Výsledky

1-1

1. $x_1 = 0$, $x_2 = 2$, $x_3 = -4$, $x_4 = 7$,
2. $x_{1,2} = 2$, $x_{3,4,5} = -3$,
3. $x_1 = 5$, $x_{2,3} = -3$, $x_{4,5} = -1 \pm i$,
4. $x_1 = 2$, $x_{2,3} = -1 \pm \sqrt{3}i$, $x_4 = -5$, $x_{5,6} = \frac{5}{2} \pm \frac{5\sqrt{3}}{2}i$,
5. $x_1 = 2$, $x_2 = -2$, $x_3 = 2i$, $x_4 = -2i$, $x_{5,6} = \sqrt{2} \pm \sqrt{2}i$, $x_{7,8} = -\sqrt{2} \pm \sqrt{2}i$,
6. $x_{1,2} = \frac{\sqrt{2+\sqrt{2}}}{2} \pm \frac{\sqrt{2-\sqrt{2}}}{2}i$, $x_{3,4} = -\frac{\sqrt{2+\sqrt{2}}}{2} \pm \frac{\sqrt{2-\sqrt{2}}}{2}i$, $x_{5,6} = \frac{\sqrt{2-\sqrt{2}}}{2} \pm \frac{\sqrt{2+\sqrt{2}}}{2}i$, $x_{7,8} = -\frac{\sqrt{2-\sqrt{2}}}{2} \pm \frac{\sqrt{2+\sqrt{2}}}{2}i$,
7. $x_1 = 2$, $x_2 = -2$, $x_{3,4} = 5$, $x_5 = -6$,
8. $x_1 = -3$, $x_2 = 7$, $x_{3,4} = i$, $x_{5,6} = -i$,
9. $x_{1,2} = -1$, $x_3 = 3$, $x_4 = -5$, $x_{5,6} = 4$.

1-2

1. $p(x) = 4x^5 + 16x^4 - 96x^3 - 184x^2 + 860x - 600$,
2. $p(x) = 2x^5 - 14x^4 + 2x^3 + 58x^2 - 400x - 728$,
3. $p(x) = x^6 + 5x^5 + 6x^4 - 8x^3 - 36x^2 - 44x - 24$,
4. $p(x) = x^5 - (6 + 2\sqrt{2})x^4 + (15 + 12\sqrt{2})x^3 - (26 + 24\sqrt{2})x^2 + (36 + 16\sqrt{2})x - 24$.

1-3

1. $p(x) = (x + 3)(x - 2)(x + 1)$,
2. $p(x) = (x - 2)(x - 1)^2(x^2 + 3) = (x - 2)(x - 1)^2(x - \sqrt{3}i)(x + \sqrt{3}i)$,
3. $3(x - 1)(x + 1)(x^2 + 1) = 3(x - 1)(x + 1)(x - i)(x + i)$,
4. $(x^2 + 4)(x^2 - 2\sqrt{3}x + 4)(x^2 + 2\sqrt{3}x + 4) = (x - 2i)(x + 2i)(x - \sqrt{3} - i)(x - \sqrt{3} + i)(x + \sqrt{3} - i)(x + \sqrt{3} + i)$,
5. $(x^2 + 3)(x^2 + 2) = (x - \sqrt{3}i)(x + \sqrt{3}i)(x - \sqrt{2}i)(x + \sqrt{2}i)$,
6. $(x + 3)(x - 2)^3(x^2 + 3) = (x + 3)(x - 2)^3(x - \sqrt{3}i)(x + \sqrt{3}i)$,
7. $(x - 4)(x + 2)(x - 3)^2(x + 1)^3$,
8. $(x - 2)(x - 3)(x^2 + 2)^2 = (x - 2)(x - 3)(x - \sqrt{2}i)^2(x + \sqrt{2}i)^2$,
9. $(x + 2)^2(x - 3)(x^2 + 3x + 3) = (x + 2)^2(x - 3)(x + \frac{3}{2} - \frac{\sqrt{3}}{2}i)(x + \frac{3}{2} + \frac{\sqrt{3}}{2}i)$,
10. $9(x - 2)^3(x + 1)(x + \frac{2}{3})^2$,
11. $2(x + 3)^2(x + 4)(x - \frac{1}{2})$.

1-4

1. $d(x) = (x-1)(x+2)(x^2+1) = x^4+x^3-x^2+x-2$, $n(x) = (x-1)^2(x+2)^3(x-3)(x-4)(x^2+1)$,
2. $d(x) = 1$, $n(x) = (x + 3)(x - 3)^2(x - 2)^2(x + 2)(x^2 + 2)(x^2 + x + 1)$,

3. $d(x) = x + 2$, $n(x) = (x + 2)(x - 2)(x^2 + 4)(x^2 - 2x + 4)$,
4. $d(x) = x^3 + 2$, $n(x) = (x^3 + 2)(x^3 + 3)(x - 1)^2(x - 4)$,
5. $d(x) = (x - 1)(x^2 + 1) = x^3 - x^2 + x - 1$, $n(x) = (x + 2)(x - 1)^3(x + 3)^2(x^2 + 1)^2$,
6. $d(x) = (x + 2)$, $n(x) = (x + 2)^2(x - 1)(x + 1)^2(x + \frac{1}{3})^2(x + \frac{3}{2})(x + \frac{2}{3})$,
7. $d(x) = (x - 4)(x - \frac{1}{2})$, $n(x) = (x - 4)^2(x + 5)(x + 1)^3(x - \frac{1}{2})^2$,
8. $d(x) = (x + \frac{2}{3})$, $n(x) = (x - 3)(x + 3)(x - 2)^2(x + 2)(x - 4)(x + \frac{2}{3})^2(x - \frac{3}{2})$.

1-5

1. $A_1 = -8$, $p_1(x) = 4x^2 - 8x - 60$ má kořeny $x_1 = -3$, $x_2 = 5$,
   $A_2 = 8$, $p_2(x) = 4x^2 + 8x - 60$ má kořeny $x_1 = 3$, $x_2 = -5$,
2. $A = -30$, $p(x) = x^3 + 6x^2 - x - 30$ má kořeny $x_1 = 2$, $x_2 = -3$, $x_3 = -5$,
3. $A = -29$, $p(x) = x^3 - 5x^2 - 29x + 105$ má kořeny $x_1 = 7$, $x_2 = 3$, $x_3 = -5$,
4. $A_1 = 8$, $p_1(x) = 4x^3 + 8x^2 - 124x + 112$ má kořeny $x_1 = 1$, $x_2 = 4$, $x_3 = -7$,
   $A_2 = -\frac{116}{5}$, $p_2(x) = 4x^3 - \frac{116}{5}x^2 - 124x + 112$ má kořeny $x_1 = \frac{4}{5}$, $x_{2,3} = \frac{5}{2} \pm \frac{\sqrt{165}}{2}$,
5. $A = 48$, $p(x) = x^4 + x^3 - 16x^2 - 4x + 48$ má kořeny $x_1 = 4$, $x_2 = 2$, $x_3 = 3$, $x_4 = -2$,
6. $A = 31$, $p(x) = x^4 - 7x^3 + 5x^2 + 31x - 30$ má kořeny $x_1 = 1$, $x_2 = 5$, $x_3 = 3$, $x_4 = -2$,
7. $A = -13$, $p(x) = x^4 - 4x^3 - 13x^2 + 28x + 60$ má kořeny $x_1 = 3$, $x_2 = 5$, $x_3 = -2$, $x_4 = -2$,
8. $A_1 = -2$, $p_1(x) = x^4 - 2x^3 - 27x^2 + 108$ má kořeny $x_1 = 2$, $x_2 = 6$, $x_3 = -3$, $x_4 = -3$,
   $A_2 = 2$, $p_2(x) = x^4 + 2x^3 - 27x^2 + 108$ má kořeny $x_1 = 2$, $x_2 = 3$, $x_3 = 3$, $x_4 = -6$.

2-1

1. $\mathbf{A} + 2\mathbf{B} = \begin{bmatrix} 1 & 4 & 5 & -6 \\ 1 & 3 & 8 & 4 \end{bmatrix}$, $-3\mathbf{A} + 4\mathbf{B} = \begin{bmatrix} -3 & -2 & 25 & -32 \\ -23 & 1 & 6 & 8 \end{bmatrix}$,
   $\mathbf{A} + \mathbf{D}$ - nelze, $\mathbf{D} - \mathbf{C}$ - nelze.
2. $\mathbf{AB}$ - nelze, $\mathbf{AC} = \begin{bmatrix} -1 & 3 \\ 18 & 24 \end{bmatrix}$, $\mathbf{CA} = \begin{bmatrix} 13 & 8 & -5 & 12 \\ 19 & 2 & 11 & -4 \\ 27 & 9 & 4 & 8 \\ 11 & 4 & 1 & 4 \end{bmatrix}$,
   $\mathbf{BC} = \begin{bmatrix} 2 & 14 \\ 1 & 19 \end{bmatrix}$, $\mathbf{CB} = \begin{bmatrix} -4 & 5 & 18 & -11 \\ -8 & 3 & 8 & 13 \\ -10 & 7 & 23 & 0 \\ -4 & 3 & 10 & -1 \end{bmatrix}$, $\mathbf{AD}$ - nelze,
   $\mathbf{DA} = \begin{bmatrix} -3 & 3 & -8 & 8 \\ 28 & 11 & 1 & 12 \end{bmatrix}$, $\mathbf{BD}$ - nelze, $\mathbf{DB} = \begin{bmatrix} 2 & 1 & 5 & -12 \\ -10 & 8 & 27 & -5 \end{bmatrix}$,
3. $\mathbf{AB}^T = \begin{bmatrix} -30 & -1 \\ 9 & -3 \end{bmatrix}$, $\mathbf{A}^T\mathbf{B} = \begin{bmatrix} -10 & 6 & 19 & 5 \\ -2 & 3 & 11 & -8 \\ -4 & -1 & -6 & 19 \\ 0 & 4 & 16 & -20 \end{bmatrix}$,
   $\mathbf{BA}^T = (\mathbf{AB}^T)^T = \begin{bmatrix} -30 & 9 \\ -1 & -3 \end{bmatrix}$, $\mathbf{B}^T\mathbf{A} = (\mathbf{A}^T\mathbf{B})^T = \begin{bmatrix} -10 & -2 & -4 & 0 \\ 6 & 3 & -1 & 4 \\ 19 & 11 & -6 & 16 \\ 5 & -8 & 19 & -20 \end{bmatrix}$,

$$\mathbf{CD} = \begin{bmatrix} 12 & 7 \\ 10 & 21 \\ 19 & 23 \\ 8 & 9 \end{bmatrix}, \ \mathbf{DC}^T = \begin{bmatrix} 4 & -6 & -1 & 0 \\ 19 & 17 & 31 & 13 \end{bmatrix}, \ \mathbf{A} + (3\mathbf{C})^T = \begin{bmatrix} 10 & -1 & 3 & 7 \\ 11 & 13 & 17 & 6 \end{bmatrix},$$

$$2\mathbf{B}^T - 4\mathbf{C} = \begin{bmatrix} -12 & -12 \\ 6 & -14 \\ 0 & -14 \\ -14 & -4 \end{bmatrix}.$$

2-2

1. $\mathbf{A}^4 = \begin{bmatrix} -196 & 588 \\ -147 & 245 \end{bmatrix}$,
2. $\mathbf{A}^5 = \begin{bmatrix} -41 & 17 & 35 \\ 173 & 357 & 69 \\ 121 & 294 & 11 \end{bmatrix}$,
3. $\mathbf{A}^{53} = \mathbf{A}^3 = \mathbf{0}$,
4. $\mathbf{A}^3 = \begin{bmatrix} 16 & -6 & 9 & 23 \\ 33 & -7 & -23 & -11 \\ -3 & 9 & -9 & -13 \\ -9 & 31 & -2 & -6 \end{bmatrix}$,
5. $\mathbf{A}^{13} = \mathbf{A}^4 = \mathbf{0}$.

