correlation between inflation and real activity.

Persistent Output Effects

Many empirical studies suggest that business cycles arising from nominal disturbances display considerable persistence, lasting for many quarters. Taylor (1980) and other New Keynesian macroeconomists have suggested that price stickiness can lead to persistent effects of various disturbances on output.

---

[24] These conventional measurement choices will require some care when comparisons are made across the panels of the figures, as discussed further below.

[25] This value of θ obtains when $\beta = .99$ and $\varphi = .05$, which are the parameter values used in sections below.

[26] Since the inflation rate is stated at an annualized percentage rate of change, the .2 percentage point increase in the price level (shown in panel a of Figure 2) that occurs at the initial date corresponds to a $4 * .2 = .8$ rise in the annualized inflation rate at the initial date (shown in panel c of Figure 2). By contrast, all of the mathematical relationships described in the text and appendices involve the quarterly inflation rate, i.e., the percentage change in the price level between $t - 1$ and t.

[27] In particular, (17) implies that $\pi_t = \frac{1-\theta}{\theta} \frac{1-\beta\rho}{1-\theta\beta\rho}(y_t - \overline{y})$. Thus, the slope of the Phillips curve depends negatively on the persistence of nominal income growth.

If persistent business cycles arise from changes in nominal income then (3) implies that nominal income must itself be persistent.[28]

To illustrate this point, (16) can be used to recompute the solution for the price level in the case of purely temporary variations in nominal income:

$$P_t = \theta P_{t-1} + (1 - \theta)(1 - \beta\theta)(Y_t - \overline{y})$$

and the comparable solution for output is

$$\begin{aligned} y_t - \overline{y} &= [(1 - (1 - \theta)(1 - \theta\beta)][Y_t - \overline{y}] \\ &\quad + \theta[y_{t-1} - \overline{y}] - \theta[Y_{t-1} - \overline{y}]. \end{aligned}$$

Thus, the effect of a purely temporary change in aggregate demand is to raise the price level somewhat and to raise output considerably on impact. But in subsequent periods, the economy will be stuck with a price level above its long-run level and have a smaller than capacity level of output. These dynamics are the dashed lines in panels a–c of Figure 2. In the impact period, the rise in nominal income produces a large increase in real output and a small increase in the price level, because price-setters correctly understand that the increase in nominal income is temporary. These analytical and simulation results highlight the fact that the pricing dynamics underlying the New Keynesian Phillips curve (3) do not themselves make business cycles persistent.[29]

5.   LONG-RUN LIMITS ON MONETARY POLICY

While allowing for short-run effects of nominal income on output, the New IS-LM model embodies the idea—put forward by Friedman (1968) and Phelps (1967)—that the monetary authority cannot engineer permanent departures of output from its capacity level. This idea was formalized in an earlier generation of rational expectations macromodels and is sometimes described as involving a vertical long-run Phillips curve. For this reason, this section considers the long-run limits on monetary policy under some alternative specifications of aggregate supply, ending with a discussion of the relationship in the New IS-LM model.

The Price Surprise Supply Curve

A previous generation of IS-LM macromodels incorporated an alternative "expectations augmented" Phillips curve (notably, see Sargent and Wallace

---

[28] But, as discussed above, its growth rate cannot be too persistent or there will be no effect of a surprise change.

[29] Chari, Kehoe, and McGrattan (2000) question whether even permanent movements in the money stock can cause persistent movements in output. In terms of the present model, they do so by imposing restrictions on η and h.

[1975] and McCallum [1989]). In particular, these models used an aggregate supply curve of the "price surprise" form

$$y_t - \overline{y}_t = l(P_t - E_{t-1}P_t), \tag{18}$$

where l is a positive parameter that governs the influence of increases in prices on output. This aggregate supply curve was rationalized by Lucas (1972a, 1973) as arising from incomplete information on the part of suppliers and by Phelps and Taylor (1977) as arising from sticky prices.

By subtracting the past price level from both P_t and $E_{t-1}P_t$, an expectational Phillips curve quite similar to (3) can be derived:

$$\pi_t = E_{t-1}\pi_t + \frac{1}{l}(y_t - \overline{y}_t). \tag{19}$$

Modern presentations of aggregate supply theory—such as those in the textbooks referenced above—stress two implications of (18) or (19) that were developed in the 1970s. First, if there is surprise expansion of demand— taken as in Section 4.2 to be an increase in nominal output $y + P$—then there is an increase in both output and the price level, with the split between these depending on the size of the supply elasticity l. Thus, there is a positive relationship between inflation and output when there are shocks to nominal demand, i.e., a short-run correlation of the form discovered by Phillips. Second, any expected expansion of demand would raise expected and actual inflation by the same amount, thus neutralizing the real consequences.[30] Thus, there is no long-run Phillips curve and the position of the short-run Phillips curve (in π, y_t space as in Figure 1) shifts with the expected rate of inflation.

The Long-run Effect of Inflation

The analysis of Section 4.2 demonstrated a similar link between temporary movements in inflation and output for the New IS-LM model's Phillips curve (3), $\pi_t = \beta E_t\pi_{t+1} + \varphi(y_t - \overline{y}_t)$. To explore the long-run implications in the new model, suppose that the economy is in an inflationary steady-state with $\pi_t = E_t\pi_{t+1} = \pi$. Then, output will be

$$y_t = \overline{y}_t + \frac{1 - \beta}{\varphi}\pi$$

so that we can say that the "long-run slope" of the Phillips curve is $\frac{1-\beta}{\varphi}$. This slope measures the response of output to changes in the long-run rate of

---

[30] More specifically, the response of output can be calculated as follows. First, it is direct from (18) that $E_{t-1}y_t = E_{t-1}\overline{y}_t$, i.e., that the economy is expected to be at capacity each period. Second, the response of real output can be calculated by using $Y_t - E_{t-1}Y_t = (P_t - E_{t-1}P_t) + (y_t - E_{t-1}y_t)$ together with (18) to determine that $P_t - E_{t-1}P_t = \frac{1}{1+l}(Y_t - E_{t-1}Y_t)$ and $y_t - E_{t-1}y_t = \frac{l}{1+l}(Y_t - E_{t-1}Y_t)$.

inflation, after the economy has made a transition from one inflationary steady state to another. With β close to unity, then, (3) implies there is a negligible long-run slope to the Phillips curve.

Experiments with fully articulated models—such as that constructed by King and Wolman (1996)—suggest that the effect of inflation on output relative to capacity is very small.[31] Accordingly, the condition $\beta = 1$ is imposed in the remainder of this section. The fully articulated models provide this quantitative result because (i) firms do not allow sustained inflation to have much effect on their monopoly profits and (ii) households do not allow sustained inflation to have much effect on their factor supply. [32]

Estimating the Long-run Effect

Lucas (1972b) and Sargent (1971) showed that it was a subtle matter to estimate the long-run effect if the economy possessed an economy with an aggregate supply equation of the form (18) or a price equation of the form (19).[33]

Earlier, Gordon (1970) and Solow (1969) had proposed to estimate the long-run slope by specifying a hybrid model that nested expectational and nonexpectational forms of the Phillips curve. A simple form of this hybrid empirical model is

$$\pi_t = g E_{t-1}\pi_t + \varphi(y_t - \overline{y}_t).$$

With $g < 1$, this specification would imply a long effect of inflation on output, with a slope of $\frac{1-g}{\gamma} > 0$. Solow and Gordon estimated this specification using adaptive expectations proxies for $E_{t-1}\pi_t$, with the simplest variant of their procedure assigning $E_{t-1}\pi_t = \pi_{t-1}$. In general, these studies found g to be significantly less than one through the 1970s.

Lucas and Sargent argued that this procedure was flawed in a setting with rational expectations. To illustrate their point, suppose that $\pi_t = \rho\pi_{t-1} + e_t$ with $\rho < 1$. Then the rational expectations solution for inflation is $\pi_t = \rho\pi_{t-1} + \varphi(y_t - \overline{y}_t)$. Application of the Solow-Gordon method would thus estimate that $g = \rho < 1$. Therefore, as stressed by Lucas and Sargent, the reduced form relationship would indicate an exploitable long-run trade-off, with a 1 percent higher inflation rate yielding $\frac{1-\rho}{\varphi}$ percentage points higher output, even though no tradeoff was actually present.

---

[31] The closely related model of Yun (1996) eliminates effects of sustained inflation by essentially allowing firms to index their nominal prices by the trend inflation rate.

[32] There is a subtlety here, in that sticky price models built up from micro foundations can imply that there is a small effect of inflation on the volume of physical output—a quantity aggregate—while there is a larger effect of inflation on the value that households place on this output, due to relative price distortions that emerge when prices are sticky.

[33] Lucas (1972b) worked with a supply schedule, while Sargent (1971) worked with a wage equation.

The Phillips curve (3) in the New IS-LM model also implies that there is this set of problems. Supposing as above that $\pi_t = \rho \pi_{t-1} + e_t$ with $\rho < 1$ to illustrate this point, it follows that (3) implies that $\pi_t = \frac{\varphi}{1-\rho}(y_t - \overline{y}_t)$. An econometrician conducting Solow and Gordon's test would estimate $g = 0$ and calculate that a 1 percent higher inflation rate would yield $\frac{1-\rho}{\varphi}$ percentage points of output.[34]

Overall, the New IS-LM model thus embodies the consensus among macroeconomists that there is little long-run trade-off between inflation and real activity. It also suggests, as did earlier rational expectations for IS-LM models, that the existence of a short-run Phillips curve could mislead applied econometricians and central bankers into believing that there is a long-run trade-off.

Disinflation Dynamics

In terms of permanent changes in the inflation rate, such as that engineered by the Federal Reserve System during the "Volcker deflation" of 1979–1983 and more recently by other central banks around the world, there are some very classical implications of the Phillips curve, stressed by Buiter and Miller (1985), that is incorporated in the New IS-LM model. While these implications are not strictly the limits on monetary policy which are the focus of this section, they are related to the shifts in trend inflation considered here.

Within the "surprise" form of the Phillips curve, which developed from Lucas's (1973) analysis, there is only a one-time real effect of an unanticipated, permanent, and credible change in the inflation rate since (19) implies that $\pi_t = E_{t-1}\pi_t + \frac{1}{l}(y_t - \overline{y}_t)$. To illustrate this point, suppose that the inflation rate is governed by the random walk specification, $\pi_t = \pi_{t-1} + e_t$, which implies that all inflation changes are unexpected and permanent. Then, $E_{t-1}\pi_t = \pi_{t-1}$ and a decline in the date t rate of inflation causes an output decline of $(y_t - \overline{y}_t) = le_t$ with no expected consequences for future output.

The new Phillips curve (3) has a related, but stronger implication: There is no effect of an unanticipated, permanent and credible shift in the inflation rate since $\pi_t = E_t\pi_{t+1}$ in this case and the above analysis (with $\beta = 1$) that changes in the trend rate of inflation have no effect on real activity.[35] Ball (1995) emphasizes the importance of policy credibility to this implication of snap disinflation.

---

[34] The assumption of exogenous inflation is simply for analytical convenience: a similar spurious long-run tradeoff appears, as in Section 4.2, when the model is solved with exogenous nominal income.

[35] A slight modification of the structure of the current model—requiring that firms post prices prior to receiving information about date t—is employed in Bernanke and Woodford (1997). This has the implication that $\pi_t = E_{t-1}\pi_{t+1} + \varphi(y_t - \overline{y}_t)$ so its has the same implication for an unanticipated, permanent disinflation as does (19).

6.   THE NEW IS CURVE

In this section, three aspects of the new IS curve are discussed. Section 6.1 explains the role of expected future output in the new IS curve. Section 6.2 considers the implications of omitting expectational terms for traditional IS specifications. Section 6.3 discusses two key implications of the new IS curve, the natural rate of interest and the cyclical behavior of the real interest rate, which can be obtained without full solution of the New IS-LM model.

