

K-map

Canonical Expression

A boolean expression consisting entirely of either all minterms or maxterms is a canonical expression.

E.g.:

$$XY + X'Y' \text{ (Minterms)}$$

$$(X+Y) \cdot (X'+Y') \text{ (Maxterms)}$$

Type:-

1. Sum of Products (SOP) - Minterms

2. Product of sums (POS) - Maxterms

Sum of Products

A boolean expression consisting entirely of only minterms is known as a sum of products canonical expression.

Let's say F is a boolean expression having variables A and B. F is true when at least 1 variable is true.

A	B	F	Minterm	\therefore Canonical SOP expression will be minterms when $F(A,B)$ is 1
0	0	0	$A'B'$	
0	1	1	$A'B$	
1	0	1	AB'	
1	1	1	AB	
				$\therefore F = A'B + AB' + AB$

Short hand notation

From the example, $F = A'B + AB' + AB$

In short hand notation,

$$A'B = (0\ 1)_2 = m_1$$

$$AB' = (1\ 0)_2 = m_2$$

$$AB = (1\ 1)_2 = m_3$$

$$\therefore F = m_1 + m_2 + m_3$$

Shorthand to SOP

$$F = \sum(1, 2, 3)$$

$$= m_1 + m_2 + m_3$$

$$= 01 + 10 + 11$$

$$= A'B + AB' + AB$$

1 is taken as-is while 0 is taken as a complement

Maxterms

A sum term in which all variables appear once either complemented or uncomplemented is a maxterm

X	Y	Z	Minterm	Maxterm
0	0	0	$\bar{X}\bar{Y}\bar{Z} m_0$	$\bar{X}+\bar{Y}+\bar{Z} M_0$
0	0	1	$\bar{X}\bar{Y}Z m_1$	$\bar{X}+\bar{Y}+\bar{Z} M_1$
0	1	0	$\bar{X}Y\bar{Z} m_2$	$\bar{X}+\bar{Y}+Z M_2$
0	1	1	$\bar{X}YZ m_3$	$\bar{X}+\bar{Y}+\bar{Z} M_3$
1	0	0	$X\bar{Y}\bar{Z} m_4$	$\bar{X}+Y+Z M_4$
1	0	1	$X\bar{Y}Z m_5$	$\bar{X}+Y+\bar{Z} M_5$
1	1	0	$XY\bar{Z} m_6$	$\bar{X}+\bar{Y}+Z M_6$
1	1	1	$XYZ m_7$	$\bar{X}+\bar{Y}+\bar{Z} M_7$

Date: Yes

Minterm

A product term in which all variables appear once, either complemented or uncomplemented is called a Minterm. It represents exactly one combination of the binary variables in a truth table.

Product of Sums

A boolean expression consisting entirely of only maxterms is a Product of Sums canonical expression.

Let's say F is a boolean expression with variables A and B. F is true when $A \oplus B = 1$

A	B	F	Maxterm
0	0	0	$A + B$
0	1	1	$A + \bar{B}$
1	0	1	$\bar{A} + B$
1	1	0	$\bar{A} + \bar{B}$

The Product of maxterms where $F = 0$ is POS expression

$$\therefore \text{POS} = (A+B) \cdot (\bar{A}+\bar{B})$$

Shorthand notation

$$A+B = \overset{(M_0)}{00}, \quad \bar{A}+\bar{B} = \overset{(M_3)}{11}$$

$$\therefore \text{Shorthand notation} = \prod(4, 0, 3)$$

(Converse):-

$$\begin{aligned}\prod(0, 3) &= M_0 \cdot M_3 \\ &= (A+B) \cdot (\bar{A}+\bar{B})\end{aligned}$$

Date : Yes

K-map for Sum of Products.

In SOP expression, each cell in K-map is a minterm.

If there are n variable for a given boolean function then
K-map has 2^n cells.

2 variable K-map for SOP :-

$$\text{Number of cells} = 2^2 = 4$$

x	y	y'	y
x'	$x'y$	$x'y'$	x
x	xy'	xy	y

The cell with number n is minterm m_n

3 variable K-map for SOP

$$\text{Number of cells} = 2^3 = e^{\ln(2)3} = 8$$

x	yz	$\bar{y}z$	$\bar{y}\bar{z}$	yz	$y\bar{z}$	$\bar{y}\bar{z}$	$\bar{y}\bar{z}$
x'	$x'y'z'$	$x'y'z$	$\bar{x}yz$	$\bar{x}y\bar{z}$	$\bar{x}y\bar{z}$	$\bar{x}\bar{y}z$	$\bar{x}\bar{y}\bar{z}$
x	$x\bar{y}\bar{z}$	$x\bar{y}z$	$\bar{x}yz$	$\bar{x}y\bar{z}$	$\bar{x}\bar{y}z$	$\bar{x}\bar{y}\bar{z}$	$\bar{x}\bar{y}\bar{z}$

4 variable K-map for SOP

wx	yz	$y'z'$	$y'z$	yz	$y'z'$	yz'	$y\bar{z}$	$y\bar{z}'$	$\bar{y}z$	$\bar{y}z'$	$\bar{y}\bar{z}$	$\bar{y}\bar{z}'$
wx'	$wx'y'z'$	$wx'y'z$	$wx'yz$	$wx'yz'$	$wx'y\bar{z}$	$wx'y\bar{z}'$	$wxyz$	$wxy\bar{z}$	$wx\bar{y}z$	$wx\bar{y}z'$	$w\bar{x}yz$	$w\bar{x}y\bar{z}$
$w'x$	$w'xy'z'$	$w'xy'z$	$w'xyz$	$w'xy\bar{z}$	$w'x\bar{y}z$	$w'x\bar{y}z'$	$w\bar{xy}z$	$w\bar{xy}\bar{z}$	$w\bar{x}y\bar{z}$	$w\bar{x}y\bar{z}'$	$w\bar{x}\bar{y}z$	$w\bar{x}\bar{y}z'$
wx	$wx'y'z'$	$wx'y'z$	$wxy'z$	$wxy\bar{z}$	$wx\bar{y}z$	$wx\bar{y}z'$	$w\bar{xy}z$	$w\bar{xy}\bar{z}$	$w\bar{x}y\bar{z}$	$w\bar{x}y\bar{z}'$	$w\bar{x}\bar{y}z$	$w\bar{x}\bar{y}z'$
wx'	$wx'y'z'$	$wx'y'z$	$wxy'z$	$wxy\bar{z}$	$wx\bar{y}z$	$wx\bar{y}z'$	$w\bar{xy}z$	$w\bar{xy}\bar{z}$	$w\bar{x}y\bar{z}$	$w\bar{x}y\bar{z}'$	$w\bar{x}\bar{y}z$	$w\bar{x}\bar{y}z'$

K-map for POS

① In POS expression each cell in K-map is a minterm. K-map will have 2^n cells where $n = \text{number of variables}$.

2 variable K-map for POS

x	y	y'
x'	$x+y$	$x+y'$
x'	$x'+y$	$x'+y'$

The cell with number n is minterm M_n

3 variable K-map for POS

x	yz	$y\bar{z}$	$\bar{y}z$	$\bar{y}\bar{z}$
x'	$x+y+z$	$x+y+\bar{z}$	$x+\bar{y}+z$	$x+\bar{y}+\bar{z}$
\bar{x}	$\bar{x}+y+z$	$\bar{x}+y+\bar{z}$	$\bar{x}+\bar{y}+z$	$\bar{x}+\bar{y}+\bar{z}$

4 variable K-map for POS

z	$w+y$	$y+z$	$w+x$	$y+z$	$y+\bar{z}$	$\bar{y}+\bar{z}$	$\bar{y}+z$
$w+x$	$w+x+y+z$	$w+x+y+\bar{z}$	$w+x+\bar{y}+z$	$w+x+\bar{y}+\bar{z}$	$w+x+\bar{y}+z$	$w+x+\bar{y}+\bar{z}$	$w+x+\bar{y}+z$
$w+\bar{x}$	$w+\bar{x}+y+z$	$w+\bar{x}+y+\bar{z}$	$w+\bar{x}+\bar{y}+z$	$w+\bar{x}+\bar{y}+\bar{z}$	$w+\bar{x}+\bar{y}+z$	$w+\bar{x}+\bar{y}+\bar{z}$	$w+\bar{x}+\bar{y}+z$
$w+\bar{x}$	$w+\bar{x}+y+\bar{z}$	$w+\bar{x}+\bar{y}+\bar{z}$	$w+\bar{x}+\bar{y}+z$	$w+\bar{x}+\bar{y}+\bar{z}$	$w+\bar{x}+\bar{y}+z$	$w+\bar{x}+\bar{y}+\bar{z}$	$w+\bar{x}+\bar{y}+z$
$\bar{w}+\bar{x}$	$\bar{w}+\bar{x}+y+\bar{z}$	$\bar{w}+\bar{x}+\bar{y}+\bar{z}$	$\bar{w}+\bar{x}+\bar{y}+z$	$\bar{w}+\bar{x}+\bar{y}+\bar{z}$	$w+\bar{x}+\bar{y}+z$	$w+\bar{x}+\bar{y}+\bar{z}$	$w+\bar{x}+\bar{y}+z$
$\bar{w}+x$	$\bar{w}+x+y+\bar{z}$	$\bar{w}+x+\bar{y}+\bar{z}$	$\bar{w}+x+\bar{y}+z$	$\bar{w}+x+\bar{y}+\bar{z}$	$w+\bar{x}+\bar{y}+z$	$w+\bar{x}+\bar{y}+\bar{z}$	$w+\bar{x}+\bar{y}+z$
$\bar{w}+x$	$\bar{w}+x+y+z$	$\bar{w}+x+\bar{y}+z$	$\bar{w}+x+\bar{y}+\bar{z}$	$\bar{w}+x+\bar{y}+z$	$w+\bar{x}+\bar{y}+z$	$w+\bar{x}+\bar{y}+\bar{z}$	$w+\bar{x}+\bar{y}+z$
	8	9	11	10			

1A

a) Given $F(A, B, C, D) = \Sigma(0, 1, 2, 3, 4, 5, 6, 7, 10, 13, 14, 15)$

i) Reduce the expression using 4 var. K-map

ii) Draw logic gate

b) Given $P(A, B, C, D) = \prod(0, 1, 2, 3, 4, 5, 6, 7, 10, 13, 14, 15)$

i) Reduce the expression using 4 var. K-map

ii) Draw logic gate

Ans.

