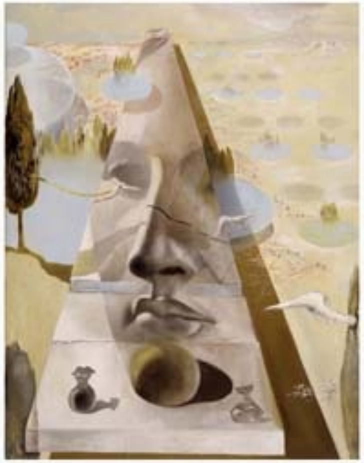


EECS 442 – Computer vision

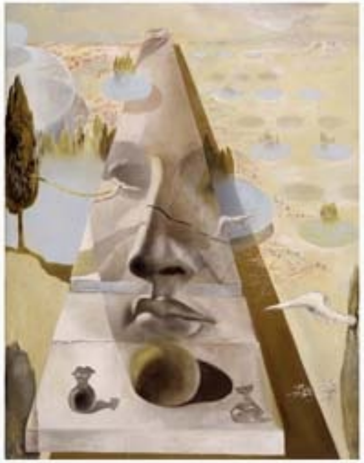
Cameras

without cameras we wouldn't have C.V. ☹

- Announcements:
  - HW1 is released today after class and it's due by 9.22 11:55PM

Reading: [FP] Chapters 1 – 3
[HZ] Chapter 6

Some slides in this lecture are courtesy to Profs. J. Ponce, S. Seitz, F-F Li



EECS 442 – Computer vision

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- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras
- Other camera models

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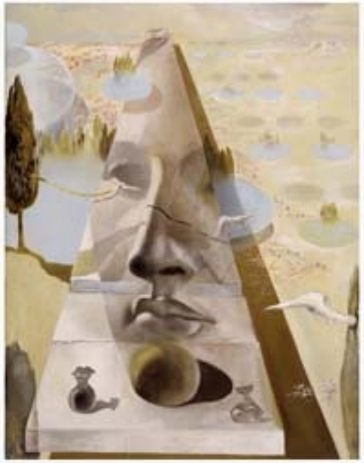

EECS 442 – Computer vision

Cameras

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- Pinhole cameras
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Reading: [FP] Chapters 1 – 3
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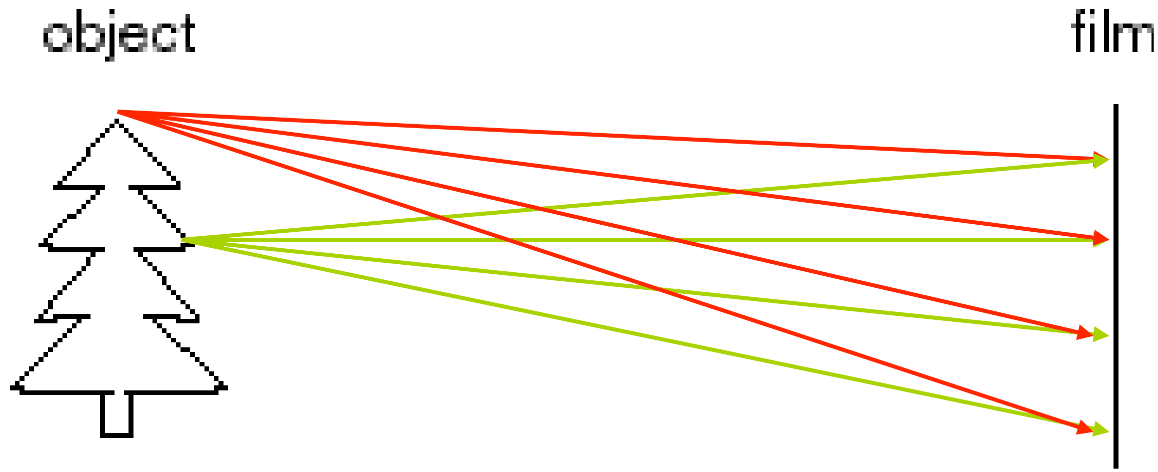
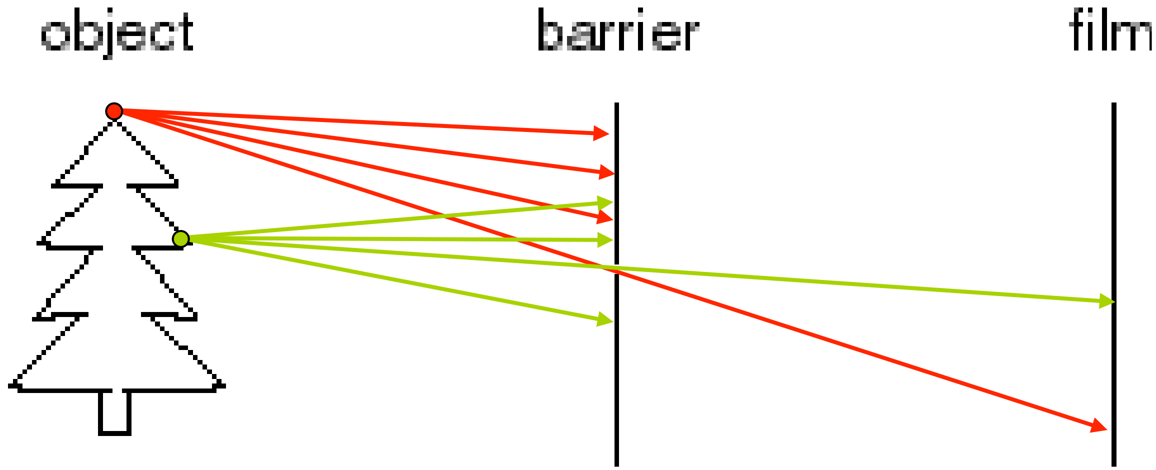
How do we see the world?



- Let's design a camera
  - Idea 1: put a piece of film in front of an object
  - Do we get a reasonable image?

Pinhole camera



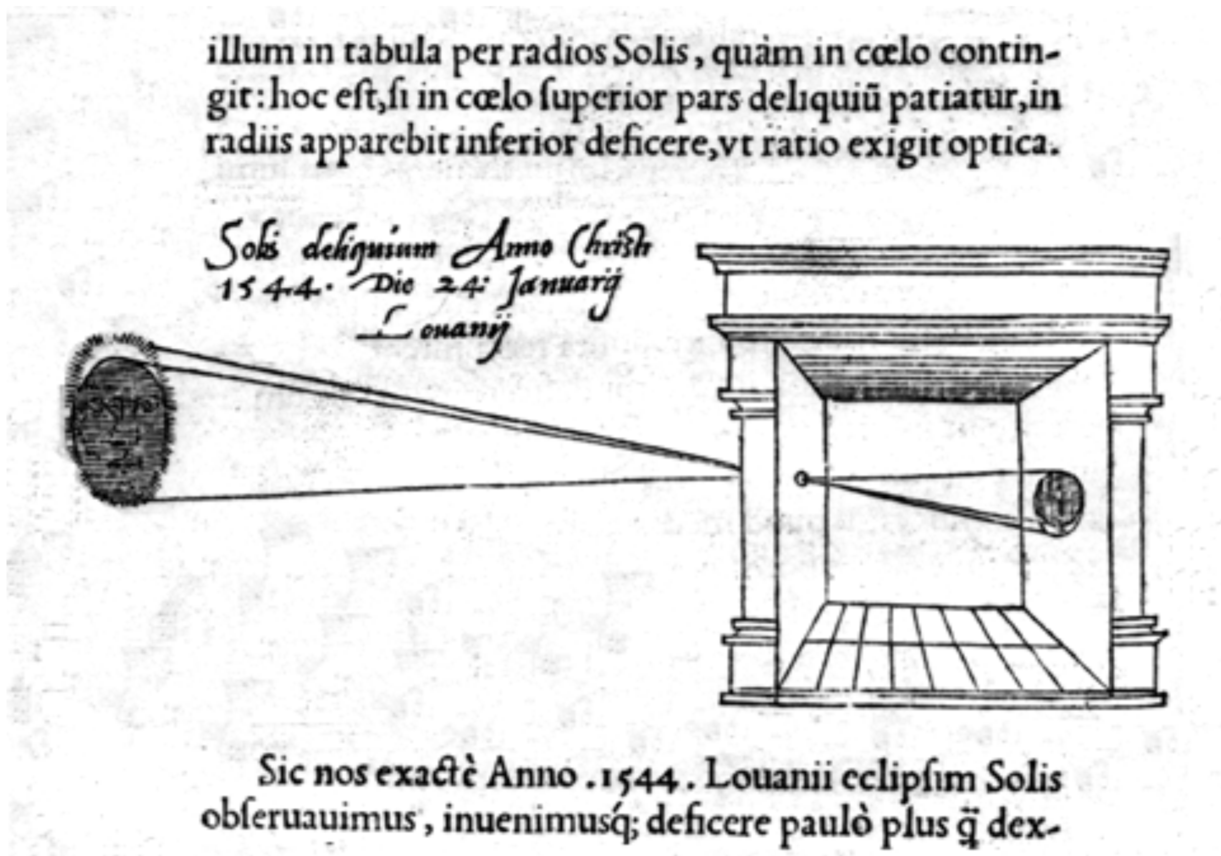
- Add a barrier to block off most of the rays
  - This reduces blurring
  - The opening known as the **aperture**