To begin, let's return to panel a of Figure 1, which may be viewed as the familiar, traditional IS curve. In this graph, a higher real interest rate leads to a lower level of aggregate demand. Given that output is demand-determined and the economy under study is closed, a higher rate thus leads to a lower level of output/income. The negative slope of this specification reflects the idea that an increase in income is partly saved by households, with a lower real interest rate required to stimulate additional investment. The traditional IS curve is viewed as fairly steep by many economists, who believe that large changes in interest rates are necessary to produce macroeconomically important changes in aggregate demand. This steep slope corresponds to a small value of s in (1).[36]

The new IS curve also implies a negative relationship between interest rates and output, holding fixed expected inflation and expected future output. In this sense, the New IS-LM model is very traditional. As stressed by McCallum and Nelson (1999b), it is also very traditional in that no asset stocks—neither the capital stock nor the quantity of real balances—enter anywhere in these specifications.

But it also predicts that shifts in expectations about future output can be a very important determinant of the level of aggregate demand. For example, if output is expected to be 1 percent higher in the future, then the new IS specification implies that aggregate demand will be 1 percent higher today.

Importance of Expected Future Output

The potential importance of this expectations effect raises two related questions. First, why is the new IS curve written as in (1), rather than as $y_t = \chi E_t y_{t+1} - s r_t + x_{dt}$ with χ being a parameter governing the size of these expectations effects? Second, is the actual behavior of income likely to mean that there is an important difference between the two specifications?

Rationalizing the unit coefficient on $E_t y_{t+1}$: Total demand in a closed economy involves consumption, investment, and government components. In

---

[36] The traditional view that the IS schedule is relatively interest-inelastic also means that many economists have downplayed the importance of shifts in expected inflation for aggregate demand, since the effect of these is captured by $s E_t \pi_{t+1}$ in (1). Without taking a stand on the interest-elasticity of aggregate demand, the present discussion therefore downplays this channel.

the United States and most other economies, consumption is by far the largest part of this demand. The modern theory of consumption, developed by Hall (1978, 1988) and others along the lines first sketched by Irving Fisher, implies that an intertemporally efficient consumption plan equates the cost of foregone consumption today and the benefits of increased future consumption. More specifically, Hall (1978) shows that efficient consumption growth should be positively related to the real interest rate. If we let c_t be the logarithm of consumption, Hall's finding suggests that the dominant component demand should obey

$$E_t c_{t+1} - c_t = s[r_t - r],$$

which alternatively implies that

$$c_t = E_t c_{t+1} - s[r_t - r].$$

To simply apply the consumption equation to total demand, it is necessary to make one of two assumptions: either consumption is assumed to be all of aggregate demand, or the residual components of demand move exactly with total demand or consumption.[37] Neither of these is likely to be true exactly, with investment being proportionately more volatile than total demand and government purchases being proportionately less volatile. While government demand may not be forward looking, neoclassical investment theory suggests that expectations about future output will be a very important determinant of current investment, with potentially much larger effects than are present in consumption. Overall, though, the consumption theory makes (1), with a unit coefficient, the natural first approximation to the forward-looking theory of aggregate demand.

Implications for the Traditional IS Curve

Suppose that there was really a new IS curve of the form (1), but that a macroeconomic analyst worked with a traditional IS curve.

Instability and lags in the traditional IS Curve: Written in terms of the nominal interest rate and organized so as to facilitate comparison with the traditional IS curve, the new IS curve is

$$y_t = -s R_t + \{[E_t y_{t+1} + s E_t \pi_{t+1}] + x_{dt}\}.$$

The term $[E_t y_{t+1} + s E_t \pi_{t+1}] + x_{dt}$ combines the actual aggregate demand shock x_{dt} with the expectational elements that are omitted in the traditional approach.

---

[37] Woodford (1996) and Dotsey, King, and Wolman (1999) are examples of economies in which (a) there is no capital or investment and (b) there are separability restrictions on preferences; these conditions guarantee that there is exactly an IS curve of the form (1). McCallum and Nelson (1999) detail the necessary separability conditions. They also argue that (1) is a good approximation to an economy with investment because there is a small cyclical variation in the capital stock.

There are thus two key implications. First, if output and inflation expectations are substantially variable, there will be large shifts in the position of the traditional IS curve. Second, variables that are useful for forecasting $E_t y_{t+1}$ and $E_t \pi_{t+1}$ will improve the empirical fit of a traditional IS curve: since both output and inflation display important persistence empirically, lagged values of these variables can enter.[38]

The long-term interest rate and the traditional IS Curve: Many economists believe that the long-term interest rate is more important for aggregate demand than the short-term interest rate (see, for example, Goodfriend [1998]). The new IS curve also helps explain why long-term interest rates can appear more important in practice even if it is the short-term interest rate that is behaviorally relevant for certain parts of aggregate demand. For this purpose, let's assume that the expectations theory of the term structure holds exactly, without a term premium, so that the n period real interest rate is $r_t^n = \frac{1}{n}[r_t + E_t r_{t+1} + \ldots E_t r_{t+n-1}]$. Let's also assume that output is expected to be equal to its capacity level after n periods. Then, iterating the new IS curve, output can be shown to be

$$
\begin{aligned}
y_t &= -s r_t + E_t y_{t+1} + x_{dt} \\
&= -s[r_t + E_t r_{t+1} + \ldots E_t r_{t+n-1}] + E_t \overline{y}_{t+n} + x_{dt} \\
&= -\sigma r_t^n + E_t \overline{y}_{t+n} + x_{dt}
\end{aligned}
$$

with $\sigma = sn$. Thus, the implied coefficient on the long rate is much larger than s and the fit of this expression should be much better because there is no longer the omitted variable $E_t y_{t+1}$. Each of these implications occurs because the long-term real interest rate "stands in" for the influence of expected future output $E_t y_{t+1}$.

Persistence of output and the importance of expectations effects: Macroeconomists agree that fluctuations in output are highly persistent, even though there is disagreement about the precise extent of this persistence. Persistence in output makes it possible to forecast output, which in turn means that there are important variations in the $E_t y_{t+1}$ term on the right hand side of (1). Yet it is only if output variations are close to temporary that there is little practical difference between the new and old IS schedules.

Interest Rate Implications

The new IS curve also embodies two modern ideas about the link between the real interest rate and real economic activity.

---

[38] The new IS curve also can explain why empirical researchers have found it hard to isolate effects of interest rates on aggregate demand. Shifts in expected income and interest rates should be correlated with nominal interest rates, leading to biased estimates of the interest sensitivity s.

The natural rate of interest: If the economy is operating at its capacity level of output, then there is a particular level of the real interest rate which one may call the natural rate of interest. The new IS curve indicates that this natural rate of interest is given by

$$\bar{r}_t = \frac{1}{s}[E_t \bar{y}_{t+1} - \bar{y}_t + x_{dt}].$$

Thus, the natural rate of interest rises when the capacity level of output is expected to grow more rapidly. It also rises if there are shocks to demand at a given real interest rate. If there is a steep IS curve (small s) then the required increase in the real interest rate for a given growth rate of capacity output or demand shock is larger.

The real interest rate and the business cycle: The new IS schedule implies that the real interest rate also rises, more generally, when output growth is expected to be higher:

$$r_t = \frac{1}{s}[E_t y_{t+1} - y_t + x_{dt}].$$

Thus, the new IS curve implies that an economy recovering from a temporarily low level of output—one which has a high expected growth rate—would have a high real interest rate. A low real interest rate would be associated with an economy experiencing a temporarily high level of output. This implication will be very useful in interpreting the comovement of the real interest rate with cyclical fluctuations in output in Section 8.

7. LIMITS ON INTEREST RATE RULES

There has been substantial recent research on interest rate rules, since these strategies appear to describe some aspects of the actual instrument choice and policy actions of the Federal Reserve System (Goodfriend 1991, Taylor 1993). Specifically, Taylor (1993) studied the properties of an interest rate rule of the form

$$T : R_t = [r + \pi] + \tau_\pi(\pi_t - \pi) + \tau_y(y_t - \bar{y}_t), \tag{20}$$

where r is the steady state real interest rate, π is the long-run inflation, and $y_t - \bar{y}_t$ is the deviation of output from capacity.[39]

Taylor proposed that a relatively aggressive response to inflation was important; in particular, he suggested that the FRS should raise the nominal interest rate more than one-for-one in response to inflation $\tau_\pi > 1$. He also suggested that the central bank should lower the nominal interest rate when output was less than capacity, thus implying a positive value for τ_y.

---

[39] In Taylor's (1993) setting, the inflation measure was a four quarter average, but the current discussion will follow the recent literature in representing this as the current quarterly inflation rate.

In work on the consequences of alternative interest rate rules within the New IS-LM model and related fully articulated models, it is common for the space of policy rule parameters to be divided into two parts. In the first part of the parameter space, which is extensively studied, there is a unique stable rational expectations equilibrium. In the second part, which is avoided, there are multiple stable equilibria. This section describes how multiple equilibria can arise under an interest rate rule. It derives some standard restrictions on the parameters of an interest rate policy rule—some of which turn out to be related to $\tau_\pi > 1$—that lead to a unique stable equilibrium.[40]

The main focus of this section, however, is on the more specific question raised in Section 2 above: What restrictions on an interest rate rule must be imposed if the central bank seeks to obtain a neutral path of economic activity—of real output, inflation, and interest rates—as a unique outcome? To aid us in answering this question, the monetary policy rule is specified as

$$R_t = \overline{R}_t + \tau_1(E_t\pi_{t+1} - E_t\overline{\pi}_{t+1}) + \tau_0(\pi_t - \overline{\pi}_t) + x_{Rt}. \qquad (21)$$

The first term in this expression is the neutral interest rate, i.e., the level of the nominal interest rate under a neutral policy. As discussed above, the neutral nominal rate involves the sum of the natural real rate of interest and the expected future inflation target, $\overline{R}_t = \overline{R}_t + E_t\overline{\pi}_{t+1}$. The rule (21) also specifies that the monetary authority adjusts the nominal interest rate relative to its neutral level $\overline{R}_t = \overline{r}_t + E_t\overline{\pi}_{t+1}$ if there are current or expected future departures of inflation from the targeted levels. This is in keeping with the spirit of Taylor's rule, involving deviations from normal values, but is appropriate for a setup with a stochastically varying neutral path of inflation and real activity. It is a convenient choice for this article because (i) it contains a number of special cases which been studied previously in the literature, and (ii) it makes it easy to determine the restrictions on an interest rate policy rule that lead to a unique equilibrium under the neutral interest rate policy, which was the key question raised in Section 2.[41]

---

[40] There are two concerns which are frequently expressed about interest rate rules. First, there is a long branch of literature in monetary economics which suggests that interest rate rules can mean that there is not a unique equilibrium in macroeconomic models. Second, there is the concern of Friedman (1982) that an interest rate rule can lead the central bank to exacerbate macroeconomic fluctuations which arise from shocks to productive opportunities, changes in money demand, and so forth. The discussion in this section will be restricted to the former concern: If the central bank is responding to inflation and output as suggested by Taylor, when do interest rate rules lead to a unique outcome? But the second question is an open and important topic.

[41] The specification of this rule leads to a subtle shift in the interpretation of the policy parameters τ_i; these involve specifying how the monetary authority will respond to deviations of inflation from target. But if these parameters are chosen so that there is a unique equilibrium, then no deviations of inflation will ever occur.