2-3

1. $\det \mathbf{A} = 22$,
2. $\det \mathbf{A} = -89$,
3. $\det \mathbf{A} = 0$,
4. $\det \mathbf{A} = -4a - 4b + 20c - 4d$,
5. $\det \mathbf{A} = 48$,
6. $\det \mathbf{A} = -12$.
7. $\det \mathbf{A} = 228$,
8. $\det \mathbf{A} = -16$,
9. $\det \mathbf{A} = 70$,
10. $\det \mathbf{A} = -30$,
11. $\det \mathbf{A} = -21$.

2-4

1. $\det \mathbf{A} = -25$, $\det \mathbf{B} = 25$, $\det \mathbf{C} = -50$, $\det \mathbf{D} = -25$,
2. $\det \mathbf{A} = -2$, $\det \mathbf{B} = -2$, $\det \mathbf{C} = -12$, $\det \mathbf{D} = -2$,
3. $\det \mathbf{A} = -19$, $\det \mathbf{B} = -19$, $\det \mathbf{C} = -152$, $\det \mathbf{D} = -19$,
4. $\det \mathbf{A} = 15$, $\det \mathbf{B} = -15$, $\det \mathbf{C} = 60$, $\det \mathbf{D} = 15$,
5. $\det \mathbf{A} = -6$, $\det \mathbf{B} = -6$, $\det \mathbf{C} = -12$, $\det \mathbf{D} = -18$.

2-5

1. $\det \mathbf{A} = -5$, $\det \mathbf{B} = -9$, $\det \mathbf{C} = 45$, $\mathbf{C} = \mathbf{A} \cdot \mathbf{B}$,
2. $\det \mathbf{A} = 6$, $\det \mathbf{B} = 24$, $\det \mathbf{C} = 30$, $\det \mathbf{C} = \det \mathbf{A} + \det \mathbf{B}$,
3. $\det \mathbf{A} = 0$, $\det \mathbf{B} = 24$, $\det \mathbf{C} = 0$, $\mathbf{C} = \mathbf{A} \cdot \mathbf{B}$,
4. $\det \mathbf{A} = 28$, $\det \mathbf{B} = 2$, $\det \mathbf{C} = 56$, $\mathbf{C} = \mathbf{A} \cdot \mathbf{B}^T$,
5. $\det \mathbf{A} = 15$, $\det \mathbf{B} = -10$, $\det \mathbf{C} = -150$, $\mathbf{C} = \mathbf{B} \cdot \mathbf{A}^T$.

2-6

1. $x_1 = 1$, $x_2 = -\frac{1}{2} + \frac{\sqrt{11}}{2} i$, $x_3 = -\frac{1}{2} - \frac{\sqrt{11}}{2} i$,
2. $x_1 = 1$, $x_2 = \frac{5}{2}$,
3. $x_{1,2} = -2$,
4. $x_1 = 1$, $x_2 = \frac{1}{12}$.

3-1

1. v_1, v_2, v_3, v_4 jsou lineárně závislé, protože $v_4 = v_1 + v_2 + v_3$, prvky v_1, v_2, v_3 jsou lineárně nezávislé,
2. p_1, p_2, p_3, p_4 jsou lineárně nezávislé,
3. $\mathbf{A_1}, \mathbf{A_2}, \mathbf{A_3}, \mathbf{A_4}, \mathbf{A_5}$ jsou lineárně nezávislé,
4. f_1, f_2, f_3 jsou lineárně závislé, protože $f_3 = \frac{1}{2} f_1 - \frac{1}{2} f_2$, prvky f_1, f_2 jsou lineárně nezávislé.

3-2

1. v_1, v_2, v_3 je báze $\mathcal{V}$, $\dim \mathcal{V} = 3$,
2. p_1, p_2, p_3, p_4 je báze $\mathcal{V}$, $\dim \mathcal{V} = 4$,
3. $\mathbf{A_1}, \mathbf{A_2}, \mathbf{A_3}$ je báze $\mathcal{V}$, $\dim \mathcal{V} = 3$,
4. f_2, f_3 je báze $\mathcal{V}$, protože $f_1 = f_2 + f_3$, $\dim \mathcal{V} = 2$.

3-3

1. V bázi v_1, v_2, v_4 je $\widehat{y} = [9, 20, -12]^T$.
2. $y \notin \mathcal{V}$.
3. V bázi $\mathbf{A_1}, \mathbf{A_2}, \mathbf{A_3}$ je $\widehat{\mathbf{Y}} = [5, 6, 4]^T$.
4. V bázi f_1, f_2 je $\widehat{y} = [-1, 1]^T$.
5. V bázi v_1, v_2 je $\widehat{y} = [5, 9]^T$.
6. V bázi p_1, p_2, p_3, p_4 je $\widehat{y} = [5, -3, 4, 7]^T$.

3-4

1. $\mathcal{V}$ je podprostor, $\dim \mathcal{V} = 3$, báze je např.: $b_1 = [1, 0, 4, -1, 0]^T$, $b_2 = [-2, 1, 1, -1, 3]^T$, $b_3 = [0, 1, -1, 0, 2]^T$, $y \in \mathcal{V}$, $\widehat{y} = [5, 2, 3]^T$,
2. $\mathcal{V}$ je podprostor, $\dim \mathcal{V} = 3$, báze je např.: $b_1 = x^5 + 2x^3 - x^2 + 3x + 1$, $b_2 = -x^5 + 4x^4 + x^2 - x + 2$, $b_3 = -x^4 + x^3 + 2x^2 - 2x$, $y \in \mathcal{V}$, $\widehat{y} = [7, 2, -5]^T$,

3. $\mathcal{V}$ je podprostor, dim$\mathcal{V} = 4$, báze je např.: $b_1 = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$, $b_2 = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 0 & 0 \\ 0 & 2 & 3 \end{bmatrix}$,
$b_3 = \begin{bmatrix} 2 & -1 & 0 \\ 0 & 1 & 2 \\ -1 & 0 & 0 \end{bmatrix}$, $b_4 = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 0 & -1 \\ 0 & 0 & -2 \end{bmatrix}$, $y \in \mathcal{V}$, $\widehat{y} = [3, 5, -6, 2]^T$,

4. $\mathcal{V}$ není podprostor, protože součet dvou prvků z $\mathcal{V}$ do podprostoru $\mathcal{V}$ nepadne z důvodu koeficientu u mocniny x^4,

5. $\mathcal{V}$ je podprostor, dim$\mathcal{V} = 4$, báze je např.: $v_1 = [2, 1, 0, 0, 1]^T$, $v_2 = [-1, 0, 1, 0, -1]^T$, $v_3 = [3, 0, -4, 2, 4]^T$, $v_4 = [0, 1, 0, -1, 0]^T$, $y \in \mathcal{V}$, $\widehat{y} = [1, 3, -2, 4]^T$,

6. $\mathcal{V}$ je podprostor, dim$\mathcal{V} = 4$, báze je např.: $b_1 = \begin{bmatrix} 1 & 0 \\ 1 & 2 \\ 0 & 0 \end{bmatrix}$, $b_2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \\ 3 & 1 \end{bmatrix}$,
$b_3 = \begin{bmatrix} 3 & -1 \\ 0 & 4 \\ 1 & 0 \end{bmatrix}$, $b_4 = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$, $y \notin \mathcal{V}$,

7. $\mathcal{V}$ je podprostor, dim$\mathcal{V} = 3$, báze je např.: $v_1 = x^5 - x^3 + 3x^2 + 2x + 1$, $v_2 = -x^5 + 3x^4 + 2x^3 - x^2 + x + 1$, $v_3 = x^5 - 2x^4 - 3x + 1$, $y \in \mathcal{V}$, $\widehat{y} = [5, 3, -7]^T$,

8. $\mathcal{V}$ je podprostor, dim$\mathcal{V} = 3$, báze je např.: $v_1 = [1, 2, -1, 3, 4]^T$, $v_2 = [2, -1, 2, -2, 0]^T$, $v_3 = [-1, 0, 2, 1, -2]^T$, $y \in \mathcal{V}$, $\widehat{y} = [7, 5, 3]^T$,

3-5

1. dim$\mathcal{U} = 3$, báze je např.: $u_1 = [3, -1, 2, 3, -2, 0, -1]^T$, $u_2 = [1, 1, -3, 8, -6, -4, 7]^T$, $u_3 = [-3, -1, 1, -1, 5, 1, -5]^T$,
dim$\mathcal{V} = 3$, báze je např.: $v_1 = [2, 1, -3, 0, 2, 0, 1]^T$, $v_2 = [-1, -4, 8, 2, 2, 2, -10]^T$, $v_3 = [3, -1, -2, 4, 9, 1, -7]^T$,
$\mathcal{U} \cap \mathcal{V}$ je podprostor, dim$(\mathcal{U} \cap \mathcal{V}) = 2$, báze je např.:
$b_1 = \frac{1}{5}(8u_1 - u_2 + 6u_3) = v_1 + v_2 = [1, -3, 5, 2, 4, 2, -9]^T$,
$b_2 = \frac{1}{5}(7u_1 + u_2 + 9u_3) = -2v_1 + v_3 = [-1, -3, 4, 4, 5, 1, -9]^T$,