Some history…

Milestones:

- Leonardo da Vinci (1452-1519):

first record of camera *obscura*



illum in tabula per radios Solis, quàm in cœlo contin-
git: hoc eſt, ſi in cœlo ſuperior pars deliquiũ patiatur, in
radiis apparebit inferior deficere, vt ratio exigit optica.

Solis deliquium Anno Christi
1544. Die 24. Januarij
Louanij

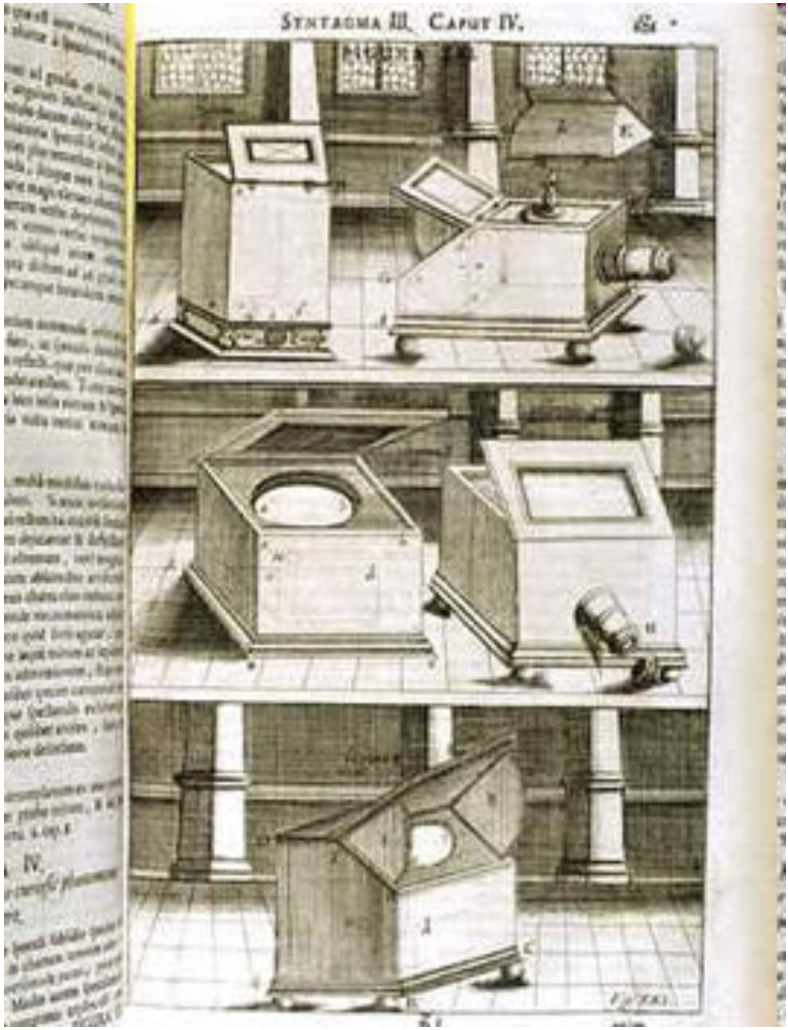
Sic nos exactè Anno .1544. Louanii eclipſim Solis
obſeruauimus, inuenimusq; deficere paulò plus q̃ dex-

Some history…

Milestones:

• Leonardo da Vinci (1452-1519): first record of camera *obscura*

• Johann Zahn (1685): first portable camera



Some history…

Milestones:
• Leonardo da Vinci (1452-1519): first record of camera *obscura*
• Johann Zahn (1685): first portable camera
• Joseph Nicephore Niepce (1822): first photo - birth of photography

Photography (Niepce, “La Table Servie,” 1822)

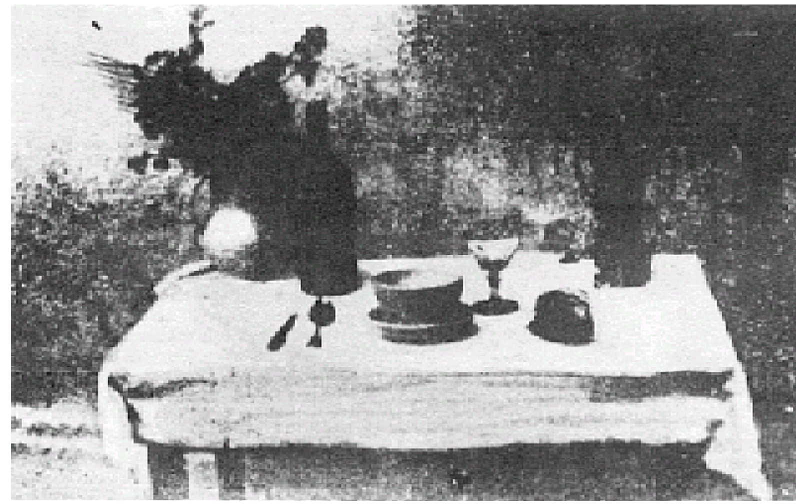
Some history…

Milestones:

• Leonardo da Vinci (1452-1519): first record of camera *obscura*

• Johann Zahn (1685): first portable camera

• Joseph Nicephore Niepce (1822): first photo - birth of photography



• Daguerréotypes (1839)

• Photographic Film (Eastman, 1889)

• Cinema (Lumière Brothers, 1895)

• Color Photography (Lumière Brothers, 1908)

Photography (Niepce, “La Table Servie,” 1822)

