Instructions for Solving the Exercises

Instructions for Solving the Exercises

The book "Particle Physics" contains exercises designed to facilitate understanding of the material. Updated hints for solving the exercises can be found on our GitHub page in the folder `Exercises`. Some exercises refer to Jupyter notebooks, which can be found in the folder `Notebooks`.

Chapter 1

1.1 As always, $E = \sqrt{\boldsymbol{p}^2 + m^2}$, $T = E - m$, and $\beta = |\boldsymbol{p}|/E$. From this, the desired values can be calculated.

1.2 If in the definition of β from exercise 1.1, E is expressed through $|\boldsymbol{p}|$, this equation only needs to be solved for $|\boldsymbol{p}|$.

1.3 For numerical solutions, it is advisable to use the constant $k = \hbar c = 197.33$ MeV fm. If all energies are used in MeV and distances in fm, $k = 197.33$ applies. The first term of equation (1.17) is then expanded with c^2 to $k^2/2E_0 r^2$ using $|\boldsymbol{p}| = \hbar/r$. E_0 is the reduced rest energy of the p, n system, approximately $2E_0 \approx 970$ or roughly 1000. In the second term, we replace g_Y^2 with ak, where a is a dimensionless constant. The resulting function $W(r)$ has a minimum for a certain value of r. One varies a until $W_{\text{min}} = 2.25$ is reached. This is most easily done by graphical study of the function $W(r)$. The solution is acceptable as the minimum lies at a distance typical for nuclear physics.

1.4 In the unit system of particle physics, the dimension of the gravitational constant (length^3 time^{-2} mass^{-1}) becomes the dimension (energy)$^{-2}$. $G/(c^5\,\hbar)$ has this dimension also in the SI system and is identical to G in the unit system of particle physics. In the SI system, $1/\sqrt{G/(c^5\hbar)} = 1.221 \times 10^{19}$ GeV applies.

1.6 Use formulas (1.23) and (1.24) to calculate E_γ. For an atom in an excited state with an energy gap Δ to the ground state, $\sqrt{s} = M + \Delta$ applies. The resulting exact equation for $|\boldsymbol{p}| = E_\gamma$ is replaced in the last step by the approximation $(M + \Delta)^{-1} = (1 - \Delta/M)/M$. Compare the derivation with the approach you probably know from a lecture on atomic physics.

1.7 You see that the correction in nuclear physics processes can already amount to 10^{-4}. This insight is very important for the interpretation of the Mössbauer effect.

1.8 We neglect the spin of the particles. Then, for the orbital angular momentum $L = Er = \sqrt{2}$, due to $J = 1$. It follows that $r = \sqrt{2}/E$. The reaction volume is a sphere with radius r, so the energy density $\rho_E \approx E^4/10$.

1.9 The virtual photon has (with our definition of the square of a four-vector) an imaginary mass.

1.10 The energy equation is $p_1 + p_2 = p_3 + p_4$. With $q^2 = (p_1 - p_3)^2$ and the definition of kinetic energy $T = E - M$, you should be able to quickly prove the important result $q^2 = -2M_2T_4$.

1.11 This is just an application of (1.23) and (1.24).

1.12 It applies that

$$s = 2(M^2 + E_1E_2 + |\boldsymbol{p}_1||\boldsymbol{p}_2|) \ .$$

From $E_1 = M_p + E_F = 968$ MeV and $E_2 = M_p + 200 = 1138$ MeV, it follows that $\sqrt{s} = 2066$ MeV.

1.13 The main decay channels are $\pi^+ \longrightarrow \mu^+\,\nu_\mu$ and $\pi^+ \longrightarrow e^+\,\nu_e$. However, decays with additional emission of photons or $e^+\,e^-$ pairs are also allowed.

1.14 With the help of (1.23), it follows that $|\boldsymbol{p}| = 0.23$ GeV.

1.15 Due to $|\boldsymbol{p}_\rho| = E_\gamma$, it applies that $E_\rho = \sqrt{E_\gamma^2 + m_\rho^2}$ and therefore $\Delta E \approx m_\rho^2/2E_\gamma$. Numerically, we find $\Delta t \approx 2.5\ 10^{-24}$ s.

1.16 We assume the base reaction to be the scattering $\nu_e\ p \longrightarrow e^-\ n$. The cross section is given by (1.33), approximately 10^{-43} cm^2. With n_{Fe} from (1.47) it follows that $n_p = Z\, n_{\mathrm{Fe}}$ and therefore $\lambda = 4.5 \times 10^{16}$ m.

1.17 With $\rho\lambda = \rho / n_0\sigma$, it follows with (1.47) an equation that can be solved for σ. The numerical value is given for $M_r \approx 27$ by about 640 mb.

1.18 The volume of a nucleus is $\propto r^3$ and $\propto M_r$. The surface is therefore $\propto M_r^{2/3}$. With the formulas from the last exercise, it follows that $\rho\lambda \propto M_r^{1/3}$.

1.19 In (1.50), we set $\dot{N}_{\mathrm{in}} = n_1 f_p$ and $n_0 = n_2/(A\ \Delta z)$.

1.20 The integral of the Bhabha cross section is $\sigma \approx 1.9 \times 10^{-30}$ cm^2. Thus, $L = 2.6 \times 10^{30}$ cm^{-2}s^{-1}. The integrated cross section of pair production is completely negligible in comparison, $\sigma \approx 5 \times 10^{-38}$ cm^2.

