Review on Differential Equation assignment

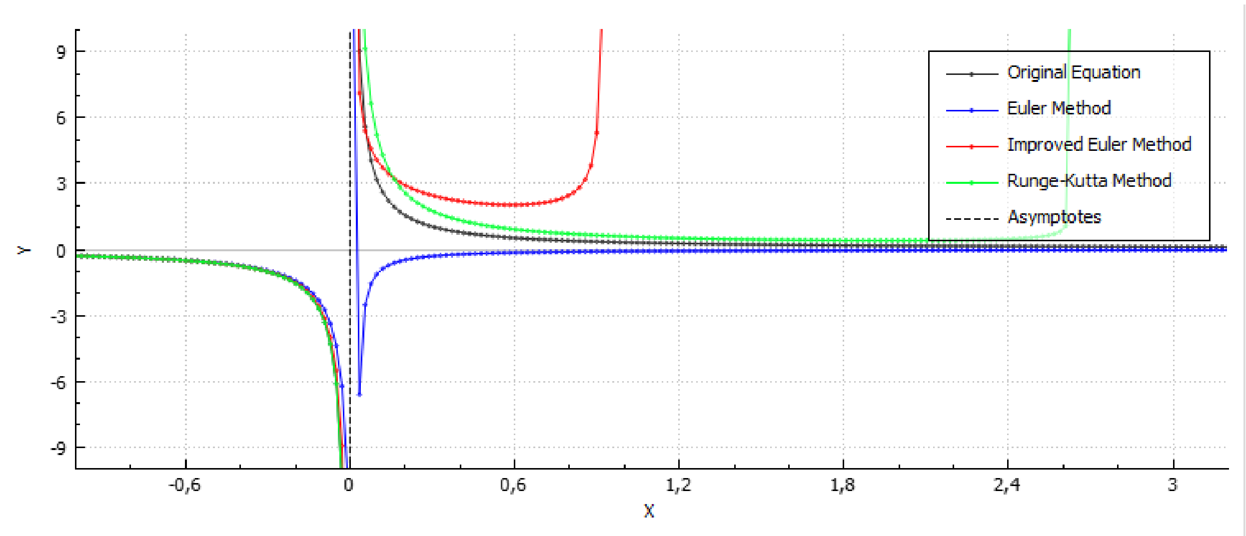
I. Methods Results

The most accurate among all methods is definitely Runge-Kutta method. With some initial data it might break after asymptotes, but this can be usually fixed with setting much enough number of points.

After Runge-Kutta goes Improved Euler, which is still can show good results from time to time, but still not comparable with others.

Euler method tends to differ from other methods, especially after asymptotes and often has the biggest total error.

I would like to analyze methods results for next example:



Here we can notice how all methods are breaking after asymptote.

Yet, Runge-Kutta shows the best results until x = 2.6 and pretty much accurate before that point.

Improved Euler method did not decreased y enough, and thus triggered its increasing to infinity again.

Finally, Euler decreased y too much, so plot follows original equation, but with different sign. In this particular example Euler will show the least total error.

II. **Software Decisions**

Program uses free QCustomPlot library to display given set of points on Euclidian space and connect them with lines.

Since each method cannot continue computation after infinity met (infinity + C = infinity for any C), program should somehow jump over asymptote. In current implementation, it simply checks for asymptote and assigns current value to the next one, multiplied by -1 (function changes sign after each asymptote).

Unfortunately, QCustomPlot doesn't give opportunity to simply disconnect two points, therefore, there are three different plots for each method, and every plot uses its own domain.

Architecture is split into MVC model – on other words, there is a .ui file with GUI markup, a controller class that catch button click and model classes for points and errors computation only.

$$y' = x^3 y^4 - \frac{y}{x}$$

Solution :

$$\frac{y'}{y^4} + \frac{1}{xy^3} = x^3$$

$$\frac{1}{y^3} = z$$
$$z' = -3\frac{y'}{y^4}$$

$$z' - 3\frac{z}{x} = -3x^3$$

$$z = uz_1$$
$$z' = u'z_1 + uz_1'$$

$$z_1' - 3\frac{z_1}{x}$$
$$\frac{dz_1}{z_1} = 3\frac{dx}{x}$$
$$z_1 = x^3$$
$$z_1' = 3x^2$$

$$u'x^2 + 3ux^2 - 3ux^2 = -3x^2$$
$$u' = -3$$
$$u = -3x + C_1$$
$$z = C_1 x^3 - 3x^4$$