Let’s also not forget…

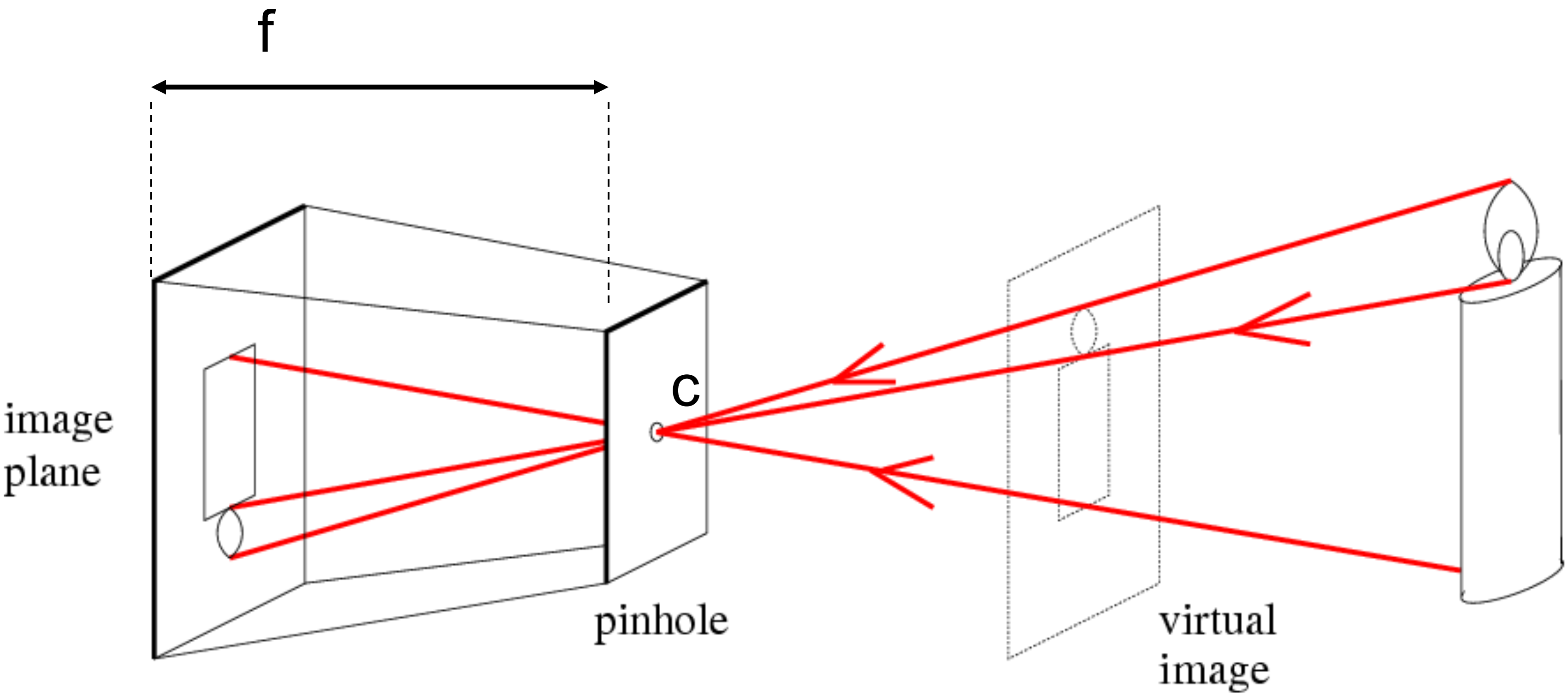
Motzu
(468-376 BC)

Oldest existent
book on geometry
in China

Aristotle
(384-322 BC)
Also: Plato, Euclid

Al-Kindi (c. 801–873)
Ibn al-Haitham
(965-1040)