1.21 It applies that

$$x_f = \boldsymbol{M}_{11} x_0 + \boldsymbol{M}_{12} x_0' \ ,$$

where the matrix elements are calculated from the product of a focusing quadrupole (1.61) and a free path with the matrix

$$\boldsymbol{M}_{\mathrm{O}} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

in section 1.4.3. From this, it follows that

$$\boldsymbol{M}_{11} = \cos\Omega - l\sqrt{k} \ ,$$

thus $l = 1.47$ m.

1.22 Multiply the matrices using SymPy. Then vary the field strength and try to find a value where x and y disappear at the same value of z. This is best done graphically, but it is already a relatively ambitious project.

1.23 The solution path is already described in detail in the exercise statement.

1.24 The solution path is given in Python example 1.2.

1.25 The solution follows Python example 1.2, but now the `scatter` routine is used. The properties of this routine can be found with a web search.

1.26 Change the values in Python example 1.1 and repeat exercises 1.24 and 1.25.

1.27 The solution follows from a simple modification of Python example 1.3.

1.28 Follow the Python example 1.3 and add another array `Za` for the third dimension. Due to the third dimension, the result must now be scaled by the factor $2^3 = 8$. Verify your result with the analytical formula for the volume of a sphere. A solution is presented in the notebook `random-sphere.ipynb`.

1.29 Follow the procedure in Exercise 1.28 and extend the number of fields to 5. Analogously, it must now be scaled with 2^5 to obtain the total volume. Compare your result with the analytical result (see, for example, `https://en.wikipedia.org/wiki/Volume_of_an_n-ball`). A solution is presented in the notebook `random-sphere.ipynb`.

1.30 Write a program based on the explanations in the text. Compare your result with the proposed solution in the notebook `random-exponential.ipynb`.

1.31 Compare your result with the proposed solution in the notebook `random-exponential.ipynb`.

1.32 The program package `scipy.stats` provides routines for each distribution for the calculation of the `pdf, cdf, ppf, etc.` and for generating random numbers (`rvs`). Try it out!

1.33 The solution is explained in the main text.

1.34 Form a sum function $s(n) = \sum_i^n h(i)/N$ over all bins of the histogram, where N is the total number of entries. If a random number Y is now generated in the range (0,1), the corresponding bin n can be identified with $Y = s(n)$. The desired random number X is then the x-value for bin n.

1.35 A rectangle of height $\sqrt{2}$ is used as an envelope in the range $0 \le x \le 1$.

1.36 Now multiple rectangles can be used as envelopes. First, a random number is generated and, based on the relative proportions of the rectangles, it is determined for which rectangle a random number should be calculated. Then, the procedure as given in the previous exercise is followed.

1.38 Random numbers are generated according to $1 + \cos^2\theta$ and uniformly distributed in ϕ, and the three momentum components of the particles are calculated with them.

Chapter 2

2.1 Play with the routine `Dalitz.ipynb`

2.3 Starting from (2.22), the solution is trivial.

2.4 The easiest way is to add a few lines to `Dalitz.ipynb` for the calculation of m_{ik}^2.

2.7 With $a_i' = R_{ik}a_k$ and $b_i' = R_{il}b_l$, it follows that $a_i'b_i' = R_{ik}R_{il}a_k b_l = a_k b_k$. In the last step, the orthogonality relation (2.35) was used.

2.8 This is nothing else than the conversion from Cartesian to polar coordinates.

2.9 Compare your result with our notebook `dfunctions.ipynb`.

2.10 Compare your result with our notebook `dfunctions.ipynb`.

2.12 To calculate (2.52), $e^{-i\sigma_2\theta}$ must be calculated. For this, (2.63) is used with the result

$$d^{1/2} = \begin{pmatrix} \cos(\theta/2) & -\sin(\theta/2) \\ \sin(\theta/2) & \cos(\theta/2) \end{pmatrix} .$$

2.13 It applies that $\hat{j}_+ = \hat{j}_{(1),+} + \hat{j}_{(2),+}$. This operator is applied to the right side of (2.68). The indices (1) and (2) refer to the first and second factor of a product of states, respectively.

2.16 To calculate the decay angle distribution of a ρ-meson with "spin up", $J = 1, J_3 = 1, \lambda = \lambda_1 = \lambda_2 = 0$ must be inserted into (2.90). It is thus given by

$$\frac{\mathrm{d}\Gamma}{\mathrm{d}\Omega} \propto \frac{1}{2}\sin^2\theta |t_{00}|^2$$

The rest of the task is handled similarly.

2.17 The formula (2.108) reduces to a term with $J = 1$. Create a corresponding table.

2.18 Only the amplitudes $f_{\frac{1}{2}\,\frac{-1}{2},\frac{1}{2}\,\frac{-1}{2}}$ and $f_{\frac{1}{2}\,\frac{-1}{2},\frac{-1}{2}\,\frac{1}{2}}$ (and their partners with mirrored helicity) remain. The corresponding functions d^1 are proportional to $(1 + \cos\theta)$ and $(1 - \cos\theta)$. The angular distribution of the cross section is thus proportional to $(1 + \cos^2\theta)$.

