

$$y' = x^3 y^4 - \frac{y}{x}$$

*Solution :*

$$\frac{y'}{y^4} + \frac{1}{xy^3} = x^3$$

$$\frac{1}{y^3} = z$$

$$z' = -3\frac{y'}{y^4}$$

$$z' - 3\frac{z}{x} = -3x^3$$

$$z = uz_1$$

$$z' = u'z_1 + uz_1'$$

$$z_1' - 3\frac{z_1}{x}$$

$$\frac{dz_1}{z_1} = 3\frac{dx}{x}$$

$$z_1 = x^3$$

$$z_1' = 3x^2$$

$$u'x^2 + 3ux^2 - 3ux^2 = -3x^2$$

$$u' = -3$$

$$u = -3x + C_1$$

$$z = C_1x^3 - 3x^4$$

$$y = \frac{1}{\sqrt[3]{C_1x^3 - 3x^4}}$$

$$y(1) = 0.5$$

$$\frac{1}{2} = \frac{1}{\sqrt[3]{C_1-3}}$$

$$C_1 = 11$$

$$y = \frac{1}{\sqrt[3]{11x^3-3x^4}}$$

y is discontinuous when:

$$\sqrt[3]{C_1 x^3 - 3x^4} = 0$$

$$x^3(C_1 - 3x) = 0$$

$$x = 0 || x = \frac{C}{3}$$