Interest Rates and Inflation

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A consensus has emerged among practitioners that the instrument of monetary policy ought to be the short-term interest rate, that policy should be focused on the control of inflation, and that inflation can be reduced by increasing short-term interest rates. At the center of this consensus is a rejection of the quantity theory. Such a rejection is a difficult step to take, given the mass of evidence linking money growth, inflation, and interest rates: increases in average rates of money growth are associated with equal increases in average inflation rates and interest rates.

These observations need not rule out a constructive role for the use of short-term interest rates as a monetary instrument. One possibility is that increasing short-term rates in the face of increases in inflation is just an indirect way of reducing money growth: sell bonds and take money out of the system. Another possibility is that, while control of monetary aggregates is the key to low long-run average inflation rates, an interest-rate policy can improve the short-run behavior of interest rates and prices. The short-run connections among money growth, inflation, and interest rates are very unreliable, so there is much room for improvement. These possibilities are surely worth exploring, but doing so requires new theory. The analysis needed to reconcile interest-rate policies with the evidence on which the quantity theory of money is grounded cannot be found in old textbook diagrams.

I. An Economy with Segmented Markets

Much recent discussion of monetary policy is centered on a class of policies known as "Taylor rules," rules that specify the interest rate set by the central bank as an increasing function of the inflation rate (or perhaps of a forecast of the inflation rate) (see John Taylor, 1993). The properties of Taylor rules can be studied within a Keynesian framework.[1] Here we examine the properties of Taylor rules using a neoclassical framework that is also consistent with the quantity theory of money and the body of evidence that confirms this theory. An essential assumption in this inquiry is that markets are incomplete, or *segmented*, in a way that is consistent with the existence of a *liquidity effect*: a downward-sloping demand for nominal bonds. The segmented-market model we use is adapted from Alvarez et al. (2001), where references to earlier work on these models can be found.

The model we develop is an exchange economy: There is no Phillips curve and no effect of monetary-policy changes on production.[2]

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[1] See Richard Clarida et al. (1999) for a helpful review.

[2] Segmented market models that have such effects include contributions by Lawrence Christiano and Martin

Think, then, of an exchange economy with many agents, all with the preferences

$$\sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t U(c_t)$$

where

$$U(c_t) = \frac{c_t^{1-\gamma}}{1-\gamma}$$

over sequences $\{c_t\}$ of a single, non-storable consumption good. All of these agents attend a goods market every period. A fraction λ of agents also attend a bond market. We call these agents "traders." The remaining $1 - \lambda$ agents (we call them "non-traders") never attend the bond market. We assume that no one ever changes status between being a trader and a non-trader.

Agents of both types have the same constant endowment of y units of the consumption good. The economy's resource constraint is thus

$$y = \lambda c_t^{\mathrm{T}} + (1-\lambda)c_t^{\mathrm{N}} \tag{1}$$

where c_t^{T} and c_t^{N} are the consumptions of the two agent types. We ensure that money is held in equilibrium by assuming that no one consumes his own endowment. Each household consists of a shopper–seller pair, where the seller sells the household's endowment for cash in the goods market, while the shopper uses cash to buy the consumption good from others in the same market. Prior to the opening of this goods market, money and one-period government bonds are traded in another market, attended only by traders.

Purchases are subject to a cash-in-advance constraint, modified to incorporate shocks to velocity. Assume, to be specific, that goods purchases $P_t c_t$ are constrained to be less than the sum of cash brought into goods trading by the household and a variable fraction v_t of *current-period* sales receipts. Think of the shopper as visiting the seller's store at some time during the trading day, emptying the cash register, and returning to shop some more.

Thus, every non-trader carries his unspent receipts from period-$(t-1)$ sales, $(1 - v_{t-1})P_{t-1}y$, into period-t trading. He adds to these balances $v_t P_t y$ from period-t sales, giving him a total of $(1 - v_{t-1})P_{t-1}y + v_t P_t y$ to spend on goods in period t. In order to keep the determination of the price level as simple as possible, we assume that every household spends all of its cash, every period.[3] Then every non-trader spends

$$P_t c_t^{\mathrm{N}} = (1 - v_{t-1})P_{t-1}y + v_t P_t y \tag{2}$$

in period t.

Traders, who attend both bond and goods markets, have more options. Like the non-traders, each trader has available the amount on the right of (2) to spend on goods in period t, but each trader also absorbs his share of the increase in the per capita money supply that occurs in the open-market operation in t. If the per capita increase in money is $M_t - M_{t-1} = \mu_t M_{t-1}$, then each trader leaves the date-t bond market with an additional $\mu_t M_{t-1}/\lambda$ dollars.[4]

Eichenbaum (1992) and Charles T. Carlstrom and Timothy S. Fuerst (1995). Our simpler model permits a discussion of inflation, but not of all of inflation's possible consequences.