a) $F(A, B, C, D) = \Sigma(0, 1, 2, 3, 4, 5, 6, 7, 10, 13, 14, 15)$

AB	CD	$C'D'$	$C'D$	CD	CD'
$A'B'$	$\overline{A'B'C'D'}$	$\overline{A'B'C'D}$	$\overline{A'B'CD}$	$\overline{A'B'CD'}$	
$A'B$	$\overline{A'BC'D'}$	$\overline{A'BC'D}$	$\overline{A'BCD}$	$\overline{A'BCD'}$	
AB	$\overline{ABC'D'}$	$\overline{ABC'D}$	$\overline{ABC'D'}$	$\overline{ABC'D}$	
$A'B'$	$\overline{AB'C'D'}$	$\overline{AB'C'D}$	$\overline{AB'CD}$	$\overline{AB'CD'}$	
$A'B$	$\overline{AB'C'D'}$	$\overline{AB'C'D}$	$\overline{AB'CD}$	$\overline{AB'CD'}$	
AB	$\overline{ABC'D'}$	$\overline{ABC'D}$	$\overline{ABC'D'}$	$\overline{ABC'D}$	

i) $F(A, B, C, D) = A' + ABD + ACD' + \cancel{AB'C'D'}$

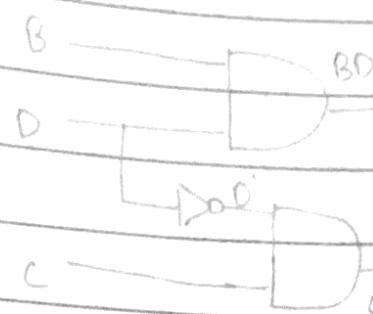
$$= A' + BD + CD'$$

1. Octet - A'

2. Quart - BD

3. Quart - CD'

ii)



$$b) P(A, B, C, D) = \overline{I}(0, 1, 2, 3, 4, 5, 6, 7, 10, 13, 14, 15)$$

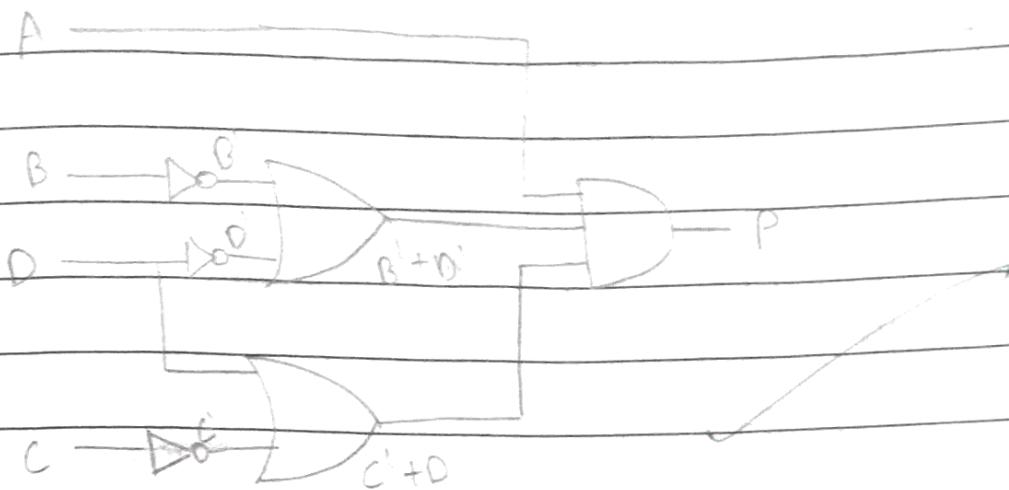
	$C+D$	$C+D'$	$C'+D$	$C'+D'$
$A+B$	$A+B+C+D$	$A+B+C+A'$	$A+B+C'+D$	$A+B+C'D$
$A+B'$	$A+B'+C+D$	$A+B'+C+D'$	$A+B'+C'+D$	$A+B'+C'D$
$A'+B'$	$A'+B'+C+D$	$A'+B'+C+D'$	$A'+B'+C'+D$	$A'+B'+C'D$
$A'+B$	$A'+B+C+D$	$A'+B+C+D'$	$A'+B+C'+D$	$A'+B+C'D$
	0	1	1	1
	1	0	1	1
	1	1	0	1
	1	1	1	0
	1	0	0	0
	0	1	0	0
	0	0	1	0
	0	0	0	1

i) 1. Octet - A

2. Quart - ~~B'D'~~ $B'+D'$ 3. Quart - $C'+D$

$$P(A, B, C, D) = (A) \cdot (B'+D') \cdot (C'+D)$$

ii)



Q1. Given a function $F(A, B, C, D) = \sum(0, 2, 4, 5, 8, 9, 10, 12, 13)$

i) Reduce the expression using K-map

ii) Draw logic gate

b) Given a function $F(P, Q, R, S) = \prod(0, 1, 3, 5, 7, 8, 9, 10, 11, 14, 15)$

i) Reduce the expression using K-map

ii) Draw logic gate

Ans.

AB	$C'D'$	$C'D$	$C'D'$	$C'D$	$C'D'$
$A'B'$	1				1
0	0	1		3	2

$A'B'$	1	1			1
4	4	5		7	6

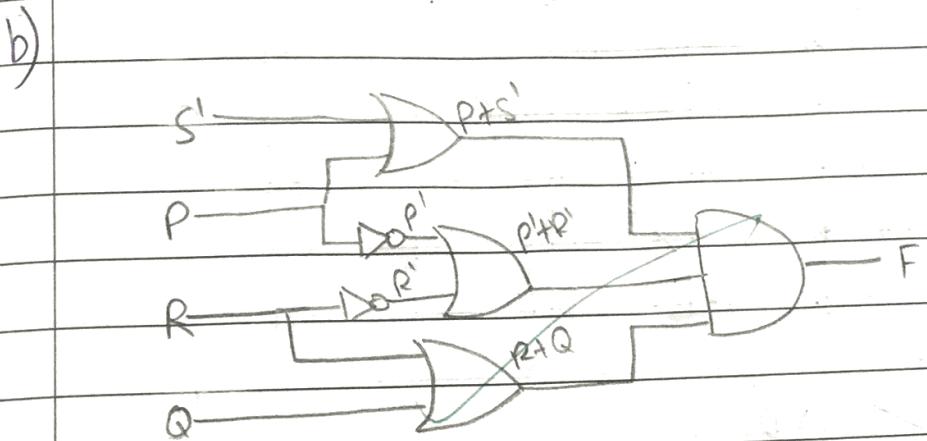
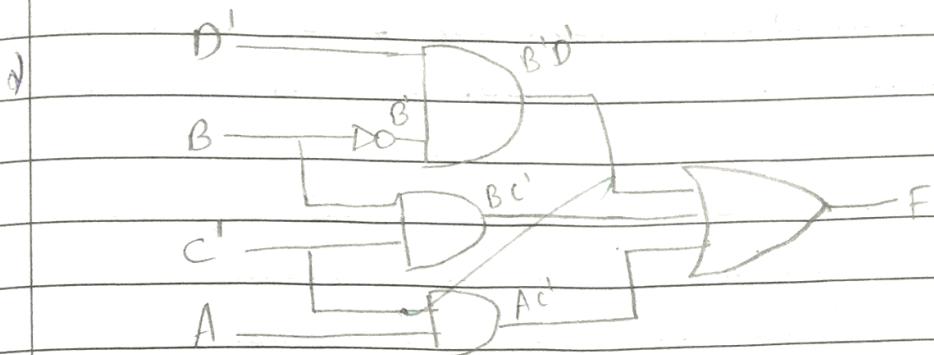
AB'	1	1	1	1	1
12	12	13	15	14	1
AB'	1	1	1	1	1

1. Quart - Bc'
2. Quart - $B'D'$
3. Quart - Ac'
4. Quart - $A'D'$

b) $F(A, B, C, D) = BC' + B'D' + AC'$
 $F(P, QR, S) = \text{IT}(0, 1, 3, 5, 7, 8, 9, 10, 11, 14, 15)$

	$P+Q$	PQ	$P+S$	$P+S'$	$P'+S$	$P'+S'$
	R^0	R^1	R^3	R^5	R^7	R^8
$P+Q$	D	D	D			
$P+Q'$	D	D	D			
$P+Q$	D	D	D			
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20.

Given the boolean function $F(A, B, C, D) = \sum(6, 2, 3, 4, 6, 7, 9, 13)$. Use k-map to reduce the function F for reduced SOP form. Draw the logic gate.

Ans.

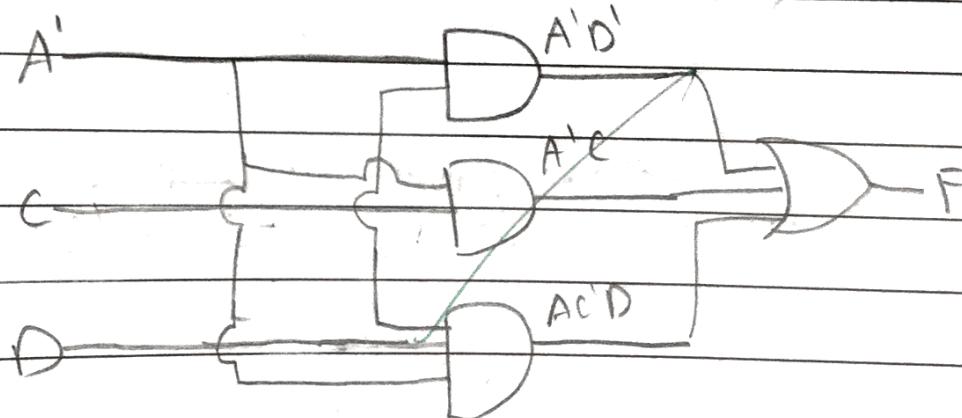
AB	CD	$C'D'$	$C'D$	CD	CD'
$A'B'$	1	0	1	1	1
$A'B$	1	1	1	1	1
AB	1	1	1	1	1
AB'	1	1	1	1	1

$$1. \text{ Quart} = A'D'$$

$$2. \text{ Quart} = A'C$$

$$3. \text{ Pair} = \cancel{A'C'D} \quad AC'D$$

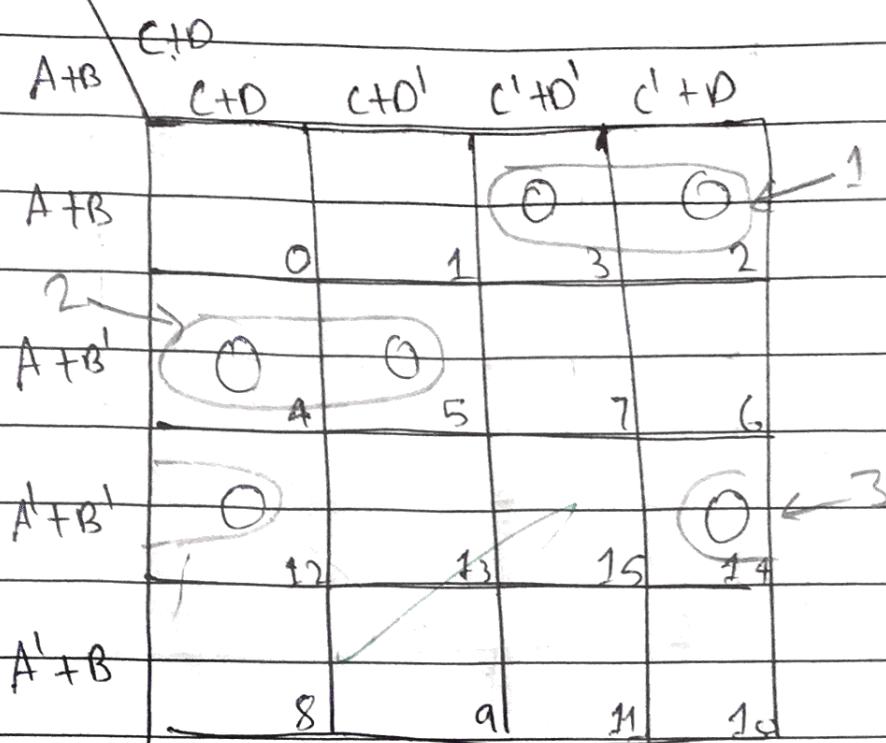
$$\therefore F(A, B, C, D) = A'D' + A'C + \cancel{A'C'D} \quad AC'D$$



21. Given the function $X(A, B, C, D) = \overline{\prod}(2, 3, 4, 5, 12, 14)$

Use K-map to reduce the function F for reduced POS form.
Draw the logic gate.