2. dim$\mathcal{U} = 3$, báze je např.: $u_1 = 3x^5 - 2x^4 + 5x^3 + 7x^2 + 13x + 2$, $u_2 = -x^5 + 2x^4 - 3x^3 - 5x^2 - 3x - 2$, $u_3 = -2x^5 + x^4 - 3x^3 - 4x^2 - 7x$,
dim$\mathcal{V} = 3$, báze je např.: $v_1 = x^5 + 2x^4 - x^3 - 3x^2 + 2x - 1$, $v_2 = 2x^5 + x^4 + x^3 + x + 1$, $v_3 = 2x^4 - 2x^3 - 4x^2 + 4x - 1$,
$\mathcal{U} \cap \mathcal{V}$ je podprostor, dim$(\mathcal{U} \cap \mathcal{V}) = 2$, báze je např.: $b_1 = \frac{1}{12}(8v_1 - 4v_2) = \frac{1}{12}(3u_1 + 9u_2) = x^4 - x^3 - 2x^2 + x - 1$, $b_2 = -\frac{1}{3}(6u_1 + 9u_3) = -\frac{1}{3}(-14v_1 + 7v_2 + 9v_3) = x^4 - x^3 - 2x^2 - 5x - 4$,

3. dim$\mathcal{U} = 5$, báze je např.: $U_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 1 \end{bmatrix}$, $U_2 = \begin{bmatrix} 0 & 2 & 1 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$, $U_3 = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 2 & -1 \\ 0 & 1 & 0 \end{bmatrix}$,
$U_4 = \begin{bmatrix} -2 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}$, $U_5 = \begin{bmatrix} 2 & 1 & 1 \\ 2 & -1 & 1 \\ 2 & 2 & 1 \end{bmatrix}$,
dim$\mathcal{V} = 3$, báze je např.: $V_1 = \begin{bmatrix} 1 & 2 & -1 \\ 0 & 1 & 2 \\ 1 & -1 & 0 \end{bmatrix}$, $V_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -1 & 0 \\ 0 & 1 & 1 \end{bmatrix}$, $V_3 = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 2 & 0 & -1 \end{bmatrix}$,
$\mathcal{U} \cap \mathcal{V}$ je podprostor, dim$(\mathcal{U} \cap \mathcal{V}) = 1$, báze je např.: $V_2 + V_3 = U_5 - U_1 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & 0 \end{bmatrix}$.

4-1

1. $\mathrm{hod}(\mathbf{A}) = 3$,
2. $\mathrm{hod}(\mathbf{A}) = 2$,
3. $\mathrm{hod}(\mathbf{A}) = 4$,
4. $\mathrm{hod}(\mathbf{A}) = 4$.

4-2

1. $\mathbf{A^{-1}} = \frac{1}{29}\begin{bmatrix} 7 & 3 \\ -5 & 2 \end{bmatrix}$,
2. $\mathbf{A^{-1}} = \frac{1}{3}\begin{bmatrix} -1-i & 2+i \\ 2-i & -1+i \end{bmatrix}$,
3. $\mathbf{A^{-1}} = \frac{1}{16}\begin{bmatrix} -27 & -2 & -17 \\ 14 & 4 & 10 \\ -5 & 2 & 1 \end{bmatrix}$,
4. $\mathbf{A^{-1}} = \frac{1}{8}\begin{bmatrix} -13 & 2 & 7 \\ -7 & -2 & 5 \\ 2 & -4 & 2 \end{bmatrix}$,
5. $\mathbf{A^{-1}} = \frac{1}{50}\begin{bmatrix} -3 & 8 & -7 \\ -10 & 10 & 10 \\ 11 & 4 & 9 \end{bmatrix}$,
6. $\mathbf{A^{-1}} = \frac{1}{107}\begin{bmatrix} 14 & 16 & -5 \\ 25 & -2 & 14 \\ -2 & 13 & 16 \end{bmatrix}$,
7. $\mathbf{A^{-1}} = \begin{bmatrix} 0 & -1 & -1 & 1 \\ -2 & 4 & 2 & -1 \\ 0 & 3 & 2 & -1 \\ -1 & -1 & -1 & 0 \end{bmatrix}$,
8. $\mathbf{A^{-1}} = \begin{bmatrix} 20 & 25 & 17 & -26 \\ -7 & -11 & -6 & 10 \\ 2 & 3 & 2 & -3 \\ -1 & 0 & -1 & 1 \end{bmatrix}$,
9. $\mathbf{A^{-1}} = \frac{1}{3}\begin{bmatrix} -4 & -1 & 2 & 0 \\ 10 & -2 & -5 & 3 \\ -7 & 2 & 5 & 0 \\ -11 & 4 & 7 & -3 \end{bmatrix}$,
10. $\mathbf{A^{-1}} = \frac{1}{22}\begin{bmatrix} -5 & 10 & 9 & 3 \\ 13 & -4 & 3 & 1 \\ 11 & 0 & -11 & -11 \\ 10 & 2 & 4 & -6 \end{bmatrix}$,
11. $\mathbf{A^{-1}} = \frac{1}{4}\begin{bmatrix} -13 & 0 & 9 & 2 & -1 \\ 5 & 2 & -3 & -2 & 1 \\ 0 & -1 & 1 & 2 & -2 \\ 5 & -3 & -2 & 4 & -1 \\ -2 & 1 & 1 & -2 & 0 \end{bmatrix}$,

12. $\mathbf{A^{-1}} = \frac{1}{18}\begin{bmatrix} 6 & 27 & -16 & -31 & 13 \\ -6 & -18 & 16 & 22 & -4 \\ 6 & 9 & -10 & -7 & 7 \\ 0 & -27 & 18 & 27 & -9 \\ 0 & 0 & 6 & 6 & -6 \end{bmatrix}.$

4-3

1. $\mathbf{X} = \mathbf{A}^{-1}\mathbf{B} = \frac{1}{18}\begin{bmatrix} 28 & -40 & 2 \\ -10 & 4 & -2 \\ -10 & 13 & 7 \end{bmatrix},$

2. $\mathbf{X} = \mathbf{B}\mathbf{A}^{-1} = \frac{1}{4}\begin{bmatrix} -8 & 2 & 8 & -2 \\ 12 & 14 & 0 & 2 \\ -14 & 3 & 8 & -1 \\ -10 & 19 & 16 & -1 \end{bmatrix},$

3. $\mathbf{X} = \mathbf{A}^{-1}\mathbf{C}\mathbf{B}^{-1} = \frac{1}{110}\begin{bmatrix} 49 & 7 \\ -26 & 12 \end{bmatrix},$

4. $\mathbf{X} = \mathbf{A}^{-1}\mathbf{C}\mathbf{B}^{-1} = \frac{1}{44}\begin{bmatrix} -102 & 60 & -46 \\ -45 & 86 & -41 \\ -153 & 90 & -69 \end{bmatrix},$

5. $\mathbf{X} = (\mathbf{A} - \mathbf{C})^{-1}\mathbf{B} = \frac{1}{7}\begin{bmatrix} -9 & 10 & -8 \\ -22 & -23 & 24 \\ -7 & -35 & 21 \end{bmatrix},$

6. $\mathbf{X} = (\mathbf{D} + \mathbf{B})(\mathbf{A} - \mathbf{C})^{-1} = \begin{bmatrix} 24 & -3 & -16 \\ 15 & -2 & -10 \\ 39 & -5 & -26 \end{bmatrix},$

7. $\mathbf{X} = \frac{1}{3}\mathbf{A}^{-1}\mathbf{B} = \frac{1}{3}\begin{bmatrix} -4 & -20 & -8 \\ 3 & 8 & 7 \\ 0 & -7 & 1 \end{bmatrix},$

8. $\mathbf{X} = (2\mathbf{D} + 3\mathbf{B})(\mathbf{A} - 7\mathbf{C})^{-1} = \frac{1}{3}\begin{bmatrix} -54 & -15 & 24 \\ -62 & -34 & -2 \\ -75 & -45 & -3 \end{bmatrix}.$

5-1

1. $\mathcal{L}$ je lineární,
2. $\mathcal{L}$ není lineární,
3. $\mathcal{L}$ je lineární,
4. $\mathcal{L}$ je lineární,
5. $\mathcal{L}$ není lineární,
6. $\mathcal{L}$ je lineární,
7. $\mathcal{L}$ není lineární,
8. $\mathcal{L}$ není lineární,
9. $\mathcal{L}$ není lineární.

5-2

1. $\dim(\mathrm{Ker}\mathcal{L}) = 1$, báze je např. $[-2, 3, 7]^T$
   $\dim(\mathrm{Im}\mathcal{L}) = 2$, báze je např. $[-1, 0, 3, 0, 2]^T, [1, 0, -1, 0, 0]^T$,

2. $\dim(\mathrm{Ker}\mathcal{L}) = 2$, báze je např. $-2x^3 + x + 2, x^3 + x^2$,
   $\dim(\mathrm{Im}\mathcal{L}) = 2$, báze je např. $[1, 0]^T, [0, 1]^T$, $\mathrm{Im}\mathcal{L} = R_2$,

3. $\dim(\mathrm{Ker}\mathcal{L}) = 3$, báze je např. $\mathbf{B_1} = \begin{bmatrix} -1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$, $\mathbf{B_2} = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & 0 \end{bmatrix}$,
   $\mathbf{B_3} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & -1 & 1 \end{bmatrix}$, $\dim(\mathrm{Im}\mathcal{L}) = 3$, báze je např. $x^2, x, 1$, $\mathrm{Im}\mathcal{L} = \mathcal{P}_2$,

4. $\dim(\mathrm{Ker}\mathcal{L}) = 2$, báze je např. $[-14, -3, 0, 5]^T$, $[-9, -3, 5, 0]^T$
   $\dim(\mathrm{Im}\mathcal{L}) = 2$, báze je např. $[1, 0]^T, [0, 1]^T$, protože $\mathrm{Im}\mathcal{L} = R_2$,

5. $\dim(\mathrm{Ker}\mathcal{L}) = 0$,
   $\dim(\mathrm{Im}\mathcal{L}) = 3$, báze je např. $[1, 4, 1, -3]^T, [2, -1, 1, 1]^T, [-3, 2, 1, 2]^T$,

6. $\dim(\mathrm{Ker}\mathcal{L}) = 1$, báze je např. $x^2 - x + 1$,
   $\dim(\mathrm{Im}\mathcal{L}) = 2$, báze je např. $[2, 0, 1, 1, 1]^T, [1, -1, -1, 0, 2]^T$.

