EECS 442 Computer Vision, Fall 2012
Homework 2 Solution

1 Fundamental Matrix

(a) Since the camera matrix M has rank 3, we can always first find 4×4 matrix H_0

$$H_0 = \begin{bmatrix} A^{-1} & -A^{-1}b \\ 0 & 1 \end{bmatrix}$$

such that

$$MH_0 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$$

Apply the same transformation to M' we will get a new matrix,

$$M'H_0 = [A'A^{-1}, -A'A^{-1}b + b'] = \begin{bmatrix} m'_{11} & m'_{12} & m'_{13} & m'_{14} \\ m'_{21} & m'_{22} & m'_{23} & m'_{24} \\ m'_{31} & m'_{32} & m'_{33} & m'_{34} \end{bmatrix}$$

As $e_3T(-A'A^{-1}b + b') \neq 0$, $m'_{34} \neq 0$. Now multiply another matrix

$$H_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -\frac{m'_{31}}{m'_{34}} & -\frac{m'_{32}}{m'_{34}} & -\frac{m'_{33}}{m'_{34}} & \frac{1}{m'_{34}} \end{bmatrix}$$

to the right of both of them, we will have

$$MH_0H_1 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \text{ and } M'H_0H_1 = \begin{bmatrix} m'_{11} - \frac{m'_{14}m'_{31}}{m'_{34}} & m'_{12} - \frac{m'_{14}m'_{32}}{m'_{34}} & m'_{13} - \frac{m'_{14}m'_{33}}{m'_{34}} & \frac{m'_{14}}{m'_{34}} \\ m'_{21} - \frac{m'_{24}m'_{31}}{m'_{34}} & m'_{22} - \frac{m'_{24}m'_{32}}{m'_{34}} & m'_{23} - \frac{m'_{24}m'_{33}}{m'_{34}} & \frac{m'_{24}}{m'_{34}} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

So a projection transformation matrix $H = H_0H_1$ will reduce M and M' to the canonical forms.

(b) Observe that $MX = (MH)(H^{-1}X)$, and similarly for M'. Thus if x and x' are matched points with respect to the pair of cameras (M, M'), corresponding to a 3D point X, then they are also matched points with respect to the pair of cameras $(MH, M'H)$, corresponding to the point $H^{-1}X$.

(c) From the conclusion of (b), the fundamental matrix of the camera pair (M, M') is the same as the fundamental matrix of the camera pair $(\hat{M}, \hat{M}')$, which is $[\hat{b}]_\times \hat{A}$ where

$$\hat{A} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 0 \end{bmatrix}, \hat{b} = \begin{bmatrix} b_1 \\ b_2 \\ 1 \end{bmatrix}$$

Thus F can be found.

$$F = [\hat{b}]_\times \hat{A} = \begin{bmatrix} 0 & -1 & b_2 \\ 1 & 0 & -b_1 \\ -b_2 & b_1 & 0 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -a_{21} & -a_{22} & -a_{23} \\ a_{11} & a_{12} & a_{13} \\ -a_{11}b_2 + a_{21}b_1 & -a_{12}b_2 + a_{22}b_1 & -a_{13}b_2 + a_{23}b_1 \end{bmatrix}$$

We can divide F by a_{21} to get a seven-parameter expression.

$$\begin{bmatrix} -1 & -\frac{a_{22}}{a_{21}} & -\frac{a_{23}}{a_{21}} \\ \frac{a_{11}}{a_{21}} & \frac{a_{12}}{a_{21}} & \frac{a_{13}}{a_{21}} \\ -\frac{a_{11}}{a_{21}}b_2 + b_1 & -\frac{a_{12}}{a_{21}}b_2 + \frac{a_{22}}{a_{21}}b_1 & -\frac{a_{13}}{a_{21}}b_2 + \frac{a_{23}}{a_{21}}b_1 \end{bmatrix}$$

The new seven parameters are $\frac{a_{11}}{a_{21}}, \frac{a_{12}}{a_{21}}, \frac{a_{13}}{a_{21}}, \frac{a_{22}}{a_{21}}, \frac{a_{23}}{a_{21}}, b_1, b_2$.

2 Epipolar Geometry

Let k represent an image line that contains point x, so $x = k \times l$, for x also lies in l. Since $l' = Fx$, so we have $l' = F(k \times l) = F[k]_x l$.

3 Programming Assignment

3.1 Fundamental Matrix

Please refer to Algorithm 11.1 in HZ pg. 282.

3.2 Stereo Rectification

Please refer to Section 11.12 in HZ pg. 302.