Ans.

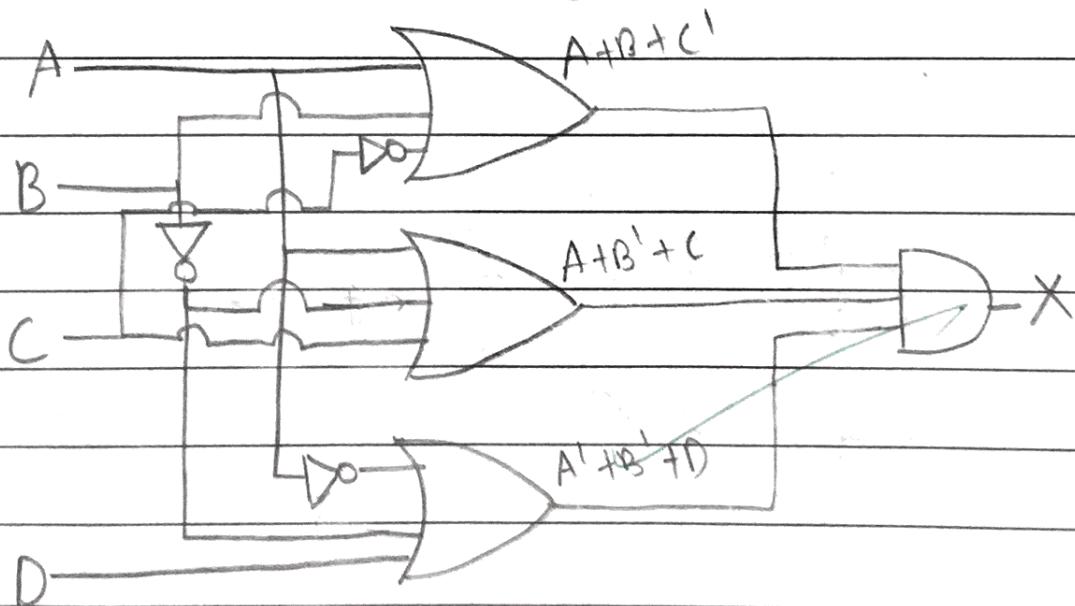


$$1. \text{ Pair} = \cancel{A\bar{B}C} \quad A + B + C'$$

$$2. \text{ Pair} = \cancel{A'\bar{B}\bar{C}} \quad A + B' + C$$

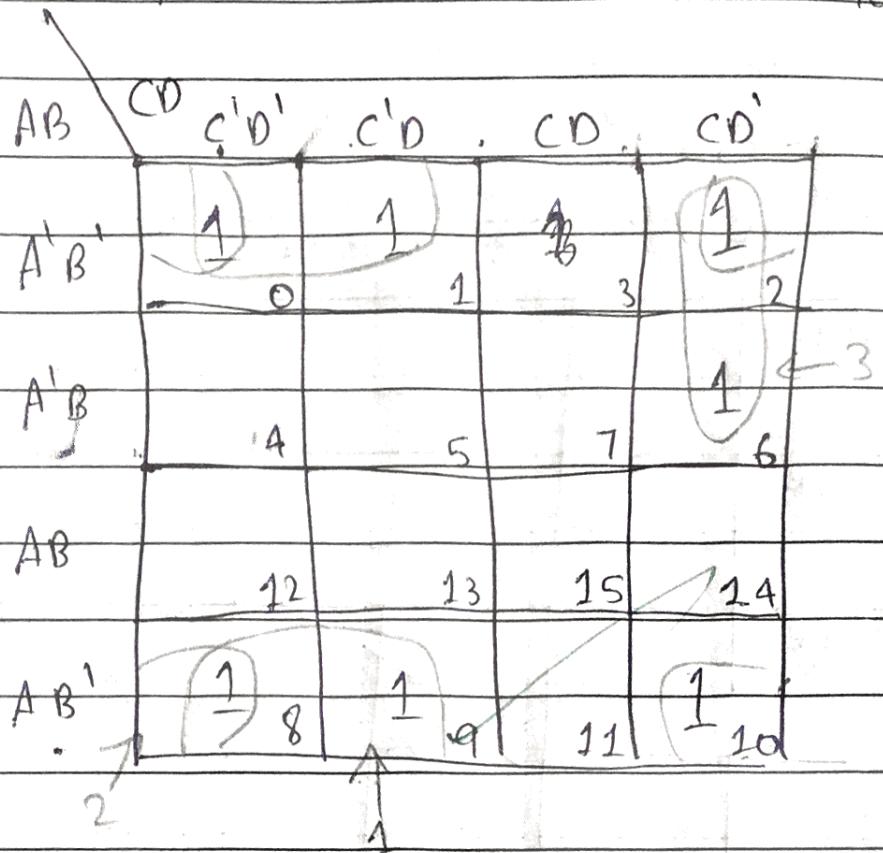
$$3. \text{ Pair} = \cancel{A\bar{B}D'} \quad A' + B' + D$$

$$\therefore X(A, B, C, D) = (\cancel{A + B + C'}) \cdot (\cancel{A + B' + C}) \cdot (\cancel{A' + B' + D})$$



30. Given the function $F(A, B, C, D) = \Sigma(0, 1, 2, 6, 8, 9, 10)$
 Use K-map to reduce this function. Draw logic gate

Ans.

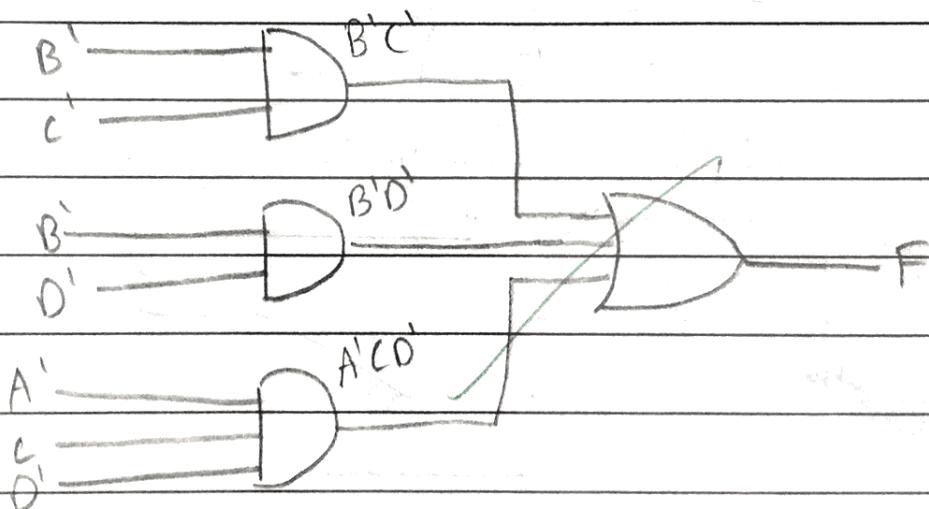


$$1. Quart = B'C'$$

$$2. Quart = B'D'$$

$$3. Pair = A'CD'$$

$$\therefore F(A, B, C, D) = B'C' + B'D' + A'CD'$$



Date : _____

Q3
Given the function $F(x, y, z, w) = \prod(0, 1, 4, 5, 8, 9, 11, 12, 13)$

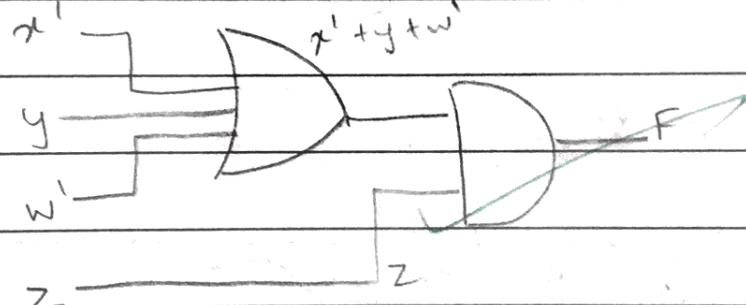
Use k-map to reduce this function. Draw logic gate

$x'y$	$z+w$	$z+w'$	$z'w'$	$z'w$
$x+y$	0	6		
$x+y'$	4	5	7	6
$x'y'$	12	13	15	14
$x'y$	8	6	6	10

$$1. \text{ Octet } = z$$

$$2. \text{ Pair } = x' + y + w'$$

$$\therefore F(x, y, z, w) = (z) \cdot (x' + y + w')$$



32. $F(U, V, W, X) = \Sigma(1, 3, 4, 6, 9, 11, 12, 14)$

i) Use K-map to reduce the function

ii) Name the gate F represents (if any) and show its symbol.

iii) Draw logic gate using only NOR gate

Ans.

UV	WX	W'X'	W'X	WX	WX'
UV	0	1	1	3	2
UV	4	5	7	6	2
UV	12	13	15	14	1
UV	8	9	11	10	1

1. Quart = ~~V'~~ $V'X$

2. Quart = VX'

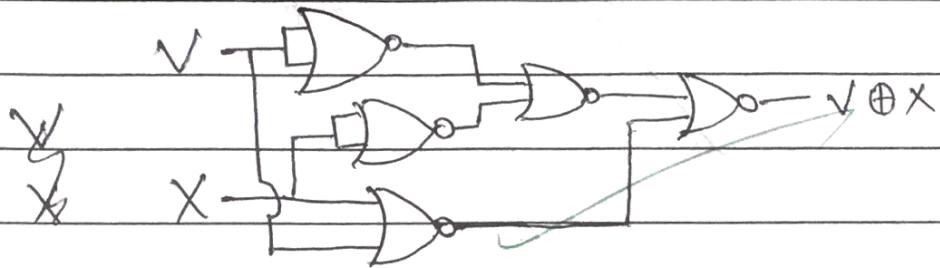
$\therefore F(U, V, W, X) = V'X + VX'$

ii) F is a XOR gate. Its operator symbol is \oplus
Its logic gate symbol is 

A	B	$A \oplus B$
0	0	1
0	1	0
1	0	1
1	1	0

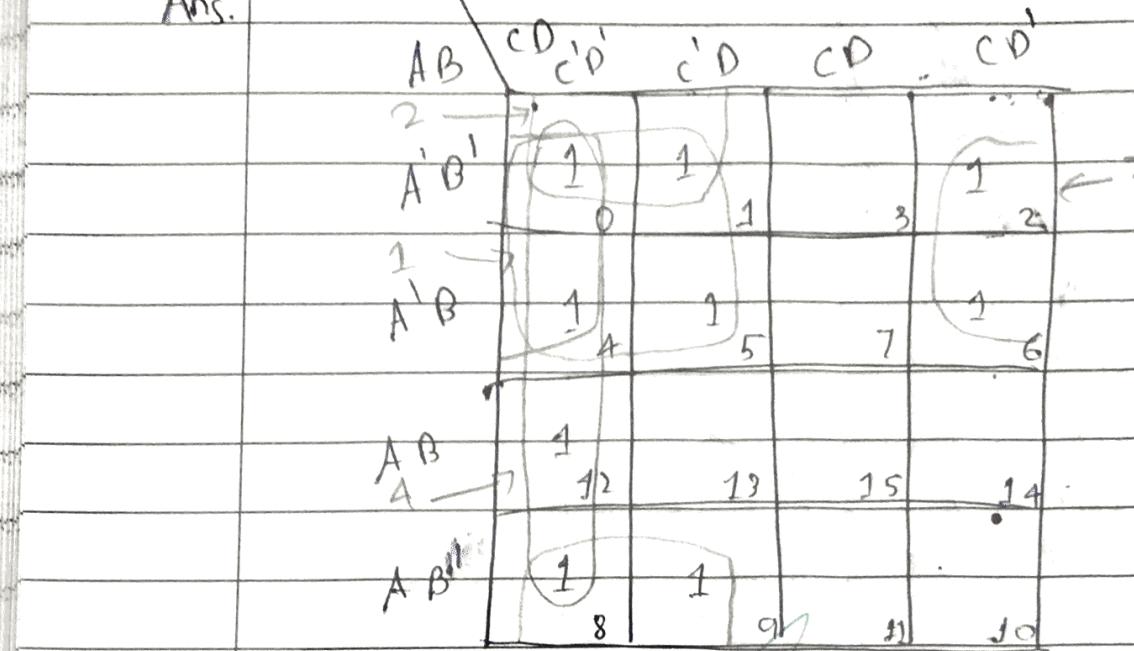
iii)
$$\begin{aligned}
 F &= V'x + Vx' \\
 &= V'x + Vx' \cancel{+ VV'} + \cancel{xx'} \\
 &\equiv (V+x) \cdot (V'+x') \\
 &= ((V+x) \cdot (V'+x'))' \\
 &\equiv (V'(V+x) + x'(V+x))' \\
 &= X
 \end{aligned}$$

iii)
$$\begin{aligned}
 F &= V'x + Vx' \\
 &= V'x + Vx' + VV' + xx' \\
 &\equiv (V+x) \cdot (V'+x') \\
 &= ((V+x)' + (V'+x')')' \\
 &\equiv ((V+x)' + ((V+v)' + (x+x')'))'
 \end{aligned}$$