5-3

1. $\mathbf{A} = \begin{bmatrix} 2 & -1 & 1 \\ 0 & 0 & 0 \\ 1 & 3 & -1 \\ 0 & 0 & 0 \\ 3 & 2 & 0 \end{bmatrix}$,

2. $\mathbf{A} = \begin{bmatrix} 1 & -1 & 2 & 0 \\ -1 & 1 & 0 & -1 \end{bmatrix}$,

3. $\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$,

4. $\mathbf{A} = \mathbf{C} = \begin{bmatrix} 1 & -2 & 3 \\ 4 & 5 & 1 \\ 0 & 1 & -2 \\ -3 & 2 & 1 \end{bmatrix}$.

5-4

1. $\mathbf{B} = \begin{bmatrix} 7 & 8 & 11 \\ -7 & -8 & -11 \\ 18 & 18 & 26 \\ -61 & -62 & -89 \\ 150 & 156 & 222 \end{bmatrix}$,

2. $\mathbf{B} = \frac{1}{3} \begin{bmatrix} -3 & 5 & 9 & -3 \\ 3 & -1 & -9 & 3 \end{bmatrix}$,

3. $\mathbf{B} = \frac{1}{9} \begin{bmatrix} 2 & 5 & 8 & 2 & -4 & 3 \\ -4 & -1 & 2 & 5 & 8 & -6 \\ 8 & 2 & -4 & -1 & 2 & 3 \end{bmatrix}$,

4. $\mathbf{B} = \frac{1}{3} \begin{bmatrix} 11 & 22 & 11 \\ 17 & 16 & 23 \\ -28 & -29 & -19 \\ 17 & 4 & -1 \end{bmatrix}$.

5-5

(a) Zobrazení je lineární,

(b) $\dim(\mathrm{Ker}\mathcal{L}) = 1$, báze je např. $[-9, 5, 1]^T$, $\dim(\mathrm{Im}\mathcal{L}) = 2$, báze je např. $[1, 0]^T$, $[0, 1]^T$,

(c) $\mathbf{A} = \begin{bmatrix} 1 & 2 & -1 \\ 2 & 3 & 3 \end{bmatrix}$,

(d) $\mathbf{B} = \frac{1}{8} \begin{bmatrix} 9 & 13 & 22 \\ 13 & -7 & -10 \end{bmatrix}$,

(e) $\mathbf{T} = \begin{bmatrix} 1 & 2 & -1 \\ 2 & -1 & 1 \\ -1 & 1 & 2 \end{bmatrix}$,

(f) $\mathbf{H}^{-1} = \frac{1}{8} \begin{bmatrix} -1 & 3 \\ 3 & -1 \end{bmatrix}$,

(g) platí.

5-6

(a) Zobrazení je lineární,

(b) $\mathrm{Ker}\mathcal{L} = 0$, $\dim(\mathrm{Im}\mathcal{L}) = 2$, báze je např. $x^4 + 2x^3 - 1$, $-x^4 + x^3 + 2x + 1$,

(c) $\mathbf{A} = \begin{bmatrix} 1 & -1 \\ 2 & 1 \\ 0 & 0 \\ 0 & 2 \\ -1 & 1 \end{bmatrix}$,

(d) $\mathbf{B} = \frac{1}{2} \begin{bmatrix} -5 & 17 \\ 11 & -5 \\ 5 & -17 \\ 1 & -7 \\ -1 & 7 \end{bmatrix}$,

(e) $\mathbf{T} = \begin{bmatrix} 1 & 2 \\ 3 & -3 \end{bmatrix}$,

(f) $\mathbf{H}^{-1} = \frac{1}{2} \begin{bmatrix} 1 & 1 & 1 & -1 & -1 \\ -1 & 1 & 1 & 1 & -1 \\ -1 & -1 & 1 & 1 & 1 \\ 1 & -1 & -1 & 1 & 1 \\ 1 & 1 & -1 & -1 & 1 \end{bmatrix}$,

(g) platí.

5-7

(a) $\mathcal{L}([a, b, c]^T) = \begin{bmatrix} 3a + b - 4c & 2a + 3b + 9c \\ 6a + 3b - 3c & 4a + 4b + 11c \end{bmatrix}$,

(b) $\mathbf{A_1} = \begin{bmatrix} 1 & 1 & 0 \\ 2 & 0 & -1 \\ 0 & 1 & 3 \\ 2 & 1 & 0 \\ 0 & 1 & 4 \end{bmatrix}$, $\mathbf{A_2} = \begin{bmatrix} 1 & 0 & 0 & 1 & -1 \\ 0 & 1 & 2 & 0 & 1 \\ 2 & 0 & -1 & 2 & 0 \\ 0 & 1 & 0 & 1 & 3 \end{bmatrix}$, $\mathbf{A} = \begin{bmatrix} 3 & 1 & -4 \\ 2 & 3 & 9 \\ 6 & 3 & -3 \\ 4 & 4 & 11 \end{bmatrix}$,

(c) $\mathbf{A_2A_1} = \mathbf{A}$.

5-8

(a) $\mathcal{L}([a,b,c,d]^T) = [b+c, -a+2b+c+d, a+b+2c-d, -a+3b+2c+d, 3b+3c]^T$,

(b) $\mathbf{A_1} = \frac{1}{14}\begin{bmatrix} 3 & -12 & -15 & 0 \\ 7 & 0 & 7 & 14 \\ -5 & -8 & -17 & -14 \end{bmatrix}$, $\mathbf{A_2} = \frac{1}{2}\begin{bmatrix} 1 & -1 & 2 \\ -1 & 1 & -2 \\ 5 & 5 & -4 \\ -1 & 17 & -2 \\ 5 & -11 & -4 \end{bmatrix}$, $\mathbf{A} = \frac{1}{2}\begin{bmatrix} -1 & -2 & -4 & -3 \\ 1 & 2 & 4 & 3 \\ 5 & -2 & 2 & 9 \\ 9 & 2 & 12 & 19 \\ -3 & -2 & -6 & -7 \end{bmatrix}$,

(c) $\mathbf{A_2A_1} = \mathbf{A}$.

5-9

(a) $\mathcal{L}(\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix}) = [a-3b+5e+f, -a+b+2c+4d+e+f, 6a-b-c-2d+3e-3f,$
$2a+7b+3c+6d-5e-3f, 4a+2b+2c+4d+2e-2f]^T$,

(b) $\mathbf{A_1} = \frac{1}{2}\begin{bmatrix} 1 & 2 & 5 & 6 & 2 & 2 \\ 4 & 3 & 5 & 4 & -1 & 3 \\ 8 & 6 & 10 & 8 & -2 & 6 \\ -1 & 1 & 0 & -6 & -1 & -5 \end{bmatrix}$, $\mathbf{A_2} = \begin{bmatrix} 6 & 5 & -8 & 0 \\ 6 & -1 & -8 & -2 \\ 2 & 2 & -3 & -1 \\ 0 & -1 & 3 & 3 \\ -4 & -4 & 10 & 2 \end{bmatrix}$,

$\mathbf{A} = \frac{1}{2}\begin{bmatrix} -38 & -21 & -25 & -8 & 23 & -21 \\ -60 & -41 & -55 & -20 & 31 & -29 \\ -13 & -9 & -10 & 2 & 9 & -3 \\ 17 & 18 & 25 & 2 & -8 & 0 \\ 58 & 42 & 60 & 28 & -26 & 30 \end{bmatrix}$,

(c) $\mathbf{A_2A_1} = \mathbf{A}$.

5-10

1. $\dim(\mathrm{Ker}\mathcal{L}) = 1$, báze je např. $[3,-2,1]^T$, $\dim(\mathrm{Im}\mathcal{L}) = 2$,
$\mathcal{L}^2([a,b,c]^T) = [3a+3b-3c, -2a-2b+2c, a+b-c]^T$, $\dim(\mathrm{Ker}\mathcal{L}^2) = 2$, báze je např. $[1,0,1]^T$, $[-1,1,0]^T$, $\dim(\mathrm{Im}\mathcal{L}^2) = 1$,
$\mathcal{L}^3([a,b,c]^T) = [0,0,0]^T$, $\dim(\mathrm{Ker}\mathcal{L}^3) = 3$, $\dim(\mathrm{Im}\mathcal{L}^3) = 0$,

2. $\dim(\mathrm{Ker}\mathcal{L}) = 1$, báze je např. $[1,0,1]^T$, $\dim(\mathrm{Im}\mathcal{L}) = 2$,
$\mathcal{L}^2([a,b,c]^T) = [0, -2a+2b+2c, -2a+2b+2c]^T$, $\dim(\mathrm{Ker}\mathcal{L}^2) = 2$, báze je např. $[1,0,1]^T$, $[1,1,0]^T$, $\dim(\mathrm{Im}\mathcal{L}^2) = 1$,
$\mathcal{L}^3([a,b,c]^T) = [0, -4a+4b+4c, -4a+4b+4c]^T$, $\dim(\mathrm{Ker}\mathcal{L}^3) = 2$, báze je např. $[1,0,1]^T$, $[1,1,0]^T$, $\dim(\mathrm{Im}\mathcal{L}^3) = 1$,

3. $\dim(\mathrm{Ker}\mathcal{L}) = 0$, $\dim(\mathrm{Im}\mathcal{L}) = 3$,
$\mathcal{L}^2([a,b,c]^T) = [5a-2b+2c, 4a-b+2c, -4a+2b-c]^T$, $\dim(\mathrm{Ker}\mathcal{L}^2) = 0$, $\dim(\mathrm{Im}\mathcal{L}^2) = 3$,

4. $\dim(\mathrm{Ker}\mathcal{L}) = 1$, báze je např. x^2+x+1, $\dim(\mathrm{Im}\mathcal{L}) = 2$,
$\mathcal{L}^2(ax^2+bx+c) = (a-2b+c)x^2 + (a+b-2c)x + (-2a+b+c)$, $\dim(\mathrm{Ker}\mathcal{L}^2) = 1$, báze je např. x^2+x+1, $\dim(\mathrm{Im}\mathcal{L}^2) = 2$,
$\mathcal{L}^3(ax^2+bx+c) = (-3b+3c)x^2 + (3a-3c)x + (-3a+3b)$, $\dim(\mathrm{Ker}\mathcal{L}^3) = 1$, báze je např. x^2+x+1, $\dim(\mathrm{Im}\mathcal{L}^3) = 2$,

5. $\dim(\mathrm{Ker}\mathcal{L}) = 1$, báze je např. $[-6,3,-1,1]^T$, $\dim(\mathrm{Im}\mathcal{L}) = 3$,
$\mathcal{L}^2([a,b,c,d]^T) = [-21a-27b+9c-36d, 10a+13b-4c+17d, -3a-4b+c-5d, 4a+5b-2c+7d]^T$,
$\dim(\mathrm{Ker}\mathcal{L}^2) = 2$, báze je např. $[3,-2,1,0]^T$, $[-3,1,0,1]^T$, $\dim(\mathrm{Im}\mathcal{L}^2) = 2$,
$\mathcal{L}^3([a,b,c,d]^T) = [-6a-6b+6c-12d, 3a+3b-3c+6d, -a-b+c-2d, a+b-c+2d]^T$,

