# CISC2006: Algorithm Design and Analysis

**Mid-term Exam** **Exam Time:** 17:35 pm - 18:35 pm

**Name**: _____ **Student Number**: _____

There are 3 problems and 3 pages in total. You can get at most 100 points if attempting all problems. Please make your answers precise and concise.

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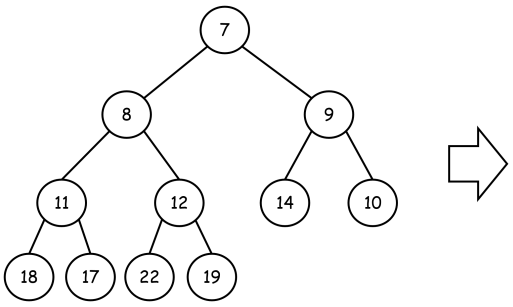
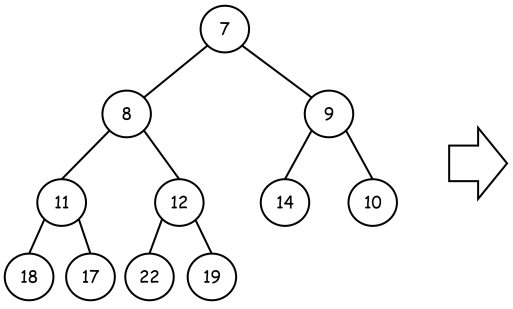
1. (40 pts) Draw the heap (as a binary tree) after executing the following operations.

   (1) (20 pts) removeMin from the following heap:

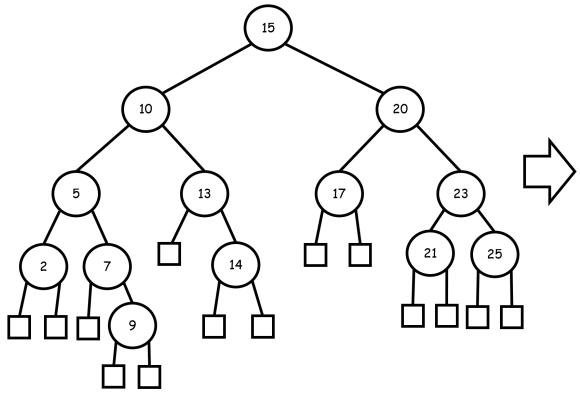



   (2) (20 pts) Insert element 5 into the following heap:

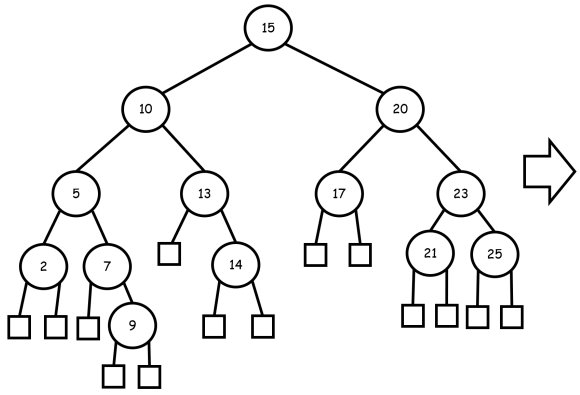


2. (40 pts) Draw the AVL tree (as a binary tree) after executing the following operations.

   (1) (20 pts) Insert element 24 into the following AVL tree:




   (2) (20 pts) Delete element 10 from the following AVL tree:



3. (30 pts) Consider a sequence of integers $(a_1, a_2, \ldots)$ defined as follows. Let $a_1 = a_2 = 1$. For all $i \geq 3$, define $a_i = 2 \cdot a_{i-1} + 3 \cdot a_{i-2}$.

   Use mathematical induction to prove the following statement.

   $$\text{For all integer } n \geq 2, \qquad a_n \geq 3^{n-2}.$$

   In the proof, please state clearly the base case and the induction hypothesis.