37. ~~BB8~~ Given the function $F(a, b, c, d) = \Sigma(0, 12, 4, 5, 6, 8, 1, 12, 13, 14)$. Simplify using K-map. Draw the logic gate

Ans.



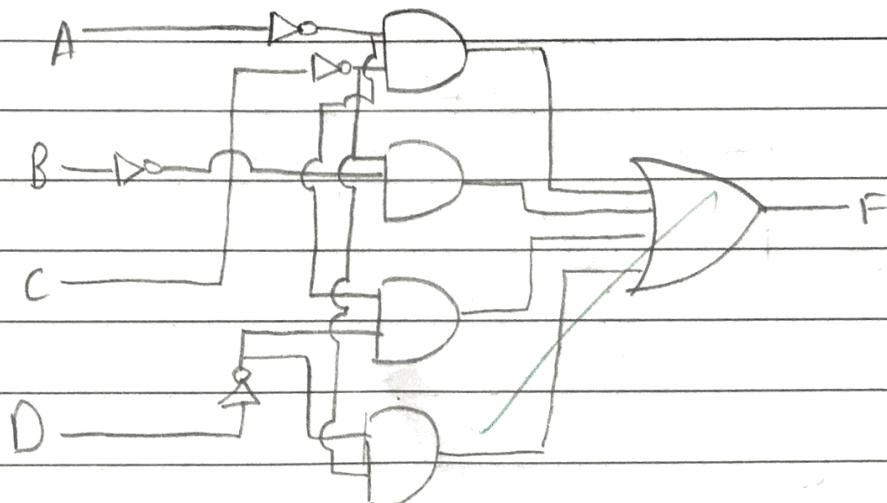
$$1. \text{ Quart} - A'c'$$

$$2. \text{ Quart} - B'c'$$

$$3. \text{ Quart} - A'd'$$

$$4. \text{ Quart} - c'd'$$

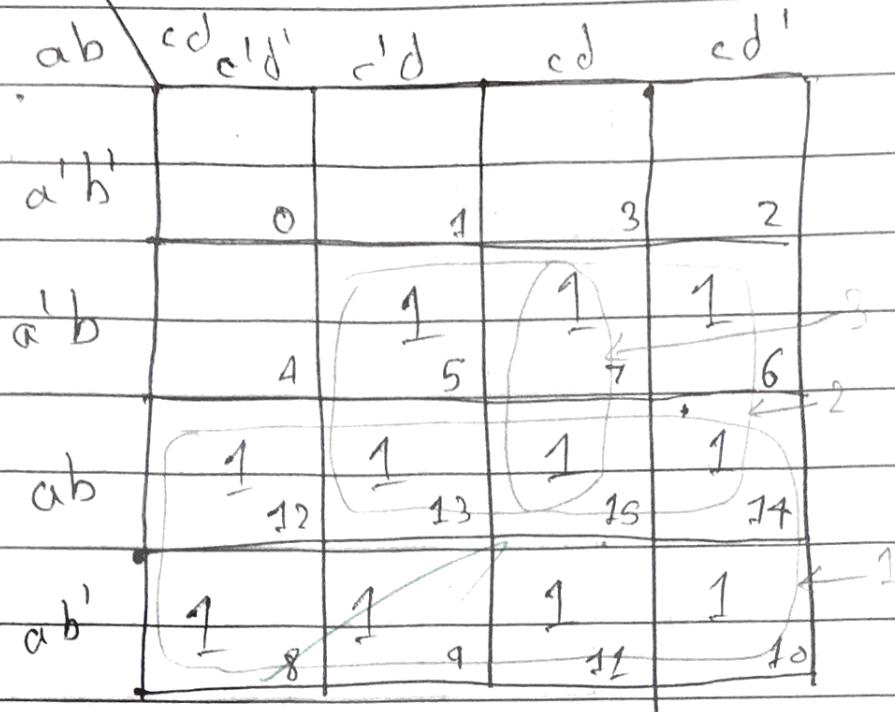
$$\therefore F = A'c' + B'c' + A'd' + c'd'$$



Date : _____

Q4. Given the function $F(a, b, c, d) = \Sigma(5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15)$. Use K-map to reduce it. Draw logic gate

Ans.

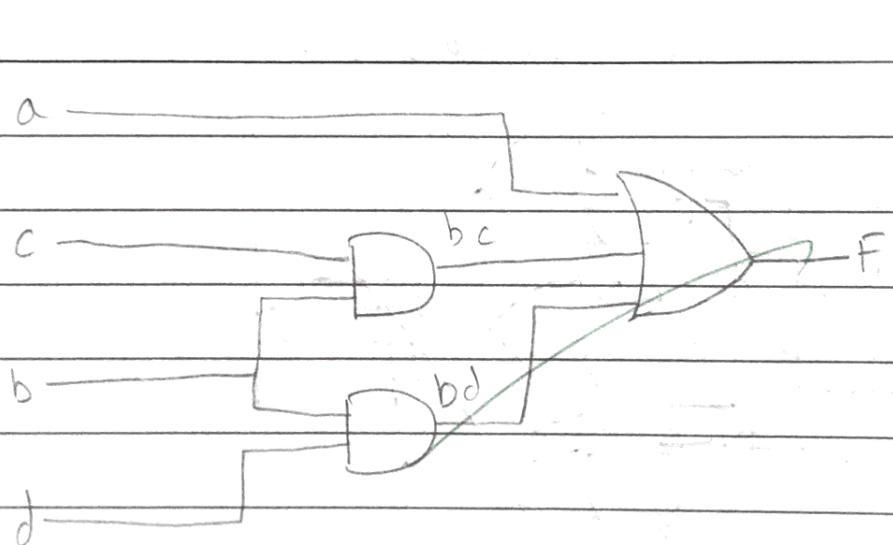


1. Octet - a

2. Quart - bc

3. Quart - bd

$$\therefore F(a, b, c, d) = a + bc + bd$$



45.

Given the function $F(a, b, c, d) = \prod(0, 1, 3, 5, 6, 7, 10, 14, 15)$. Use K-map to reduce this function. Draw logic

$a+b$	$c+d$	$c+d'$	$c'+d'$	$c'+d$
$a+b$	0	0	0	2
$a+b'$	0	0	0	1
$a'+b'$	12	13	15	14
$a'+b$	8	9	11	10

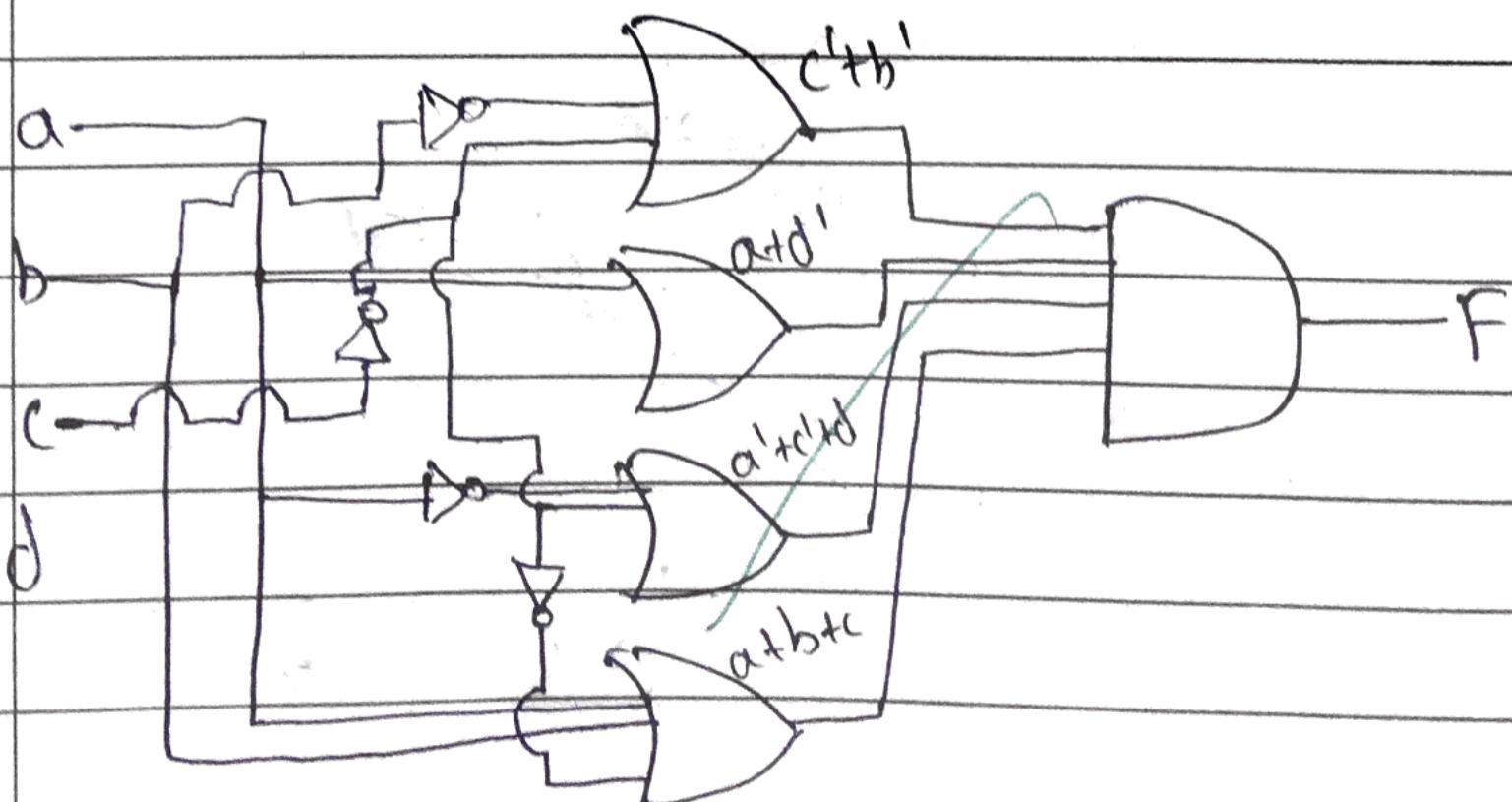
$$1. \text{ Quart} - c'+b'$$

$$2. \text{ Quart} - a+d'$$

$$3. \text{ Pair} - a'+c'+d$$

$$4. \text{ Pair} - a+b+c$$

$$\therefore F(a, b, c, d) = (c'+b') \cdot (a+d') \cdot (a'+c'+d) \cdot (a+b+c)$$