$\dim(\mathrm{Ker}\mathcal{L}^3) = 3$, báze je např. $[-1,1,0,0]^T$, $[1,0,1,0]^T$, $[-2,0,0,1]^T$, $\dim(\mathrm{Im}\mathcal{L}^3) = 1$,
$\mathcal{L}^4([a,b,c,d]^T) = [0,0,0,0]^T$, $\dim(\mathrm{Ker}\mathcal{L}^4) = 4$, $\dim(\mathrm{Im}\mathcal{L}^4) = 0$,
$\mathcal{L}^5([a,b,c,d]^T) = [0,0,0,0]^T$, $\dim(\mathrm{Ker}\mathcal{L}^5) = 4$, $\dim(\mathrm{Im}\mathcal{L}^5) = 0$,

6. $\dim(\mathrm{Ker}\mathcal{L}) = 1$, báze je např. x^3+2x^2+x+1, $\dim(\mathrm{Im}\mathcal{L}) = 3$,
$\mathcal{L}^2(ax^3+bx^2+cx+d) = (-17a+8b+3c-2d)x^3 + (-28a+13b+5c-3d)x^2 + (-23a+11b+4c-3d)x + (a-b+d)$,
$\dim(\mathrm{Ker}\mathcal{L}^2) = 2$, báze je např. x^3+x^2+3x, $2x^3+5x^2+3$, $\dim(\mathrm{Im}\mathcal{L}^2) = 2$,
$\mathcal{L}^3(ax^3+bx^2+cx+d) = (-6a+3b+c-d)x^3 + (-6a+3b+c-d)x^2 + (-12a+6b+2c-2d)x + (12a-6b-2c+2d)$,
$\dim(\mathrm{Ker}\mathcal{L}^3) = 3$, báze je např. x^3+2x^2, x^3+6x, $-x^3+6$, $\dim(\mathrm{Im}\mathcal{L}^3) = 1$,
$\mathcal{L}^4(ax^3+bx^2+cx+d) = (-6a+3b+c-d)x^3 + (-6a+3b+c-d)x^2 + (-12a+6b+2c-2d)x + (12a-6b-2c+2d)$,
$\dim(\mathrm{Ker}\mathcal{L}^4) = 3$, báze je např. x^3+2x^2, x^3+6x, $-x^3+6$, $\dim(\mathrm{Im}\mathcal{L}^4) = 1$,
$\mathcal{L}^5(ax^3+bx^2+cx+d) = (-6a+3b+c-d)x^3 + (-6a+3b+c-d)x^2 + (-12a+6b+2c-2d)x + (12a-6b-2c+2d)$,
$\dim(\mathrm{Ker}\mathcal{L}^5) = 3$, báze je např. x^3+2x^2, x^3+6x, $-x^3+6$, $\dim(\mathrm{Im}\mathcal{L}^5) = 1$,
$\mathcal{L}^6(ax^3+bx^2+cx+d) = (-6a+3b+c-d)x^3 + (-6a+3b+c-d)x^2 + (-12a+6b+2c-2d)x + (12a-6b-2c+2d)$,
$\dim(\mathrm{Ker}\mathcal{L}^6) = 3$, báze je např. x^3+2x^2, x^3+6x, $-x^3+6$, $\dim(\mathrm{Im}\mathcal{L}^6) = 1$,

7. $\dim(\mathrm{Ker}\mathcal{L}) = 0$, $\dim(\mathrm{Im}\mathcal{L}) = 4$,
$\mathcal{L}^2(\begin{bmatrix} a & b \\ c & d \end{bmatrix}) = \begin{bmatrix} 38a-15b+4c-9d & -8a+7b-2c \\ -118a+51b-12c+27d & 110a-48b+14c-23d \end{bmatrix}$,
$\dim(\mathrm{Ker}\mathcal{L}^2) = 0$, $\dim(\mathrm{Im}\mathcal{L}^2) = 4$.

5-11

(a) Zobrazení $\mathcal{L}$ je izomorfismus, protože zobrazení je lineární, $\mathrm{Ker}\mathcal{L} = 0$, $\dim(\mathrm{Im}\mathcal{L}) = 2 = \dim\mathcal{P}_1$, proto $\mathrm{Im}\mathcal{L} = \mathcal{P}_1$.

(b) $\mathbf{A} = \begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$,

(c) $\mathbf{A}^{-1} = \frac{1}{7}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$,

(d) $\tilde{v} = \mathbf{A}^{-1}\widehat{q} = \frac{1}{7}[3a+b, -a+2b]^T$,

(e) $v = \frac{1}{7}(3a+b)e_1 + \frac{1}{7}(-a+2b)e_2 = \frac{1}{7}[3a+b, -a+2b]^T$,

(f) $\mathcal{L}^{-1}(ax+b) = \frac{1}{7}[3a+b, -a+2b]^T$.

6-1

1. $\mathbf{T} = \frac{1}{3}\begin{bmatrix} 7 & 1 \\ 1 & 7 \end{bmatrix}$, $\mathbf{T^{-1}} = \frac{1}{16}\begin{bmatrix} 7 & -1 \\ -1 & 7 \end{bmatrix}$,

2. $\mathbf{T} = \frac{1}{4}\begin{bmatrix} 2 & 0 & 2 \\ 0 & 1 & -2 \\ 2 & -1 & 0 \end{bmatrix}$, $\mathbf{T^{-1}} = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 2 & -2 \\ 1 & -1 & -1 \end{bmatrix}]$,

3. $\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 \\ 3 & -1 & 1 & 0 & 0 & 0 \\ -5 & 3 & -1 & 1 & 0 & 0 \\ 11 & -5 & 3 & -1 & 1 & 0 \\ -21 & 11 & -5 & 3 & -1 & 1 \end{bmatrix}$, $\mathbf{T^{-1}} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 & 0 \\ -2 & 1 & 1 & 0 & 0 & 0 \\ 0 & -2 & 1 & 1 & 0 & 0 \\ 0 & 0 & -2 & 1 & 1 & 0 \\ 0 & 0 & 0 & -2 & 1 & 1 \end{bmatrix}$,

4. $\mathbf{T} = \frac{1}{4}\begin{bmatrix} 0 & -1 & -3 & 0 \\ 0 & -3 & -2 & 1 \\ -2 & -2 & -1 & 1 \\ 2 & 2 & 2 & 2 \end{bmatrix}$, $\mathbf{T^{-1}} = \frac{1}{5}\begin{bmatrix} -1 & 7 & -9 & 1 \\ 7 & -9 & 3 & 3 \\ -9 & 3 & -1 & -1 \\ 3 & -1 & 7 & 7 \end{bmatrix}$,

5. $\mathbf{T} = \frac{1}{33}\begin{bmatrix} 3 & -6 & 3 & 29 & 13 \\ 6 & 21 & 6 & -8 & -7 \\ 24 & 18 & -9 & -32 & 5 \\ -12 & -9 & 21 & 27 & 3 \\ 3 & -6 & 3 & 18 & 24 \end{bmatrix}$, $\mathbf{T^{-1}} = \frac{1}{2}\begin{bmatrix} 5 & -3 & 5 & 3 & -5 \\ -1 & 5 & -3 & -3 & 3 \\ -1 & -1 & 3 & 5 & -1 \\ 3 & 1 & -1 & -1 & -1 \\ -3 & 1 & -1 & -1 & 5 \end{bmatrix}$,

6. $\mathbf{T} = \frac{1}{24}\begin{bmatrix} 4 & 9 & -2 & 1 \\ 4 & 1 & 2 & 13 \\ 4 & 3 & 10 & 19 \\ 4 & 7 & 14 & -5 \end{bmatrix}$, $\mathbf{T^{-1}} = \frac{1}{3}\begin{bmatrix} -4 & 33 & -20 & 9 \\ 9 & -15 & 9 & -3 \\ -3 & -3 & 3 & 3 \\ 1 & -3 & 5 & -3 \end{bmatrix}$,

7. $\mathbf{T} = \frac{1}{18}\begin{bmatrix} -22 & -21 & 42 \\ 56 & -3 & -12 \\ -28 & 33 & -12 \end{bmatrix}$, $\mathbf{T^{-1}} = \frac{1}{134}\begin{bmatrix} 24 & 63 & 21 \\ 56 & 80 & 116 \\ 98 & 73 & 69 \end{bmatrix}$].

6-2

1. $\mathbf{A} = \frac{1}{5}\begin{bmatrix} 18 & 7 \\ -7 & 7 \end{bmatrix}$, $\mathbf{B} = \frac{1}{2}\begin{bmatrix} 0 & -7 \\ 4 & 10 \end{bmatrix}$, $\mathbf{T} = \frac{1}{15}\begin{bmatrix} -2 & 2 \\ 8 & 7 \end{bmatrix}$, $\mathbf{T^{-1}AT} = \mathbf{B}$,

2. $\mathbf{A} = \begin{bmatrix} -3 & 3 & 2 \\ 12 & -7 & -5 \\ -24 & 19 & 13 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 0 & 0 & 0 \\ 4 & 0 & 2 \\ 0 & 1 & 3 \end{bmatrix}$, $\mathbf{T} = \begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & -2 \\ 3 & -1 & 6 \end{bmatrix}$, $\mathbf{T^{-1}AT} = \mathbf{B}$,

3. $\mathbf{A} = \frac{1}{28}\begin{bmatrix} 40 & -77 & 1 \\ 76 & 35 & -3 \\ -116 & 7 & 121 \end{bmatrix}$, $\mathbf{B} = \frac{1}{14}\begin{bmatrix} 50 & -29 & 9 \\ -40 & 19 & 39 \\ 32 & -67 & 29 \end{bmatrix}$, $\mathbf{T} = \frac{1}{2}\begin{bmatrix} -1 & 1 & 1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$,
$\mathbf{T^{-1}AT} = \mathbf{B}$,

4. $\mathbf{A} = \frac{1}{6}\begin{bmatrix} 17 & 14 & 11 & 17 & 7 \\ -7 & 8 & 5 & 11 & 13 \\ 11 & 2 & 11 & 5 & 1 \\ -25 & -16 & -13 & -19 & -5 \\ 5 & 2 & -7 & -1 & -5 \end{bmatrix}$, $\mathbf{B} = \frac{1}{2}\begin{bmatrix} 0 & -2 & -1 & -7 & 2 \\ 4 & -2 & -1 & -1 & 0 \\ 2 & -2 & 1 & -1 & 0 \\ 2 & 2 & -1 & 7 & -2 \\ -2 & 6 & -3 & 9 & -2 \end{bmatrix}$,
$\mathbf{T} = \frac{1}{3}\begin{bmatrix} 0 & 0 & 0 & -3 & 2 \\ 0 & 0 & -3 & 3 & -1 \\ 0 & -3 & 3 & 0 & -1 \\ -3 & 3 & 0 & 0 & -1 \\ 3 & 0 & 0 & 0 & 2 \end{bmatrix}$, $\mathbf{T^{-1}AT} = \mathbf{B}$,

5. $\mathbf{A} = \begin{bmatrix} 1 & 1 & 0 & 2 \\ 0 & 0 & -1 & -1 \\ -1 & 0 & 3 & -2 \\ -1 & -1 & 1 & -1 \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} 2 & 1 & 0 & 0 \\ -1 & -2 & -1 & -1 \\ -1 & 2 & 2 & 1 \\ 1 & 0 & -1 & 1 \end{bmatrix}$, $\mathbf{T} = \frac{1}{2}\begin{bmatrix} 2 & 1 & 2 & 1 \\ 0 & 1 & 0 & -1 \\ 2 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix}$,
$\mathbf{T^{-1}AT} = \mathbf{B}$.

