Are interest rate regressions evidence for a Taylor rule?

Patrick Minford, Francesco Perugini, Naveen Srinivasan[*]

Cardiff Business School, Aberconway Building, Colum Drive, Cardiff CF10 3EU, UK

Abstract

In a variety of recent papers, researchers have found that interest rate behaviour approximately follows a Taylor rule. We show that such interest rate behaviour results when the central bank may be following quite different monetary policy rules from the one proposed by Taylor. In other words, an interest rate relation with output and inflation does not identify a central bank reaction function. © 2002 Elsevier Science B.V. All rights reserved.

1. Introduction

In recent years, interest rate rules have attracted increased attention of analysts, policymakers, and the financial press.[1] For example Taylor's (1993) rule, which is said to 'represent' central banks' policy, recommends a setting for the level of the federal funds rate based on the state of the economy. Interest rate rules have become more appealing recently because of the alleged instability of monetary aggregates due to institutional changes. Yet it remains unclear stochastically whether they outperform money supply rules, which have different properties (see Bryant et al., 1993; Henderson and McKibbin, 1993, who first examined these types of rules; also Minford et al., 2001).[2]

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[*]Corresponding author. Tel.: +44-29-2087-6545; fax: +44-29-2087-4419.

E-mail address: srinivasannk@cf.ac.uk (N. Srinivasan).

[1]In this spirit, Clarida et al. (1998, 2000) estimate a forward looking rule for the Bundesbank, the Bank of Japan, and the Federal Reserve.

[2]We mean by rules quarterly-data representations of central bank behaviour, not very short-term operating procedures which could be used to implement a variety of different rules in our sense. For example, a central bank could be targeting an interest rate on a daily basis but adjusting its target regularly in order to achieve a *k*% money supply target over a longer period.

The focus in this paper is on the empirical question of whether such rules are actually followed. We show that estimates of reduced forms alone for the interest rate, the main evidence adduced, will not permit one to discriminate between a Taylor rule and Taylor-type rules. In effect, then, this paper poses the question: How do we know that a central bank is following a Taylor rule or any other rule? By looking at the behaviour of nominal interest rate one cannot distinguish a Taylor rule from any other rule. Indeed, Taylor (1999) himself emphasised that his rule mimicked the interest rate behaviour one would expect from a $k\%$ money supply rule.

Section 2 is devoted to specification of the macroeconomic model to be utilised.[3] In Section 3 we construct a pseudo Taylor rule. The final section summarises our main conclusions.

2. Theoretical structure

2.1. Aggregate demand

Following McCallum and Nelson (1999) and Clarida et al. (1999) we use a dynamic general equilibrium model with money. In recent years this framework has been widely used for theoretical analysis of monetary policy. It is well known that, in such a framework, we can derive as loglinear approximations IS and LM curves of the following form:

$$\log C_t = -\alpha_0 - \alpha_1 \log(1 + r_t) + E_t \log C_{t+1}, \quad \alpha_0, \alpha_1 > 0, \tag{1}$$

$$\log\left(\frac{M_t}{P_t}\right) = \gamma_0 + \gamma_1 E_t \log C_{t+1} + \gamma_2 R_t, \quad \gamma_1 > 0, \gamma_2 < 0. \tag{2}$$

After re-arranging (1) in terms of $E_t \log C_{t+1}$ we can express the representative household's lifetime budget constraint [given that all output (GDP) except government expenditure and investment expenditure is consumed] as

$$\sum_{j=0}^{\infty}\left(\frac{1}{1 + r_t^*}\right)^j C_{t+j} = \sum_{j=0}^{\infty}\left(\frac{1}{1 + r_t^*}\right)^j (Y_{t+j} - G_{t+j} - I_{t+j})$$

or

$$\sum_{j=0}^{\infty}\left(\frac{1}{1 + r_t^*}\right)^j \{\alpha_0[\alpha_1(1 + r_t^*)]\}^j C_t = \sum_{j=0}^{\infty}\left(\frac{1}{1 + r_t^*}\right)^j (1 + g)^j (\overline{Y}_t - \overline{G}_t - \overline{I}_t),$$

where 'g' denotes steady-state growth of consumption, r_t^* is long-run interest rate, and $\overline{Y}_t$, $\overline{G}_t$ and $\overline{I}_t$ denote steady-state values for output, government expenditure, and investment expenditure, respectively.[4] Leading the above equality one-period (expressing it in natural logarithms) and taking expectations at time 't' yields a variable made up of slow-moving steady-state elements:

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[3] Rather than work through the details of the derivation, which are readily available elsewhere, we instead directly introduce the key aggregate relationships.