39. Given the function $\Pi(3, 4, 5, 7, 11, 12, 13, 14, 15) = G(A, B, C, D)$
 Reduce the expression in POS form. Draw logic gate

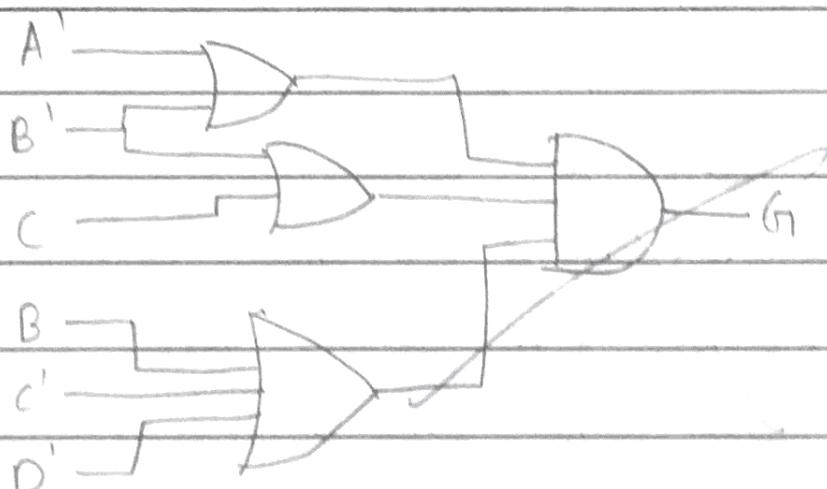
	$A+B$	$C+D$	$C+D'$	$C'D'$	$C'D$
$A+B$	0	1	0	3	2
$A'B$	0	0	0	5	7
$A'+B'$	6	0	0	11	14
$A'+B$	8	9	0	11	10

1. Quart - $(A'+B')$

2. Quart - $(B'+C)$

3. Pair - $(B+C'+D')$

$$\therefore G(A, B, C, D) = (A'+B')(B'+C) \cdot (B+C'+D')$$



34. There are four parallel railway tracks at a place.
 If 3+ trains pass, a warning signal flashes on
 1 indicates presence of train
 0 indicates absence of train
 1 indicates warning signal on
 0 indicates warning signal off

- Design truth table and write SOP expression
- Reduce SOP expression using K-map
- Draw logic gate

Ans.

	A	B	C	D	W	
0.1.	0	0	0	0	0	Canonical representation
1.	0	0	0	1	0	$= \Sigma(7, 11, 13, 14, 15)$
2.	0	0	1	0	0	$= m_7 + m_{11} + m_{13} + m_{14} + m_{15}$
3.	0	0	1	1	0	$= \Sigma(7, 11, 13, 14, 15)$
4.	0	1	0	0	0	
5.	0	1	0	1	0	
6.	0	1	1	0	0	
7.	0	1	1	1	1	
8.	1	0	0	0	0	
9.	1	0	0	1	0	
10.	1	0	1	0	0	
11.	1	0	1	1	1	
12.	1	1	0	0	0	
13.	1	1	0	1	1	
14.	1	1	1	0	1	
15.	1	1	1	0	1	

Date : _____

	CD	$C'D'$	$c'D$	CD	CD'
$A'B'$	0	1	3	2	
$A'B$	4	5	7	6	
AB	12	13	15	14	
AB'	8	9	11	10	

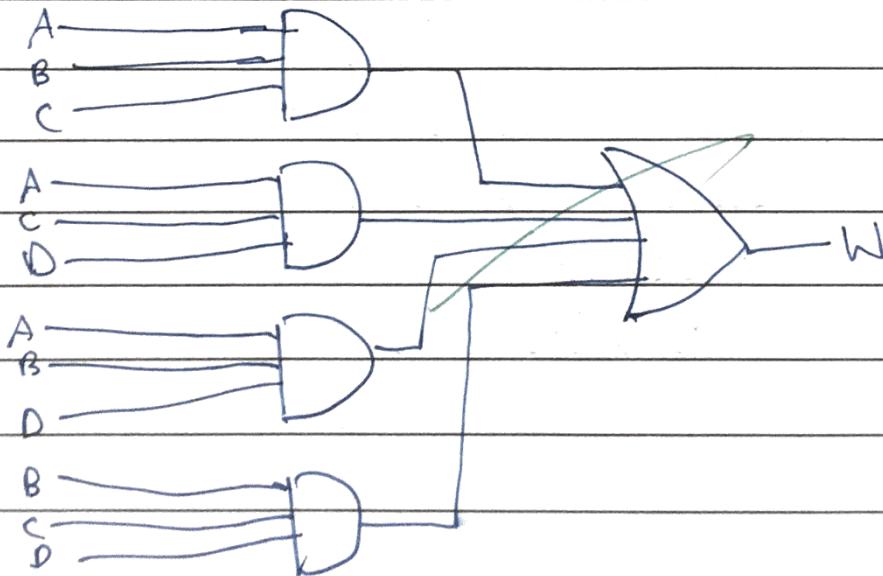
1. Pair - ABC

2. Pair - ACD

3. Pair - ~~BCD~~ ABD

4. Pair - ~~ACD~~ BCD

$$\therefore W(A, B, C, D) = ABC + ACD + ABD + BCD$$



Date: 7/6/2022

- Q. An insurance company issues a policy to an applicant when atleast one of these conditions are satisfied
- Applicant is a male, married, ≥ 25 years
 - Applicant is a female, no car accident
 - Applicant is a female, married, no car accident
 - Applicant is a male, < 25 years
 - Applicant ≥ 25 years, no car accident
 - Applicant is a male $\rightarrow < 25$ years
 - Applicant is ≥ 25 years, no car accident

Inputs:-

M - Married , 1 is yes, 0 is no

C - Car accident, 1 is yes, 0 is no

S - Male , 1 is yes, 0 is no

Y - Below 25 years, 1 is yes, 0 is no

Output :-

I - Whether the policy will be issued. 1 is yes, 0 is no

a) Draw the truth table for inputs and outputs. Write

SOP expression

b) Reduce expression using K-map.

c) Draw logic gate using AND and OR gates.

Date: 7/6/2022

Ans.	M	C	S	Y	I
------	---	---	---	---	---

0. 0 0 0 0 1

Canonical representation

1. 0 0 0 1 1

$$= m_0 + m_1 + m_2 + m_3 + m_7 + m_8$$

2. 0 0 1 0 1

$$+ m_{10} + m_{11} + m_{12} + m_{14} + m_{15}$$

3. 0 0 1 1 1

$$= \Sigma(0, 1, 2, 3, 7, 8, 10, 11, 12, 14, 15)$$

4. 0 1 0 0 0

5. 0 1 0 1 0

~~SY SY'~~ ~~SY~~ ~~SY~~ ~~SY~~ ~~SY'~~

6. 0 1 1 0 0

~~M'P~~ ~~Y'~~ ~~Y'~~ ~~Y'~~ ~~Y'~~ ~~Y'~~

7. 0 1 1 1 1

~~1~~ ~~1~~ ~~1~~ ~~1~~ ~~1~~ ~~1~~

8. 1 0 0 0 1

~~M'C'P~~ ~~1~~ ~~0~~ ~~1~~ ~~3~~ ~~2~~

9. 1 0 0 1 1

~~M'C'~~ ~~1~~ ~~0~~ ~~1~~ ~~6~~ ~~3~~

10. 1 0 1 0 1

~~M'C'~~ ~~4~~ ~~5~~ ~~1~~ ~~7~~ ~~6~~

11. 1 0 1 1 1

~~M'C'~~ ~~1~~ ~~1~~ ~~1~~ ~~1~~ ~~2~~

12. 1 1 0 0 1

~~M'C'~~ ~~12~~ ~~13~~ ~~25~~ ~~24~~ ~~1~~

13. 1 1 0 1 1

~~M'C'~~ ~~1~~ ~~2~~ ~~1~~ ~~1~~ ~~1~~

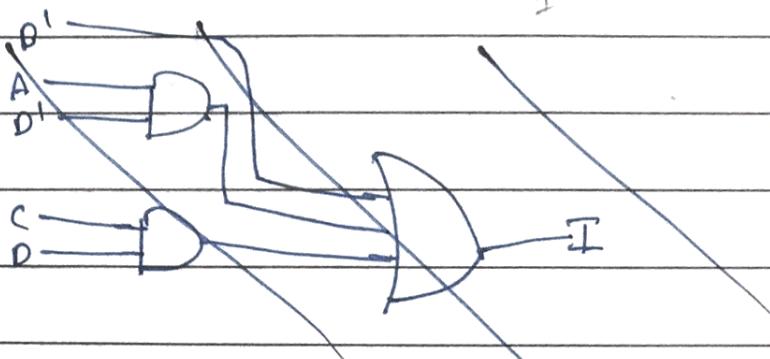
14. 1 1 1 0 1

~~M'C'~~ ~~8~~ ~~9~~ ~~21~~ ~~40~~ ~~1~~

15. 1 1 1 1 1

~~1~~ ~~1~~ ~~1~~ ~~1~~ ~~1~~

1. Octet - B'



2. Quart - AD'

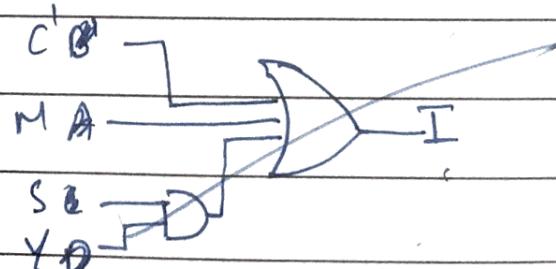
3. Quart - CD

$$I(ABCD) = B' + AD' + CD$$

1. Octet - C'

2. Octet - M

3. Quart - ~~SY~~ ~~SY'~~ SY



$$I(ABCD) = \cancel{B'} + \cancel{A} + \cancel{C} + \cancel{D}$$

$$C' + M + SY$$

42. A committee has three general members and a group head, Mr. Amazing. The general members are Ms. ~~Amazing~~^{Creative}, Mr. Big and Ms. Dynamic. A motion passes when either
- The group head and at least one member votes yes
 - All three general members vote yes

Inputs -

A - Mr. Amazing vote

C - Ms. Creative vote

B - Mr. Big vote

D - Ms. Dynamic vote

Output -

M - Whether the motion passed (1 is yes, 0 is no)

Draw a truth table for the expression $M(A, B, C, D)$.

Reduce it using K-map. Draw logic gate

Assume that their variables and components are available as inputs. Use

Use AND and OR gates to draw the logic gate.