At the same time, the parameter restrictions developed here would also apply to a rule of the general form originally studied by Taylor, i.e.,

$$R_t = r + \pi + \tau_1(E_t\pi_{t+1} - \pi) + \tau_0(\pi_t - \pi) + x_{Rt}.$$

Potential Multiple Equilibria

It is useful to start by considering a simple, flexible price setup in which the monetary authority can affect the behavior of inflation but not the behavior of the real rate of interest. Suppose that the authority adopts the rule

$$R_t = r_t + \pi + \tau(\pi_t - \pi) + x_{Rt}, \tag{22}$$

where π is a constant trend rate of inflation and τ governs the response of inflation to deviations from this level, which is a simplification of the two rules discussed above. Since the Fisher equation specifies that $R_t = r_t + E_t\pi_{t+1}$, it follows that inflation is constrained by

$$\tau[\pi_t - \pi] + x_{Rt} = [E_t\pi_{t+1} - \pi]. \tag{23}$$

If $\tau > 1$, which is the case normally considered, then the unique stable rational expectations solution to this difference equation can be obtained by recursively solving the difference equation forward

$$
\begin{aligned}
\pi_t - \pi &= \frac{1}{\tau}\{[E_t\pi_{t+1} - \pi] - x_{Rt}\} \\
&= \frac{1}{\tau}\{[\frac{1}{\tau}((E_t\pi_{t+2} - \pi) - x_{R,t+1})] - x_{Rt}\}
\end{aligned}
$$

and so forth until one concludes that[42]

$$\pi_t - \pi = -\{\sum_{j=0}^{\infty}(\frac{1}{\tau})^{j+1}E_t x_{t+j}\}. \tag{24}$$

This unique stable solution makes inflation into a present value of expected monetary policy shocks.[43]

This is because the difference between these two rules is

$$\bar{R}_t - (r + \pi) + \tau_1(E_t\bar{\pi}_{t+1} - \pi) + \tau_0(\bar{\pi}_t - \pi)$$

which is just a complicated "shock" term that depends on exogenous variables.

[42] At the end of this process, one uses $\lim_{j\to\infty}(\frac{1}{\tau})^j E_t x_{R,t+j} = 0$, which surely obtains because $\tau > 1$ and $x_{R,t}$ is stationary.

[43] There are some puzzling aspects of this flexible price solution, which implies that the behavior of the nominal interest rate is

$$R_t = r_t + \pi - \{\sum_{j=0}^{\infty}(\frac{1}{\tau})^j E_t x_{t+j}\} + x_{Rt}.$$

That is, when an x_t shock occurs so that the central bank's chosen path is autonomously increased, then inflation must move to offset this response. For example, if x_t is serially uncorrelated, then inflation moves just enough so that the nominal rate is unresponsive to the shock (in this case $-\{\sum_{j=0}^{\infty}(\frac{1}{\tau})^j E_t x_{t+j}\} = -x_t$ so that the interest rate is just $R_t = r_t + \pi$). For another example, if x_t is autoregressive with persistence parameter ρ, then the nominal interest rate must actually fall in response to a positive policy shock.

By contrast, if $0 < \tau < 1$, there are multiple stable rational expectations solutions, which take the form

$$\pi_{t+1} - \pi = \tau[\pi_t - \pi] + x_{Rt} + \xi_{t+1} \tag{25}$$

with ξ_{t+1} being an arbitrary random variable with $E_t\xi_{t+1} = 0$. These non-fundamental stochastic elements are sometimes referred to as "sunspots" or "animal spirits."[44] Mathematically, they can enter in (25) because the perfect foresight solution displays an indeterminacy: any initial value of π_0 can be an equilibrium with the remainder of the stable perfect foresight equilibrium path being $\pi_{t+1} - \pi = \tau^{t+1}(\pi_0 - \pi)$. From this perspective, the ξ_{t+1} can be interpreted as a randomly shifting set of initial conditions for the stochastic difference equation.

Economically, the equilibria described by (25) can be too volatile relative to the fundamental forces in the model economy. For example, even if the x_{Rt} shocks are absent, inflation under such a policy rule can be arbitrarily volatile since the variance of ξ is arbitrary.[45] These multiple equilibria arise for a basic economic reason introduced in Section 2, which is the that the central bank's policy rule does not provide a sufficient nominal anchor.

Therefore, a simple flexible price model indicates that there could be a good reason for interest rate rules to be restricted to aggressive values of parameters, in line with Taylor's (1993) suggestion that $\tau > 1$. The simple model also indicates, however, that there are other parameter choices which will lead to uniqueness. In particular, if the monetary authority aggressively lowers the rate in response to inflation (makes $\tau < -1$), then there will also be a unique equilibrium since the same logic employed in the derivation of (24) may be employed. Thus, in the simple flexible price model there is a "zone of indeterminacy" which includes all policy rules with $-1 < \tau < 1$.[46]

Limits in the New IS-LM Model

In models with sticky prices, it is sometimes argued that there is a greater latitude for interest rate policies than in flexible price models. The New IS-LM model is simple enough that one can characterize analytically the parts of the parameter space in which there are unique equilibria and the parts in which

---

[44] Farmer (1999) has recently argued that understanding the effects of nonfundamental uncertainties of this form is very important for macroeconomics, echoing earlier assertions of Jevons and Keynes.

[45] Another, less stressed, implication is that a shock which increases the nominal interest rate will raise the inflation rate under this solution.

[46] With recent interest in the analysis of alternative interest rate rules under rational expectations, within the New IS-LM model and related fully articulated models, economists are beginning to explore new territory in terms of coefficients in interest rate rules within quantitative models (as in the recent volume of studies edited by Taylor [1999]).

Figure 3

a. Current Inflation Rule

b. Expected Inflation Rule

c. Composite Rule
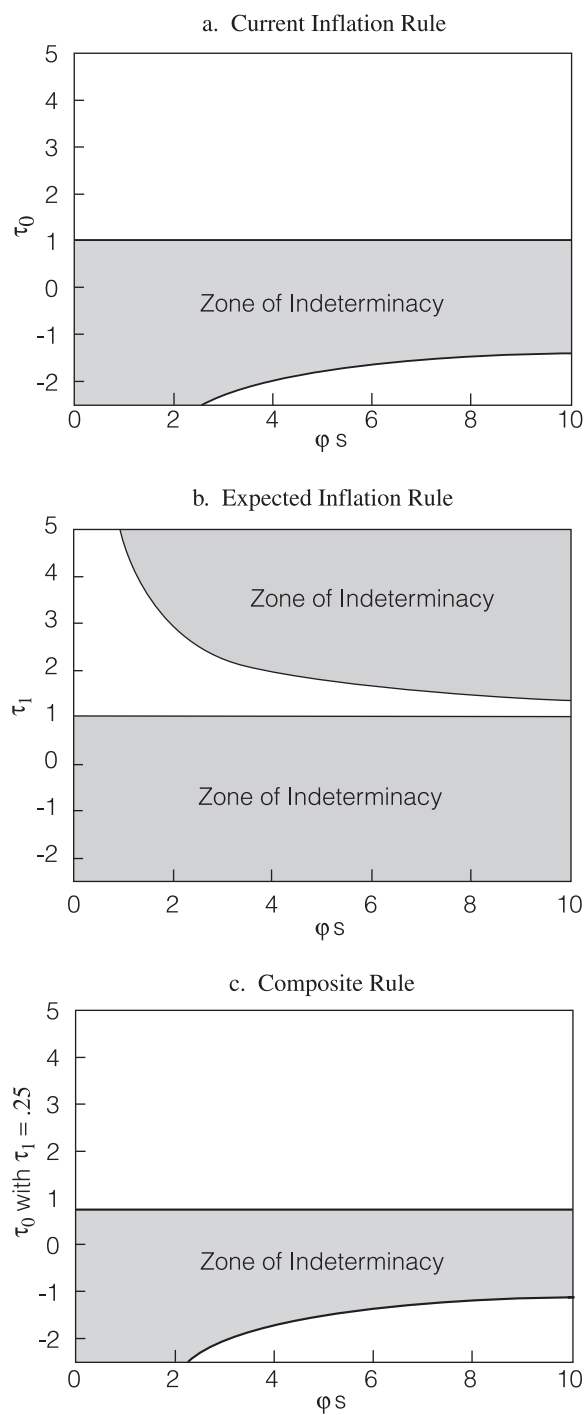
there are multiple equilibria.[47] However, modern literature on the design of monetary policy rules, as exemplified by the recent volume edited by Taylor (1999), typically proceeds by using graphical presentations of these rules, with some regions blocked out as "zones of indeterminacy." Figure 3 is an example of this approach for the New IS-LM model, with some various versions of the general policy rule.

Response to the current inflation rate: Kerr and King (1996) used the New IS-LM model to study the case in which the central bank responds only to the current inflation rate.[48] In panel a, the shaded region is the set of inadmissable settings for the response to current inflation (τ_0) given that there is no response to expected inflation ($\tau_1 = 0$). As suggested by Taylor (1993) and the analysis of the flexible price model above, one boundary of the zone of indeterminacy is given by $\tau_0 = 1$, which was the restriction also focused on by Kerr and King. The figure implies that any rule of the form (21) with $\tau_1 = 0$ and $\tau_1 > 1$ is consistent with neutral behavior of output and inflation. Thus, in terms of the answer to the question raised in Section 2, the analysis indicates that there will be a unique equilibrium if the monetary says, "If inflation deviates from the neutral level, then the nominal interest rate will be increased by more than one-for-one relative to the level which it would be at under a neutral monetary policy."

In the New IS-LM model, in contrast to conventional wisdom, the stickiness of prices implies that there is a larger zone of indeterminacy than in the flexible price model. This feature of the model was not stressed by Kerr and King because they did not focus on the lower boundary of the zone, which can be determined to be $\tau_0 = -\frac{2(1+\beta)}{\varphi s} - 1$. Hence, as prices become more flexible or the IS curve becomes flatter—there is a larger value of φs—then the result approaches the boundary in the flexible price model of $\tau_0 = -1$, but the zone of indeterminacy is always larger with sticky prices. The monetary authority, however, may also insure a unique equilibrium by saying that it will very aggressively lower the inflation rate in response to deviations of inflation from its target.

Response to the expected inflation rate: Bernanke and Woodford (1997) studied a purely forward-looking rule in which $\tau_0 = 0$, which is the case illustrated in panel b of Figure 2. With a response to expected inflation (but no response to current inflation), there are two zones of indeterminacy. All policy responses with $\tau_1 < 1$ are precluded, so it is necessary for policy to be

---

[47] Appendix C contains a detailed discussion of these regions. The approach is to (i) find the boundaries of the regions by learning when there are roots which are ±1 and (ii) determine which regions are zones of indeterminacy.

[48] Comparison with Kerr and King (1996) highlights a feature of the current analysis. The earlier paper was concerned with rules of the form $R_t = r + \pi + \tau(\pi_t - \pi)$ so that the focus was on how the central bank should respond to deviations of inflation from a constant target. The current analysis focuses on deviations from a neutral inflation target.

aggressive in Taylor's sense if it is forward looking. It is important, though, that it not be too aggressive, since the figure shows that some larger values are also ruled out because these lead to indeterminacies (the precise boundary is $\tau_1 > 1 + \frac{2(1+\beta)}{\varphi s}$).[49] Forward-looking rules, then, suggest a very different pattern of restrictions are necessary to assure that there is a neutral level of output.

Response to both current and expected inflation: When the policy rule combines a mixture of current and expected inflation responses, there is a more complicated set of possibilities. In general, the results are closer to those in panel a when the forward-looking part of policy is not aggressive ($\tau_1 < 1$) and closer to panel b when it is aggressive ($\tau_1 > 1$).

For example, suppose that policy is mildly forward-looking, which is illustrated in panel c under the assumption that τ_1 is set equal to .25. The key implication of the figure is that policy can then respond less aggressively to current inflation. There is now a larger range of admissable positive τ_0 values, in the sense that values of $\tau_0 < 1$ lead to unique equilibria when they did not in panel a.