$$y = \frac{1}{\sqrt[3]{C_1 x^3 - 3x^4}}$$

$$y(1) = 0.5$$
$$\frac{1}{2} = \frac{1}{\sqrt[3]{C_1 - 3}}$$
$$C_1 = 11$$

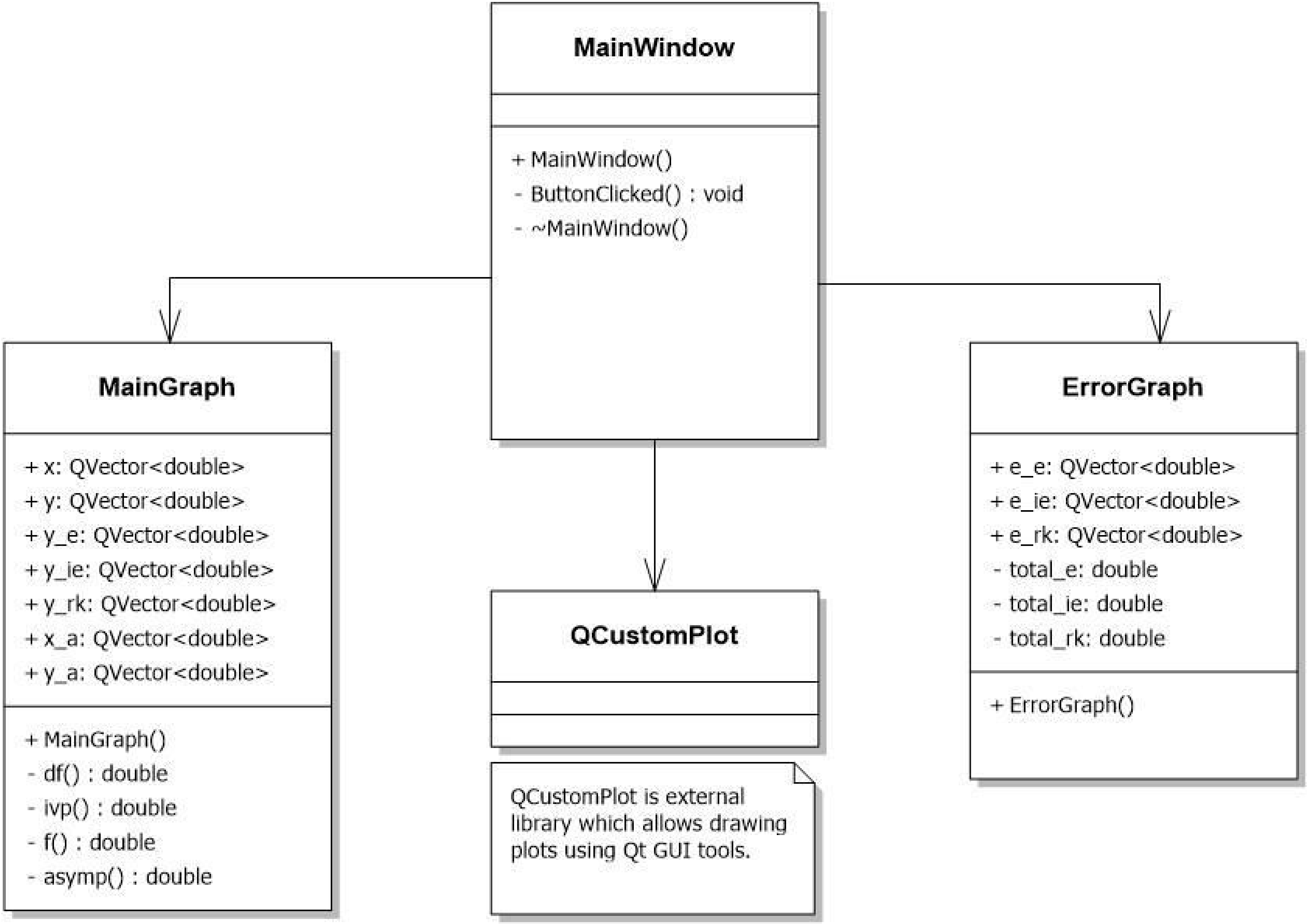
$$y = \frac{1}{\sqrt[3]{11x^3 - 3x^4}}$$

y is discontinious when:

$$\sqrt[3]{C_1x^3 - 3x^4} = 0$$

$$x^3(C_1 - 3x) = 0$$

$$x = 0 || x = \frac{C}{3}$$


MainWindow
+ MainWindow()
- ButtonClicked() : void
- ~MainWindow()
MainGraph
+ x: QVector<double>
+ y: QVector<double>
+ y_e: QVector<double>
+ y_ie: QVector<double>
+ y_rk: QVector<double>
+ x_a: QVector<double>
+ y_a: QVector<double>
+ MainGraph()
- df() : double
- ivp() : double
- f() : double
- asymp() : double
QCustomPlot
QCustomPlot is external library which allows drawing plots using Qt GUI tools.
ErrorGraph
+ e_e: QVector<double>
+ e_ie: QVector<double>
+ e_rk: QVector<double>
- total_e: double
- total_ie: double
- total_rk: double
+ ErrorGraph()