Date : _____

~~Ans.~~ A B C D M

0. 0 0 0 0 0

1. 0 0 0 1 0

2. 0 0 1 0 0

3. 0 0 1 1 0

4. 0 1 0 0 0

5. 0 1 0 1 0

6. 0 1 1 0 0

7. 0 1 1 1 1

8. 1 0 0 0 10

9. 1 0 0 1 1

10. 1 0 1 0 1

11. 1 0 1 1 1

12. 1 1 0 0 1

13. 1 1 0 1 1

14. 1 1 1 0 1

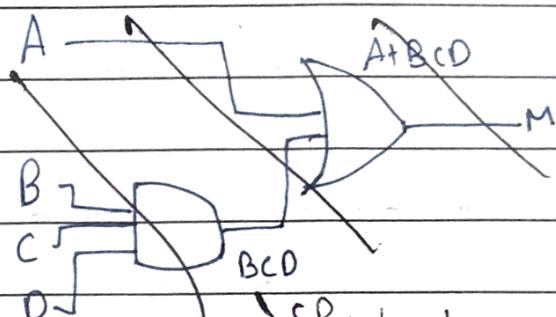
15. 1 1 1 1 1

	AB	CD	C'D'	CD'	CD'
	A'B'	0	1	3	2
	A'B	4	5	1	6
	AB	1	1	1	1
	AB'	1	1	1	1

1. Octet - A

2. Pair - BCD

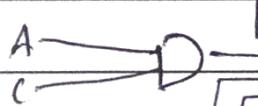
$$\therefore M(A, B, C, D) = A + BCD$$



	AB	CD	C'D'	CD'	CD'
	A'B'	0	1	3	2
	A'B	4	5	1	6
	AB	1	1	1	1
	AB'	1	1	1	1



1. Quart - $A^3 B$



2. Quart - AC



3. Quart - AD



4. Pair - BCD

$$\therefore M(A, B, C, D) = AB + AC + AD + BCD$$

4a.

The Panchayat Union of a particular village consists of 4 members - 1 president and 3 officers. Any decision taken can be implemented if and only if either

- The President and at least 1 officer votes yes
- All officers vote yes

Inputs -

A - President's vote, 1 is Yes, 0 is no

B - 1st officer's vote, 1 is yes, 0 is no

C - 2nd officer's vote, 1 is yes, 0 is no

D - 3rd officer's vote, 1 is yes, 0 is no

Output:-

X - Whether the plan can be implemented [1 is yes, 0 is no]

Draw a truth table for the above expression and write SOP expression. Reduce $X(A, B, C, D)$ using K-map.

~~Draw logic gate diagram for the reduced SOP~~

expression using AND and OR gates. Assume that variables and their complements are available as input.

Date : _____

	A	B	C	D	X
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	1
10	1	0	1	0	1
11	1	0	1	1	1
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	1

$\bar{A}\bar{B}$ $\bar{C}\bar{D}$ $\bar{C}D$ $C\bar{D}$ CD

$\bar{A}\bar{B}$	0	1	3	2
$\bar{A}B$	4	5	7	6
$A\bar{B}$	2	1	12	1
AB	11	13	15	14
$\bar{A}B'$	8	1	2	10

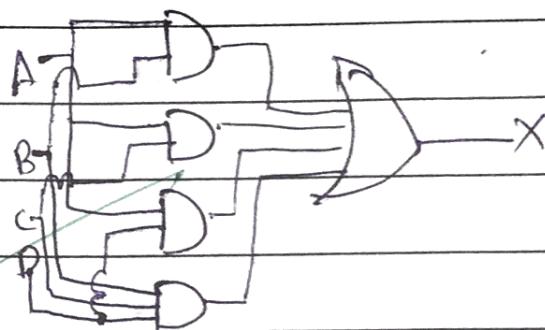
1. Quart - AB

2. Quart - AC

3. Quart - AD

4. Pair - BCD

$$\therefore X(A,B,C,D) = AB + AC + AD + BCD$$



51.

The immigration rules of a ~~company~~ country allows the issue of work-visa permits to an applicant only if the applicant satisfies any one of the following conditions.

- The spouse of the applicant is a permanent resident of that country having lived there for at least 5 years.

OR

- The applicant possesses certain special skills in the Skill requirement list of the country and is sponsored by a permanent resident of that country.

The inputs are:

- A - the spouse has permanent residence status
- B - The spouse has lived in the country for 5 or more years
- C - The applicant possesses the required special skills
- D - Sponsorship by a permanent resident

Output

X - Denotes eligibility for permit issue

Draw the truth table for the inputs and outputs given above and write SOP expression for $X(A,B,C,D)$. Reduce $X(A,B,C,D)$

using K-map. Draw logic gate diagram using AND and OR gates. Assume that variables and their complements are available as inputs.

Date : _____

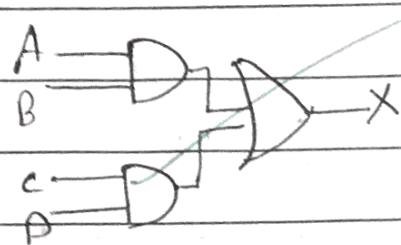
	A	B	C	D	X
0	0	0	0	0	0
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	1
4	0	1	0	0	0
5	0	1	0	1	0
6	0	1	1	0	0
7	0	1	1	1	1
8	1	0	0	0	0
9	1	0	0	1	0
10	1	0	1	0	0
11	1	0	1	1	1
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	1

AB	C^0	$C'D'$	CD	CD	$C'D$
$A'B'$	0	1	1	1	2
$A'B$	1	5	1	1	6
AB	3	1	1	1	1
AB'	8	9	1	1	2

1. Quart - AB

2. Quart - CD

$$\therefore X(A,B,C,D) = AB + CD.$$



42.

A committee has 3 general members and a head, Mr. Amazing. The general members are Mr. Big, Ms. Creative and Ms. Dynamit. A motion passes only when

- The group head and at least one general member votes
- All three general members vote

Inputs :-

- A - Mr. Amazing's vote
- B - Mr. Big's vote
- C - Ms. Creative's vote
- D - Ms. Dynamit's vote

Output - M - Whether the motion passes

1 is yes, 0 is no

Draw the truth table for the inputs and output given above. Write SOP expression. Reduce it using K-map. Draw logic gate using AND / OR gates. Variable and complements are available as input

A	B	C	D	M	
0	0	0	0	0	AB'CD'
1	0	0	0	1	0
2	0	0	1	0	0
3	0	0	1	1	0
4	0	1	0	0	A'B
5	0	1	0	1	0
6	0	1	1	0	AB
7	0	1	1	1	AB'
8	1	0	0	0	
9	1	0	0	1	
10	1	0	1	0	1
11	1	0	1	1	1
12	1	1	0	0	1
13	1	1	0	1	1
14	1	1	1	0	1
15	1	1	1	1	1

Canonical expression = $\Sigma (7, 9, 10, 11, 12, 13, 14, 15)$

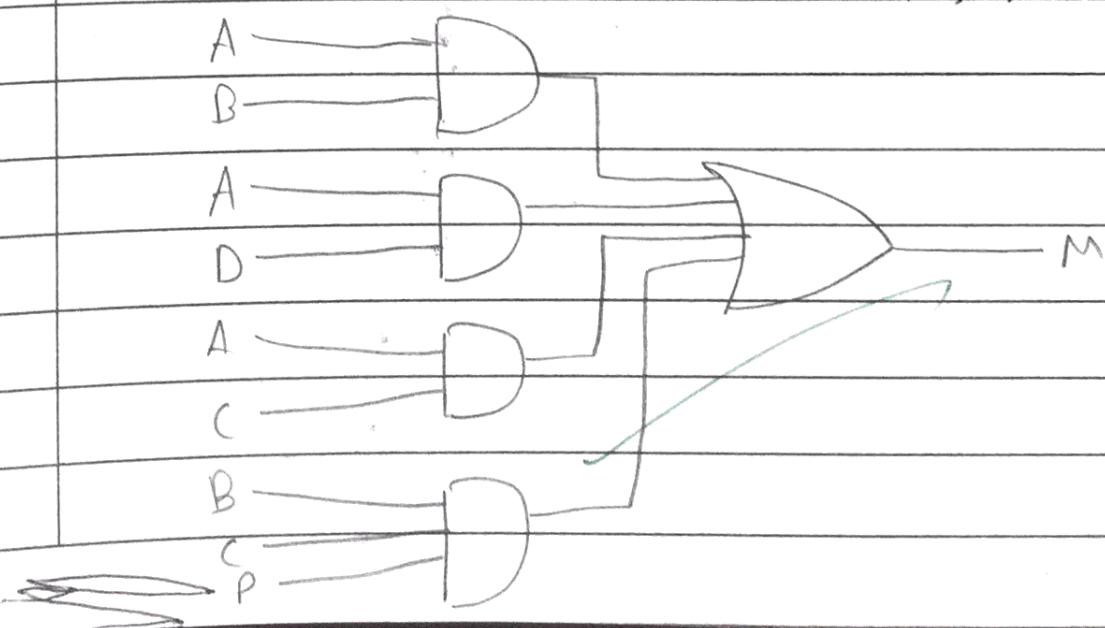
1. Quart - AB

2. Quart - AD

3. Quart - AC

4. Pair - BCD

$$\therefore M(A,B,C,D) = AB + AD + AC + BC(D)$$



Date: 8/6/2022

AQ

51. The immigration rules of a country allow issuing permits to an applicant only if at least 1 condition is satisfied.

- Spouse must be a permanent resident, and lived there for 5 years.

- Applicant must have special skills and must be sponsored by permanent resident.

Inputs -

A - Spouse is a permanent resident

B - Spouse has lived in country for 5+ years

C - Applicant possesses the required special skills

D - Applicant is sponsored by a permanent resident

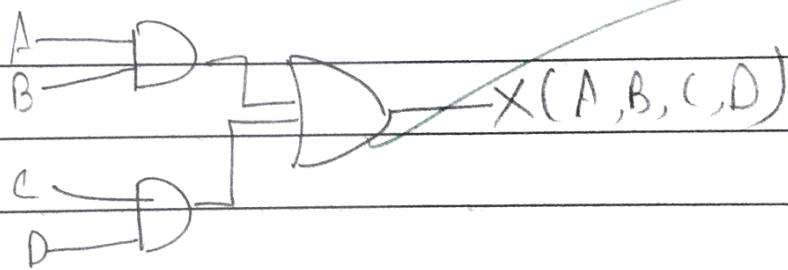
Output - X - Whether permit is issued, 1 is yes, 0 is no

Draw truth table for the inputs and output's above.

Write SOP expression for $X(A, B, C, D)$. Draw logic gate diagram. Variables and complements are available as inputs.

Date : 8/6/2022

	A	B	C	D	X					
0	0	0	0	0	0	AB	CD			
1	0	0	0	1	0	AB	CD	CD	CD	CD
2	0	0	1	0	0					
3	0	0	1	1	1	A'B'				
4	0	1	0	0	0	A'B				
5	0	1	0	1	0					
6	0	1	1	0	0	AB				
7	0	1	1	1	1					
8	1	0	0	0	0	AB'				
9	1	0	0	1	0					
10	1	0	1	0	0	1. Quart - AB				
11	1	0	1	1	1	2. Quart - (D)				
12	1	1	0	0	1	X(A, B, C, D) = AB + CD				
13	1	1	0	1	1	Canonical representation = E(3, 7, 11, 12)				
14	1	1	1	0	1	(13, 14, 15)				
15	1	1	1	1	1					



Date : _____

52. The main safe in a bank can be opened by a 3 part password. Parts are held by Chairman, Regional manager, Bank manager and Head cashier.

