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# Comment

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### 1. PROBLEM OF WEAK IDENTIFICATION

Under the restriction that the time discount factor is one, the new Keynesian Phillips curve is given by

$$\pi_t = \lambda s_t + \gamma_f E_t[\pi_{t+1}] + (1 - \gamma_f)\pi_{t-1}, \quad (1)$$

where  $\pi_t$  is inflation and  $s_t$  is the output gap at time  $t$ . The underlying structural parameters are  $\theta \in [0, 1]$ , the frequency of price adjustment, and  $\omega \in [0, 1]$ , the fraction of backward-looking price setters. They are related to the parameters in the Phillips curve through

$$\lambda = \frac{(1 - \omega)(1 - \theta)^2}{\theta + \omega},$$

$$\gamma_f = \frac{\theta}{\theta + \omega}.$$

Galí and Gertler (1999, p. 212) recognized a "small sample normalization problem in GMM." Because of this problem, they estimate the Phillips curve based on two alternative moment restrictions,

$$E_t[(\theta + \omega)(\pi_t - \lambda s_t - \gamma_f \pi_{t+1} - (1 - \gamma_f)\pi_{t-1})z_t] = 0, \quad (2)$$

$$E_t[(\pi_t - \lambda s_t - \gamma_f \pi_{t+1} - (1 - \gamma_f)\pi_{t-1})z_t] = 0. \quad (3)$$

Table 1 reports their estimates.

Based on moment restriction (2), the 95% asymptotic confidence intervals are  $\lambda \in [0.02, 0.04]$  and  $\gamma_f \in [0.74, 0.80]$ . Based on moment restriction (3), the 95% asymptotic confidence intervals are  $\lambda \in [0.00, 0.01]$  and  $\gamma_f \in [0.58, 0.65]$ . Quite disturbingly, the confidence intervals produced by the two alternative normalizations are disjoint.

The fact that the asymptotic confidence interval can be disjoint, depending on the normalization of the moment restriction, has been emphasized by Hahn and Hausman (2002) and Hamilton, Waggoner, and Zha (2007). Another economic application for which this same problem arises is the estimation of the elasticity of intertemporal substitution (Neely, Roy, and Whiteman 2001). In that application, however, precise estimates can be obtained using methods that are robust to weak identification (Stock and Wright 2000; Yogo 2004).

### 2. ESTIMATES ROBUST TO WEAK IDENTIFICATION

Kleibergen and Mavroeidis (2009) estimate the Phillips curve using the methodology developed by Kleibergen (2005), which leads to valid confidence sets even in the presence of weak identification. Table 2 reports their estimates.

The 95% confidence intervals are  $\lambda \in [0.00, 0.17]$  and  $\gamma_f \in [0.56, 1.00]$ . These confidence intervals are much wider than those reported by Galí and Gertler (1999), which indicates that weak identification is a serious issue in estimation of the Phillips curve. To get a sense of the imprecision of these estimates, the implied 95% confidence intervals for the underlying structural parameters are  $\omega \in [0.00, 0.80]$  and  $\theta \in [0.50, 1.00]$ . The confidence interval for the frequency of price adjustment implies that the duration is between six months and forever! Because of weak identification, structural parameters cannot be identified based on macrodata alone.

### 3. COMPARISON TO MICROESTIMATES

In order to understand the Phillips curve, it is important to examine its microfoundations through firms' pricing behavior. In a recent study, Nakamura and Steinsson (2008) estimate the frequency of price adjustment for various goods and services using microdata from the Bureau of Labor Statistics. Table 3 reports some relevant numbers from their study.

The median duration between price adjustment is 11 months for all sectors, which is remarkably close to the point estimate of 13 months obtained by Kleibergen and Mavroeidis (2009) for the 1960–2007 sample. There is substantial cross-sectional variation in the duration between price adjustment across sectors, ranging from 0.5 months for vehicle fuel to 27.3 months for apparel.