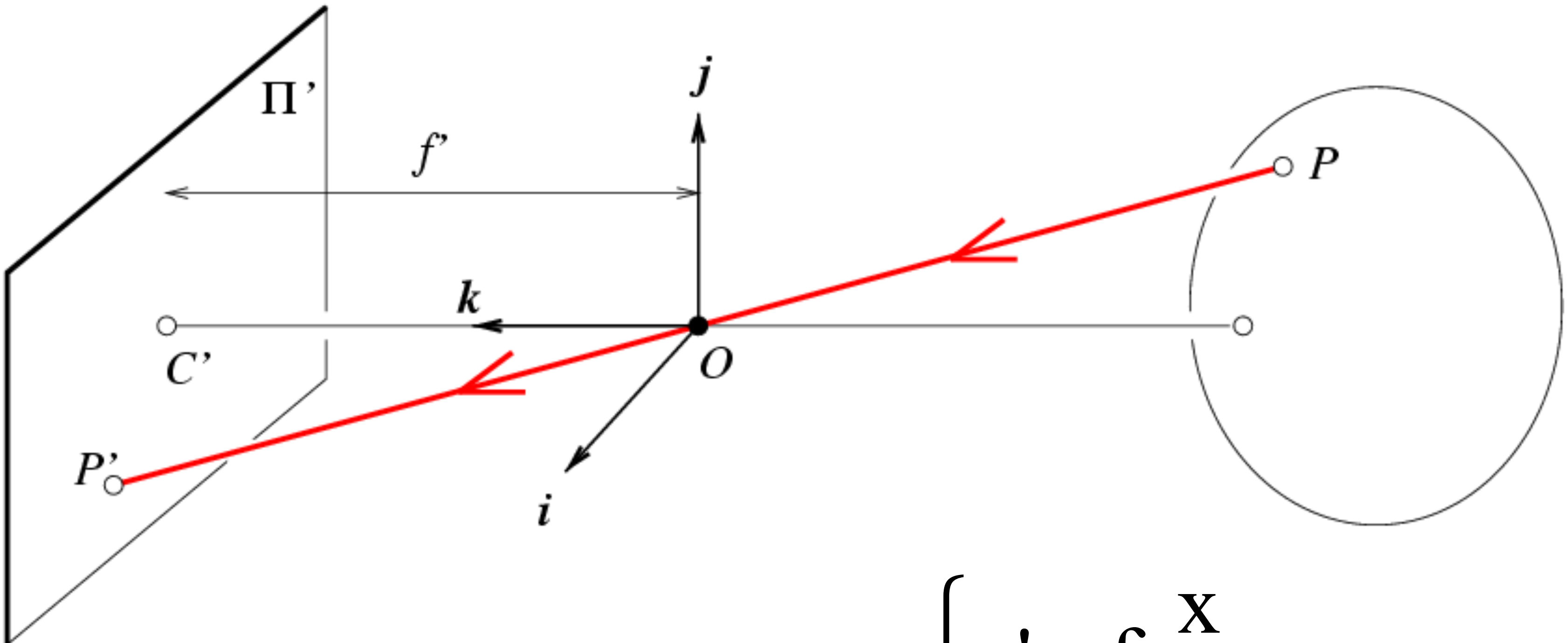
Pinhole camera



f = focal length

c = aperture = pinhole = center of the camera

Pinhole camera

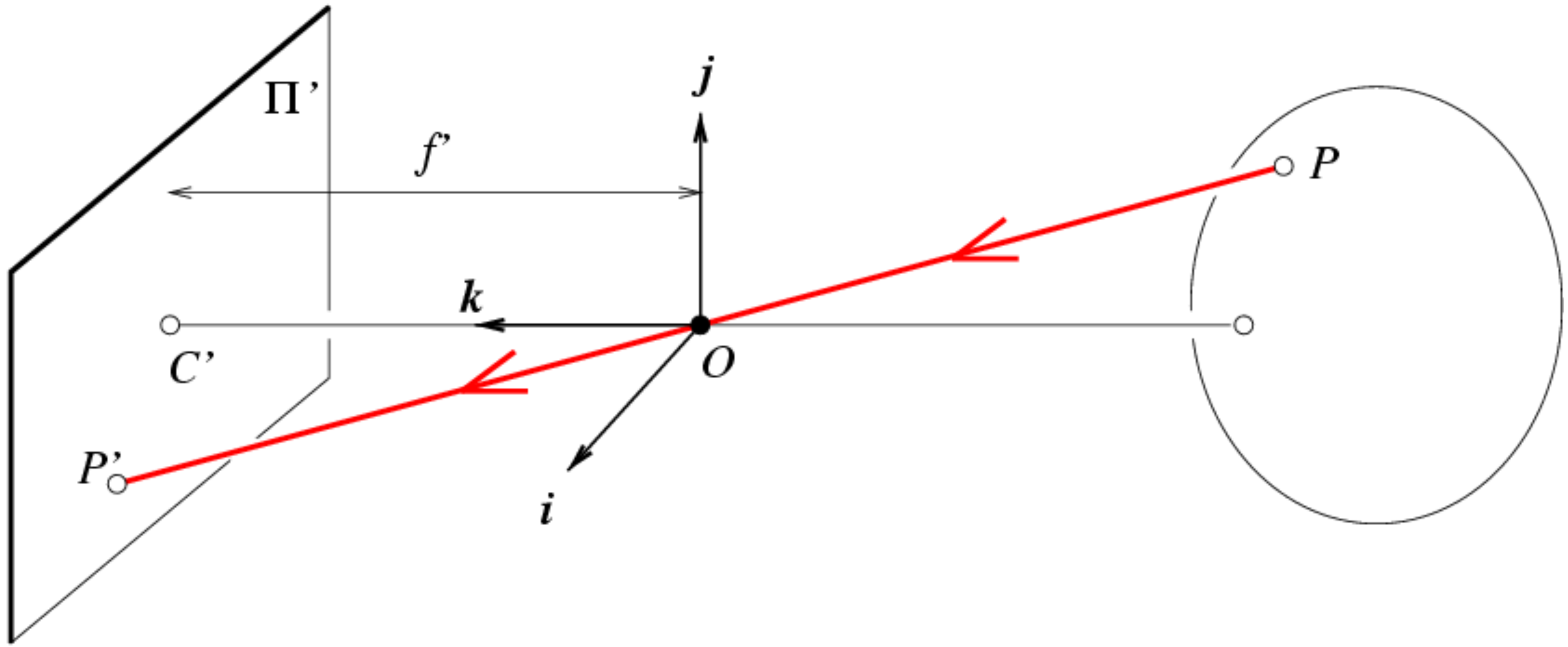


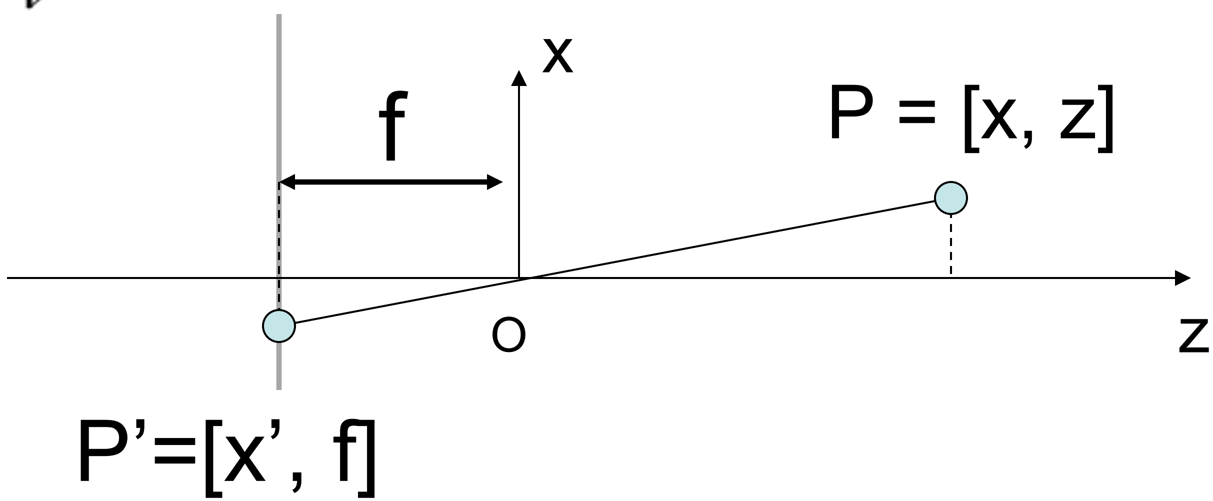
$$P = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \rightarrow P' = \begin{bmatrix} x' \\ y' \end{bmatrix}$$