To open the safe

- The password of chairman and that of two officials

- Password of all three officials must be entered

Input -

A - Chairman's password

B - Regional manager's password

C - Bank Manager's password

D - Head cashier password

Output - X - Whether safe can be opened

Draw truth table.

Write SOP expression

Reduce $X(A, B, C, D)$ using k-map

Draw logic gate. Variables and complements are available as inputs

Date: _____

	A	B	C	D	X	AB	$C'D$	$C'D'$	$C'D$	CD	CD'
0	0	0	0	0	0	$A'B'$	0	1	3	2	
1	0	0	0	0	1	$A'B$	4	5	1	6	
2	0	0	1	0	0	AB	12	13	1	10	1
3	0	0	1	1	0	AB'	8	9	1	11	10

4 0 1 0 0 0 1. Pair - ABC

5 0 1 0 1 0 2. Pair - BCD

6 0 1 1 0 0 3. Pair - ACD

7 0 1 1 1 1 4. Pair - ABD

$$8 \ 1 \ 0 \ 0 \ 0 \ 0 \therefore X(A, B, C, D) = ABC + BCD + ACD + ABD$$

9 1 0 0 1 0

10 1 0 1 0 0

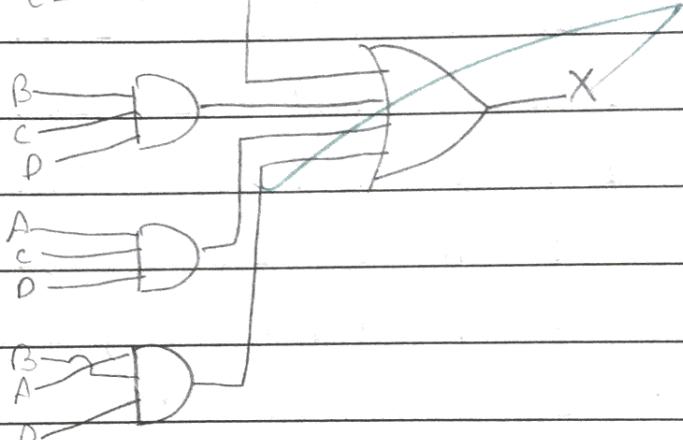
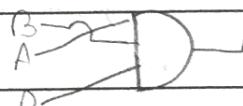
11 1 0 1 1 1

12 1 1 0 0 0

13 1 1 0 1 1

14 1 1 1 0 1

15 1 1 1 1 1



70. A Government Institution awards a medal to a person who -

- i) ~~is~~^{is} an Indian citizen, lost their life in war,
had not completed 25 years of service
- ii) is an Indian citizen, ~~not~~^{lost} their life in war,
completed 25 years of service
- iii) is not an Indian citizen but has taken part
in activities for nation upliftment

Inputs -

- A - Indian citizen
- B - More than 25 years service
- C - Lost their life in war
- D - Uplifted the nation

Output - whether they are eligible for the medal

Draw the truth table. Write POS expression.

Reduce it using K-map. Draw logic gate.

Variables and their complements are available as inputs.

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	A	B	C	D	X	AB	AB'	A'B	AB - AB'	
0	0	0	0	0	0					
1	0	0	0	1	1					
2	0	0	1	0	0					
3	0	0	1	1	1					
4	0	1	0	0	0					
5	0	1	0	1	1	$A+B$	$A+B'$	$A+B'$	$A+B'$	
6	0	1	1	0	0					
7	0	1	1	1	1					
8	1	0	0	0	0					
9	1	0	0	1	0					
10	1	0	1	0	1	$A+B$	$C+D$	$C+D'$	$C'+D'$	$C'+D$
11	1	0	1	1	1	$A+B$	$\textcircled{0}_b$	1	3	$\textcircled{0}_2$
12	1	1	0	0	1	$A+B'$	$\textcircled{0}_4$	5	7	$\textcircled{0}_6$
13	1	1	0	1	1	$A+B'$	$\textcircled{0}_{12}$	$\textcircled{0}_{13}$	$\textcircled{0}_{15}$	$\textcircled{0}_{14}$
14	1	1	1	0	0	$A'+B$	$\textcircled{0}_8$	$\textcircled{0}_9$	11	10
15	1	1	1	1	0	1. Quart - $\textcircled{0}_1$ ($A+D$)				

2. Pair - $(A'+B'+C')$

3. Parr - $(A'+B+C)$

$$X = (A+D)(A'+B'+C)(A'+B+C)$$

5. Find the complement of

$$a) (A+B) \cdot (B+C) \cdot (A+C)$$

$$= [(A+B)(B+C)(A+C)]'$$

$$= [(\overline{A}+B)(\overline{B}+C)]' + (\overline{A}+C)'$$

$$= (\overline{A}+B)' + (\overline{B}+C)' + (\overline{A}+C)'$$

$$= \overline{A} \cdot \overline{B} \cdot \overline{C} + \overline{B} \cdot \overline{C} + \overline{A} \cdot \overline{C}$$

$$b) A \cdot B + (\overline{A} \cdot \overline{B}) \cdot (B \cdot C + \overline{B} \cdot \overline{C}) = X$$

A	B	C	X	X'	
0	0	0	0	1	$X' = A \cdot \overline{B} \cdot \overline{C}$
1	0	0	1	0	A'
2	0	1	0	0	A
3	0	1	1	0	1. $A' \cdot B$
4	1	0	0	0	2. $A' \cdot C$
5	1	0	1	0	3. $B' \cdot C$
6	1	1	0	1	4. $A \cdot B'$
7	1	1	1	1	$\therefore X' = A' \cdot B + A' \cdot C + B' \cdot C + A \cdot B'$

$$= \overline{A} \cdot \overline{B} + (\overline{A} \cdot \overline{B}) \cdot (B \cdot C + \overline{B} \cdot \overline{C})$$

$$= (\overline{A} \cdot \overline{B}) \cdot [(\overline{A} \cdot \overline{B}) \cdot (B \cdot C + \overline{B} \cdot \overline{C})]$$

$$= (\overline{A} + \overline{B}) \cdot (\overline{A} \cdot \overline{B} + B \cdot C + \overline{B} \cdot \overline{C})$$

$$= (\overline{A} + \overline{B}) \cdot (A + B) \cdot (\overline{B} + \overline{C}) \cdot (B + C)$$

$$= (\overline{A} + \overline{B}) \cdot (A + B) \cdot (\overline{B} + \overline{C}) \cdot (B + C)$$

7. Reduced :-

$$a) F(a,b,c,d) = \sum (1, 2, 3, 11, 12, 14, 15)$$

	$a'b'c'd'$	$a'b'c'd$	$a'b'cd'$	$a'b'cd$	$a'bcd'$	$a'bcd$	$a'b'c'd'$
$a'b'$	1	2	3	4	5	6	7
$a'b$	8	9	10	11	12	13	14
$a'b'c$							
$a'b'cd$							

1. Pair - $a'b'd$ 2. Pair - $a'b'c$ 3. Pair - $ab'd'$ 4. Pair - ~~$ab'c$~~ acd 5. Pair - ~~$b'cd$~~

$$F(a,b,c,d) = a'b'd + a'b'c + abd' + \cancel{ab'c} + \cancel{b'cd}$$

$$b) F(A,B,C,D) = \sum (0, 1, 2, 3, 12, 13, 14, 15)$$

1. Quart - $A'B'$ 2. Quart - AB

$$\therefore F(A,B,C,D) = A'B' + AB$$

	$A'B'c'd'$	$A'B'c'd$	$A'B'cd'$	$A'B'cd$	$A'Bcd'$	$A'Bcd$	$A'B'c'd'$
$A'B'$	1	2	3	4	5	6	7
AB	8	9	10	11	12	13	14
AB'							
$A'B'c$							
$A'B'cd$							

$$c) F(a,b,c,d) = \sum (0, 1, 4, 5, 6, 8, 9, 12, 13, 14)$$

1. Octet - C' 2. Quart - BD' 3. Quart - $A'D'$

$$\therefore F(A,B,C,D) = C' + BD' + A'D'$$

	$A'B'c'd'$	$A'B'c'd$	$A'B'cd'$	$A'B'cd$	$A'Bcd'$	$A'Bcd$	$A'B'c'd'$
$A'B'$	1	2	3	4	5	6	7
AB	8	9	10	11	12	13	14
AB'							
$A'B'c$							
$A'B'cd$							

8. Write max term expression

	A	B	C	D	$A+B+C+D$	$C+D$	$C+D'$	$C'+D$	$C'+D'$	
0	0	0	0	0	1	0	1	3	0 ₂	
1	1	0	0	1	0	1	5	7	6	
2	0	1	0	0	1	12	13	15	14	
3	0	1	1	1	1	8	9	11	10	
4	1	0	0	0	1	2	3	4	5	
5	1	0	1	1	2	0	0	1	3	0 ₂
7	1	1	0	1	1	4	1	5	7	6
6	1	1	1	1	1	1.	$B+C$			

2. $A+C$

3. $A+B$

$$= (B+C)(A+C)(A+B)$$

9. M is a function such that it is 0 if there are more 0s or 1 if there are more 1s. Draw SOP expression k-map and find expression

	A	B	C	M
0	0	0	0	0
1	0	0	1	0
2	0	1	0	0
3	0	1	1	1
4	1	0	0	0
5	1	0	1	1
6	1	1	0	1
7	1	1	1	1

	$B'C$	$B'C'$	$B'C$	$BC'*$	BC'
A'	0	1	3	2	
A	4	5	7	6	

$$M = AC + BC + AB$$

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12. Write POS expression

	A	B	C	F	A	$\bar{B} + C$	$B + C$	$B + C'$	$B' + C$	$B' + C'$
0	0	0	0	0	A	0	1	3	0	2
1	0	0	1	1	A'	0	1	5	7	0
2	0	1	0	0						
3	0	1	1	1						
4	1	0	0	0						
5	1	0	1	1						
6	1	1	0	0						
7	1	1	1	1						

$$F(A, B, C) = C$$

13. Prove

a) ~~LHS~~ RHS = $(A \cdot \bar{B} + B \cdot \bar{A})$

$$LHS = (A + B) \cdot (A \cdot B)$$

~~$A \bar{B}(A + B)$~~

	A	B	$(A+B)$	$A \cdot B$	$\bar{A} \bar{B}$	$\bar{B} \bar{A}$	LHS	RHS
0	0	0	0	0	0	0	0	0
1	1	0	1	0	0	1	0	1
2	0	1	1	0	1	0	0	1
3	1	1	1	1	0	0	1	0

$$LHS = RHS$$

-By

$$3) (A+B)' + (A+\bar{B})' = \bar{A}$$

RHS = ~~A~~ A'

$$\begin{aligned}
 \text{LHS} &= (A+B)' + (A+\bar{B})' \\
 &= ((A+B)' + (A+B'))' \quad (\text{Involution}) \\
 &= (A+B' + A+B)' \\
 &= \cancel{B}(A+B' + B)' \quad (1\text{-dempotent}) \\
 &\cancel{=} \cancel{A+B}(A+1)' \quad (\text{Complementary}) \\
 &= A' \quad (\text{Property of } 1)
 \end{aligned}$$

14.