Table 1. Estimates from Galí and Gertler (1999)

	$\lambda$	$\gamma_f$	$\omega$	$\theta$	Duration
Panel A: Estimate based on moment restriction (2)					
Point estimate	0.03	0.77	0.24	0.80	15
95% CI	[0.02, 0.04]	[0.74, 0.80]	[0.19, 0.30]	[0.77, 0.84]	[13, 18]
Panel B: Estimate based on moment restriction (3)					
Point estimate	0.01	0.62	0.52	0.84	19
95% CI	[0.00, 0.01]	[0.58, 0.65]	[0.44, 0.61]	[0.79, 0.89]	[14, 28]

NOTE: This table is taken from Galí and Gertler (1999, table 2). Estimation is by GMM with the Newey–West covariance matrix. The instruments include four lags of inflation, labor income share, long-short yield spread, output gap, wage inflation, and commodity price inflation. The brackets contain asymptotic 95% confidence intervals. The last column reports the implied duration between price adjustment in months. The data are quarterly for the sample period 1960:1–1997:4.

Table 2. Estimates from Kleibergen and Mavroeidis (2009)

	$\lambda$	$\gamma_f$	$\omega$	$\theta$	Duration
Panel A: 1960–2007 sample					
Point estimate	0.04	0.77	0.23	0.77	13
95% CI	[0.00, 0.17]	[0.56, 1.00]	[0.00, 0.80]	[0.50, 1.00]	[6, $\infty$ ]
Restricted 95% CI	[0.02, 0.10]	[0.56, 1.00]	[0.00, 0.58]	0.73	11
Panel B: 1960–1983 sample					
Point estimate	0.21	0.81	0.14	0.58	7
95% CI	[0.00, 0.62]	[0.46, 1.00]	[0.00, 1.00]	[0.27, 1.00]	[4, $\infty$ ]
Restricted 95% CI	[0.01, 0.10]	[0.46, 1.00]	[0.00, 0.87]	0.73	11
Panel C: 1984–2007 sample					
Point estimate	0.01	0.79	0.23	0.87	24
95% CI	[0.00, 0.23]	[0.57, 1.00]	[0.00, 0.77]	[0.46, 1.00]	[6, $\infty$ ]
Restricted 95% CI	[0.03, 0.10]	[0.57, 1.00]	[0.00, 0.56]	0.73	11

NOTE: This table is taken from Kleibergen and Mavroeidis (2009, tables 2 and 3). Estimation is by continuous updating GMM with the Newey–Weight covariance matrix. The instruments include three lags of first differenced inflation and output gap. The brackets contain 95% confidence intervals based on the test in Kleibergen (2005). The restricted confidence interval, which imposes  $\theta = 0.73$ , is based on the author's calculations. The last column reports the implied duration between price adjustment in months. The data are quarterly for the sample period 1960:1–2007:3.

Table 3. Frequency of price adjustment

Measure	Frequency	Duration
All sectors	8.7	11.0
Processed food	10.5	9.0
Unprocessed food	25.0	3.5
Household furnishing	6.0	16.1
Apparel	3.6	27.3
Transportation goods	31.3	2.7
Recreation goods	6.0	16.3
Other goods	15.0	6.1
Utilities	38.1	2.1
Vehicle fuel	87.6	0.5
Travel	41.7	1.9
Services (excluding travel)	6.1	15.8

NOTE: This table is taken from Nakamura and Steinsson (2008, table 2), for median estimates that exclude sales and substitutions. The frequency of price adjustment is reported as percent per month, and the implied duration is reported in months. The sample period is 1998–2005.

One way to improve the precision of the estimates in the Phillips curve is to impose the frequency of price adjustment measured in microdata. A duration of 11 months implies a quarterly frequency of price adjustment of  $\theta = 0.73$ . The last row of each panel in Table 2 reports the 95% confidence intervals for  $\lambda$  and  $\gamma_f$  under the restriction that  $\theta = 0.73$ . For the 1960–2007 sample, the 95% confidence interval for  $\lambda$  shrinks to [0.02, 0.10], compared to [0.00, 0.17] without the restriction.

However, the restriction does not improve the confidence interval for  $\gamma_f$ . Intuitively, it is difficult to identify whether firms are responding to  $E_t[\pi_{t+1}]$  or  $\pi_{t-1}$  due to the persistence in inflation.

In conclusion, Kleibergen and Mavroeidis (2009) have made a convincing case that weak identification is a serious issue in estimation of the Phillips curve. Future studies should strive to achieve better identification through the use of rich microdata.