$$\begin{cases} x' = f\,\dfrac{x}{z} \\ y' = f\,\dfrac{y}{z} \end{cases}$$

Derived using similar triangles

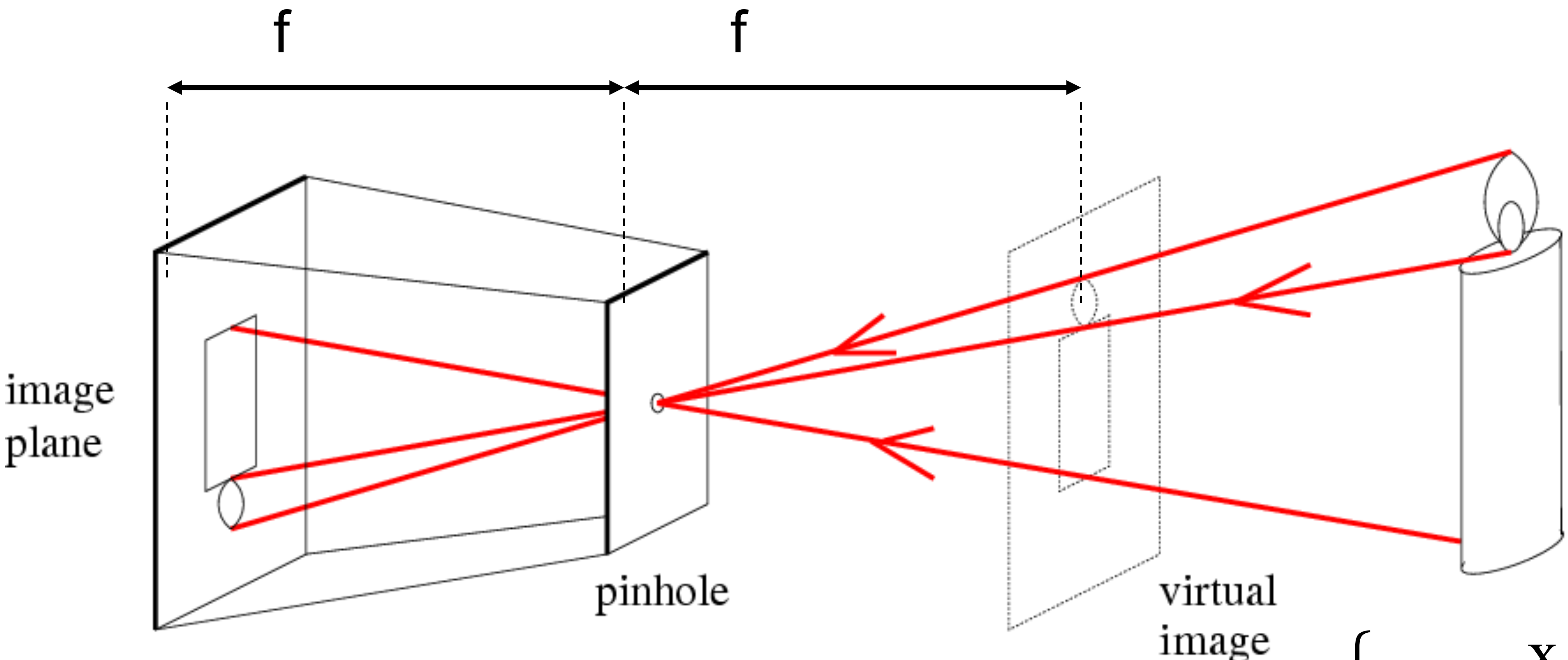
Pinhole camera





$$\frac{x'}{f} = \frac{x}{z}$$

Pinhole camera



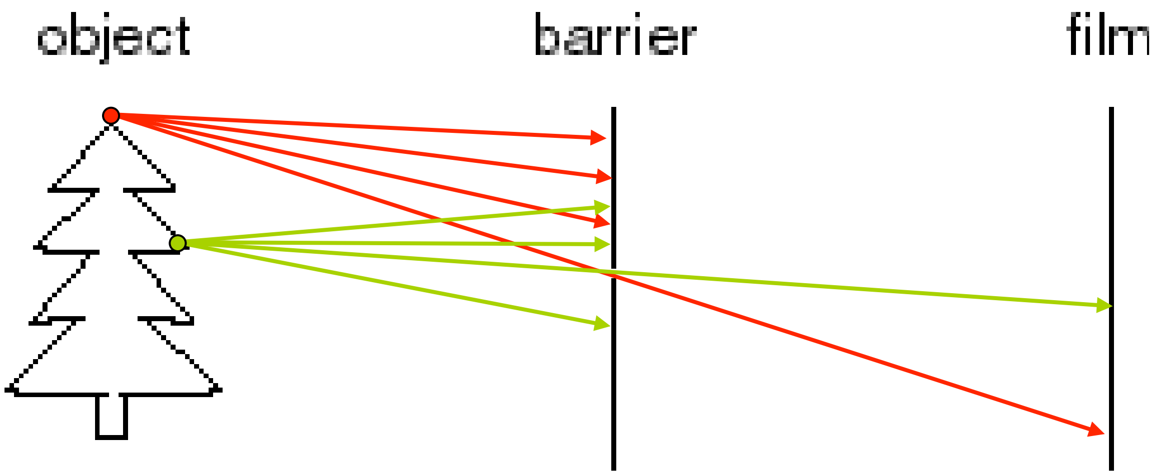
$$\begin{cases} x' = f\dfrac{x}{z} \\ y' = f\dfrac{y}{z} \end{cases}$$

Common to draw image plane *in front* of the focal point.

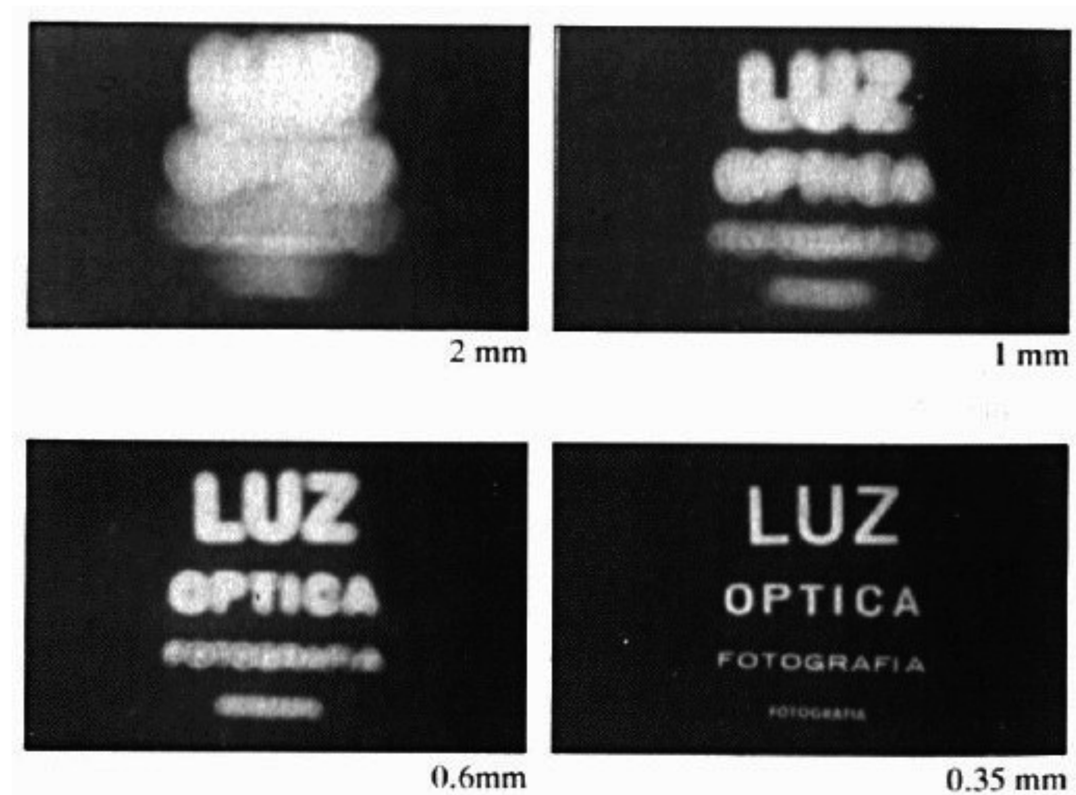
What's the transformation between these 2 planes?

Pinhole camera

Is the size of the aperture important?



Shrinking aperture size

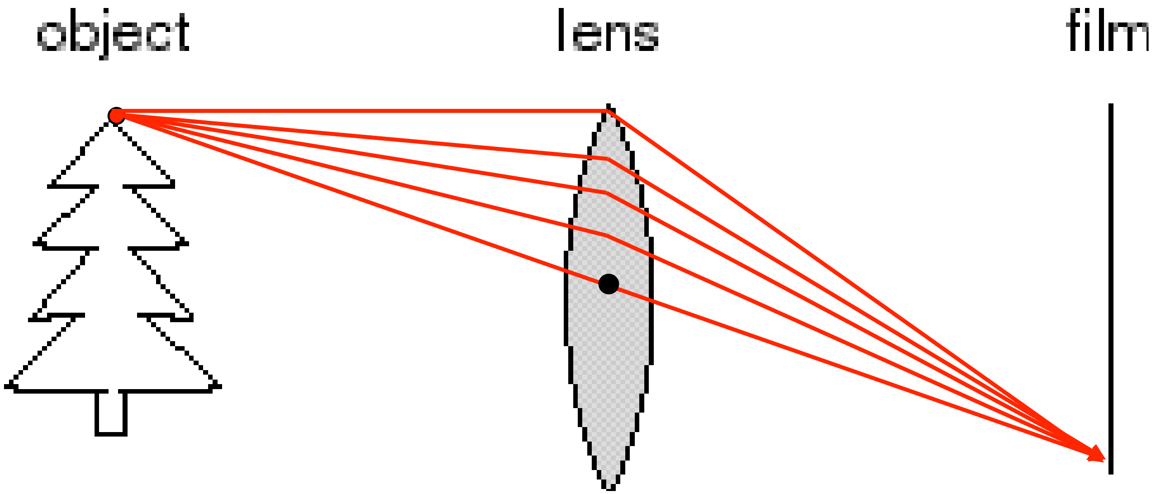


-Why the aperture cannot be too small?

- -Less light passes through
- -Diffraction effect

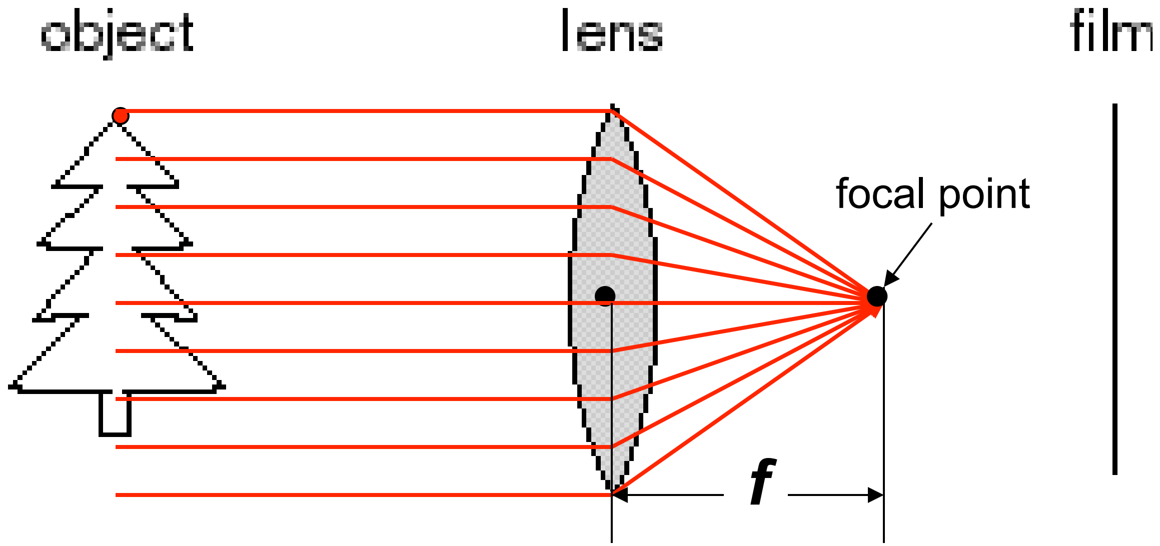
Adding lenses!