If monetary policy is to respond positively to both current and expected inflation, however, then it is necessary that the overall policy be aggressive. The upper boundary of the zone of indeterminacy is given by $\tau_0 + \tau_1 = 1$, so that $\tau_0 > .75$ leads to a unique equilibrium in the graph.[50] Still, by responding partially to expected future inflation, monetary policy makes it less necessary to respond aggressively to current inflation.

If one takes all of these results together, one can see that the New IS-LM model suggests that there are important limits on interest rate rules if there is to be a unique equilibrium. There are important differences in the zones of indeterminacy for rules that respond to current inflation and prospective inflation.

An Alternative Nominal Anchor

While there is a substantial limit on the coefficients in "inflation" rules such as those put forward by Taylor (1993), it is important to note that interest rate rules with an alternative nominal anchor—a relationship to the price level—also

---

[49] Michael Dotsey has stressed to me that there are no unique equilibria with forward-looking rules in a flexible price model, since the Fisher equation and policy rule are each equations linking the nominal rate to expected inflation. The restriction on inflation, analogous to (23), is $\tau_1[E_t\pi_{t+1} - \pi] + x_{Rt} = [E_t\pi_{t+1} - \pi]$ and there is no possibility of a unique equilibrium. Hence, as described in the text discussion of the current inflation rule, an increase in φs leads to a shrinking zone of admissable rules. But in this case the range of admissable rules is asymptotically negligible.

[50] The appendix analysis also indicates that the lower boundary is given by $\tau_0 = \tau_1 - \frac{2(1+\beta)}{\varphi s} - 1$. Hence, a positive value of τ_1 requires that even more negative values of τ_0 are necessary to assure uniqueness relative to those shown in panel a.

can be used to insure neutral output under an interest rate rule. In particular, suppose that the nominal interest rate rule takes the form

$$R_t = \bar{r}_t + E_t\bar{\pi}_{t+1} + f(P_t - \widetilde{P}_t) + x_{Rt}, \qquad (26)$$

which involves three components. First, as above, the nominal interest rate moves with the underlying neutral interest rate $\bar{R}_t = \bar{r}_t + E_t\bar{\pi}_{t+1}$ as above. Second, there are interest rate shocks x_{Rt} as above. Third, the nominal rate is adjusted whenever the price level deviates from a target path $\widetilde{P}_t$. Then, it is possible to show that there is a unique stable rational expectations equilibrium so long as $f > 0$, i.e., the nominal rate is raised whenever the price level exceeds the target path.[51] This theoretical conclusion corresponds to an idea sometimes presented in discussions of monetary policy—for example, Goodfriend and King (1997)—that a central bank can have a greater degree of freedom in the short-run dimensions of its policy rule if it adopts a specification which recognizes the importance of the price level.

8.   POLICY: SHOCKS, RULES, AND TRADE-OFFS

The New IS-LM model suggests that monetary policy may influence real economic activity in two distinct ways. First, the central bank may itself be a source of shocks, with the effects of monetary policy disturbances also depending on the form of the monetary policy rule in place. Second, by the choice of its monetary policy rule, the central bank can affect how macroeconomic activity responds to shocks originating elsewhere in the economy. The various influences of monetary policy may be summarized by a graph, as employed by Taylor (1979) and many subsequent studies, of the relationship between the variability of inflation and the variability of real activity. This section considers each of these ideas in turn.

Dynamic Response to an Interest Rate Shock

Increases in the target range for a short-term interest rate, such as the federal funds rate in the United States, are a monetary policy shock of sorts. These changes are typically suggested to lower the rate of inflation and to temporarily decrease real output as well.

In order to study the effects of such a shock within the New IS-LM model, it is necessary to choose parameters of the model—including those of the private economy (β, s, φ) and of the policy rule $(\tau_0, \tau_1$ and the process governing

---

[51] The derivation in Appendix C assumes that the target path is the neutral price level path $\bar{P}_t$ for ease of mathematical analysis. However, nearly any target path can be accommodated since the rule can be rewritten as $R_t = \bar{R}_t + f(P_t - \widetilde{P}_t) + x_{Rt} = \bar{R}_t + f(P_t - \bar{P}_t) + \{x_{Rt} + f(\bar{P}_t - \widetilde{P}_t)\}$ with the deviation $f(\bar{P}_t - \widetilde{P}_t)$ being an additional shock of sorts.

x_{Rt})—and solve for the dynamic responses to the shock. As an example, Figure 4 displays the paths that arise when there is a simple rule that mandates a response to current, but not expected, inflation. The specific rule is

$$R_t = r + \pi + \tau(\pi_t - \pi) + x_{Rt}$$

with τ set equal to 1.05 so as to assure uniqueness. It is also assumed that there is an interest rate shock process that is first order autoregressive, $x_t = \rho_R x_{t-1} + e_t$, and that $\rho_R = .75$. The policy shock is a rise in the nominal rate, $e_0 = 1$ with $e_t = 0$ for $t > 0$.[52]

As discussed above, the time unit is taken to be one quarter of a year, which is a conventional macroeconomic modeling interval. The shock shown in the figure is a 100-basis-point rise in the annualized interest rate ($e_0 = 1$) as shown in panel a of the figure. Readers may find these graphs are most easily interpreted as representing the deviation from an initial zero inflation steady state in which the economy is operating at capacity output, although since the model is linear they also describe the effects of shocks on the economy more generally. This increase in interest rate is assumed to be followed by a 50-basis-point increase in the subsequent year, a 25-basis-point increase in the year after that, and so forth.

Response of output: The interest rate shock causes an immediate decline in output, with output reduced about 1/2 percent below capacity in the initial period (date 0) in panel b of Figure 4. The vertical axis can be interpreted as measuring the percentage deviation from the capacity level of output, so that it is about .45 in period 0, about .34 in period one, and so forth.[53]

Response of inflation: The period of reduced output shown in panel b is accompanied by a similar interval of reduced inflation in panel c. As in Figure 2, the inflation rate is stated at an annualized perentage rate, so that it is four times the percentage change in the price level between $t - 1$ and t. There is a relatively small reduction in inflation in the near term.[54]

Response of the nominal interest rate: The behavior of inflation also is important for the path of the nominal interest rate in Figure 4; there is an important difference between the policy shock component of the interest rate (the 'o' path in panel a) and the actual behavior of the nominal interest rates. While there is a 100-basis-point increase in the policy shock component of the interest rate (x_{R0}), the decline in inflation means that this is not fully reflected

---

[52] In terms of the private sector parameters, $\beta = .99$, $s = .5$, and $\varphi = .05$. The value of s is in line with estimates of the intertemporal elasticity of substitution, which typically exceed unity. The value of β is a conventional quarterly discount rate. The value of $\varphi = .05$ is one of those employed by Taylor (1980).

[53] Since output is depressed below capacity in period 0, it is expected to grow back toward its capacity level, with the one period growth rate being about $.11 = (-.34) - (-.45)$. At an annualized rate of growth, this is .45 percent.

[54] There is a decrease in the annualized inflation rate of .35 percent in the initial period and a decrease of about .27 percent in the subsequent period.

Figure 4

a. Nominal Rate

b. Output
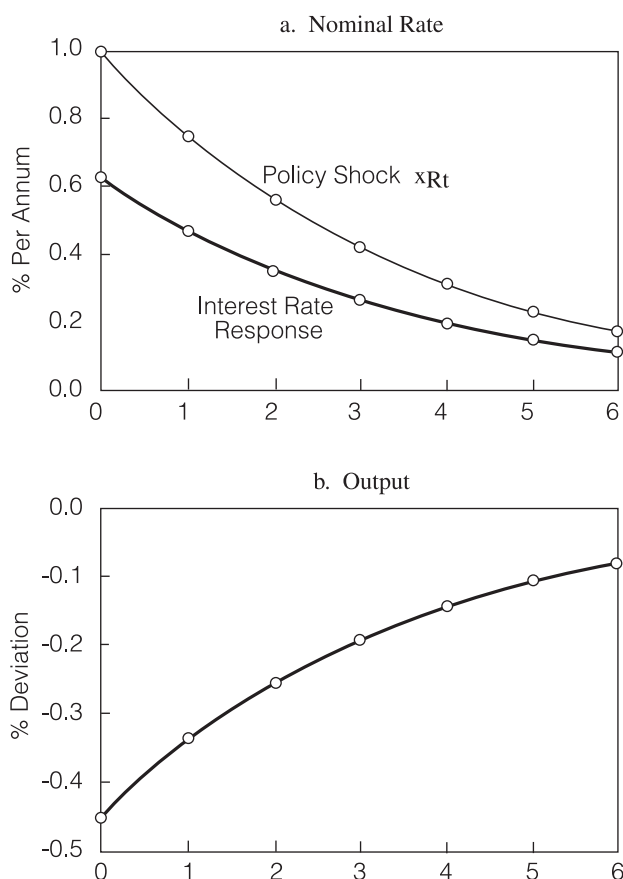
in the nominal rate.[55] The New IS-LM model therefore suggests that there may be a quantitatively large difference between monetary policy shocks and the innovations in the path of the interest rate.

Response of the real rate: There are two complementary ways of looking at the path for the real interest rate. One highlights the fact that the real interest rate rises by more than the nominal interest rate since there is a temporary period of expected deflation.[56] The other derives from the link between the

---

[55] The response of the nominal interest rate is given by $R_0 = \tau_0 \pi_0 + x_{R0}$
$= 1.05 * (-0.35) + 1 = .63$.

[56] In fact, at date 0, the nominal interest rate rises by 63 basis points and the real interest rate rises by 90 basis points (since inflation is expected to be $-.27$ percent next period).

Figure 4

c. Inflation

d. Real Rate
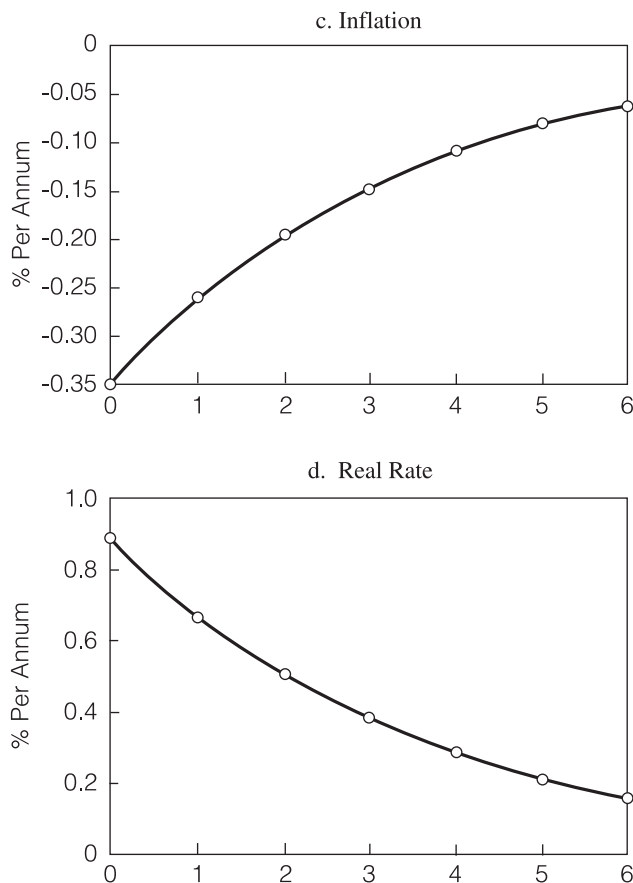
real interest rate and the growth rate of output, based on the specification of $r_t = \frac{1}{s}[E_t y_{t+1} - y_t + x_{dt}].^{57}$ Each of these complementary descriptions of the real interest rate is a partial explanation of the workings of this simple dynamic general equilibrium model, but each also helps understand its operation.