2.19 With Euler's formula $\mathrm{e}^{\mathrm{i}\delta_l} = \cos\delta_l + \mathrm{i}\sin\delta_l$, it is quite simple.

2.20 With the assumption that the resonance is generated via a partial wave with $l = J$, one compares (2.103) with the formula (2.105) using $\Gamma_i = \Gamma_{\mathrm{el}}$.

2.21 Consider (2.108). Due to $\sigma = \int |f|^2 \mathrm{d}\Omega$, it applies with $|t|_{\max} = 1$ for each contributing helicity amplitude

$$\sigma_{\max} = \frac{12\pi}{p^2} \quad .$$

Here, the orthogonality relation (2.53) was used. If the amplitude is real, the limit is reduced to half (Exercise 2.19).

2.24 One decomposes the partial wave amplitude (2.99) into real part x and imaginary part y and then plots y against x in a parametric plot as functions of $\sqrt{s}$. What curve shape do you find? More about this in the short SymPy-notebook `Argand.ipynb`.

2.26 The Z boson can also have the helicity $\lambda = 0$.

2.27 Is the phase factor the same as in (2.133)?

2.28 From (2.108) follows

$$f_{-\lambda_3-\lambda_4,-\lambda_1-\lambda_2}(\theta,\phi) = \frac{1}{|\boldsymbol{p}|}\sum_J (2J+1) t^J_{-\lambda_3-\lambda_4,-\lambda_1-\lambda_2}(\sqrt{s}) d^J_{-\lambda-\mu}(\theta) \mathrm{e}^{-\mathrm{i}(\lambda-\mu)\phi} \quad .$$

With the transformation equation (2.134) it follows due to parity conservation

$$t^J_{-\lambda_3-\lambda_4,-\lambda_1-\lambda_2} = \eta_g t^J_{\lambda_3\lambda_4,\lambda_1\lambda_2}$$

with

$$\eta_g = \eta_1\eta_2\eta_3\eta_4(-1)^{2J-j_{(1)}-j_{(2)}-j_{(3)}-j_{(4)}} \quad .$$

Since $m = J - j_{(1)} - j_{(2)}$ is always an integer, we use $(-1)^m = (-1)^{-m}$ to derive

$$\eta_g = \eta_1\eta_2\eta_3\eta_4(-1)^{j_{(1)}+j_{(2)}-j_{(3)}-j_{(4)}} \quad .$$

With the help of (2.54) and the subsequent formula, the proof of (2.135) is completed.

2.29 The sum of the distances to the three sides is equal for every point within an equilateral triangle to the height of the triangle. This corresponds to the energy conservation $T_1 + T_2 + T_3 = Q$. We place a rectangular coordinate system at the intersection of the three angle bisectors. Then it applies $y = T_1 - Q/3$ and $x = (T_2 - T_3)/\sqrt{3}$ and therefore

$$x^2 + y^2 = \frac{Q^2}{9} + T_1^2 + \frac{1}{3}(T_2^2 + T_3^2 - 2T_2T_3 - 2T_1Q) \ .$$

In the bracket, we replace Q with $T_1 + T_2 + T_3$ and obtain

$$x^2 + y^2 = \frac{Q^2}{9} + \frac{1}{3}\left[(T_1 + T_2 + T_3)^2 - 4T_1T_2\right] \ .$$

The expression in the square bracket vanishes on the boundary line, as can be immediately seen from (2.21) for $\cos\theta_2 = 1$. Thus, it applies $x^2 + y^2 = Q^2/9$, which is a circle with the radius $Q/3$.

2.30 With the usual methods of linear algebra one determines

$$R = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & \gamma \\ 1 & -\gamma \end{pmatrix}$$

with $\gamma = \sqrt{B/C}$.

2.31 It is advisable to write the probability P_{K^0} for example as a function of (t/τ_S) and adjust the exponential functions, i.e., $\exp(-\Gamma_L t) \longrightarrow \exp(-\tau_S/\tau_L(t/\tau_S))$.

2.33 Here, the conversion from the rest system to the laboratory system is queried once again.

2.34 With $p \propto 1 + \epsilon$ etc., this goes very quickly.

2.35 Here we deliberately argue in detail and step by step: The most general matrix with 4 complex numbers is

$$U = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \tag{B.1}$$

We find 4 unitarity conditions:

$$\begin{aligned} aa^* + bb^* &= 1 && (1) \\ cc^* + dd^* &= 1 && (2) \\ ca* + db^* &= 0 && (3) \\ ac* + bd^* &= 0 && (4) \end{aligned}$$

Condition (4) is satisfied by $c^* = -\lambda b, \quad d^* = \lambda a$, where λ is an arbitrary real number. Thus, (3) is also satisfied, while (2)

$$\lambda^2(aa^* + bb^*) = 1$$

leads to the condition $\lambda = \pm 1$ due to (1). Thus, we have

$$U = \begin{pmatrix} a & b \\ -\lambda b^* & \lambda a^* \end{pmatrix} \tag{B.2}$$

proven. Now, the condition of unimodularity $\det U = 1$ is evaluated, $\det U = \lambda(aa^* + bb^*)$. Due to (1), it applies $\det U = \lambda$ and thus $\lambda = 1$.