Cameras & Lenses



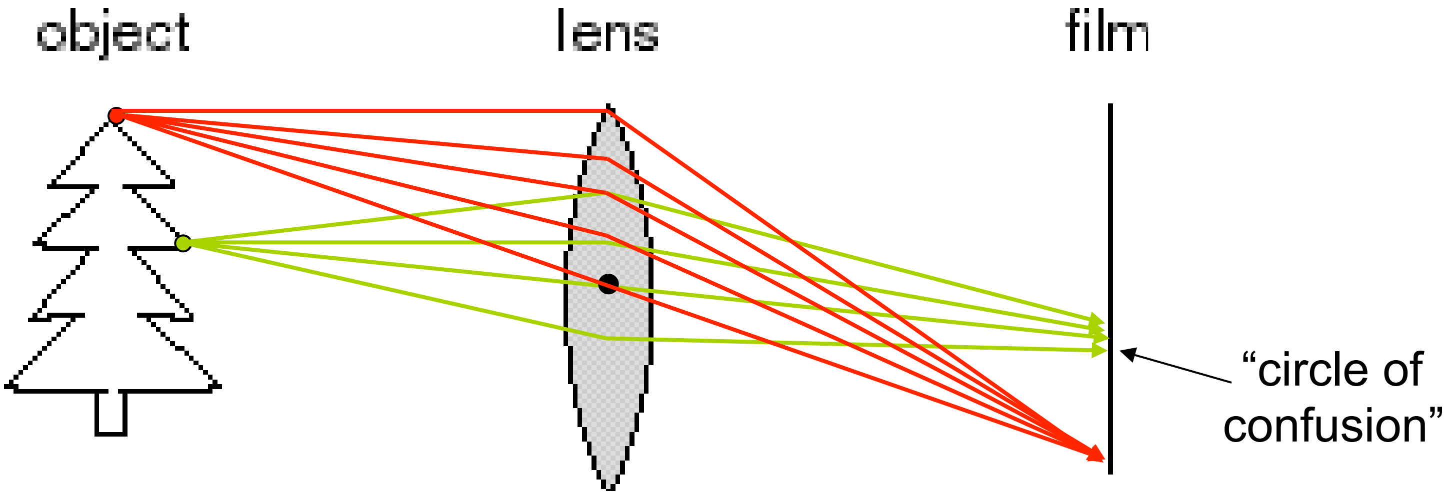
- A lens focuses light onto the film

Cameras & Lenses



- A lens focuses light onto the film
  - Rays passing through the center are not deviated
  - All parallel rays converge to one point on a plane located at the *focal length f*

Cameras & Lenses



- A lens focuses light onto the film
  - There is a specific distance at which objects are “in focus”
  - Related to the concept of depth of field

Cameras & Lenses

- A lens focuses light onto the film
  - There is a specific distance at which objects are “in focus”
  - Related to the concept of depth of field

Cameras & Lenses

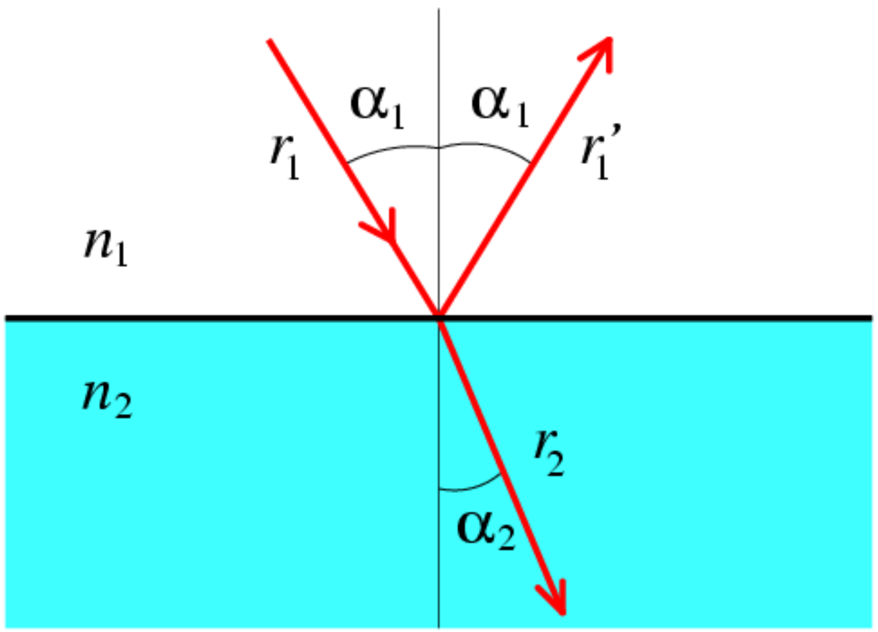
Relationship between points in 3D and images

Snell's law

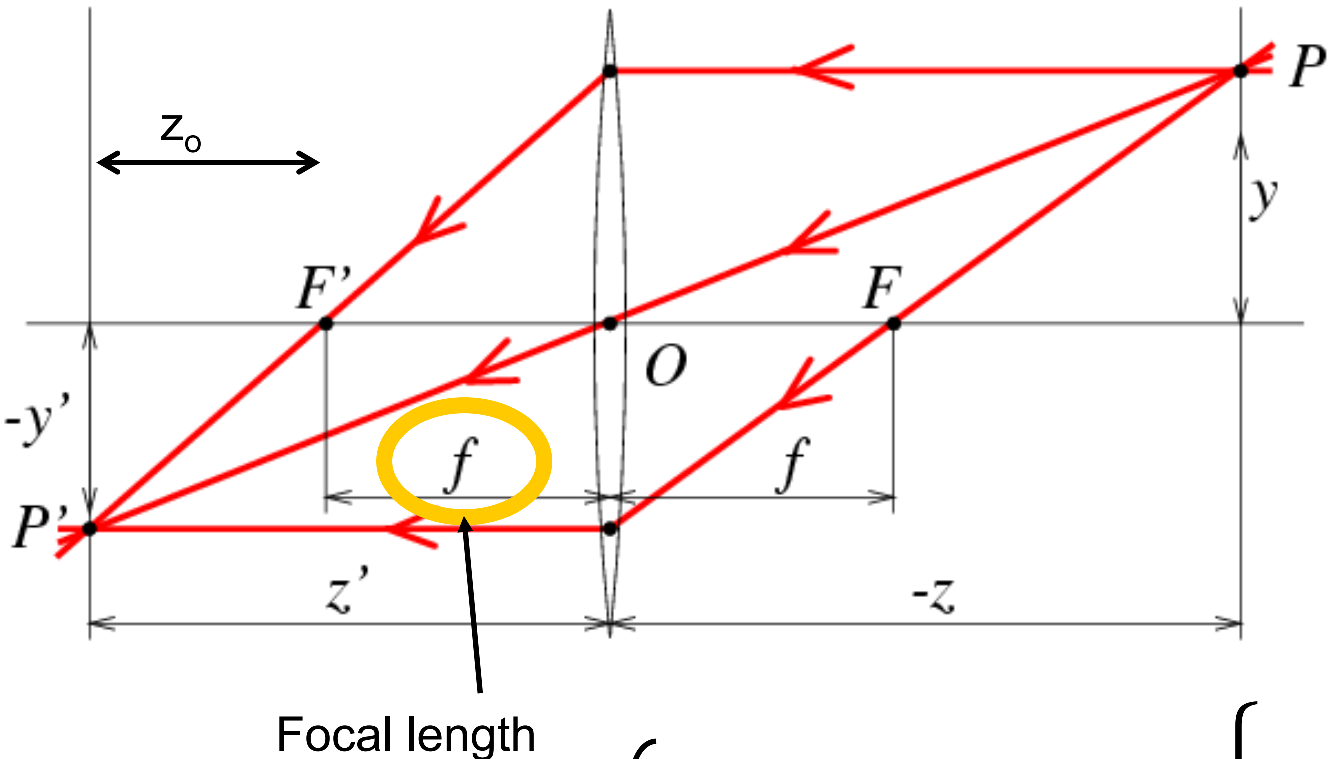
$$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$$