[3] After solving for equilibrium prices and quantities under the assumption that cash constraints always bind, one can go back to individual maximum problems to find the set of parameter values under which this provisional assumption will hold (see Alvarez et al., 2001 appendix A).

[4] If B_t is the value of bonds maturing at date t and if T_t is the value of lump-sum tax receipts at t, the market-clearing condition for this bond market becomes

$$B_t - \left(\frac{1}{1+r_t}\right)B_{t+1} - T_t = M_t - M_{t-1}.$$

We assume that all taxes are paid by the traders, so Ricardian equivalence will apply, and the timing of taxes will be immaterial. These taxes play no role in our discussion, except to give us a second way to change the money supply besides open-market operations. With this flexibility, any monetary policy can be made consistent with the real debt remaining bounded. The arithmetic that follows will be

Consumption spending per trader is thus given by

$$(3)\quad P_t c_t^{\mathrm{T}} = (1 - v_{t-1})P_{t-1}y + v_t P_t y + (M_t - M_{t-1})/\lambda.$$

Now, using the cash flow equations (2) and (3) and the market-clearing condition (1) we obtain

$$\begin{aligned} P_t y &= (1 - v_{t-1})P_{t-1}y + v_t P_t y + M_t - M_{t-1} \\ &= M_{t-1} + v_t P_t y + M_t - M_{t-1} \end{aligned}$$

since $M_{t-1} = (1 - v_{t-1})P_{t-1}y$ is total dollars carried forward from $t - 1$. Thus, a version

$$(4)\qquad M_t\left(\frac{1}{1 - v_t}\right) = P_t y$$

of the equation of exchange must hold in equilibrium, and the fraction v_t can be interpreted (approximately) as the log of velocity.

Introducing shocks to velocity captures the short-run instability in the empirical relationship between money and prices. In addition, it allows us to study the way interest rates react to news about inflation for different specifications of monetary policy. In the formulation of the segmented-markets model that we use here, there are no possibilities for substituting against cash, so the interest rate does not appear in the money demand function [in (4)], and velocity is simply given. Given the behavior of the money supply, then prices are entirely determined by (4). This is the quantity theory of money in its very simplest form.

The exogeneity of velocity in the model is, of course, easily relaxed without altering the essentials of the model, but at the cost of complicating the solution method. In the version we study here, the two cash-flow equations (2) and (3) describe the way the fixed endowment is distributed to the two consumer types. The three equations (1)–(3) thus completely determine the equilibrium resource allocation and the behavior of the price level. No maximum problem has been studied, and no derivatives have been taken!

To study the related behavior of interest rates, however, we need to examine bond-market equilibrium, and there the real interest rate will depend on the current and expected future consumption of the traders only. Solving (1), (2), and (4), we derive the formula for c_t^{T}:

$$c_t^{\mathrm{T}} = \left[\frac{1 + \mu_t v_t + \mu_t(1 - v_t)/\lambda}{1 + \mu_t}\right] y = c(v_t, \mu_t)y$$

where the second equality defines the relative consumption function $c(v_t, \mu_t)$. Then the equilibrium nominal interest rate must satisfy the familiar marginal condition,

$$(5)\quad \frac{1}{1 + r_t} = \left(\frac{1}{1 + \rho}\right) \times E_t\left[\frac{U'(c(v_{t+1}, \mu_{t+1})y)}{U'(c(v_t, \mu_t)y)}\left(\frac{1}{1 + \mu_{t+1}}\right) \times \left(\frac{1 - v_{t+1}}{1 - v_t}\right)\right]$$

where $E_t(\cdot)$ means an expectation conditional on events dated t and earlier.

We use two approximations to simplify equation (5). The first involves expanding the function $\log[c(v_t, \mu_t)]$ around the point $(\bar{v}, 0)$ to obtain the first-order approximation

$$\log[c(v_t, \mu_t)] \cong (1 - \bar{v})\left(\frac{1 - \lambda}{\lambda}\right)\mu_t.$$

both monetarist and pleasant in the sense of Thomas J. Sargent and Neil Wallace (1985).