2.36 The proof is already contained in the detailed comment on exercise 2.35.

2.37

$$\begin{aligned}
\left|\pi^+ p\right\rangle &= \left|\tfrac{3}{2};\tfrac{3}{2}\right\rangle \\
\left|\pi^0 p\right\rangle &= \sqrt{\tfrac{2}{3}}\left|\tfrac{3}{2};\tfrac{1}{2}\right\rangle - \sqrt{\tfrac{1}{3}}\left|\tfrac{1}{2};\tfrac{1}{2}\right\rangle \\
\left|\pi^0 n\right\rangle &= \sqrt{\tfrac{2}{3}}\left|\tfrac{3}{2};\tfrac{-1}{2}\right\rangle + \sqrt{\tfrac{1}{3}}\left|\tfrac{1}{2};\tfrac{-1}{2}\right\rangle \\
\left|\pi^- p\right\rangle &= \sqrt{\tfrac{1}{3}}\left|\tfrac{3}{2};\tfrac{-1}{2}\right\rangle - \sqrt{\tfrac{2}{3}}\left|\tfrac{1}{2};\tfrac{-1}{2}\right\rangle
\end{aligned}$$

Since only the amplitude $T_{3/2}$ is supposed to contribute, it applies

$$\begin{aligned}
\left\langle \pi^+ p\right| T \left|\pi^+ p\right\rangle &= a \\
\left\langle \pi^- p\right| T \left|\pi^- p\right\rangle &= \frac{a}{3} \\
\left\langle \pi^0 n\right| T \left|\pi^- p\right\rangle &= \frac{\sqrt{2}a}{3} \\
\left\langle \pi^0 n\right| T \left|\pi^0 n\right\rangle &= \frac{2a}{3}
\end{aligned}$$

and therefore, for example, $\sigma(\pi^+ p)/\sigma(\pi^- p) = 3$ (Figure 2.14).

2.38 Of course, J_3 is given by

$$J_3 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}$$

Due to $J_+|1;-1\rangle = \sqrt{2}|1;0\rangle$ and $J_+|1;0\rangle = \sqrt{2}|1;1\rangle$, it applies

$$J_+ = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix} .$$

Similarly, we derive

$$J_- = \begin{pmatrix} 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix}$$

With $J_\pm = J_x \pm \mathrm{i}J_y$ it follows

$$J_x = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix}$$

and

$$J_y = \frac{\mathrm{i}}{\sqrt{2}} \begin{pmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{pmatrix} \ .$$

2.39 We supplement (2.199) with

$$\begin{aligned} \langle \pi^+\pi^- | T | K_1 \rangle &= \sqrt{\frac{1}{3}}(T_2 - T_2^*) + \sqrt{\frac{2}{3}}(T_0 - T_0^*) \\ \left\langle \pi^0\pi^0 \right| T | K_1 \rangle &= \sqrt{\frac{2}{3}}(T_2 - T_2^*) - \sqrt{\frac{1}{3}}(T_0 - T_0^*) \ . \end{aligned}$$

The specifications given in the text about T_2 and T_0 then lead directly to (2.200).

Chapter 3

3.2 The first part of the task can be solved by explicitly calculating the sum or by applying the commutation relation (3.9). The second part is best solved using the commutation relation.

3.5 Due to $\gamma_5^2 = 1$, it applies that $(1 + \gamma_5)(1 - \gamma_5) = 0$ and $(1 + \gamma_5)(1 + \gamma_5) = 2(1 + \gamma_5)$ and so on.

3.6 The SymPy-notebook `Dirac.ipynb` is always helpful!

3.8 Use the commutation relation (3.9) and the fact that matrices are commutable under the trace.

3.9 Instead of (3.48), the much simpler integral

$$\mathrm{Sp} \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \gamma_\mu \frac{\not{k} + m}{k^2 - m^2}$$

must be evaluated. With the help of the results from the last task, only an integral with an odd integrand in the components of k remains, which therefore vanishes in the limits from $-\infty$ to $+\infty$.

3.10 (3.56) satisfies the differential equation

$$q^2 \frac{\mathrm{d}\alpha}{\mathrm{d}q^2} = \frac{\alpha^2}{3\pi} \ .$$

3.11 It is even easier if you directly study the SymPy-notebook `eemumu.ipynb`.

3.12 There is also a notebook for this (`eeee.ipynb`)

3.13 You only need to evaluate q^2 as the square of the difference of the proton's momenta.

3.14 In the center-of-mass system, it applies that $t = 2|\boldsymbol{p}^*|^2(1 - \cos\theta)$ and therefore $t_{\max} = 4|\boldsymbol{p}^*|^2$. This invariant is calculated as usual with the help of (1.24). After some transformations, one gets

$$S_{12}^2 = s_0^2 - 4m^2M^2 = s_0^2 \left(1 - \frac{4m^2M^2}{s_0^2}\right)$$

This formula can be verified with SymPy. In the laboratory system, for scattering on stationary electrons, $s_0 = 2E_\mu m$ and therefore $S_{12}^2 = s_0^2\beta_\mu^2$. The rest of the calculation follows from the information on the topic "a) Energy loss of charged particles" in section 3.2.4.