[4] In this general equilibrium framework we introduce a government that spends current output according to a non-negative stochastic process (G_t) that satisfies $G_t \leq Y_t$ for all 't'. The government finances its expenditure by a stream of lump-sum taxes, issuing debt, and via seigniorage revenue.

$$E_t \log C_{t+1} \equiv \log \tilde{C}_{t+1}$$
$$= \log(1 - \alpha_0\alpha_1) + \log(\overline{Y}_{t+1} - \overline{G}_{t+1} - \overline{I}_{t+1}) + \log(1 + r^*_{t+1}) - \log(r^*_{t+1} - g). \tag{3}$$

2.2. Aggregate supply

The technology available to the economy is described by the production function $Y_t = Z_t f(N_t, \overline{K})$, where Y_t is aggregate output, $\overline{K}$ is capital stock which is assumed to be fixed, N_t is labour supply and Z_t reflects the state of technology.[5] Here, firms operate in competitive markets and therefore take prices as given when solving their own constrained maximisation problem.

2.3. Introduction of overlapping non-contingent wage contracts

Following Fischer (1977), wages are set for two periods so as to maintain expected real wages constant. The actual wage rate at any given point in time would be an average of the wages that have been set at various dates in the past. Hence, nominal wage at time 't' in natural logarithms would be

$$W_t = 0.5(_{t-1}W_t + {}_{t-2}W_t)$$

or

$$\log(W_t) = \log(w^*) + 0.5 \cdot \sum_{i=1}^{2} E_{t-i}[\log(P_t)],$$

where w^* denotes the equilibrium real wage.[6] If we let output supply be a declining function of the real wage (from firms maximising profits subject to a production function with labour inputs and some fixed overheads) then one can derive the Phillips curve (generalised version of Fischer, 1977) which is expressed in natural logarithms as follows:

$$\log Y_t = \log Y^* + q\left(\log P_t - \frac{1}{N}\sum_{i=1}^{N} E_{t-i}[\log(P_t)]\right) + \lambda_0(\log Y_{t-1} - \log Y^*),$$

where $q > 0$, λ_0 denotes persistence, Y^* is potential output and N is the contract length (here $N = 2$); we assume this to be growing at the steady rate 'g', while the lagged term captures persistence due to the capital stock overshooting or undershooting the steady-state growth path.

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[5] We assume that $f(N,K)$ is smooth and concave and satisfies Inada-type conditions. Capital stock is assumed to be fixed because we are interested in the derivation of the short-run aggregate supply curve. If we assume that $f(\cdot) = N_t^\alpha \overline{K}^{1-\alpha}$, i.e. a Cobb–Douglas function, then we can express the demands for the two factors as functions of the optimal output choice, i.e. $N_t = \alpha Y_t/w_t$ and $\overline{K} = (1 - \alpha)Y_t/r_t$. In order to solve for the optimal choice of output we substitute N_t into the Cobb–Douglas production function which essentially yields the supply function of the firm, where output supply is a declining function of the real wage.

[6] Note that $_{t-i}W_t = w^* + E_{t-i}P_t$, where $i = 1,2$. Ideally, the equilibrium real wage is a time-varying constant, i.e. it would move with taste and technology shocks. Treating w^* as a constant there does not in any way affect our final conclusion.

3. Constructing the Taylor pseudo-rule

To construct a relationship that looks like a Taylor rule, or 'pseudo-rule', we substitute for $\log P_t$ (from our Phillips curve) and substitute (3) for $E_t \log C_{t+1}$ in (2) the LM curve to get

$$
\begin{aligned}
R_t ={}& -\frac{\gamma_0}{\gamma_2} - \frac{\gamma_1}{\gamma_2} \log \tilde{C}_{t+1} - \frac{1}{q\gamma_2}(\log Y_t - \log Y^*) \\
&- \frac{1}{\gamma_2}[(\log P_t - \log P_{t-1}) - (\log M_t - \log P_{t-1})] + \frac{1}{\gamma_2}\left(\log P_t - \frac{1}{N}\sum_{i=1}^{N} E_{t-i}[\log(P_t)]\right) \\
&+ \frac{\lambda_0}{q\gamma_2}(\log Y_{t-1} - \log Y^*).
\end{aligned}
$$

We can now consider how a money supply reaction function could be embedded in the above relationship for the nominal interest rate to mimic a Taylor rule.