(Note that the first-order effect of velocity changes on consumption is zero.) With the constant-relative-risk-aversion (CRRA) preferences we have assumed, the marginal utility of traders is then approximated by

$$U'(c(v_t, \mu_t)y) = \exp(-\phi\mu_t)y^{-\gamma}$$

where

$$\phi = \gamma(1 - \bar{v})\left(\frac{1 - \lambda}{\lambda}\right) > 0.$$

Taking logs of both sides of (5), we have

$$r_t = \rho - \log\left(E_t\left[\exp\{-\phi(\mu_{t+1} - \mu_t)\} \times \left(\frac{1}{1 + \mu_{t+1}}\right)\left(\frac{1 - v_{t+1}}{1 - v_t}\right)\right]\right).$$

We apply a second approximation to the right-hand side to obtain

$$r_t = \hat{\rho} + \phi(E_t[\mu_{t+1}] - \mu_t) + E_t[\mu_{t+1}] + E_t[v_{t+1}] - v_t \tag{6}$$

where $\hat{\rho} - \rho > 0$ is a risk correction factor.[5]

From equation (6) one can see that the immediate effect of an open-market-operation bond purchase, $\mu_t > 0$, is to reduce interest rates by $\phi\mu_t$. This is the liquidity effect that the segmented-market models are designed to capture. If we drop the segmentation and let everyone trade in bonds, then $\lambda = 1$, $\phi = 0$, and the liquidity effect vanishes. In this case, open-market operations can only affect interest rates through information effects on the inflation premium. Interest-rate increases can only reflect expected inflation: monetary ease. With $\phi > 0$, the model combines quantity-theoretic predictions for the long-run behavior of money growth, inflation, and interest rates, with a *potential* role for interest rates as an instrument of inflation control in the short run. We explore this potential in the next section.

[5] The risk correction $p - \hat{p}$ depends on conditional variances, which are constant in the following applications.

II. Inflation Control with Segmented Markets

In this section, we work through a series of thought experiments based on the equilibrium condition (6) that illuminate various aspects of monetary policy. These examples all draw on the fact, obtained by differencing the equation of exchange (4), that the inflation rate is the sum of the money growth rate and the rate of change in velocity:

$$\pi_t = \mu_t + v_t - v_{t-1}. \tag{7}$$

Example 1 (Constant Velocity and Money Growth): Let v_t be constant at $\bar{v}$, and let μ_t be constant at μ. Then (6) becomes

$$r = \rho + \mu.$$

We can view this equation interchangeably as fixing money growth, given the interest rate, or as fixing the interest rate given money growth and inflation. This Fisher equation must always characterize long-run average money growth, inflation, and interest rates.

Example 2 (Constant Money Growth and i.i.d. Shocks): Let the velocity shocks be independently and identically distributed (i.i.d.) random variables, with mean $\bar{v}$ and variance σ_v^2. Let μ_t be constant at μ. Under these conditions, (6) implies

$$r_t = \hat{\rho} + \mu - (v_t - \bar{v}).$$

A transient increase in velocity raises the current price level, reducing expected inflation. This induces a transient decrease in interest rates. In this example, r_t is i.i.d., with mean $\hat{\rho}$ +

μ and variance σ_v^2; the inflation rate has mean μ and variance $2\sigma_v^2$.

Example 3 (Exact Inflation-Targeting): It is always possible to attain a target inflation rate $\bar{\pi}$ exactly. Just set the money growth rate according to

$$\mu_t = \bar{\pi} - v_t + v_{t-1}.$$

Then interest rates will be given by

$$r_t = \hat{\rho} + \phi(-E_t[v_{t+1}] + 2v_t - v_{t-1}) + \bar{\pi}.$$

If the velocity shocks are i.i.d., as in Example 2, then $\mathrm{Var}(\mu_t) = 2\sigma_v^2$, and r_t has mean $\hat{\rho} + \bar{\pi}$ and variance $5\phi^2\sigma_v^2$.

Example 4 (An Interest-Rate Peg): Assume i.i.d. v_t, with mean $\bar{v}$ and variance σ_v^2. Let μ_t satisfy

$$\mu_t - \mu = B(v_t - \bar{v})$$

where the constant B is chosen to make r_t constant at $\hat{\rho} + \mu$. Then (6) implies

$$\hat{\rho} + \mu = \hat{\rho} - B\phi(v_t - \bar{v}) + \mu - (v_t - \bar{v}).$$

If this equality holds for all realizations of v_t, it follows that $B = -1/\phi$. In this case, $\mathrm{Var}(\mu_t) = (\sigma_v/\phi)^2$. Using (7), the variance of the inflation rate is,

$$\sigma_\pi^2 = \mathrm{Var}(\mu_{t+1} + v_{t+1} - v_t)$$
$$= \left[1 + \left(\frac{\phi - 1}{\phi}\right)^2\right]\sigma_v^2.$$

Comparing this case to Example 2, one sees that pegging the interest rate is inflation-stabilizing, relative to constant money growth, if and only if $\phi > 1/2$.