3.16 We choose an atomic model with an elastically bound electron of natural frequency ω_0, which begins to oscillate under the influence of an electromagnetic wave of frequency ω and amplitude E_0. The radiated power is determined in the SI system by

$$\bar{P} = \frac{1}{12\pi\epsilon_0 c^3}\omega^4 p_0^2$$

where the electric dipole moment p_0 is given, neglecting damping, by

$$p_0 = \frac{e^2 E_0}{m(\omega^2 - \omega_0^2)} \ .$$

The cross section is defined by $\sigma = \bar{P}/I$. After substituting the intensity $I = c\,\epsilon_0\,E_0^2/2$, it follows in the unit system of particle physics

$$\sigma = \frac{8\pi\alpha^2}{3m^2}\frac{\omega^4}{(\omega^2 - \omega_0^2)^2} \ .$$

This formula contains the two limiting cases of Thomson scattering ($\omega_0 \approx 0$) and Rayleigh scattering ($\omega_0 \gg \omega$).

3.17 The lifetime is calculated from the mean free path as $\tau = 1/(c\,\sigma\,n_\gamma)$. For σ, we use the Thomson cross section. For the spectral distribution of the photon number density, it applies

$$\frac{\mathrm{d}n_\gamma}{\mathrm{d}\omega} = \frac{\mathrm{d}u}{\mathrm{d}\omega}\frac{1}{\hbar\omega} \ ,$$

where the first factor on the right side is the spectral energy density of the Planck formula. After integration, we obtain n_γ with the numerical result $n_\gamma = 20.2\,T^3\ \mathrm{cm}^{-3}\mathrm{K}^{-3}$. This results in a lifetime of 25.5 h.

3.18 With $\lambda = 630$ nm ($\omega = 2$ eV), the evaluation of the formula for ω' given on the previous page yields the value 75.5 GeV.

3.19 Since the mass of the electron cannot be neglected, one must evaluate (3.97). This yields

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} \propto \frac{1 + \beta^2\cos^2\theta}{1 - \beta^2\cos^2\theta} + 2K + 2K^2 \ ,$$

where β is the velocity of the electron and K is calculated from

$$K = \frac{m^2}{E\omega(1 - \beta^2\cos^2\theta)} \ .$$

E and ω denote the energies of the electron and photon. For $\beta \longrightarrow 0$, the angular distribution becomes isotropic.

3.20 One replaces the magnitudes $|\boldsymbol{p}|$ with the energies E and expands the roots for large E. By neglecting all terms $\propto M^4$, the desired result follows quickly.

Chapter 4

4.1 A routine for calculating the $SU3$ structure constants is also included in `heppackv0.py`. Compare your result!

4.2 We start from equation (4.7). For small Θ, it applies with $\theta = \Theta/2$

$$U = \begin{pmatrix} 1 & -i\theta & 0 \\ -i\theta & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

and $U^{-1} = U^*$ therefore

$$T_1'^1 = T_1^1 + i\theta T_1^2 - i\theta(T_1^2 + i\theta T_2^2) \ .$$

Corresponding expressions result for $T_2'^2$ and $T_3'^3$. By neglecting quadratic terms in θ, it follows that $T_i'^i = T_i^i$.

4.3 With $q^1 = p$ and $q^2 = n$, it is immediately apparent that (4.15) generates the states (2.189) and (2.190). Similarly, (4.8) generates the states (2.194) and (2.195) up to a phase.

4.5 As an alternative to the methods in the text of the book, we use a method known from atomic physics. The color factor $c_\mathrm{F} = \langle \boldsymbol{F}_1 \boldsymbol{F}_2 \rangle$ is defined by the interaction of the $\boldsymbol{F}$-spins. With $\boldsymbol{F} = \boldsymbol{F}_1 + \boldsymbol{F}_2$, c_F can be expressed by the magnitude squares $\boldsymbol{F}^2, \boldsymbol{F}_1^2, \boldsymbol{F}_2^2$ of octet, triplet, and antitriplet, whose eigenvalues are defined in equation (4.4). This results in $c_\mathrm{F} = 1/6$ in the octet and as usual $c_\mathrm{F} = -4/3$ in the singlet.

4.6 In the notebook `colorfactors.ipynb`, routines for complicated color factors are included. Extend the notebook with a small program for color factors with fixed colors of the quarks.

4.7 In the notebook `colorfactors.ipynb` and in `heppackv0.py`, the λ-matrices are defined. You can try examples there or write a routine that checks all possibilities.

4.9 It is actually already explained in the text that only $T_{fi}^{3g}(g_-g_- \to q_-\bar{q}_+)$, $T_{fi}^{3g}(g_-g_- \to q_+\bar{q}_-)$, $T_{fi}^{3g}(g_+g_+ \to q_-\bar{q}_+)$, $T_{fi}^{3g}(g_+g_+ \to q_+\bar{q}_-)$ remain.

4.10 f^{abc} is calculated in `heppackv0.py` with the function `fsu3(a,b,c)`.

4.12 The result can be found in `heppackv0.py`.

4.14 Compare your result with Table 9.1 in (Barger 1991). Only after calculating the large cross section of the reaction $gg \to gg$ did the experiments for the production of two jets agree with the theory.

4.16 From the wave function of the flavor-singlet, it follows

$$\langle \eta_1 | M | \eta_1 \rangle = \frac{1}{3}(2m_u + 2m_d + 2m_s) \ .$$

In the additive quark model, this mass is identical to $(2\,M_K + M_\pi)/3$.