6-3

(a) $\mathbf{A} = \begin{bmatrix} 8 & 7 \\ -3 & -1 \end{bmatrix}$,

(b) $\mathbf{B} = \frac{1}{40}\begin{bmatrix} 211 & 79 \\ -79 & 69 \end{bmatrix}$,

(c) $\mathbf{C} = \begin{bmatrix} 3 & -1 \\ 1 & 4 \end{bmatrix}$,

(d) $\mathbf{T} = \begin{bmatrix} 11 & -1 \\ -4 & 4 \end{bmatrix}$, $\mathbf{B} = \mathbf{T}^{-1}\mathbf{A}\mathbf{T}$,

(e) $\mathbf{H} = \begin{bmatrix} 1 & 2 \\ 1 & 1 \end{bmatrix}$, $\mathbf{A} = \mathbf{H}^{-1}\mathbf{C}\mathbf{H}$,

(f) $\mathbf{K} = \begin{bmatrix} 3 & 7 \\ 7 & 3 \end{bmatrix}$, $\mathbf{B} = \mathbf{K}^{-1}\mathbf{C}\mathbf{K}$,

(g) $\det(\lambda\mathbf{I} - \mathbf{A}) = \det(\lambda\mathbf{I} - \mathbf{B}) = \det(\lambda\mathbf{I} - \mathbf{C}) = \lambda^2 - 7\lambda + 13 = (\lambda - \frac{7}{2} - \frac{\sqrt{3}}{2}i)(\lambda - \frac{7}{2} + \frac{\sqrt{3}}{2}i)$.

6-4

(a)$\mathbf{A} = \begin{bmatrix} 17 & 17 & 13 \\ -20 & -21 & -18 \\ 10 & 11 & 10 \end{bmatrix}$,

(b) $\mathbf{B} = \frac{1}{3}\begin{bmatrix} -50 & 4 & 48 \\ -70 & 11 & 60 \\ -54 & 0 & 57 \end{bmatrix}$,

(c) $\mathbf{C} = \begin{bmatrix} 0 & 4 & 13 \\ 1 & 1 & -5 \\ 0 & 2 & 5 \end{bmatrix}$,

(d) $\mathbf{T} = \begin{bmatrix} 1 & 1 & -1 \\ 0 & -3 & 2 \\ -3 & 3 & 0 \end{bmatrix}$, $\mathbf{B} = \mathbf{T}^{-1}\mathbf{A}\mathbf{T}$,

(e) $\mathbf{H} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{bmatrix}$, $\mathbf{A} = \mathbf{H}^{-1}\mathbf{C}\mathbf{H}$,

(f) $\mathbf{K} = \begin{bmatrix} 1 & 1 & -1 \\ 1 & -2 & 1 \\ -2 & 1 & 1 \end{bmatrix}$, $\mathbf{B} = \mathbf{K}^{-1}\mathbf{C}\mathbf{K}$,

(g) $\det(\lambda\mathbf{I} - \mathbf{A}) = \det(\lambda\mathbf{I} - \mathbf{B}) = \det(\lambda\mathbf{I} - \mathbf{C}) = \lambda^3 - 6\lambda^2 + 11\lambda - 6 = (\lambda - 1)(\lambda - 2)(\lambda - 3)$.

6-5

(a)$\mathbf{A} = \frac{1}{2}\begin{bmatrix} 0 & 4 & 5 & -7 \\ 0 & 2 & 5 & -1 \\ 0 & -2 & -3 & 3 \\ 0 & -2 & -1 & 5 \end{bmatrix}$,

(b) $\mathbf{B} = \frac{1}{2}\begin{bmatrix} 4 & 2 & 2 & 4 \\ -6 & -4 & -6 & -8 \\ 4 & 2 & 2 & 4 \\ 1 & 1 & 2 & 2 \end{bmatrix}$,

(c) $\mathbf{C} = \begin{bmatrix} 1 & 0 & -1 & 1 \\ 0 & 1 & 0 & -1 \\ 1 & 1 & -1 & 0 \\ 2 & 1 & -2 & 1 \end{bmatrix}$,

(d) $\mathbf{T} = \frac{1}{2}\begin{bmatrix} 3 & 2 & 3 & 2 \\ 3 & 4 & 3 & 2 \\ -1 & -2 & -3 & -2 \\ -1 & 0 & -1 & -2 \end{bmatrix}$, $\mathbf{B} = \mathbf{T}^{-1}\mathbf{A}\mathbf{T}$,

(e) $\mathbf{H} = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \end{bmatrix}$, $\mathbf{A} = \mathbf{H}^{-1}\mathbf{C}\mathbf{H}$,

(f) $\mathbf{K} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 2 & 2 & 2 & 2 \end{bmatrix}$, $\mathbf{B} = \mathbf{K}^{-1}\mathbf{C}\mathbf{K}$,

(g) $\det(\lambda\mathbf{I} - \mathbf{A}) = \det(\lambda\mathbf{I} - \mathbf{B}) = \det(\lambda\mathbf{I} - \mathbf{C}) = \lambda^4 - 2\lambda^3 = \lambda^3(\lambda - 2)$.

7-1

1. $x = k_1[-7, 0, 2, 1, 0]^T + k_2[1, 1, 0, 0, 0]^T$,
2. $x = [0, 0, 0, 0]^T$,
3. $x = k[18, 13, -5, 4, 37]^T$.

7-2

1. $x = [10, -4, -3, 0]^T + k[4, -3, 1, 1]^T$,
2. $x = [14, 0, 7, -2, 0]^T + k_1[-2, 1, 0, 0, 0]^T + k_2[17, 0, 9, -4, 1]^T$,
3. nemá řešení,
4. $x = [1, 1, 1, 1, 1]^T$,
5. $x = [3, 0, 1, 0, 3]^T + k_1[-2, 1, 0, 0, 0]^T + k_2[-5, 0, -2, 1, 0]^T$,
6. $x = [22, -\frac{9}{2}, 0, -\frac{3}{2}]^T + k[5, -2, 1, 0]^T$,
7. $x = [-7, 1, 0, 3, 0]^T + k_1[5, -2, 1, 0, 0]^T + k_2[-5, -1, 0, 2, 1]^T$,
8. $x = [10, -1, 0, -4, 0, 0]^T + k_1[-1, -2, 1, 0, 0, 0]^T + k_2[7, -5, 0, -3, 1, 0]^T + k_3[-7, 3, 0, 2, 0, 1]^T$.

7-3

1. $x = \frac{1}{6}[113, 28, -59]^T$,
2. $x = [1, 1, 1]^T$,
3. $x = \frac{1}{2}[-11, 5, 2, 3]^T$.

7-4

1. pro $a \neq 2$ má soustava jedno řešení $x = [-2, -1, 2]^T$,
   pro $a = 2$ má soustava nekonečně mnoho řešení $x = [-5, 0, 4]^T + k[-3, 1, 2]^T$,
2. pro $a = 0$ soustava nemá řešení,
   pro $a \neq 0$ má soustava nekonečně mnoho řešení
   $x = \frac{1}{5a}[5a + 15, 37a - 6, 3 - 6a, 0, -15a]^T + k[0, -5, 1, 1, 2]^T$,
3. pro $a = 8$ soustava nemá řešení,
   pro $a = 2$ má soustava nekonečně mnoho řešení $x = [0, 0, -1, 0]^T + k[1, -3, 1, 2]^T$,
   pro $a \neq 2$, $a \neq 8$ má soustava jedno řešení $x = \frac{1}{2(a-8)}[3 - a, a - 5, 15 - a, 2]^T$,

4. pro $a = 1$ má soustava nekonečně mnoho řešení $x = [-8, 5, -7, 0]^T + k[3, -1, 2, 1]^T$,
   pro $a \neq 1$ má soustava jedno řešení $x = [-5, a+3, a-6, 1]^T$,

5. pro $a = -1$ má soustava nekonečně mnoho řešení $x = [1, 0, 0, 0]^T + k_1[1, 0, 1, 0]^T + k_2[0, 1, 0, 1]^T$,
   pro $a \neq -1$ má soustava jedno řešení $x = \frac{1}{a^2-2a+5}[a^2 - 3a + 4, a^2 - a + 2, a + 1, 3 - a]^T$.

8-1

1. $\lambda_1 = 2$, $\lambda_2 = 4$, $h_1 = [-1, 3]^T$, $h_2 = [-2, 5]^T$, $\mathbf{J} = \begin{bmatrix} 2 & 0 \\ 0 & 4 \end{bmatrix}$,

2. $\lambda_1 = i$, $\lambda_2 = -i$, $h_1 = [-13 - i, 34]^T$, $h_2 = [-13 + i, 34]^T$, $\mathbf{J} = \begin{bmatrix} i & 0 \\ 0 & -i \end{bmatrix}$,

3. $\lambda_1 = -1$, $\lambda_2 = 2$, $\lambda_3 = 3$, $h_1 = [-1, 4, 3]^T$, $h_2 = [1, -1, 1]^T$, $h_3 = [2, -1, 3]^T$,
   $\mathbf{J} = \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$,

4. $\lambda_{1,2} = 0$, $\lambda_3 = 2$, $h_1 = [3, 2, 0]^T$, $h_2 = [1, 0, 2]^T$, $h_3 = [-3, 7, -26]^T$, $\mathbf{J} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$,

5. $\lambda_{1,2} = 0$, $\lambda_3 = 2$, $h_1 = [1, -2, 1]^T$, $h_2 = [-1, 7, 5]^T$, $\mathbf{J} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 2 \end{bmatrix}$,

6. $\lambda_1 = 0$, $\lambda_2 = 1$, $\lambda_3 = -2$, $h_1 = [-3, 7, -26]^T$, $h_2 = [1, -1, 5]^T$, $h_3 = [2, -1, 7]^T$
   $\mathbf{J} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{bmatrix}$,

7. $\lambda_1 = 2$, $\lambda_2 = 1 + i$, $\lambda_3 = 1 - i$, $h_1 = [2, -1, 3]^T$, $h_2 = [-2 + 4i, 11 - 7i, 10]^T$,
   $h_3 = [-2 - 4i, 11 + 7i, 10]^T$ $\mathbf{J} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1+i & 0 \\ 0 & 0 & 1-i \end{bmatrix}$,

8. $\lambda_1 = -4$, $\lambda_{2,3} = 3$, $h_1 = [0, 0, 1]^T$, $h_2 = [1, 1, 1]^T$, $\mathbf{J} = \begin{bmatrix} -4 & 0 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 3 \end{bmatrix}$.