---

[57] Recall from a previous footnote that $s = .5$. The real interest rate at date 0 is .90 percent higher because the economy is expected to grow about .45 percent between period 1 and period 0, so that the response of the real interest rate is $.90\% = \frac{1}{s}[E_t y_{t+1} - y_t] = 2 * .45\%$.

Policy Rules and Macroeconomic Activity

To illustrate that alternative monetary policy rules can have a potentially important effect on how the macroeconomy responds to various shocks, it is easiest to modify the example studied in Section 4.2 above, which was used to trace out the dynamic response of prices and output to a change in nominal income. This is an interesting example from the standpoint of the design of monetary policy rules because some economists have suggested that the central bank should conduct monetary policy so that there is a target path of nominal output (see McCallum and Nelson [1999a] for one recent discussion of such nominal GDP rules).

One case for nominal GDP rules: It is sometimes argued that nominal GDP rules are desirable because they insulate output from various shocks. In the New IS-LM model, if monetary policy is structured so that nominal income is exogenous, then the analysis of Section 4 can be used to discuss the determination of output in the absence of price shocks or changes in capacity. In this case, with constant nominal income, the level of output would remain at capacity even if there were changes in the position of the IS curve, the LM curve, and so forth.

The case against nominal GDP rules if capacity changes: There is an important cost of such rules, which is that when there is an expansion of capacity output, the economy cannot immediately expand up to the new capacity level since the price level must gradually fall through time.[58] By contrast, under the neutral monetary policy discussed earlier, a monetary expansion would have permitted an immediate output expansion while leaving the price level unaffected by the expansion of capacity.

The case against nominal GDP rules if there are price shocks: There is a similar case against nominal GDP rules if there are price shocks. In Section 3, it was shown that a neutral monetary policy would accommodate those disturbances, so that nominal income would change according to $\Delta Y_t = \Delta \overline{\pi}_t + \Delta \overline{y}_t$ under a neutral policy. Price shocks would therefore also cause departures from capacity output if a nominal GDP rule were in place. For example, a positive price shock would raise the price level and lower output relative to capacity.

The relevance of alternative monetary rules for macroeconomic activity was originally stressed by Phelps and Taylor (1977), working in a loglinear macromodel with nominal stickiness. Dotsey (1999) has recently highlighted

---

[58] Calculations similar to those in Section 4 can illustrate this point. Suppose that the path of nominal output ($Y_t = y_t + P_t$) is constant through time at Y and that capacity output is a random walk, $\overline{y}_t = \overline{y}_{t-1} + e_t$ with e_t being white noise. The solution for the price level is $P_t = \theta P_{t-1} + (1 - \theta)(Y - \overline{y}_t)$. Output is then $y_t - \overline{y}_t = \theta(y_{t-1} - \overline{y}_{t-1}) - (1 - \theta)(\overline{y}_t - \overline{y}_{t-1})$. Mechanically, this solution says that an increase in capacity output of $\Delta \overline{y}_t$ only affects current output by $\theta \Delta \overline{y}_t$: as discussed above, the stickiness of prices—as captured by θ—implies that the economy cannot immediately expand up to the new level of capacity output.

Figure 5
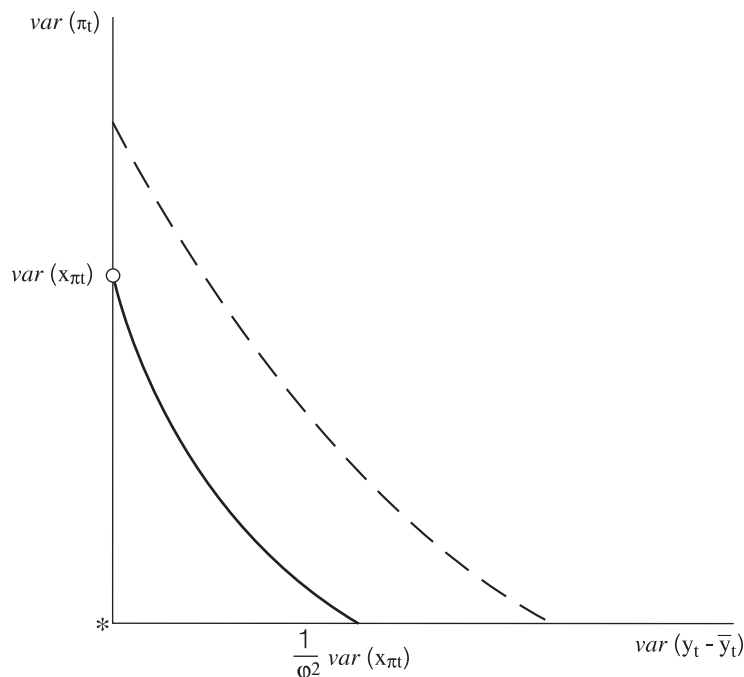
this relevance, working in fully articulated models with a slightly different specification of price stickiness from that considered here. The fact that the form of the monetary policy rule matters for the response of the economy to real and nominal shocks is motivating many economists to study the performance of alternative monetary policy rules in forward-looking macromodels.

The Variability Trade-off

Taylor (1979) introduced the idea of summarizing the effects of alternative monetary policy rules in terms of their implications for the variability of inflation and real activity. He also suggested that there would typically be trade-offs between these two variability measures. Within the New IS-LM framework as developed here, both the internal logic of the model and a close reading of Taylor indicates that the natural trade-off to explore is that between inflation π_t and the output deviation $z_t = y_t - \overline{y}_t$.

As an example, suppose that there are only inflation shocks $x_{\pi t}$ and that these are serially uncorrelated random variables. Suppose additionally that the monetary authority can respond directly to inflation shocks and does so to

make inflation equal to $\pi_t = fx_{\pi t}$, where f is a parameter that governs the extent of the inflation response. Making use of the Phillips curve (3) and the fact that expected future inflation is zero, it follows that

$$
\begin{aligned}
var(\pi_t) &= f^2 var(x_{\pi t}) \\
var(z_t) &= (\frac{f-1}{\varphi})^2 var(x_{\pi t}),
\end{aligned}
$$

where $var(\pi_t)$ is the variance of inflation, $var(z_t)$ is the variance of the deviation of output from capacity and $var(x_{\pi t})$ is the variance of $x_{\pi t}$.

To look at policies that minimize output variance given inflation variance, it is sufficient to restrict attention to values of f between zero and one. Over the range between zero and one, there is indeed a trade-off. If there is a larger value of f, then there is more inflation variability but less output variability. This trade-off is illustrated with the downward sloping solid line in Figure 5. The neutral monetary policy discussed in Section 3 above corresponds to minimizing the variance of output deviations by setting f equal to 1.

If inflation responds to another shock that is serially uncorrelated and uncorrelated with the inflation shock—for example, to productivity or money demand disturbances—according to a rule $\pi_t = fx_{\pi t} + ge_t$, then the frontier would shift upward, as illustrated by the dashed line in Figure 5. Proceeding as above, this alternative frontier is

$$
\begin{aligned}
var(\pi_t) &= f^2 var(x_{\pi t}) + g^2 var(e_t) \\
var(z_t) &= (\frac{f-1}{\varphi})^2 var(x_{\pi t}) + (\frac{g}{\varphi})^2 var(e_t)
\end{aligned}
$$

so that monetary policies allowing these influences would produce more inflation variability for a given amount of output variability.

9.   SUMMARY AND CONCLUSIONS

The distinguishing characteristic of the New IS-LM model is that its key behavioral relations can be derived from underlying choice problems of households and firms and that these relations consequently involve expectations about the future in a central manner. The IS curve relates expected output growth to the real interest rate, which is a central implication of the modern theory of consumption. The aggregate supply/Phillips curve component of the model relates inflation today to expected future inflation and an output gap. This relationship can be derived from a monopoly pricing decision that is constrained by stochastic opportunities for price adjustment together with a consistent definition of the price level.

The New IS-LM model is increasingly being utilized to illustrate macroeconomic concepts that are robust across a variety of more detailed models and to exposit the implications of alternative monetary policy rules. This article has provided a description of this framework, highlighting its language and

logic. The article has also derived certain key implications of the framework for the conduct of monetary policy, which are summarized in the introduction: the case for inflation targets, the importance of adjustment of real and nominal interest rates to underlying real disturbances, the relevance of alternative monetary rules for the determination of output, and the potential consequences of monetary policy shocks.

Three aspects of this article may strike some readers as curious choices since my recent research has been aimed at developing small-scale fully articulated models of nominal frictions[59] and exploring the implications of optimal and alternative policy rules within these models.[60] First, the New IS-LM model is laid out with many free parameters and no attempt is made to compare its key predictions to the experience of the United States or other countries. Second, the New IS-LM model can be derived from first principles as a fully articulated model that arises from specifying preferences, technologies, and market institutions; poses and solves household and firm optimization problems; and finally imposes market equilibrium and other aggregate consistency conditions. This article, however, does not derive the behavioral relations from first principles. Instead, it follows the traditional IS-LM approach of postulating behavioral relations, with some background rationalization in terms of optimizing, and then manipulates these to study various monetary policy issues. Third, the New IS-LM model abstracts from investment and capital, while most of my research has placed these features at center stage.

At one level, this approach reflects the limited goal of the article—to provide a simple exposition of the New IS-LM model and to exemplify how it is currently being used to discuss monetary policy topics. This goal was itself chosen, however, because it is my belief that many macroeconomists will use the New IS-LM model without all of its background detail to discuss monetary policy and, in particular, to communicate results from other, more complicated macroeconomic models.

Yet the microeconomic foundations are not to be dismissed. In the course of this article, there were many critical junctures at which the New IS-LM model was silent on central questions because microfoundations were absent. For example, in the Section 3 discussion of why a neutral monetary policy—defined as one that stabilized output at a capacity level—was desirable, it was necessary to step outside the New IS-LM model to draw on alternative studies in which the concept of capacity output was carefully defined and in which the monetary policy conclusion was derived as one that maximized the welfare of the citizens of the economy. Otherwise, the neutral monetary policy, which is marked as the point 'o' in Figure 5, would simply be one of a menu of choices that the monetary authority might consider desirable, given some

---

[59] King and Watson (1996), King and Wolman (1996), and Dotsey, King, and Wolman (1999).

[60] King and Wolman (1996), Goodfriend and King (1997), and King and Wolman (1999).

posited preferences of its own. Further, the analysis of neutral and alternative monetary policies suggests that the case for inflation targets, as opposed to a policy of full price level stabilization, depends entirely on the existence of inflation shocks. If these shocks were absent, the Taylor frontier in Figure 5 would collapse to the origin, with no trade-off between the variability of inflation and the variability of output relative to capacity (this possibility is marked as * in the figure). Yet there is an increasing use of the New Keynesian Phillips curve and the New IS-LM model for monetary policy analysis without detailed consideration of a question which seems central: What are inflation shocks?[61] Popular discussions sometimes point to changes in capacity output (supply shocks) or energy price variations as price shocks. Nevertheless, to study whether these are price shocks of the form incorporated in this model, it is necessary to develop additional microeconomic underpinnings of the New IS-LM model, working in detail with the pricing decisions of firms, the consumption decisions of households, and so forth. Changes in capacity output induced by fluctuations in productivity or the prices of inputs such as energy are not price shocks according to such a detailed analysis because these affect prices by shifting marginal cost, which is a key economic determinant included in the pricing equation. Within the basic framework of sticky price models, it is difficult to find price shocks that are not interpretable as behavioral errors on the part of price-setters, although perhaps the addition of a sector with flexible price firms would lead to changes in relative prices that might be interpreted in this manner.[62] This issue illustrates well, I believe, an inevitable limitation of IS-LM style analysis, which is that it may be useful for illustrating new results but it will certainly not be useful for deriving them. Finally, my suspicion is that the omission of investment and capital from the New IS-LM model may be an important, if not fatal, flaw. But determining whether this suspicion is warranted will again require a more detailed analysis that builds up from the microfoundations.