4.17 a_0 and $f_0(500)$ should have approximately the same mass, possibly 100 to 200 MeV heavier than ρ, ω due to the orbital angular momentum $L = 1$. The $f_0(980)$ should be heavier, as it should consist only of s-quarks. None of these predictions are fulfilled.

4.18 From the fifth line of Table 4.4, we read – replacing the colors with the quark types –

$$|\Lambda\rangle = \frac{1}{2}(|usd\rangle - |uds\rangle + |dsu\rangle - |dus\rangle)\,.$$

4.19 The wave function of the Σ^+ is obtained from (4.57) by replacing the d quark with an s quark. Thus, it follows

$$\mu_{\Sigma^+} = \frac{e}{2m_u}\left(\frac{8}{9} + \frac{1}{9}\frac{m_u}{m_s}\right)\,.$$

The predicted magnetic moment is therefore slightly smaller (2.7 μ_{K}) than the magnetic moment of the proton (2.79 μ_{K}). Measured was 2.5 μ_{K}.

4.20 This is a task for trying out. $m_{u,\,d}$ = 360 MeV, m_s = 510 MeV, b' = 3.8 × 10^7 MeV^2 and b = 2 × 10^7 MeV^2 give good results for the baryon octet and decuplet as well as the ρ, ω- and Φ-mesons, but poor values for the K- and K*-mesons.

4.21 We apparently need to calculate $T_{fi} = f_{\rho\gamma} e\, \varepsilon_{\rho,\mu} j^\mu / M_\rho^2$ for the possible helicities of the ρ-meson and the electron-positron pairs. We restrict ourselves to massless electrons and positrons. For (e^-_-, e^+_+) we take j^μ from (3.59) with $E = M_\rho/2$. The polarization vectors $\varepsilon_{\rho,\pm}$ are given by (2.122) with $\theta, \phi = 0$, while $\varepsilon_{\rho,0}$ is given by (3.99), i.e., $[0, 0, 0, 1]$ for a stationary ρ-meson. Thus, $|T_{fi}|^2$ can be calculated and it should not surprise you that the angular distribution is isotropic. To also consider (e^-_+, e^+_-), the result is multiplied by 2. After that, everything proceeds as usual.

4.22 In the text of the chapter (4.4), some examples are calculated. Note the relative signs of the $q\bar{q}$ pairs in the π^0 wave function.

4.23 For the ϕ-meson, we obtain $|R_S(0)|$ = 0.24 GeV$^{3/2}$. The evaluation of J/ψ and Υ shows that $|R_S(0)|^2/M_V^2$ changes by only a few percent.

4.24 First, set up the formulas corresponding to (4.91) for the η- and η'-mesons considering a mixing angle. The decay constant is always set to f_π. To become independent of the mixing angle, it is then proven that

$$\frac{\Gamma^{\eta'}_{\gamma\gamma}}{m^3_{\eta'}} = \frac{3\Gamma^{\pi^0}_{\gamma\gamma}}{m^3_{\pi^0}} - \frac{\Gamma^{\eta}_{\gamma\gamma}}{m^3_{\eta}}$$

For example, $\Gamma^{\eta'}_{\gamma\gamma}$ can be calculated. The evaluation shows that the prediction ($\Gamma^{\eta'}_{\gamma\gamma}$ = 5.7 keV) is only fulfilled to about 30 %.

4.25 The integration over (2.113) results in no difference in the formulas for the decay width.

4.26 From (4.87) we deduce $|Q_b| = 1/3$. From (4.94) we determine with α_s = 0.196 a hadronic decay width of 57 keV!

4.27 In the oscillator potential, the ground state energy is $E = 3\,\omega/2$. From the spectrum, we have read off ω = 315 MeV for charmonium. Since the potential and kinetic energy are on average equal, $3\,\omega/2 = 2(m_c\gamma - m_c)$, where the expression in the parentheses on the right is for the relativistic kinetic energy. With $\beta^2 = 1 - 1/\gamma^2$, we get $\beta^2 = 0.25$. The same calculation gives $\beta^2 = 0.12$ in the case of bottomonium.

4.28 The evaluation of the formula with typical constituent masses of the u and b quarks results in $M_{\eta_b} \approx M_\Upsilon - 3$ MeV.

Chapter 5

5.1 Proof again with illustrative argumentation: As an example, we take blue u quarks. There are the processes $u_B \longrightarrow u_G g_{B\bar{G}}$, $u_B \longrightarrow u_R g_{B\bar{R}}$ and $u_B \longrightarrow u_B g_{B\bar{B}}$. The amplitudes of the first two have the weight 1 due to the wave functions (4.11), while the third reaction receives a weighting factor of $2/\sqrt{6}$ via the wave function (4.14). These factors must be squared and added. After multiplication with the usual factor 1/2, one obtains $c_F = 4/3$.

5.2 The energies are arranged according to $E_1 > E_2 > E_3$. Then it applies

$$\sum |\boldsymbol{p}_i \boldsymbol{n}| = |E_1 \cos\theta_1| + |E_2 \cos\theta_2| + |E_3 \cos\theta_3| \ .$$

The right side is, however, $\geq |E_1 \cos\theta_1| + |E_2 \cos\theta_2 + E_3 \cos\theta_3|$ and due to momentum conservation $\geq 2|E_1 \cos\theta_1|$. At the maximum ($\cos\theta_1 = 1$), the equality sign applies. Thus, the proof is complete.