In Examples 2, 3, and 4, the economy is subjected to unavoidable velocity shocks. The variability of these shocks must show up somewhere, either in interest rates, money-growth rates, or inflation rates. The way it is distributed over these three variables can, in the presence of a liquidity effect, be determined by policy. However this is done, the long-run connections between money growth, inflation, and interest rates are entirely quantity-theoretic.

Our next two examples consider versions of Taylor rules. Suppose, to be specific, that interest rates are set according to the formula

$$r_t = \hat{\rho} + \bar{\pi} + \theta(\pi_t - \bar{\pi}) \tag{8}$$

where $\theta > 0$ means that if the current inflation rate π_t is to exceed the target rate $\bar{\pi}$, we raise this period's interest rate above its target level, $\hat{\rho} + \bar{\pi}$. To study the dynamics implied by the rule (8), we eliminate r_t and π_t between (6), (7), and (8) to obtain the difference equation

$$\mu_t - \bar{\pi} = \left(\frac{1+\phi}{\theta+\phi}\right)(E_t[\mu_{t+1}] - \bar{\pi}) + \left(\frac{1}{\theta+\phi}\right)[E_t[v_{t+1}] - v_t - \theta(v_t - v_{t-1})]. \tag{9}$$

We can solve this difference equation "forward" to get

$$\mu_t - \bar{\pi} = \sum_{i=0}^{\infty}\left(\frac{1+\phi}{\theta+\phi}\right)^i E_t[s_{t+i}] \tag{10}$$

where

$$s_t = \frac{1}{\theta+\phi}[v_{t+1} - v_t - \theta(v_t - v_{t-1})]$$

provided that the series on the right-hand side of

(10) converges.[6] We now use (10) to study two more examples.

Example 5 (A Taylor Rule with i.i.d. Velocity): Let v_t be i.i.d., with mean $\bar{v}$ and variance σ_v^2. Inserting the corresponding values of $E_t[s_{t+j}]$ into (10) gives

$$(11)\quad \mu_t - \bar{\pi} = -\left(\frac{\phi + \theta^2}{(\theta + \phi)^2}\right)(v_t - \bar{v}) + \left(\frac{\theta}{\theta + \phi}\right)(v_{t-1} - \bar{v}).$$

The interest-rate consequences of these open-market operations can then be calculated from the Taylor rule, (8):

$$(12)\quad r_t = \hat{\rho} + \bar{\pi} + \left(\frac{\theta\phi}{(\theta + \phi)^2}\right) \times (2\theta + \phi - 1)(v_t - \bar{v}) - \left(\frac{\theta\phi}{\theta + \phi}\right)(v_{t-1} - \bar{v}).$$

The money-supply response to a temporary increase in velocity, described in (11), is to reduce money growth initially, increase it in the next period, and return to the target growth rate thereafter. This will smooth the inflationary impact of the velocity increase, whether or not there is a positive liquidity effect ϕ. If $\phi > 0$ and $2\theta + \phi > 1$, (12) implies that these open-market operations will raise the interest rate initially in response to a velocity increase, then reduce it below the target, and then return it to $\hat{\rho} + \bar{\pi}$.

[6] If $\theta > 1$, the right-hand side of (10) is the only solution to (9) with bounded expected values. This case is referred to as an "active" Taylor rule. If $\theta < 1$ (a "passive" Taylor rule) and the series in (10) converges, (10) gives one solution to (9), but there will be others (which we do not examine here) as well.

Example 6 (A Taylor Rule with Random-Walk Velocity): Assume that the changes, $v_t - v_{t-1}$, in velocity are i.i.d. random variables with mean 0 and variance σ_v^2. Then, for any t, calculating the terms $E_t[s_{t+k}]$ and substituting (10) yields

$$\mu_t - \bar{\pi} = \left(\frac{\theta}{\theta + \phi}\right)(v_t - v_{t-1}).$$

Again, the interest-rate consequences can be calculated from the Taylor rule, (8):

$$(13)\quad r_t = \hat{\rho} + \bar{\pi} + \left(\frac{\theta\phi}{\theta + \phi}\right)(v_t - v_{t-1}).$$

As in the case of i.i.d. shocks in Example 5, (13) implies that open-market bond sales in response to a velocity increase will increase interest rates only if $\phi > 0$.