Ultimately, the case for (or against) the New IS-LM model and its fully articulated relatives must involve a systematic exploration of their empirical implications. There is much recent progress on this important front that involves the evaluation of components of the models—notably the pricing and aggregate demand specifications—and full system implications. But a great deal of work remains to be done before we understand whether this new small model captures the reality of the choices facing monetary policy decisionmakers of major economies.

---

[61] See, for example, Clarida, Gali, and Gertler (1999).

[62] I have benefited from discussion of this topic with John Taylor.

APPENDIX A: DERIVING THE NEW PHILLIPS CURVE

Start with the equations describing the price level (11) and the optimal price (12), which are repeated from the main text as

$$A - 1 : P_t = \eta P_{t-1} + (1 - \eta) P_t^*$$

$$A - 2 : P_t^* = \eta \beta E_t P_{t+1}^* + (1 - \beta \eta)[\psi_t + P_t] + z_{pt},$$

where $z_{pt} = x_{Pt} - \beta \eta E_t x_{P,t+1}$. Update the first equation, take expectations, multiply by $\eta \beta$ and subtract the result from (A-1), then rearrange the result to

$$P_t - \eta \beta E_t P_{t+1} = \eta(P_{t-1} - \eta \beta P_t) + (1 - \eta)(P_t^* - \eta \beta E_t P_{t+1}^*).$$

Substitute in A-2:

$$P_t - \eta \beta E_t P_{t+1} = \eta(P_{t-1} - \eta \beta P_t) + (1 - \eta)(1 - \eta \beta)[\psi_t + P_t] + (1 - \eta) z_{pt}.$$

Rearrange the result and substitute in the marginal cost specification.

$$
\begin{aligned}
P_t - P_{t-1} &= \beta(E_t P_{t+1} - P_t) + \frac{(1 - \eta)(1 - \eta \beta)}{\eta} \psi_t + \frac{1 - \eta}{\eta} z_{pt} \\
&= \beta(E_t P_{t+1} - P_t) + [h \frac{(1 - \eta)(1 - \eta \beta)}{\eta}](y_t - \overline{y}_t) + \frac{1 - \eta}{\eta} z_{pt}
\end{aligned}
$$

so $x_{\pi t} = \frac{1 - \eta}{\eta} z_{pt} = \frac{1 - \eta}{\eta}[x_{Pt} - \beta \eta E_t x_{P,t+1}]$.

## APPENDIX B: EXOGENOUS NOMINAL INCOME

The analysis begins by combining (3) with the definition link between nominal and real income, ignoring inflation shocks for mathematical simplicity.

$$P_t - P_{t-1} = \beta(E_t P_{t+1} - P_t) + \varphi(Y_t - P_t - \overline{y}_t)$$

This can be written as the expectational difference equation

$$-E_t P_{t+1} + (1 + \frac{1}{\beta} + \frac{\varphi}{\beta}) P_t - \frac{1}{\beta} P_{t-1} = \frac{\varphi}{\beta}(Y_t - \overline{y}_t),$$

which has a polynomial $\Phi(z) = [-\beta z^2 + (1 + \beta + \varphi)z - 1]$. The product of the roots of this polynomial is $\frac{1}{\beta}$ and the sum of roots is $(1 + \frac{1}{\beta} + \frac{\varphi}{\beta})$. If θ is the smaller of the roots, then the larger of the roots is $\frac{1}{\beta \theta} > 1$.

Figure B-1
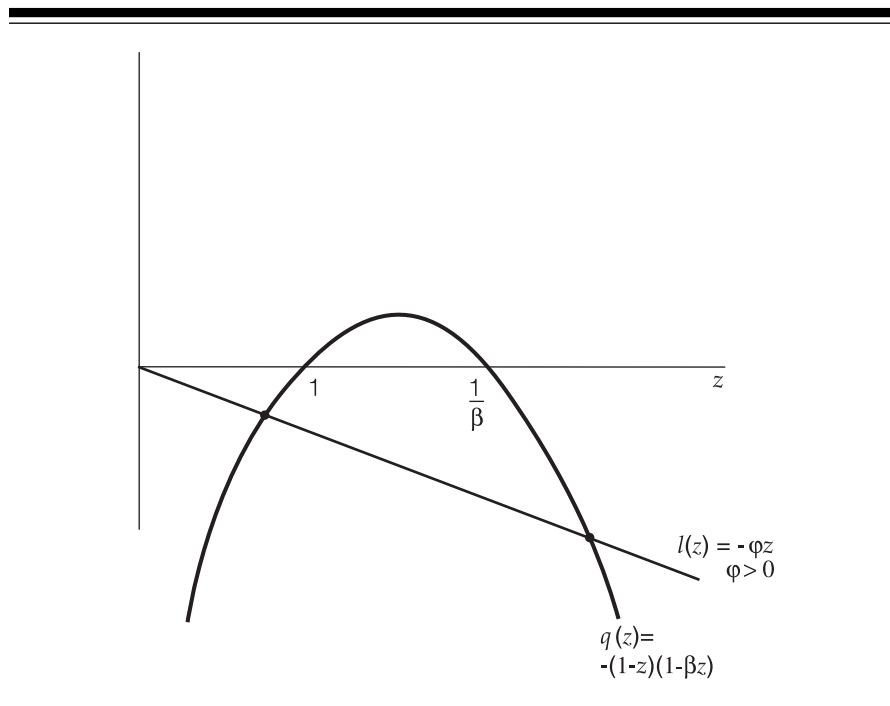
A graphical analysis of the roots of this familiar difference equation will provide some useful background for the analysis of more complicated models below. The graph is based on decomposing $\Phi(z) = 0$ into $\Phi(z) = q(z) - l(z)$ with $l(z) = -\varphi z$ and $q(z) = [-\beta z^2 + (1 + \beta)z - 1] = -(1 - z)(1 - \beta z)$. Figure B-1 displays the quadratic equation $q(z)$, which has roots of 1 and $1/\beta$, and the line $l(z)$, which is negatively sloped if $\varphi > 0$ and passes through the origin. The intersection of these two curves implies that $l(z) = q(z)$ and thus the values of z at the intersection points are the solutions to $\Phi(z) = 0$.

With $\varphi = 0, l(z) = 0$ and the solutions are thus 1 and $1/\beta$. For any $\varphi > 0$ the solution must be as displayed in Figure B-1, which is that there is one root less than 1 and one root that is greater than $1/\beta$. Finally, increases in φ will lower the smaller root θ.

With this information about the magnitude of the roots, the next task is to determine the solution, following Sargent (1978). Using the operator $\mathbf{F}$ which shifts the dating of the variable, but not the conditional expectation so that $\mathbf{F}^j E_t x_{t+k} = E_t x_{t+k+j}$, we can deduce that

$$\frac{\varphi}{\beta}E_t(Y_t - \bar{y}_t) = -E_t P_{t+1} + (1 + \frac{1}{\beta} + \frac{\varphi}{\beta})P_t - \frac{1}{\beta}P_{t-1}$$

$$= -(\mathbf{F} - \theta)(\mathbf{F} - \frac{1}{\theta\beta})E_t P_{t-1}$$

$$= (\frac{1}{\theta\beta})(\mathbf{F} - \theta)(1 - \theta\beta\mathbf{F})E_t P_{t-1}.$$

The general solution to the difference equation can be produced by unwinding the unstable root forward, so that

$$P_t - \theta P_{t-1} = \theta\varphi\frac{1}{(1 - \theta\beta\mathbf{F})}E_t(Y_t - \bar{y}_t)$$

$$= (1 - \theta)(1 - \beta\theta)\sum_{j=0}^{\infty}(\theta\beta)^j E_t(Y_{t+j} - \bar{y}_{t+j})$$

with one step in this derivation using the fact that $(1 + \frac{1}{\beta} + \frac{\varphi}{\beta}) = \theta + \frac{1}{\beta\theta}$ means that $\theta\varphi = (1 - \theta)(1 - \beta\theta)$.

Under the assumed driving process, it follows that

$$\sum_{j=0}^{\infty}(\theta\beta)^j E_t(Y_{t+j})$$

$$= \sum_{j=0}^{\infty}(\theta\beta)^j E_t[Y_{t-1} + (Y_t - Y_{t-1}) + \dots (Y_{t+j} - Y_t)]$$

$$= \sum_{j=0}^{\infty}(\theta\beta)^j[Y_{t-1} + (1 + \rho + \rho^2 \dots + \rho^j)(Y_t - Y_{t-1})]$$

$$= \frac{1}{1 - \theta\beta}Y_{t-1} + [(\frac{1}{1 - \theta\beta})(\frac{1}{1 - \theta\beta\rho})(Y_t - Y_{t-1})]$$

so that the specific solution for the price level is

$$P_t = \theta P_{t-1} + (1 - \theta)(Y_{t-1} - \bar{y}) + [\frac{1 - \theta}{1 - \theta\beta\rho}](Y_t - Y_{t-1}).$$

To find the behavior of output, we use the relationship between nominal and real income followed by some algebra:

$$\begin{aligned}
y_t - \overline{y} &= Y_t - \overline{y} - P_t \\
&= [Y_t - \overline{y}] - \\
&\quad [\theta P_{t-1} + (1 - \theta)(Y_{t-1} - \overline{y}) + (\frac{1 - \theta}{1 - \theta\beta\rho})(Y_t - Y_{t-1})] \\
&= \theta[Y_{t-1} - P_{t-1} - \overline{y}] + (\frac{1 - \theta}{1 - \theta\beta\rho} - 1)(Y_t - Y_{t-1}) \\
&= \theta(y_{t-1} - \overline{y}) + (\theta\frac{1 - \beta\rho}{1 - \theta\beta\rho})(Y_t - Y_{t-1}).
\end{aligned}$$

APPENDIX C: UNIQUENESS UNDER INTEREST RATE RULES

To analyze the system dynamics under interest rate rules, it is convenient to subtract its neutral counterpart from each of the equations of the model. For example, the IS equation is $y_t = E_t y_{t+1} - s r_t + x_{dt}$ and its neutral counterpart is $\overline{y}_t = E_t \overline{y}_{t+1} - s\overline{r}_t + x_{dt}$ so that the result is

$$IS : y_t - \overline{y}_t = E_t(y_{t+1} - \overline{y}_{t+1}) - s(r_t - \overline{r}_t).$$

Similarly, the Fisher equation is

$$F : r_t - \overline{r}_t = (R_t - \overline{R}_t) - E_t(\pi_{t+1} - \overline{\pi}_{t+1})$$

and the Phillips curve is

$$PC : (\pi_t - \overline{\pi}_t) = \beta E_t(\pi_{t+1} - \overline{\pi}_{t+1}) + \varphi(y_t - \overline{y}_t).$$

The monetary policy rules can similarly be transformed, by simply subtracting $R_t = \overline{r}_t + E_t\overline{\pi}_{t+1}$ from both sides of the equation.

For example, with the general specification (text ref) we have that

$$R_t - \overline{R}_t = \tau_1(E_t\pi_{t+1} - E_t\overline{\pi}_{t+1}) + \tau_0(\pi_t - \overline{\pi}_t) + x_{Rt}.$$

Thus, the analysis of system dynamics can be performed as if all shocks had been dropped—except for the policy shock—and the capacity output level had been treated as constant.

Similarly, with the price level specification (text ref) we have that

$$R_t - \overline{R}_t = f(P_t - \overline{P}_t) + \{f(\overline{P}_t - \widetilde{P}_t) + x_{Rt}\}$$

so that the term in braces can be treated as a complicated interest rate shock.