α_1 = incident angle
α_2 = refraction angle
n_i = index of refraction

Air ~ 1.0 (1.0 in Vacuum)
Water ~ 1.3
Glass ~ 1.5



Thin Lenses



$$z' = f + z_o$$

$$f = \frac{R}{2(n-1)}$$

Snell's law:

$n_1 \sin \alpha_1 = n_2 \sin \alpha_2$

Small angles:

$n_1 \alpha_1 \approx n_2 \alpha_2$

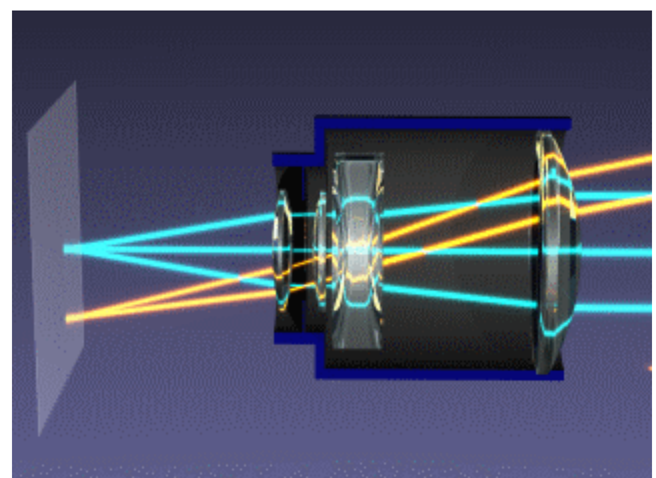
$n_1 = \text{n (lens)}$

$n_1 = 1 \text{ (air)}$

$$\begin{cases} x' = z' \dfrac{x}{z} \\ y' = z' \dfrac{y}{z} \end{cases}$$

Lenses are combined in various ways…

- Adjust the focus
- Change the field of view



Source wikipedia

Effect of the focal length

28 mm lens

50 mm lens

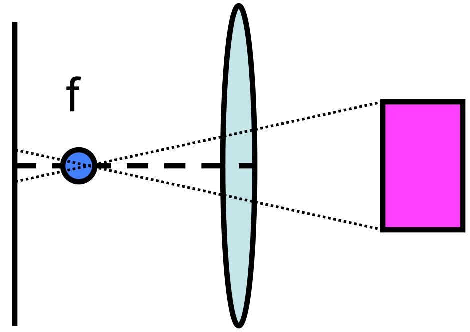
70 mm lens

210 mm lens

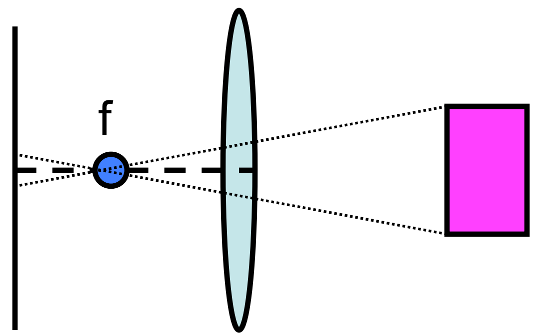
Source: wikipedia

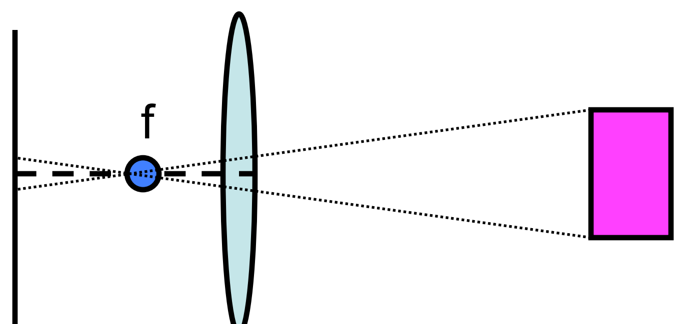
Dolly zooms

Source wikipedia



Exaggerated perception of depth





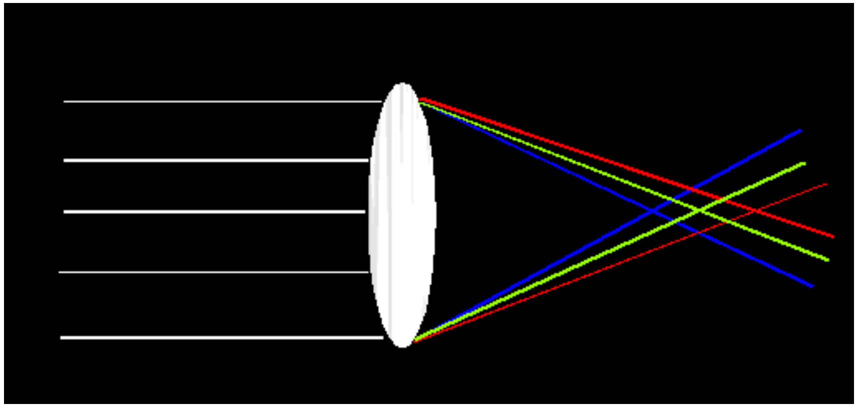
Compressed perception of depth

La Haine

Issues with lenses: Chromatic Aberration

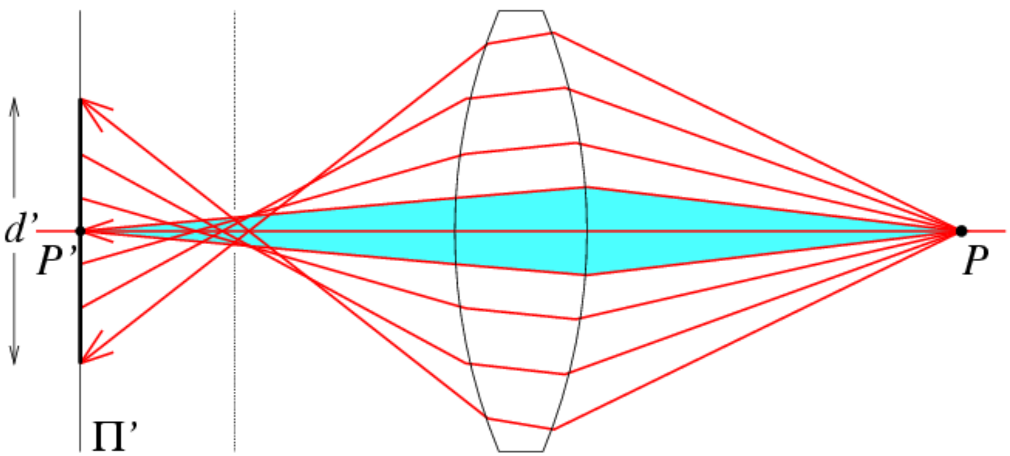
- Lens has different refractive indices for different wavelengths: causes color fringing