5.3 In the isotropic decay of a system into particles of equal mass, all momenta are of equal magnitude. Thus, T is proportional to $\int |\cos\theta| \mathrm{d}\cos\theta$. This integral has the value 1/2.

5.4 Write down all the terms allowed in (5.12) and then use the conditions of the following equation.

5.5 The experiments were all conducted on stationary protons. It applies $q^2 = -2\,E\,E'(1-\cos\theta)$ thus

$$EE' = \frac{-q^2}{1-\cos\theta} = a$$

and

$$E - E' = \frac{-q^2}{2M} = b \ .$$

The solution for E' is

$$E' = -\frac{b}{2} + \sqrt{\frac{b^2}{4} + a} \ .$$

Thus, we obtain $E = 1.022$ GeV, $E' = 0.489$ GeV for $\theta = 90°$ and $E = 11.732$ GeV, $E' = 11.199$ GeV for $\theta = 5°$.

5.6 A fairly small proportion should result.

5.7 How the formula is derived is described in detail in the text.

5.9 We place the z-axis of the coordinate system in the direction of the incoming proton. With $x = -q^2/(2\,q \cdot P)$ and $y = q \cdot P/(e \cdot P)$ it follows

$$x = \frac{2EE'(1+\cos\theta)}{4E_pE - 2E_pE'(1-\cos\theta)}$$

and

$$y = 1 - \frac{E'}{E}\frac{1-\cos\theta}{2} \ .$$

5.11 This is a typical application of the Jacobian determinant. We remain in the HERA system. With the results of the last task, it applies

$$\begin{aligned}
\frac{\partial q^2}{\partial \cos\theta} &= -2EE' \\
\frac{\partial q^2}{\partial E'} &= -2E(1+\cos\theta) \\
\frac{\partial y}{\partial \cos\theta} &= +\frac{E'}{2E} \\
\frac{\partial y}{\partial E'} &= -\frac{1}{2E}(1-\cos\theta) \ .
\end{aligned}$$

Thus, it applies

$$\frac{\mathrm{d}q^2\mathrm{d}y}{\mathrm{d}E'\mathrm{d}\Omega} = \frac{E'}{\pi}$$

and with $q^2 = -x\,y\,s$

$$\frac{\mathrm{d}q^2\mathrm{d}x}{\mathrm{d}E'\mathrm{d}\Omega} = \frac{E'x}{\pi y} \ .$$

The proof for stationary protons is even simpler.

5.12 The necessary steps are described in the text.

5.13 Starting from (5.36), you will easily find that the points lie at the extreme right edge of the curves in Figure 5.24.

5.15 Note the symmetry conditions of the fragmentation functions on page 329.

5.16 The evaluation of (5.67) with values for $\Gamma_{\gamma\gamma}$ from the tables of the PDG results in a cross section of 2365, 1568, and 2360 pb for the production of π^0-, η- and η'-mesons. This expresses that $\Gamma_{\gamma\gamma}$ for these mesons is approximately proportional to M^3.

5.17 The log term must be > 1.

Chapter 6

6.1 The scattering amplitude (6.9) becomes in the convention of nuclear physics

$$f = \frac{1}{\sqrt{2}\pi} G_\mathrm{F}\sqrt{s} \ .$$

Since there is no angular dependence, this corresponds to the term with $J = 0$ in (2.108)

$$f = \frac{2}{\sqrt{s}} t^0_{\frac{-1}{2}\frac{-1}{2},\frac{-1}{2}\frac{-1}{2}} \ .$$

The amplitude is real, the real part of t_0 can reach a maximum value of 1/2. This results in $s_\mathrm{max} = \sqrt{2}\pi/G_\mathrm{F}$.

6.5 With $\boldsymbol{p} \longrightarrow 0$, from (3.20) it becomes

$$u_r = \sqrt{2m}\begin{pmatrix}\chi_r\\ 0\end{pmatrix} ,$$

and similarly for $u_{r'}$. Thus, it applies $j_{\mathrm{V}}^0 = 2m\chi_{r'}^\dagger\chi_r$, $j_{\mathrm{V}}^i = 0$ and $j_{\mathrm{A}}^0 = 0$, $j_{\mathrm{A}}^i = 2m\chi_{r'}^\dagger\sigma^i\chi_r$,

6.7 Thus, one is apparently quite close to the correct result!

6.8 Counting the contributing families of quarks and leptons, remembering that quarks appear as color triplets.

6.9 According to the pattern of the decay $\mu \longrightarrow \nu_\mu e\bar{\nu}_e$, besides the decay into $\nu_\tau e\bar{\nu}_e$, only the decays into $\nu_\tau\mu\bar{\nu}_\mu$ and $\nu_\tau d\bar{u}$ are possible.

6.10 Proceed analogously to the proof for the quark densities $q(x)$.

6.11 Then probably s and $\bar{c}$ do not contribute.

6.12 In addition to supplementing Figure 6.16 with diagrams with c and s quarks, the connection of Section 5.3.3 between the quark densities in the proton and neutron must be considered.