Hence, in the remainder of this appendix, attention is restricted to analysis of a deterministic system—without any shocks or time variation in capacity—for the purpose of studying uniqueness issues.

The text discussion of interest rate rules involved the idea that there was a unique equilibrium so long as the central bank was willing to raise the real rate in specified circumstances, which suggests focusing on the real interest rate. To derive one restriction on the real rate, multiply IS by φ and then eliminate output using the Phillips curve:

$$\varphi s * r_t = [-\pi_t + (1+\beta)E_t\pi_{t+1} - \beta E_t\pi_{t+2}] = -[(1-\mathbf{F})(1-\beta\mathbf{F})]E_t\pi_t,$$

where $\mathbf{F}$ is the forward operator as in the main text. This is a private sector restriction on the behavior of the real interest rate, which links it to the inflation rate.

Uniqueness with the Interest Rate Rule (21)

Combining the Fisher equation (2) and the monetary policy rule (21), it is possible to determine an additional restriction on the real interest rate:

$$r_t = \tau_0\pi_t + (\tau_1 - 1)E_t\pi_{t+1} = [\tau_0 + (\tau_1 - 1)\mathbf{F}]E_t\pi_t.$$

Combining this expression with the private sector restriction on the real rate leads to

$$l(\mathbf{F})E_t\pi_t = \varphi s[\tau_0 + (\tau_1 - 1)\mathbf{F}]E_t\pi_t$$
$$= -[(1-\mathbf{F})(1-\beta\mathbf{F})]E_t\pi_t = q(\mathbf{F})E_t\pi_t.$$

The left-hand side of this expression is a linear function l, and the right hand side of this expression is a quadratic function q.

The nature of the system dynamics will depend on the roots of the quadratic polynomial $q(z) - l(z)$, which may be written as

$$-\beta z^2 + [\beta + 1 - \varphi s(\tau_1 - 1)]z - [1 + \varphi s\tau_0] = -\beta(z - \mu_1)(z - \mu_2).$$

This expression makes clear that the sum of the roots is $[1 + \frac{1}{\beta} + \frac{\varphi s(\tau_1 - 1)}{\beta}]$ and that the product of the roots is $[\frac{1 + \varphi s\tau_0}{\beta}]$. Since there are no predetermined variables in this system, there is a unique equilibrium only if there are two unstable roots, i.e., values of μ_i that are both larger than unity in absolute value. To study the magnitude of these, it is convenient to use a mixture of graphical and analytical techniques.

Determining the boundaries: The boundaries of the policy parameter regions can be determined by requiring that there is a root of exactly positive or negative one. Taking the positive unit root first,

$$l(1) = q(1)$$
$$\Rightarrow \varphi s[\tau_0 + (\tau_1 - 1)\mathbf{1}] = -[(1 - \mathbf{1})(1 - \beta\mathbf{1})] = 0 \Rightarrow \tau_0 + \tau_1 = 1$$

so that there is a restriction that the sum of the policy rule coefficients must equal one from this source. Taking the negative unit root next,

$$l(-1) = q(-1) \Rightarrow \varphi s[\tau_0 + (\tau_1 - 1)(-\mathbf{1})] =$$

$$- [(1 - (-\mathbf{1}))(1 - \beta(-\mathbf{1}))] = -2(1 + \beta) \Rightarrow \tau_0 - \tau_1 = -1 - \frac{2(1 + \beta)}{\varphi s}$$

so that there is a restriction on the difference between the coefficients from this source.

Graphing the functions l(z) and q(z) to determine the nature of the regions: A graph of the functions, similar to that used in Appendix B above, provides the easiest way of determining the nature of the roots in the regions defined by the above boundaries. Figure C-1 shows the nature of this pair of functions. The form of the quadratic equation $q(z)$ is invariant to the nature of the policy rule; as is clear from the fact that $q(z) = -[(1 - z)(1 - \beta z)] = 0$ the two zeros are 1 and $1/\beta$. The figure is drawn for the case of a simple rule which involves only response to current, not expected inflation ($\tau_1 = 0$) so that it corresponds to panel a of Figure 3 in the text. The function $l(z)$ is downward sloping in this case since $l(z) = s\varphi(\tau_0 - z)$ and $s\varphi > 0$. If $\tau_0 = 1$ then $l(z)$ intersects with the quadratic at $z = 1$; this possibility is shown by the dashed line in B-1. If $\tau_0 > 1$, then this intersection is shifted to the right, i.e., all roots are greater than 1. In this case, there are two unstable roots and there is thus a unique stable rational expectations equilibrium. Hence, as τ_0 is increased from the boundary region in panel a of Figure 3 in the main text, the region of unique equilibria is entered.

This graphical analysis can also be used to (i) confirm that a reduction in τ_0 from the other boundary also produces an entry into the region of stability in panel a of Figure 3 of the text, and (ii) to determine that the other aspects of panels b and c are as described in the text.

Uniqueness with the Interest Rate Rule (26)

By combining the Fisher equation (2) and the monetary policy rule (26), it is possible to determine an additional restriction on the real interest rate:

$$r_t = f P_t - E_t \pi_{t+1} = [\mathbf{F} - \mathbf{F}(\mathbf{F} - 1)] E_t P_{t-1}.$$

Combining this expression with the private sector restriction on the real rate leads to

$$a(\mathbf{F}) E_t P_{t-1} = \varphi s[f\mathbf{F} - \mathbf{F}(\mathbf{F} - 1)] E_t P_{t-1}$$
$$= -[(1 - \mathbf{F})(1 - \beta\mathbf{F})][\mathbf{F} - 1] E_t P_{t-t} = b(\mathbf{F}) E_t P_{t-1}.$$

The left-hand side of this expression is a quadratic function, $a(\mathbf{F})$, and the right-hand side of this expression is a cubic function $b(\mathbf{F})$.

The nature of the system dynamics will depend on the roots of the polynomial $c(z) = b(z) - a(z)$. To study the magnitude of these, it is again convenient to use a mixture of graphical and analytical techniques.

Determining the roots of a(z) and b(z): It turns out to be a simple matter to determine the roots of these expressions. The quadratic function q(z) has two roots, one of which is zero and the other of which is $f + 1$. The cubic equation $b(z)$ has a root of $\frac{1}{\beta}$ and two roots of 1.

Graphing the functions a(z) and b(z) to determine the stability condition: A graph of the functions provides the easiest way of determining the nature of the roots of the cubic polynomial $c(z) = b(z) - a(z) = 0$.

Figure C-2 contains three functions. One of the solid lines is the cubic $b(z)$, which highlights the fact that it has two repeated roots at $z = 1$ and a single root at $z = 1/\beta$.

The dashed line is the quadratic $a(z)$ with the parameter $f = 0$. There are two roots of this equation, one which is zero and the other which is unity. Hence, with $f = 0$, the graph highlights the fact—which can easily be determined using the definitions of $a(z)$ and $b(z)$—that there is an exact root of unity in c(z). It also shows only one other intersection of the two lines, so that there is one unstable root and two unit roots of $c(z) = b(z) - a(z)$.

The solid line which lies below the dashed line in the range $0 < z < 1$ is an example of the quadratic $a(z)$ with the parameter $f > 0$. Note that there is a zero root to this quadratic and a root greater than one (which was earlier determined to be $1 + f$). Hence, with $f > 0$ there are three distinct roots, one which is positive and less than unity and the other two which are unstable. This is the configuration that insures uniqueness given that there is a single predetermined variable P_{t-1}.
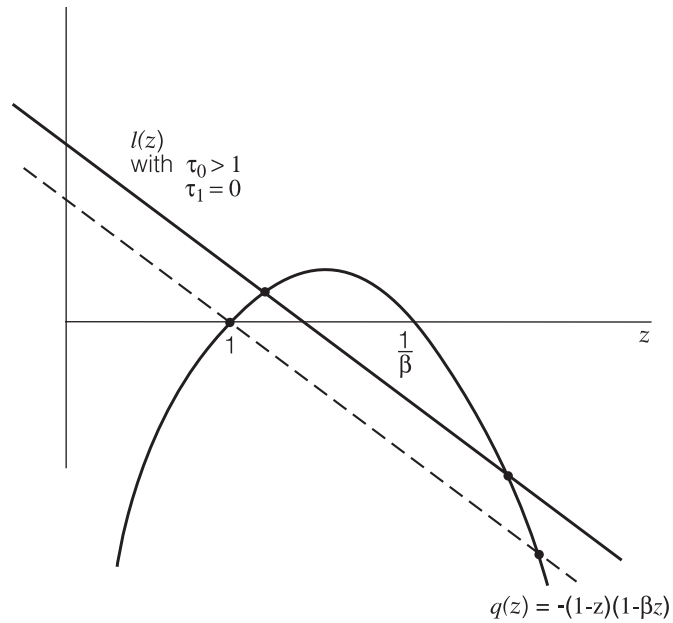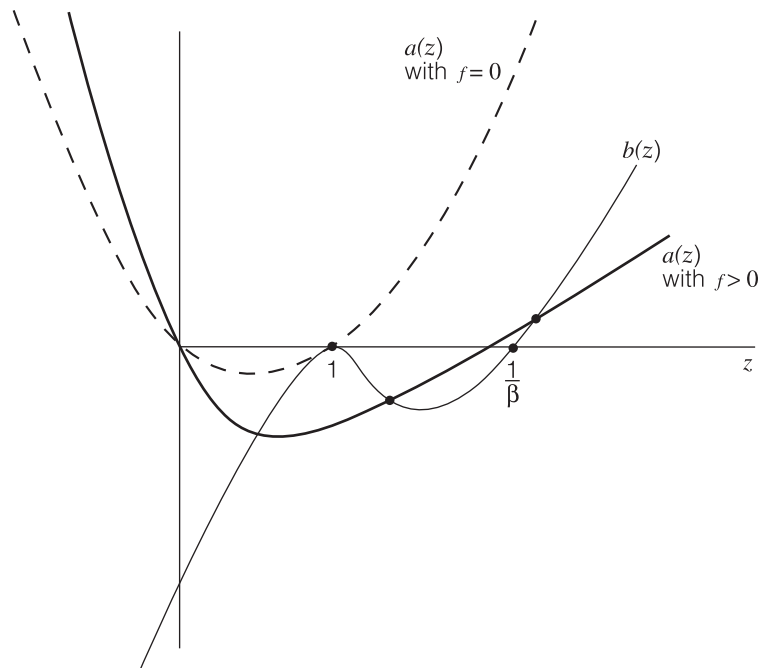
Figure C-1

$l(z)$
with $\tau_0 > 1$
$\tau_1 = 0$

$\frac{1}{\beta}$

z

1

$q(z) = -(1-z)(1-\beta z)$

Figure C-2

$a(z)$
with $f = 0$

$b(z)$

$a(z)$
with $f > 0$

1

$\frac{1}{\beta}$

z

REFERENCES

Ball, Lawrence. "Disinflation with Imperfect Credibility," *Journal of Monetary Economics,* vol. 35 (February 1995), pp. 5–23.

Bernanke, Ben S., and Michael Woodford. "Inflation Forecasts and Monetary Policy," *Journal of Money, Credit, and Banking,* vol. 29 (November 1997), pp. 653–84.

Buiter, Willem H., and Marcus Miller. "Cost and Benefits of an Anti-Inflationary Policy: Questions and Issues," in Victor E. Argy and John W. Neville, eds., *Inflation and Unemployment: Theory, Experience and Policy Making*. London: Allen and Unwin, 1985.

Calvo, Guillermo. "Staggered Prices in a Utility-Maximizing Framework," *Journal of Monetary Economics,* vol. 12 (September 1983), pp. 383–98.