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# Comment

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## 1. INTRODUCTION

The paper by Kleibergen and Mavroeidis (2009), hereafter KM, is an excellent survey of the current state of the art in the weak instrument robust econometrics for testing subsets of parameters in the generalized method of moments (GMM), and provides an important and relevant application of the econometric theory to the analysis of the new Keynesian Phillips curve. We are extremely grateful to have the opportunity to comment on this very nice paper. Our comments will focus on the weak instrument robust tests for subsets of parameters, and in particular on the projection-based test that KM refer to as the Robins (2004) test.

We show that KM's implementation of the Robins test is inefficient, and provide an efficient implementation that performs nearly as well as the MQLR test recommended by KM. Our comment proceeds as follows. Section 2 reviews the tests used for inference on subsets of parameters in GMM and discusses in detail the implementation of the Robins test, which we call the new method of projection. Section 3 reports the results of a small simulation study to demonstrate that the new method of projection performs nearly as well as the tests recommended by KM. Section 4 contains our concluding remarks.

## 2. INFERENCE ON SUBSETS OF PARAMETERS IN GMM

In this section we describe inference on subsets of parameters in the GMM framework. We follow the notation and assumptions of KM regarding the GMM framework. Interest centers on a  $p$ -dimensional vector of parameters  $\theta$  identified by a set of  $k \geq p$  moment conditions

$$E[f_i(\theta)] = 0.$$

Let  $\theta = (\alpha', \beta')'$ , where  $\alpha$  is  $p_\alpha \times 1$  and  $\beta$  is  $p_\beta \times 1$ . The parameters of interest are  $\beta$ , and  $\alpha$  are considered nuisance parameters. The weak identification robust methods of inference on

$\theta$  are based on the (efficient) continuous updating (CU) GMM objective function

$$Q(\theta) = T f_T(\theta)' \hat{V}_{ff}(\theta)^{-1} f_T(\theta), \quad (1)$$

where  $f_T(\theta) = T^{-1} \sum_{i=1}^T f_i(\theta)$  and  $\hat{V}_{ff}(\theta)$  is a consistent estimator of the  $k \times k$  dimensional covariance matrix  $V_{ff}(\theta)$  of the vector of sample moments. Let  $q_i(\theta) = \text{vec}(\frac{\partial f_i(\theta)}{\partial \theta'})$  and define  $\bar{f}_i(\theta) = f_i(\theta) - E[f_i(\theta)]$  and  $\bar{q}_i(\theta) = q_i(\theta) - E[q_i(\theta)]$ . Assumption 1 of KM states that

$$\frac{1}{\sqrt{T}} \sum_{i=1}^T \begin{bmatrix} \bar{f}_i(\theta) \\ \bar{q}_i(\theta) \end{bmatrix} \xrightarrow{d} \begin{bmatrix} \psi_f(\theta) \\ \psi_\theta(\theta) \end{bmatrix} \sim N(0, V(\theta)),$$

$$V(\theta) = \begin{bmatrix} V_{ff}(\theta) & V_{f\theta}(\theta) \\ V_{\theta f}(\theta) & V_{\theta\theta}(\theta) \end{bmatrix}.$$

The gradient of (1) with respect to  $\theta$  is given by

$$\nabla_\theta Q(\theta) = \frac{\partial Q(\theta)}{\partial \theta'} = 2 f_T(\theta)' \hat{V}_{ff}(\theta)^{-1} \hat{D}_T(\theta),$$

where  $\hat{D}_T(\theta) = \sum_{i=1}^T D_i(\theta)$  and  $D_i(\theta) = \text{devec}_k[q_i(\theta) - \hat{V}_{\theta f}(\theta) \hat{V}_{ff}(\theta)^{-1} f_i(\theta)]$ . For the definition of the devec operator see Chaudhuri (2008).

### 2.1 Tests for the Full Parameter Vector

Valid tests of the hypothesis  $H_0: \theta = \theta_0$  were developed in Stock and Wright (2000) and Kleibergen (2005). Stock and Wright's  $S$ -statistic is a generalization of the Anderson–Rubin statistic (see Anderson and Rubin 1949) and is given by  $S(\theta) = Q(\theta)$ . Kleibergen's  $K$ -statistic is a score-type statistic based on  $Q(\theta)$  and may be expressed as

$$K(\theta) = \frac{1}{4} (\nabla_\theta Q(\theta)) [\hat{D}_T(\theta)' \hat{V}_{ff}(\theta)^{-1} \hat{D}_T(\theta)]^{-1} (\nabla_\theta Q(\theta))'. \quad (2)$$