8-2

1. $\mathbf{J} = \begin{bmatrix} 3 & 0 \\ 0 & 5 \end{bmatrix}$, $\mathbf{T} = \begin{bmatrix} -3 & -5 \\ 1 & 2 \end{bmatrix}$,

2. $\mathbf{J} = \begin{bmatrix} -1+2i & 0 \\ 0 & -1-2i \end{bmatrix}$, $\mathbf{T} = \begin{bmatrix} 1+i & 1-i \\ 2 & 2 \end{bmatrix}$,

3. $\mathbf{J} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -5 \end{bmatrix}$, $\mathbf{T} = \begin{bmatrix} 1 & -1 & 2 \\ -1 & 4 & -1 \\ 1 & 3 & 3 \end{bmatrix}$,

4. $\mathbf{J} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, $\mathbf{T} = \begin{bmatrix} 3 & 1 & 3 \\ 2 & 0 & -7 \\ 0 & 2 & 26 \end{bmatrix}$,

5. $\mathbf{J} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -2 \end{bmatrix}$, $\mathbf{T} = \begin{bmatrix} -1 & 1 & 2 \\ 4 & -1 & -1 \\ 3 & 1 & 3 \end{bmatrix}$,

6. $\mathbf{J} = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 8+4i & 0 \\ 0 & 0 & 8-4i \end{bmatrix}$, $\mathbf{T} = \begin{bmatrix} 1 & -1 & -1 \\ 2 & 2i & -2i \\ 1 & 1 & 1 \end{bmatrix}$.

8-3

1. $\mathbf{A} = \begin{bmatrix} 1 & -3 \\ 3 & -9 \end{bmatrix}$, $\lambda_1 = 0$, $\lambda_2 = -8$,

2. $\mathbf{A} = \begin{bmatrix} 45 & -26 \\ 78 & -46 \end{bmatrix}$, $\lambda_1 = 6$, $\lambda_2 = -7$,

3. $\mathbf{A} = \begin{bmatrix} 1 & 3 \\ -4 & 5 \end{bmatrix}$, $\lambda_1 = 3 + 2\sqrt{2}i$, $\lambda_2 = 3 - 2\sqrt{2}i$,

4. $\mathbf{A} = \begin{bmatrix} 7 & -2 & 1 \\ -3 & 2 & -3 \\ -11 & 2 & -5 \end{bmatrix}$, $\lambda_1 = 2$, $\lambda_2 = 6$, $\lambda_3 = -4$,

5. $\mathbf{A} = \begin{bmatrix} 17 & -6 & 5 \\ 4 & 0 & 4 \\ -17 & 6 & -5 \end{bmatrix}$, $\lambda_{1,2} = 0$, $\lambda_3 = 12$,

6. $\mathbf{A} = \begin{bmatrix} -5 & -1 & 9 \\ 0 & 1 & 2 \\ 0 & -1 & 4 \end{bmatrix}$, $\lambda_1 = -5$, $\lambda_2 = 2$, $\lambda_3 = 3$,

7. $\mathbf{A} = \begin{bmatrix} -1 & -2 & 1 \\ 3 & -4 & 3 \\ 3 & 2 & 1 \end{bmatrix}$, $\lambda_1 = -2$, $\lambda_2 = 2$, $\lambda_3 = -4$,

8. $\mathbf{A} = \begin{bmatrix} 20 & -1 & -8 & -3 \\ 52 & -3 & -17 & -9 \\ 30 & -2 & -13 & -4 \\ 42 & -2 & -17 & -6 \end{bmatrix}$, $\lambda_1 = 1$, $\lambda_2 = 2$, $\lambda_3 = -2$, $\lambda_4 = -3$,

9. $\mathbf{A} = \begin{bmatrix} -6 & 4 & 6 & -2 \\ 8 & -1 & -6 & 1 \\ -12 & 3 & 10 & -3 \\ 4 & -7 & -6 & 3 \end{bmatrix}$, $\lambda_1 = 0$, $\lambda_{2,3} = 4$, $\lambda_4 = -2$,

10. $\mathbf{A} = \begin{bmatrix} 0 & 4 & -8 \\ -3 & 9 & -15 \\ -1 & 3 & -5 \end{bmatrix}$, $\lambda_1 = 0$, $\lambda_2 = 2$, $\lambda_3 = 2$.

8-4

1. $\mathbf{B} = \begin{bmatrix} -5 & 3 \\ 5 & -3 \end{bmatrix}$, $\lambda_1 = 0$, $\lambda_2 = -8$,

2. $\mathbf{B} = \begin{bmatrix} -7 & 65 \\ 0 & 6 \end{bmatrix}$, $\lambda_1 = 6$, $\lambda_2 = -7$,

3. $\mathbf{B} = \frac{1}{10}\begin{bmatrix} 43 & 51 \\ -19 & 17 \end{bmatrix}$, $\lambda_1 = 3 + 2\sqrt{2}i$, $\lambda_2 = 3 - 2\sqrt{2}i$,

4. $\mathbf{B} = \frac{1}{3}\begin{bmatrix} -17 & -15 & 5 \\ 10 & 18 & -10 \\ -23 & -15 & 11 \end{bmatrix}$, $\lambda_1 = 2$, $\lambda_2 = 6$, $\lambda_3 = -4$,

5. $\mathbf{B} = \frac{1}{5}\begin{bmatrix} -51 & -158 & -111 \\ -36 & -48 & -36 \\ 99 & 222 & 159 \end{bmatrix}$, $\lambda_{1,2} = 0,\ \lambda_3 = 12$,

6. $\mathbf{B} = \begin{bmatrix} 1 & 0 & 2 \\ -1 & 2 & 2 \\ 6 & 0 & -3 \end{bmatrix}$, $\lambda_1 = -5,\ \lambda_2 = 2,\ \lambda_3 = 3$,

7. $\mathbf{B} = \begin{bmatrix} -4 & 6 & -2 \\ 0 & 2 & 0 \\ 0 & 6 & -2 \end{bmatrix}$, $\lambda_1 = -2,\ \lambda_2 = 2,\ \lambda_3 = -4$,

8. $\mathbf{B} = \begin{bmatrix} 18 & -17 & -14 & -3 \\ 10 & -3 & -7 & -3 \\ 6 & -22 & -7 & 2 \\ 26 & 8 & -15 & -10 \end{bmatrix}$, $\lambda_1 = 1,\ \lambda_2 = 2,\ \lambda_3 = -2,\ \lambda_4 = -3$,

9. $\mathbf{B} = \frac{1}{3}\begin{bmatrix} 24 & -8 & -6 & 32 \\ -6 & 28 & 12 & -28 \\ -6 & -32 & -12 & 20 \\ -15 & 4 & 3 & -22 \end{bmatrix}$, $\lambda_1 = 0,\ \lambda_{2,3} = 4,\ \lambda_4 = -2$,

10. $\mathbf{B} = \frac{1}{11}\begin{bmatrix} 64 & 10 & 48 \\ -408 & -72 & -372 \\ 40 & 9 & 52 \end{bmatrix}$, $\lambda_1 = 0,\ \lambda_2 = 2,\ \lambda_3 = 2$.

9-1

1. $v_1 = [1,0,0]^T,\ v_2 = [0,1,0]^T,\ v_3 = [0,0,1]^T$,
2. $v_1 = [1,2,2]^T,\ v_2 = [-2,5,-4]^T,\ v_3 = [-2,0,1]^T$,
3. $v_1 = [2,-1,4,1,3]^T,\ v_2 = [-5,80,21,75,-23]^T,\ v_3 = [83,184,105,-237,-55]^T$,
4. $v_1 = 1,\ v_2 = x + \frac{1}{2},\ v_3 = x^2 + x - \frac{1}{2}$,
5. $v_1 = 1,\ v_2 = x,\ v_3 = x^2 - 3$.

9-2

1. $e_1 = [1,0,0,0]^T,\ e_2 = [0,1,0,0]^T,\ e_3 = [0,0,1,0]^T,\ e_4 = [0,0,0,1]^T$,
2. $e_1 = \frac{1}{\sqrt{14}}[1,2,3]^T,\ e_2 = \frac{1}{\sqrt{35}}[-1,5,-3]^T,\ e_3 = \frac{1}{\sqrt{10}}[-3,0,1]^T$,
3. $e_1 = \frac{1}{2}[1,-1,1,0,1]^T,\ e_2 = \frac{1}{2\sqrt{39}}[-9,1,3,-4,7]^T$,
   $e_3 = \frac{1}{\sqrt{5694}}[12,29,-30,40,47]^T$,
4. $e_1 = \frac{1}{\sqrt{3}},\ e_2 = \frac{1}{3}(2x+1),\ e_3 = \frac{\sqrt{5}}{3\sqrt{3}}(2x^2+2x-1)$,
5. $e_1 = \frac{1}{\sqrt{6}},\ e_2 = \frac{1}{3\sqrt{2}}x,\ e_3 = \frac{\sqrt{5}}{6\sqrt{6}}(x^2-3),\ e_4 = \frac{\sqrt{7}}{6\sqrt{486}}(5x^3-27x)$.