Example 7 (A Change in the Inflation Target): Holding the distribution of velocity shocks fixed, suppose that the inflation target is moved permanently from $\bar{\pi}$ to $\hat{\pi}$. This retargeting changes nothing on the right-hand side of (10), so (10) implies simply an immediate, permanent change in the money-growth rate from $\bar{\pi}$ to $\hat{\pi}$. Of course, this implies an immediate, permanent change in the interest rate of $\hat{\pi} - \bar{\pi}$. Neither the size ϕ of the liquidity effect nor the responsiveness θ of the Taylor rule has any bearing on these changes.

III. Conclusions

Using a model of segmented markets, we have shown that a policy of increasing short-term interest rates to reduce inflation can be rationalized with essentially quantity-theoretic models of monetary equilibrium. In the model we used to generate all of our specific examples, production is a given constant, velocity is an exogenous random shock, and the equation of exchange determines the equilibrium price level, given the money supply. In this theory of inflation, consistent with much of the evidence, interest rates play no role whatsoever.

To this simple model we have added segmented markets. With this added feature, we can describe a monetary policy action interchangeably as a change in the money supply or as a change in interest rates. In this context, we considered a series of examples under different assumptions on the behavior of velocity shocks and on the specification of a policy rule.

In the first two stochastic examples, Examples 2 and 3, a policy at any date is set in advance of the realization of the velocity shock in that period: One can commit to a given rate of money growth, leaving interest rates free to vary with the velocity shock (Example 2), or one can commit to an interest rate, leaving money growth to be adjusted later to maintain this rate (Example 3). Neither policy can reduce the variance of inflation to zero. The larger is the liquidity effect, the higher is the relative effectiveness of the interest-rate rule in stabilizing inflation rates about a target rate.

In the remaining examples we consider, policy (however specified) is permitted to respond to contemporaneous velocity shocks. In Example 4, we show that under this assumption an inflation target can be hit *exactly* by a money-supply rule that is conditioned on the shock, and that this is true whatever is the shock process. In our context, inflation-targeting cannot be done any better than this.

The remaining examples in the paper consider Taylor rules: policies in which the interest rate is set so as to deviate from its long-run (Fisherian) target in proportion to the deviation of the inflation rate from its target. Such rules use the same information as the rule in Example 4 that attains the inflation target perfectly. From the viewpoint of inflation-targeting, then, committing to a Taylor rule amounts to tying the hands of the monetary authority in a way that can only limit its effectiveness. As our examples illustrate, the importance of this limitation varies with assumptions on the time-series character of the velocity shocks.

To rationalize the use of any of the interest-rate rules we have examined, it would be necessary to use an objective function that assigns weight to some other objective besides the attainment of an inflation target. We have in fact considered variations on the model presented here in which relative endowments of agents fluctuate, giving rise to gains from pooling endowment risk. In the absence of a monetary-policy design to offset these shocks, they will increase interest-rate variability. In a model with segmented markets where such pooling cannot take place, there can be real gains from policies that smooth real interest rates. We leave the analysis of this question, the issue of what the founders of the Federal Reserve System called an "elastic currency," to another paper.

REFERENCES

Alvarez, Fernando; Atkeson, Andrew, and Kehoe, Patrick. "Money, Interest Rates, and Exchange Rates with Endogenously Segmented Asset Markets." *Journal of Political Economy*, 2001 (forthcoming).

Carlstrom, Charles T. and Fuerst, Timothy S. "Interest Rate Rules vs. Money Growth Rules: A Welfare Comparison in a Cash-in-Advance Economy." *Journal of Monetary Economics*, November 1995, *36*(2), pp. 246–67.

Christiano, Lawrence J. and Eichenbaum, Martin. "Liquidity Effects and the Monetary Transmission Mechanism." *American Economic Review*, May 1992 (*Papers and Proceedings*), *82*(2), pp. 346–53.

Clarida, Richard; Galí, Jordi and Gertler, Mark. "The Science of Monetary Policy: A New Keynesian Perspective." *Journal of Economic Literature*, December 1999, *37*(4), pp. 1661–1707.

Sargent, Thomas J. and Wallace, Neil. "Some Unpleasant Monetarist Arithmetic." *Federal Reserve Bank of Minneapolis Quarterly Review*, Winter 1985, *9*(1), pp. 15–31.

Taylor, John B. "Discretion versus Policy Rules in Practice." *Carnegie-Rochester Conference Series on Public Policy*, December 1993, *39*(0), pp. 195–214.