i) Find complement of $XY'Z + XY + YZ'$

$$\begin{aligned}
 &= (XY'Z + XY + YZ')' \\
 &= (XY'Z)' \cdot (XY)' \cdot (YZ')' \\
 &= (X'+Y+Z) \cdot (X'+Y') \cdot (Y'+Z') \quad (\text{De Morgan's}) \\
 &= (X'+Y+Z)(X'Y' + Y'Y' + X'Z' + Y'Z') \quad (\text{Distributive}) \\
 &= (X'+Y+Z)(X'Y' + Y' + X'Z' + YZ') \quad (1\text{-dempotent}) \\
 &= X'X'Y' + X'Y'Y + X'Y'Z + Y'X' + Y'Y + Y'Z + X'Z'X' \\
 &\quad + X'Z'Y + X'Z'Z + X'YZ' + YYZ' + ZY'Z \\
 &= X'Y' + 0 + X'Y'Z + X'Y' + 0 + Y'Z + X'Z' \\
 &\quad + X'YZ' + X' + X'YZ' + YZ' + 0 \\
 &= X'Y' + X'Y'Z + Y'Z + X'Z' + X'YZ' + X' + YZ' \\
 &= X'Y' + X'Y' + Y'Z + X'Z + X'YZ' + X' + YZ' \\
 &= X'Y' + Y'Z + X'Z + YZ' + X' \\
 &= \cancel{\cancel{X}} + X' + Y'Z + YZ'
 \end{aligned}$$

13. Prove the following

~~a) $(A+B)(A \cdot B) = A \cdot \bar{B} + B \cdot \bar{A}$~~

~~$RHS = AB' + BA'$~~

~~$LHS = (A+B) \cdot AB$~~

~~e) $(A+C) \cdot (A' + B + C) = (A+C) \cdot (B+C)$~~

~~$RHS = (A+C) \cdot (B+C)$~~

~~$LHS = (A+C) \cdot (A' + B + C)$~~

~~$= A(A' + B + C) + C(A' + B + C) \quad (\text{Distributive})$~~

~~$= AA' + AB + AC + A'C + BC + CC \quad (\text{Distributive})$~~

~~$= 0 + AB + AC + A'C + BC + CC \quad (\text{Complementary})$~~

~~$= AB + AC + A'C + BC + C \quad (\text{Idempotent})$~~

~~$= AB + C(A+A') + BC + C \quad (\text{Distributive})$~~

~~$= AB + C \cdot 1 + BC + C \quad (\text{Complementary})$~~

~~$= AB + BC + C \quad (\text{Idempotent})$~~

~~$= AB + BC + AC + C \quad (\text{Absorption})$~~

~~$= AB + BC + AC + CC \quad (\text{Idempotent})$~~

~~$= B(A+C) + C(A+C) \quad (\text{Distributive})$~~

~~$= (B+C)(A+C) \quad (\text{Distributive})$~~

~~$= RHS$~~

Hence Proven

14.

Convert the following expression into POS form

$$F(A, B) = (A+B) \cdot A'$$

	A	B	F
0	0	0	0
1	0	1	1
2	1	0	0
3	1	1	0

$$\begin{aligned} \text{POS form} &= \overline{\Pi}(M_0, M_2, M_3) = \overline{\Pi}(0, 2, 3) \\ &= (A+B) \cdot (A'+B) \cdot (A'+B') \end{aligned}$$

A	B	B'
A	0	0
A'	0	1

$$= A' B (A') (B)$$

15.

Minimize using K-map

$$F(A, B, C) = A'B'C' + A'B'C + A'BC' + ABC$$

	A	B	C	F
0	0	0	0	0
1	0	0	1	0
2	0	1	0	1
3	0	1	1	1
4	1	0	0	0
5	1	0	1	0
6	1	1	0	1
7	1	1	1	1

A	B	C	B'C'	B'C	BC	BC'
A'	0	1	1	1	1	0
A	1	0	1	1	0	1

Quart - B

$$\therefore F(A, B, C) = B$$

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16. Reduce $F(A, B, C, D) = (A' + C)(A' + C')(A' + B + C' + D)$

$$\begin{aligned}
 &= (A' + C)(A' + C')(A' + B + C' + D) \\
 &= A' \cancel{(A' + C)(A' + C' + A'B + BC' + A'C' + C'C + A'D + C'D)} \\
 &= (A' + C)(A' + A'C + A'B + A'C' + A'D + BC' + C' + C'D) \\
 &= (A' + C)(A' + C') \quad (\text{Absorption}) \\
 &= \cancel{A'A' + A'C + A'C' + CC'} \\
 &= A' + A' + 0 \\
 &= A'
 \end{aligned}$$

17. State the principle of duality. Write the dual of
 $(P + Q') \cdot R \cdot I = P \cdot R + Q' \cdot R$

Replacing each plus with dot and each dot with $\cancel{P^{plus}}$ and each zero with one and

each one with zero without changing the variables gives
 an equivalent expression on both sides.

$$PQ' + R + 0 \cancel{\Rightarrow (P+R)(Q' + R)}$$

18. Minimize $(A' + B')(B + CD)'$

$$\begin{aligned}
 &= (A' + B')(B'CCD)' = (A' + B')(B'C' + B'D') \\
 &= A'B'C' + B'B'C' + A'B'D' + B'B'D' \\
 &= A'B'C' + B'C' + A'B'D' + B'D' \\
 &= B'C' + B'D'
 \end{aligned}$$

19. Convert $F(P, Q, R) = \bar{P}(\bar{Q}, \bar{R})$ from cardinal form to canonical form

~~$$(P + Q + R) \cdot (P + Q' + R')$$~~

Input			Output	
	x	y	B	D
0 0	0	0	0	0
0 1	0	1	1	1
1 0	1	0	0	1
1 1	1	1	0	0

- i) Write SOP expression for D
- ii) Write POS expression for B

i)

x	y	y'	y
x'		0	1
xy	1	2	3

$D = xy' + x'y$

ii)

x	y	y'
x'	0 0	1
xy	0 0	0

Pair - y
Pair - x'

$$B = (y) \cdot (x')$$

21. $F(P, Q, R)$ is true if there are odd numbers of 0s.

Draw truth table

Derive canonical expression

Find the complement and check whether it is equivalent to its POS form or not.

P Q R F

0 0 0 0 1

1 0 0 1 0

2 0 1 0 0

3 0 1 1 1

4 1 0 0 0

5 1 0 1 1

6 1 1 0 1

7 1 1 1 0

$F(P, Q, R) = P'Q'R' + PQ'R + P'QR + PQR'$

P	Q	R	$P'Q'R'$	$PQ'R$	$P'QR$	PQR
0	0	0	1	0	0	0
1	0	0	0	1	1	0
2	0	1	0	0	1	0
3	0	1	1	0	0	1

$$F' = (P'Q'R')' + (PQ'R)' + (P'QR)' + (PQR)'$$

$$= (P'Q'R')' \cdot (PQ'R)' \cdot (P'QR)' \cdot (PQR)'$$

$$= (P+Q+R) \cdot (P'+Q+R') \cdot (P+Q'+R) \cdot (P'+Q'+R)$$

P	Q	R	$P+Q+R$	$P'+Q+R'$	$Q+R$	$Q'+R'$	$Q'+R$	$P+Q+R'$
0	0	0	0	1	0	1	0	0
1	0	0	1	0	1	0	1	1

$$F(P, Q, R) = (P+Q'+R) \cdot (P'+Q+R) \cdot (P'+Q'+R') \cdot (P+Q+R')$$

∴ POS expression is not the same as complement of SOP

expression. POS expression can be obtained by complementing each variable in the POS form of complement of SOP expression.

23. Given $F(X, Y, Z) = (X' + Y')(Y + Z')$

Write in canonical POS form

$$(X' + Y' + Z) \cdot (X' + Y' + Z') \cdot (X + Y + Z')$$

$$A=1, B=0, C=1, D=1$$

Find Maxterm and Minterm

$$\text{Maxterm} = (A' + B + C' + D')$$

$$\text{Minterm} = AB'CD$$

25. RHS = $(X' + Y')Z'$

$$\text{LHS} = X'Y'Z' + X'Y'Z' + X'YZ' \quad 30. \text{ Convert } F(L, M, O, P) = \overline{IJ}(0, 2, 8, 20)$$

$$+ X'YZ' + XY'Z' + XY'Z' + XY'Z'$$

Prove RHS = LHS

$$= X'Y'Z' + X'Y'Z' + X'YZ'$$

~~+ X'YZ'~~ ~~+ XY'Z'~~ $X'Z' + XY'Z'$

$$+ XY'Z'$$

$$= X'Y'Z' + X'Y'Z' + XY'Z'$$

~~(Absorption)~~ ~~(Idempotent)~~

=

~~$X'Z' + XY'Z'$ (Absorption)~~

$$= X'Y'Z' + X'YZ' + XY'Z'$$

$$= Y'Z' + X'YZ' \quad (\text{Absorption})$$

$$= Z'(Y' + X'Y) \quad (\text{Distribution})$$

$$= Z'(Y' + X')(Y' + Y) \quad (\text{Distribution})$$

$$= Z'(Y' + X') \quad (\text{Complementarity})$$

$$F(L, M, O, P) = (L + M + O + P) \cdot (L' + M + O' + P)$$

$$\cdot (L' + M + O + P) \cdot (L' + M + O' + P)$$

~~cancel~~

$$= ((L + M + P) + O) \cdot ((L + M + P) + O')$$

$$(L' + M + O + P) (L' + M + O' + P)$$

$$= (L + M + P) \cdot (L + M + P)$$

$$\text{Let } L + M + P = x$$

$$= (x + O)(x + O') (L' + M + O + P) (L' + M + O' + P)$$

$$= (x \cdot x + O \cdot x + O' \cdot x + O \cdot O') (L' + M + O + P) (L' + M + O' + P)$$

$$= (x + x + O) (L' + M + O + P) (L' + M + O' + P)$$

$$= x (L' + M + O + P) (L' + M + O' + P)$$

$$\text{Let } L' + M + P = y$$

$$= x(y + O)(y + O')$$

$$= x(y + OO') \quad (\text{Commutative or Distributive})$$

$$= x(y + O) = xy \quad (\text{Identity})$$

$$= (L + M + P) (L' + M + P) \quad (\text{Undo substitution})$$

$$= (L + (M + P)) (L' + (M + P)) \quad (\text{Associativity})$$

$$= (M+P) + LL' \quad (\text{Complement of Distributive})$$

$$= M+P$$

	y^2	$y+2$	$y+z'$	$y+z$	$y+2$
x	0	0	1	0	0
x'	1	1	0	1	0

$$= (x+y')(x+z)$$

∴ \star

∴ The expressions are equal

34. $F(x, y, z) = \Sigma(1, 4, 5, 6, 7)$

Prove that $F(x, y, z) = T(0, 2, 3)$

~~$$F(x, y, z) = x'y'z + xyz' + xy'z + x'yz + x'y'z + xyz$$~~

~~$$= x'y'z + xyz + xy'z' + xyz' + xyz + xyz' + xyz (Commutative)$$~~

~~$$= y'z + xz' + xyz (Absorption)$$~~

~~$$= y'z + x(z' + yz) (Distribution)$$~~

~~$$= y'z + x(z' + y)(z' + y) (Distributive)$$~~

~~$$= y'z + x(z' + y) (Property of 1)$$~~

~~$$= y'z + xz' + xy$$~~

~~$$\therefore \Sigma(0, 2, 3) = x'y'z' + x'yz$$~~

~~$$+ x'y'z$$~~

~~$$= x'z' + x'y'z$$~~

~~good~~ ~~don't~~ ~~divide~~

~~Please~~ ~~the~~ ~~page~~

	y^2	yz'	$y'z$	yz	y^2
x'	0	1	1	0	0
x	1	1	0	1	0

$$= x + y'z'$$

$$= (x+y')(x+z) \quad (\text{Distribution})$$