3.1. The Taylor pseudo-rule when a Friedman rule is operating

Consider a Friedman money growth rule:

$$
\log M_t - \log M_{t-1} = \mu + \frac{\varepsilon_t}{(1 - (1 - \lambda_1)L)},
$$

where $0 < \lambda_1 < 1$ and $\mu = \pi^* + g_{\mathrm{m}}$ (the k in Friedman's $k\%$ rule) is assumed to be split between an inflation target, π^*, and an allowance for growth in real money demand, g_{m}; λ_1 denotes speed of adjustment with which the rule comes back on track, and L is the lag operator. Substituting the Friedman money growth rule above for $\log M_t$ yields

$$
R_t = \phi_1 + a_0(\log Y_t - \log Y^*) - b_0(\pi_t - \pi^*) + u_{1t},
$$

where

$$
\phi_1 = -\frac{\gamma_0}{\gamma_2} + \frac{1}{\gamma_2}g_{\mathrm{m}}, \quad a_0 = -\frac{1}{q\gamma_2}, \quad b_0 = -\frac{1}{\gamma_2},
$$

and

$$
\begin{aligned}
u_{1t} ={}& -b_0\left((\log M_{t-1} - \log P_{t-1}) + \frac{\varepsilon_t}{(1 - (1 - \lambda_1)L)} + \left(\log P_t - \frac{1}{N}\sum_{i=1}^{N} E_{t-i}[\log(P_t)]\right)\right) \\
&+ b_0\gamma_1 \log \tilde{C}_{t+1} - a_0\lambda_0(\log Y_{t-1} - \log Y^*).
\end{aligned}
$$

This gives us the pseudo-rule in a simple form. To create a more complex rule where the lagged interest rate enters we extract the term in lagged real money balance from the error term and replace it with its value from Eq. (2) to obtain

$$
R_t = \phi_{11} + R_{t-1} + a_0(\log Y_t - \log Y^*) - b_0(\pi_t - \pi^*) + u_{2t},
$$

where

$$\phi_{11} = \phi_1 - b_1 \gamma_0,$$

and

$$u_{2t} = -b_0 \left(\frac{\varepsilon_t}{(1 - (1 - \lambda_1)L)} + \left(\log P_t - \frac{1}{N} \sum_{i=1}^{N} E_{t-i}[\log(P_t)] \right) \right) + b_0 \gamma_1 (\log \tilde{C}_{t+1} - \log \tilde{C}_t)$$
$$- a_0 \lambda_0 (\log Y_{t-1} - \log Y^*).$$

We can also write

$$R_t = \phi_{11} + \rho R_{t-1} + a_0 (\log Y_t - \log Y^*) - b_0 (\pi_t - \pi^*) + u_{3t},$$

where

$$u_{3t} = -b_0 \left(\frac{\varepsilon_t}{(1 - (1 - \lambda_1)L)} + \left(\log P_t - \frac{1}{N} \sum_{i=1}^{N} E_{t-i}[\log(P_t)] \right) \right) + b_0 \gamma_1 (\log \tilde{C}_{t+1} - \log \tilde{C}_t)$$
$$+ (1 - \rho) R_{t-1} - a_0 \lambda_0 (\log Y_{t-1} - \log Y^*).$$

Thus we have a full set of dynamic pseudo-rules, with the error and constant term adjusting according to the dynamics.[7] Notice that, in a monetary regime with an inflation target, nominal interest rates will be stationary; hence the error term will also be stationary. It will also be serially correlated and will contain endogenous variables. In these respects it will not be distinguishable from the error in a Taylor rule which represents central bank reactions to other developments; and is appropriately dealt with by the same techniques for stationary processes, such as the use of instruments and lagged errors.

We could go on in this manner with other money supply rules. Essentially, as our example shows, we would substitute the contents of a money supply rule for $\log M_t$ and then sweep the components that are not in the Taylor rule into the error term or the constant when the equation should in principle pass the usual time-series tests. What this shows is that money supply rules give rise to interest rate behaviour that looks like a Taylor rule. Yet the behaviour of the economy will be different under these money supply rules from what it would be under the Taylor rule they resemble; the different rules constitute different monetary regimes.[8] In other words, what we are seeing is that all these quite different rules have a Taylor rule representation.

4. Conclusion

In a variety of recent papers, researchers have found that interest rate behaviour approximately follows a Taylor rule. From this they have concluded that the central bank is following a Taylor rule as its monetary policy reaction function. However, we have shown in this paper that such interest rate

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[7] Note that if we replace our Fischer-style Phillips curve by the widely used Calvo–Rotemberg Phillips curve, none of our conclusions are affected. We would expect future inflation instead of expected current inflation to appear in the error term.

[8] The difference in behaviour arises due to the problems analysed in a seminal paper by Poole (1970). The point is that, in a stochastic world, one policy rule may be superior to the other depending on the values of the structural parameters and on the variances of the disturbances.

behaviour results when the central bank may be following quite different monetary policy rules from the one proposed by Taylor. In other words, an interest rate relation with output and inflation does not identify a central bank reaction function. Other information about the model structure must be used for identification: for example, statements by the central bank itself (though these are often ambiguous about the role of the money supply, as exemplified by the ECB's 'two pillars'). We conclude that, while from a normative viewpoint it is well known that these rules have different stochastic properties which are the subject of other research, the main evidence for their actual existence from interest rate equations is basically flawed.

Acknowledgements

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