Clarida, Richard, Jordi Gali, and Mark Gertler. "The Science of Monetary Policy: A New Keynesian Perspective," *Journal of Economic Literature*, vol. 37 (December 1999), pp. 1661–1707.

Chari, V. V., Patrick J. Kehoe, and Ellen R. McGrattan, "Sticky Price Models of the Business Cycle: Can the Contract Multiplier Solve the Persistence Problem," *Econometrica*, vol. 68 (September 2000), pp. 1151-79.

Dotsey, Michael. "The Importance of Systematic Monetary Policy for Economic Activity," Federal Reserve Bank of Richmond *Economic Quarterly,* vol. 85 (Summer 1999), pp. 41–59.

_____. "Structure from Shocks," Working Paper 99-6. Richmond: Federal Reserve Bank of Richmond, August 1999.

_____, Robert G. King, and Alexander L. Wolman. "State-Dependent Pricing and the General Equilibrium Dynamics of Money and Output," *Quarterly Journal of Economics,* vol. 114 (May 1999), pp. 655–90.

Farmer, Roger E.A. *The Macroeconomics of Self-Fulfilling Prophecies*. Cambridge, Mass.: MIT Press, 1999.

Fischer, Stanley. "Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule," *Journal of Political Economy,* vol. 85 (February 1977), pp. 191–205.

Friedman, Milton. "Monetary Policy: Theory and Practice," *Journal of Money, Credit, and Banking,* vol. 14 (February 1982), pp. 98–118.

_____. "A Theoretical Framework for Monetary Analysis," *Journal of Political Economy,* vol. 78 (March–April 1970), pp. 193–238.

_____. "The Role of Monetary Policy," *American Economic Review,* vol. 58 (March 1968), pp. 1–17.

Gali, Jordi, and Mark Gertler. "Inflation Dynamics: A Structural Econometric Analysis," *Journal of Monetary Economics,* vol. 44 (October 1999), pp. 195–222.

Goodfriend, Marvin. "Using the Term Structure of Interest Rates for Monetary Policy," Federal Reserve Bank of Richmond *Economic Quarterly,* vol. 84 (Summer 1998), pp. 13–30.

_____. "Interest Rate Policy and the Inflation Scare Problem," Federal Reserve Bank of Richmond *Economic Quarterly*, vol. 79 (Winter 1993), pp. 1–24.

_____. "Interest Rates and the Conduct of Monetary Policy," *Carnegie-Rochester Conference Series on Public Policy,* vol. 34 (Spring 1991), pp. 7–30.

_____, and Robert G. King. "The New Neoclassical Synthesis and the Role of Monetary Policy," in Ben Bernanke and Julio Rotemberg, eds., *NBER Macroeconomics Annual 1997*. Cambridge, Mass.: MIT Press, 1997, pp. 231–82.

Gordon, Robert J. "The Recent Acceleration of Inflation and Its Lessons for the Future," *Brookings Papers on Economic Activity,* 1:1970, pp. 8–41.

Hall, Robert E. "Stochastic Implications of the Life Cycle-Permanent Income Hypothesis: Theory and Evidence," *Journal of Political Economy* vol. 86 (December 1978), pp. 971-87.

_____. "Intertemporal Substitution in Consumption," *Journal of Political Economy,* vol. 96 (April 1988), pp. 339–57.

Hicks, John R. "Mr. Keynes and the Classics: A Suggested Interpretation," *Econometrica,* vol. 7 (April 1937), pp. 147–59.

Ireland, Peter N. "The Role of Countercyclical Monetary Policy," *Journal of Political Economy,* vol. 104 (August 1996), pp. 704–23.

Kerr, William, and Robert G. King. "Limits on Interest Rate Rules in the IS Model," Federal Reserve Bank of Richmond *Economic Quarterly*, vol. 82 (Spring 1996). pp. 47–75.

Kimball, Miles S. "The Quantitative Analytics of the Basic Neomonetarist Model," *Journal of Money, Credit, and Banking*, vol. 27 (November 1995), pp. 1241–77.

King, Robert G. "Will the New Keynesian Macroeconomics Resurrect the IS-LM Model?" *The Journal of Economic Perspectives*, vol. 7 (Winter 1993), pp. 67–82.

_____, and Mark W. Watson. "Money, Prices, Interest Rates and the Business Cycle," *Review of Economics and Statistics,* vol. LXXVIII, (February 1996), pp. 35–53.

_____, and Alexander L. Wolman. "What Should the Monetary Authority Do If Prices Are Sticky?" in John B. Taylor, ed., *Monetary Policy Rules*. Chicago: University of Chicago Press, 1999.

_____. "Inflation Targeting in a St. Louis Model of the 21st Century," Federal Reserve Bank of St. Louis *Review,* vol. 78 (May/June 1996), pp. 83–107.

Koenig, Evan. "Rethinking the IS in IS-LM: Adapting Keynesian Tools to NonKeynesian Economies: Part 2," Federal Reserve Bank of Dallas *Economic Review* (4th quarter 1993), pp.17–35.

_____. "Rethinking the IS in IS-LM: Adapting Keynesian Tools to NonKeynesian Economies: Part 1," Federal Reserve Bank of Dallas *Economic Review* (3rd quarter 1993), pp. 33–49.

Lucas, Robert E., Jr. "Methods and Problems in Business Cycle Theory," *Journal of Money, Credit, and Banking,* vol. 12 (November 1980), pp. 696–715.

_____. "Econometric Policy Evaluation: A Critique," in Karl Brunner and Allan H. Meltzer, eds., *The Phillips Curve and Labor Markets*, Carnegie-Rochester Conference Series on Public Policy, vol. 1 (1976), pp. 19–46.

_____. "Some International Evidence on Output-Inflation Tradeoffs," *The American Economic Review*, vol. 63 (June 1973), pp. 326–34.

_____. "Econometric Testing of the Natural Rate Hypothesis," in Otto Eckstein, ed., *The Econometrics of Price Determination*. Washington: Board of Governors of the Federal Reserve System, 1972.

_____. "Expectations and the Neutrality of Money," *Journal of Economics Theory*, vol. 4 (April 1972), pp. 103–24.

Mankiw, N. Gregory. "A Quick Refresher Course in Macroeconomics," *Journal of Economic Literature,* vol. 28 (December 1990), pp. 1645–60.

_____, and David Romer, eds. *New Keynesian Economics*. Cambridge, Mass.: MIT Press, 1991.

McCallum, Bennett T. *Monetary economics: Theory and policy.* London: Collier Macmillan, 1989.

_____. "Price Level Determinacy with an Interest Rate Policy Rule and Rational Expectations," *Journal of Monetary Economics*, vol. 8 (November 1981), pp. 319–29.

——————, and Edward Nelson. "An Optimizing IS-LM Specification for Monetary Policy and Business Cycle Analysis," *Journal of Money, Credit, and Banking,* vol. 31 (August 1999), pp. 296–316.

——————."Nominal Income Targeting in an Open-Economy Optimizing Model," *Journal of Monetary Economics*, vol. 43 (June 1999), pp. 553–78.

Modigliani, Franco. "Liquidity Preference and the Behavior of Interest and Money," *Econometrica,* vol. 12 (January 1944), pp. 45–88.

Muth, John F. "Rational Expectations and the Theory of Price Movements," *Econometrica*, vol. 29 (July 1961), pp. 315–35.

Parkin, Michael. "A Comparison of Alternative Techniques of Monetary Control under Rational Expectations," *Manchester School of Economic and Social Studies*, vol. 46 (September 1978), pp. 252–87.

Phelps, Edmund S. "Phillips Curves, Expectations of Inflation and Optimal Unemployment Over Time," *Economica,* vol. 34 (August 1967), pp. 254–81.

——————, and John B. Taylor. "Stabilizing Powers of Monetary Policy under Rational Expectations," *Journal of Political Economy,* vol. 85 (February 1977), pp. 163–90.

Phillips, A.W. "The Relationship Between Unemployment and the Rate of Price Change of Money Wage Rates in the United Kingdom, 1862–1957," *Economica*, vol. 25 (November 1958), pp. 283–99.

Roberts, John M. "New Keynesian Economics and the Phillips Curve," *Journal of Money, Credit, and Banking,* vol. 27 (November 1995), pp. 975–84.

Romer, David. "The New Keynesian Synthesis," *Journal of Economic Perspectives,* vol. 7 (Winter 1993), pp. 5–22.

Rotemberg, Julio J. "Inflation Targeting in a St. Louis Model of the 21st Century: Commentary," Federal Reserve Bank of St. Louis *Review,* vol. 78 (May/June 1996), pp. 108–11.

——————. "The New Keynesian Microfoundations," in Stanley Fischer, ed., *NBER Macroeconomics Annual 1987.* Cambridge, Mass.: MIT Press, 1987, pp. 63–129.

——————. "Sticky Prices in the United States," *Journal of Political Economy,* vol. 90 (December 1982), pp. 1187–1211.

——————, and Michael Woodford. "Interest Rate Rules in an Estimated Sticky Price Model," in John B. Taylor, ed., *Monetary Policy Rules*. Chicago: University of Chicago Press, 1999.

_____. "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy," in Ben Bernanke and Julio Rotemberg, eds., *NBER Macroeconomics Annual 1997*. Cambridge, Mass.: MIT Press, 1997, pp. 297–345.

Sargent, Thomas J. "Interpreting Economic Time Series," *Journal of Political Economy,* vol. 89 (April 1981), pp. 213–48.

_____. "Rational Expectations, the Real Rate of Interest, and the Natural Rate of Unemployment," *Brookings Papers on Economic Activity*, 2:1973, pp. 429–72, reprinted in Sargent, Thomas J. *Macroeconomic Theory*. New York: Academic Press, 1978.

_____. "A Note on The Accelerationist Controversy," *Journal of Money, Credit, and Banking*, vol. 3 (August 1971), pp. 721–25.

_____, and Wallace, Neil. "'Rational' Expectations, the Optimal Monetary Instrument, and the Optimal Money Supply Rule," *Journal of Political Economy,* vol. 83 (April 1975), pp. 241–54.

Sbordonne, Argia. "Prices and Unit Labor Costs: A New Test of Price Stickiness," Rutgers University Working Paper, October 1998.

Sims, Christopher A. "Macroeconomics and Reality," *Econometrica*, vol. 48 (Jan. 1980), pp. 1–48.

Solow, Robert M. *Price Expectations and The Phillips Curve*. Manchester, U.K.: Manchester University Press, 1969.

Taylor, John B., ed. *Monetary Policy Rules*. Chicago: University of Chicago Press for NBER, 1999.

_____. *Macroeconomic policy in a world economy: from econometric design to practical operation*. London: Norton, 1993.

_____. "Discretion versus Rules in Theory and Practice," *Carnegie-Rochester Conference Series on Public Policy,* vol. 39 (1993), pp. 195–214.

_____. "Aggregate Dynamics and Staggered Contracts," *Journal of Political Economy,* vol. 88 (February 1980), pp. 1–23.

_____. "Estimation and Control of an Econometric Model with Rational Expectations," *Econometrica,* vol. 47 (September 1979), pp. 1267–86.

Tinsley, Peter A, "Fitting Both Data and Theories: Polynomial Adjustment Costs and Error-Correction Decision Rules," Board of Governors of the Federal Reserve System Finance and Economics Discussion Paper Series 93-21, June 1993.

Wolman, Alexander L. "Sticky Prices, Marginal Cost, and the Behavior of Inflation," Federal Reserve Bank of Richmond *Economic Quarterly,* vol. 85 (Fall 1999), pp. 29–48.

Woodford, Michael. "Control of the Public Debt: A Requirement for Price Stability?" NBER Working Paper 5684, July 1996.

Yun, Tack. "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles," *Journal of Monetary Economics*, vol. 37 (April 1996), pp. 345–70.