9-3

1. $v_0 = \frac{1}{13}[16,77,87]^T$,
2. $v_0 = [-7,6,5,-3,-1,6]^T$,
3. $v_0 = (\frac{3}{2}\pi - 6 + 3\ln 2)x + (3 - \frac{\pi}{2} - 2\ln 2)$,
4. $v_0 = \frac{\pi}{2} - \frac{4}{\pi}\cos x$,

5. $v_0 = 0,2 + \frac{3,85}{x} - \frac{2,16}{x^2}$,
soustava má tvar:
$3\lambda_1 + \ln 4\lambda_2 + \frac{3}{4}\lambda_3 = \frac{13}{3}$,
$\ln 4\lambda_1 + \frac{3}{4}\lambda_2 + \frac{15}{32}\lambda_3 = 3\ln 4 - 2$,
$\frac{3}{4}\lambda_1 + \frac{15}{32}\lambda_2 + \frac{21}{64}\lambda_3 = \frac{5}{4}$.

9-4

1. $f(x) = -x^2 + 2x - 3$,
2. $f(x) = -1.014x^2 + 2.001x - 2.923$,
3. $f(x) = 0.5x + 3$,
4. $f(x) = 5 - \frac{2}{x-1} + \frac{1}{(x-1)^2}$,
5. $f(x) = 2x^3 - 3x^2 + 4x + 2$.

9-5

1. $\dim(\mathcal{U}^\perp) = 2$, báze je např. $v_1 = [-16, 0, -5, 3, 1]^T$, $v_2 = [-4, 1, -2, 0, 0]^T$,
2. $\dim(\mathcal{U}^\perp) = 1$, báze je např. $v_1 = [-19, 3, 29, 4]^T$,
3. $\dim(\mathcal{U}^\perp) = 3$, báze je např. $v_1 = 900x^4 - 1225x^3 + 378x^2$, $v_2 = 3456x^4 - 3528x^3 + 378x$, $v_3 = 14580x^4 - 13230x^3 + 378$.

10-1

1. $\kappa(x) = x^T\mathbf{A}x = -2x_1^2 + x_2^2 + 4x_1x_2$,
2. $\kappa(x) = x^T\mathbf{A}x = 7x_1^2 + 5x_2^2 + 6x_3^2 - 4x_1x_3 - 4x_2x_3$,
3. $\kappa(x) = x^T\mathbf{A}x = 7x_1^2 + 7x_2^2 + 7x_3^2 + 7x_4^2 - 2x_1x_2 - 2x_1x_3 - 10x_1x_4 - 10x_2x_3 - 2x_2x_4 - 2x_3x_4$.

10-2

1. $\mathbf{A} = \begin{bmatrix} -1 & -5 & 4 \\ -5 & -1 & -4 \\ 4 & -4 & 8 \end{bmatrix}$,
2. $\mathbf{A} = \begin{bmatrix} -3 & 2 & -2 \\ 2 & -4 & 0 \\ -2 & 0 & -2 \end{bmatrix}$,
3. $\mathbf{A} = \begin{bmatrix} -85 & 5 & 10 \\ 5 & -61 & -2 \\ 10 & -2 & -64 \end{bmatrix}$.

10-3

1. $\mathrm{in}(\kappa) = (1,1,0)$, $\kappa(x)$ je indefinitní, $\kappa(x) = 2(\frac{1}{\sqrt{5}}(x_1 + 2x_2))^2 - 3(\frac{1}{\sqrt{5}}(-2x_1 + x_2))^2$,
2. $\mathrm{in}(\kappa) = (2,0,0)$, $\kappa(x)$ je pozitivně definitní,$\kappa(x) = 10(-\frac{3}{\sqrt{10}}x_1 + \frac{1}{\sqrt{10}}x_2)^2 + 20(\frac{1}{\sqrt{10}}x_1 + \frac{3}{\sqrt{10}}x_2)^2$,
3. $\mathrm{in}(\kappa) = (3,0,0)$, $\kappa(x)$ je pozitivně definitní, $\kappa(x) = 3(\frac{1}{3}(x_1 + 2x_2 + 2x_3))^2 + 6(\frac{1}{3}(2x_1 - 2x_2 + x_3))^2 + 9(\frac{1}{3}(-2x_1 - x_2 + 2x_3))^2$,
4. $\mathrm{in}(\kappa) = (1,1,1)$, $\kappa(x)$ je indefinitní, $\kappa(x) = 12(\frac{1}{\sqrt{6}}(x_1 - x_2 + 2x_3))^2 - 6(\frac{1}{\sqrt{2}}(x_1 + x_2))^2$,

5. $\mathrm{in}(\kappa) = (0,2,1)$, $\kappa(x)$ je negativně semidefinitní, $\kappa(x) = -3(\frac{1}{3}(x_1+2x_2+2x_3))^2 - 6(\frac{1}{3}(2x_1 - 2x_2 + x_3))^2$,

6. $\mathrm{in}(\kappa) = (0,3,0)$, $\kappa(x)$ je negativně definitní,
$\kappa(x) = -60(\frac{1}{\sqrt{26}}(x_1+5x_2))^2 - 60(\frac{1}{\sqrt{195}}(5x_1 - x_2 + 13x_3))^2 - 90(\frac{1}{\sqrt{30}}(-5x_1 + x_2 + 2x_3))^2$
nebo $\kappa(x) = -60(\frac{1}{\sqrt{5}}(-2x_2+x_3))^2 - 60(\frac{1}{\sqrt{6}}(x_1+x_2+2x_3))^2 - 90(\frac{1}{\sqrt{30}}(-5x_1+x_2+2x_3))^2$,

7. $\mathrm{in}(\kappa) = (2,0,1)$, $\kappa(x)$ je pozitivně semidefinitní,
$\kappa(x) = (\frac{1}{\sqrt{2}}x_1 + \frac{1}{\sqrt{2}}x_2)^2 + 3(\frac{1}{\sqrt{6}}x_1 - \frac{1}{\sqrt{6}}x_2 + \frac{2}{\sqrt{6}}x_3)^2$,

8. $\mathrm{in}(\kappa) = (3,0,1)$, $\kappa(x)$ je pozitivně semidefinitní,
$\kappa(x) = 4(\frac{1}{2}(x_1 - x_2 - x_3 + x_4))^2 + 12(\frac{1}{\sqrt{2}}(-x_2+x_3))^2 + 12(\frac{1}{\sqrt{2}}(-x_1+x_4))^2$,

9. $\mathrm{in}(\kappa) = (4,0,0)$, $\kappa(x)$ je pozitivně definitní,
$\kappa(x) = 6(\frac{1}{\sqrt{3}}(-x_1-x_3+x_4))^2 + 9(\frac{1}{\sqrt{3}}(x_2+x_3+x_4))^2 + 12(\frac{1}{\sqrt{3}}(-x_1-x_2+x_3))^2 + 15(\frac{1}{\sqrt{3}}(x_1 - x_2 + x_4))^2$,

10. $\mathrm{in}(\kappa) = (0,4,0)$, $\kappa(x)$ je negativně definitní,
$\kappa(x) = -3(\frac{1}{\sqrt{6}}(x_1+x_2+x_3))^2 - 3(\frac{1}{\sqrt{6}}(-x_1+x_2+x_4))^2 - 9(\frac{1}{\sqrt{3}}(-x_1-x_2+x_3))^2 - 15(\frac{1}{\sqrt{3}}(x_1 - x_2 + x_4))^2$.

10-4

1. $\mathrm{in}(\kappa) = (2,0,0)$, $\kappa(x)$ je pozitivně definitní,
$\kappa(x) = 15(\frac{1}{\sqrt{5}}(-2x_1+x_2))^2 + 5(\frac{1}{\sqrt{5}}(x_1+2x_2))^2$,

2. $\mathrm{in}(\kappa) = (1,1,0)$, $\kappa(x)$ je indefinitní,
$\kappa(x) = 6(\frac{1}{\sqrt{2}}(x_1+x_2))^2 - 4(\frac{1}{\sqrt{2}}(-x_1+x_2))^2$,

3. $\mathrm{in}(\kappa) = (0,3,0)$, $\kappa(x)$ je negativně definitní,
$\kappa(x) = -18(\frac{1}{3}(2x_1 - 2x_2 + x_3))^2 - 9(\frac{1}{3}(x_1+2x_2+2x_3))^2 - 27(\frac{1}{3}(-2x_1 - x_2 + 2x_3))^2$,

4. $\mathrm{in}(\kappa) = (2,1,0)$, $\kappa(x)$ je indefinitní,
$\kappa(x) = 3(\frac{1}{\sqrt{5}}(x_1+2x_2))^2 + 3(\frac{1}{\sqrt{30}}(-2x_1+x_2+5x_3))^2 - 3(\frac{1}{\sqrt{6}}(2x_1 - x_2 + x_3))^2$,

5. $\mathrm{in}(\kappa) = (2,0,1)$, $\kappa(x)$ je pozitivně semidefinitní,
$\kappa(x) = 6(\frac{1}{\sqrt{2}}(x_2+x_3))^2 + 12(\frac{1}{\sqrt{6}}(2x_1 - x_2 + x_3))^2$,

6. $\mathrm{in}(\kappa) = (2,0,1)$, $\kappa(x)$ je pozitivně semidefinitní,
$\kappa(x) = 5(\frac{1}{\sqrt{5}}(-2x_1+x_2))^2 + 140(\frac{1}{\sqrt{70}}(3x_1+6x_2+5x_3))^2$,

7. $\mathrm{in}(\kappa) = (1,1,1)$, $\kappa(x)$ je indefinitní,
$\kappa(x) = -10(\frac{1}{\sqrt{5}}(-2x_1+x_2))^2 + 14(\frac{1}{\sqrt{14}}(-x_1 - 2x_2 + 3x_3))^2$,

8. $\mathrm{in}(\kappa) = (3,1,0)$, $\kappa(x)$ je indefinitní,
$\kappa(x) = -8(\frac{1}{\sqrt{2}}(-x_1+x_4))^2 + 6(\frac{1}{\sqrt{2}}(x_2+x_3))^2 + 4(\frac{1}{2}(x_1+x_2-x_3+x_4))^2 + 20(\frac{1}{2}(x_1 - x_2 + x_3 + x_4))^2$,

9. $\mathrm{in}(\kappa) = (3,1,0)$, $\kappa(x)$ je indefinitní,
$\kappa(x) = 2(\frac{1}{\sqrt{2}}(x_2+x_3))^2 + 2(\frac{1}{2}(x_1+x_2-x_3+x_4))^2 - 2(\frac{1}{\sqrt{2}}(-x_1+x_4))^2 + 6(\frac{1}{2}(x_1-x_2+x_3+x_4))^2$,

10. $\mathrm{in}(\kappa) = (0,3,1)$, $\kappa(x)$ je negativně semidefinitní,
$\kappa(x) = -3(\frac{1}{\sqrt{3}}(x_2+x_3+x_4))^2 - 6(\frac{1}{3}(-x_1-x_3+x_4))^2 - 9(\frac{1}{\sqrt{3}}(-x_1-x_2+x_3))^2$.