6.13 Note the connection of the structure functions with the cross section of photon-hadron scattering in Section 5.3.2

6.15 Only the leading order in λ is taken, i.e., $V_{u\,d} = 1 - \lambda^2/2$, $V_{u\,s} = \lambda$, $V_{u\,b} = A\,\lambda^3(\rho - \mathrm{i}\eta)$. The last equation is derived using the identities $\cos\delta_{13} = \rho/\sqrt{\eta^2 + \rho^2}$ and $\sin\delta_{13} = \eta/\sqrt{\eta^2 + \rho^2}$. The other matrix elements are then determined in a similar manner.

6.16 The products of the matrix elements for the combinations $j, k = (u, u),\ (u, c),\ (c, u),\ (c, c)$ are written down and the values of the Cabibbo matrix are inserted. For equal masses, $f(m_j, m_k)$ is only a common factor.

6.17 Like the last task, only a bit more complicated.

6.18 Draw a line segment CB of length 1, which represents the product $|\,V_{cb}V_{cd}\,|$. Then, draw a circle around C with a radius of 0.38 (corresponding to $|\,V_{ub}V_{ud}\,|$) and a circle around B with a radius of 0.93 (corresponding to $|\,V_{tb}V_{td}\,|$). The intersection point is the apex of the triangle A.

6.19 Write down the unitarity condition (6.47) for $k = 2,\ j = 1$. Then you will notice that the corresponding triangle is very flat and practically becomes a line.

6.20 c is the side opposite the point C. Therefore, you only need to justify $c = |c| \exp \mathrm{i}(\pi - \beta)$ using the representation of complex numbers in a plane.

6.21 In the text, the measurement result for $\sin(2\beta)$ is given with an error of 1σ. Calculate the error for CL=99% and determine the allowed range of values for β.

6.22 To prove this, evaluate

$$T_{fi} = \frac{g^2}{2}\bar{u}_{\mathrm{R}}(k)\ \not{\epsilon}_0^*(p')\frac{\not{q}}{q^2}\ \not{\epsilon}_0^*(k')u_{\mathrm{L}}(p)$$

This can be done by hand or by extending a notebook like `Compton(HE).ipynb`.

6.25 Here, too, you only need to use the table, counting the (colored) quarks three times.

6.26 For massless fermions, the necessary amplitudes are included in Table 6.3. With more ambition, you can also extend one of our notebooks, such as `eemumu.ipynb`.

6.27 Hints for the solution are plentiful in the text.

6.29 Here, too, you only need to carry out the intermediate steps of the conversion discussed in the text.

Chapter 7

7.1 Hints for the solution are given in the text.

7.2 Calculate the product, taking into account that the individual terms are generally not commutative, but mixed partial derivatives can be exchanged. See also the notebook `gauge.ipynb`.

7.5 Just calculate the product. Take into account that each term is a 2×2 matrix.

7.13 Use the Python module `matplotlib` and ignore the theoretical uncertainties.

Chapter 8

8.1 With the definition of the Jacobi determinant in Section 2.4, this is a simple mathematical calculation. Alternatively, the calculation can also be performed with SymPy.

8.2 This task is more extensive than others. The solution path is explained in the task description. Try it before consulting the solution in `Drell-Yan.ipynb`.

8.3 Compare your result with the notebook `Drell-Yan-numerical.ipynb`.

8.4 The notebook explains all steps, from the installation to the usage of LHAPDF PDF sets.

8.5 Compare your result with the notebook `gluon-gluon.ipynb`.

8.6 Integrate the elastic cross section of the last formula over t to verify the claim.

8.7 Square (2.28) and use (2.13) at the point $t = 0$.

8.8 With a total cross section of 110 mb, the ratio $\sigma_{\text{el}}/\sigma_t$ becomes less than 1.4×10^{-4} even for $\rho = 1$.

8.9 Compare your result with our notebook `WZrapidity.ipynb`.

8.10 Compare your result with our notebook `WZrapidity.ipynb`.

8.12 For this calculation, you can integrate the normal distribution or use the `cdf` function of the `scipy.stats.norm` routine.

Chapter 9

9.1 The result is obtained directly by using the PMNS matrix.

9.2 The solution results from a direct application of Equation 9.9 in the text, where all neutrino masses m_i are expressed through the lowest mass m_0 and the mass differences.

9.3 The solution path is given in the task description. Compare your calculation with the solution in the notebook `neutrino-vacuum.ipynb`.

9.4 First, calculate the eigenvalues of the matrix

$$M = \begin{pmatrix} a & b \\ b & c \end{pmatrix}$$

In the next step, replace a, b, c with the matrix elements in (9.28), expressing all masses through m_1 and m_2 using (9.21). This is a somewhat tedious calculation, where SymPy can help. Compare your result with the notebook `neutrino-matter.ipynb`.

9.6 It applies that $N_\nu = 2S/Q$, where S is the solar constant and $Q = 26,731$ MeV is the heat of reaction defined in the text after (9.35). This results in a flux of $6.54 \times 10^{14} m^{-2} s^{-1}$.

Chapter 10

10.1 In Figure 10.8, the photons are replaced by W bosons. The horizontal line in the loop is then, for example, a ν_μ, and the slanted lines belong to a muon that couples to the Z.

10.2 Corresponding to the Z decay, there is the decay into a sfermion-fermion pair $\tilde{Z} \rightarrow \tilde{f}\bar{f}$, or $\tilde{Z} \rightarrow \bar{\tilde{f}} f$ with the subsequent decay of the sfermion into the LSP $\tilde{f} \rightarrow f\tilde{\gamma}$.