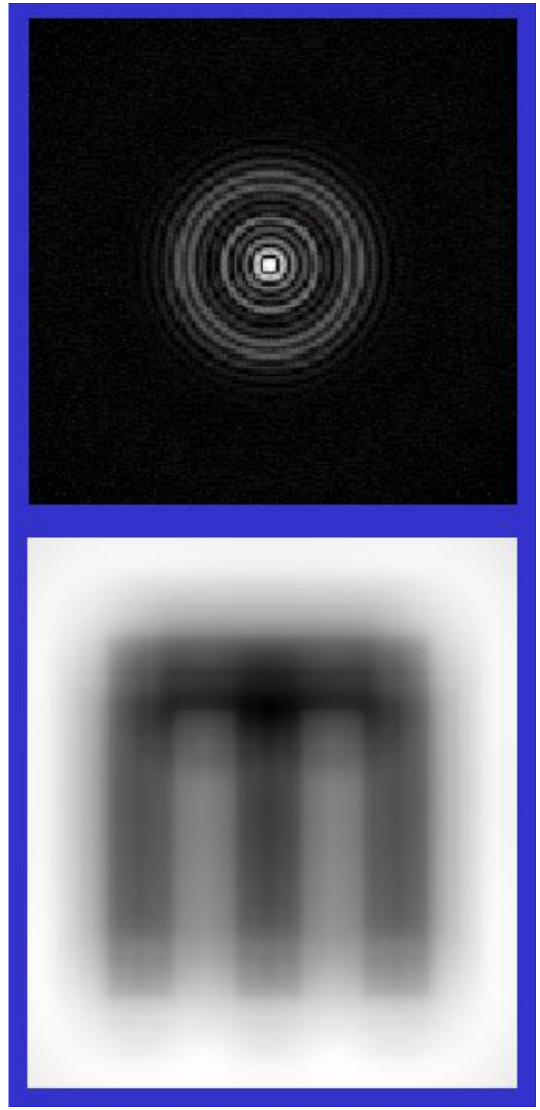
$$f = \frac{R}{2(n-1)}$$



Issues with lenses: Spherical aberration

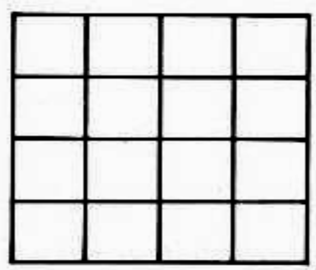
- Rays farther from the optical axis focus closer



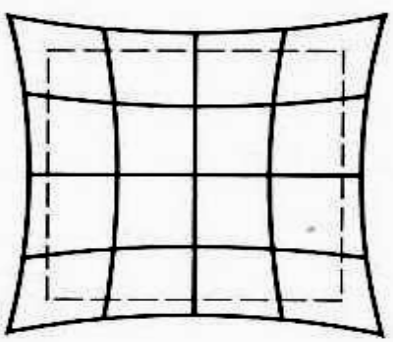


Issues with lenses: Radial Distortion

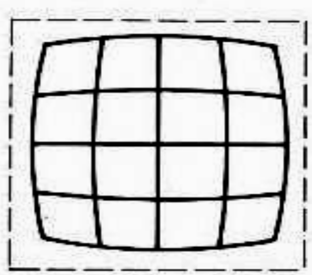
– Deviations are most noticeable for rays that pass through the edge of the lens



No distortion



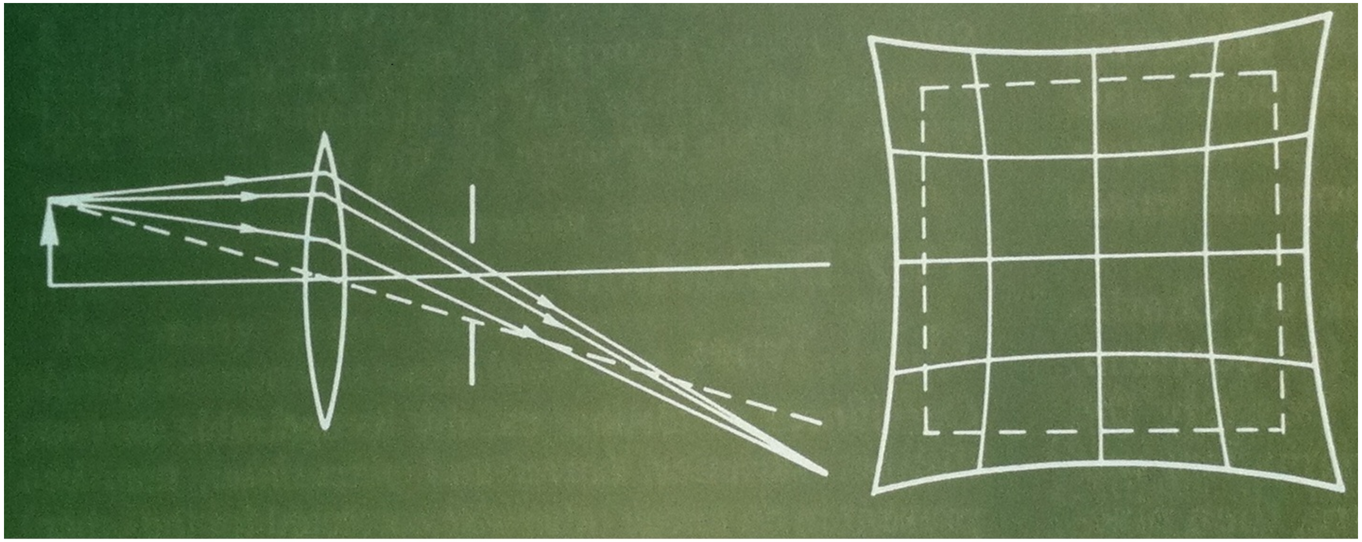
Pin cushion



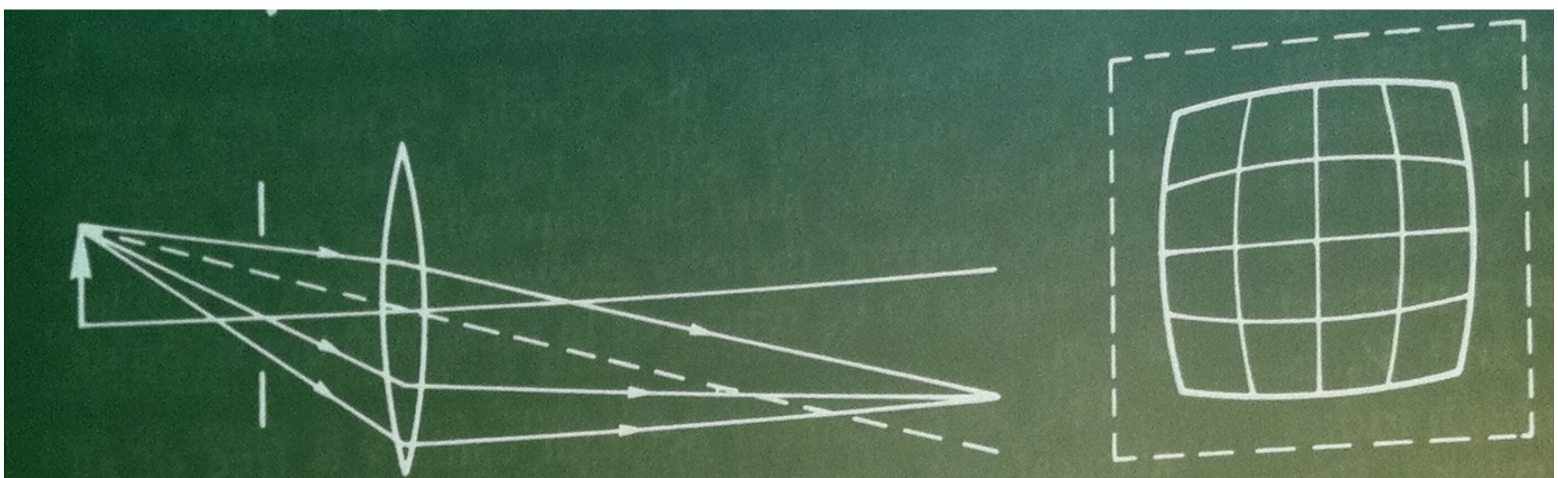
Barrel (fisheye lens)

Image magnification decreases with distance from the optical axis

Issues with lenses: Radial Distortion



Pin cushion

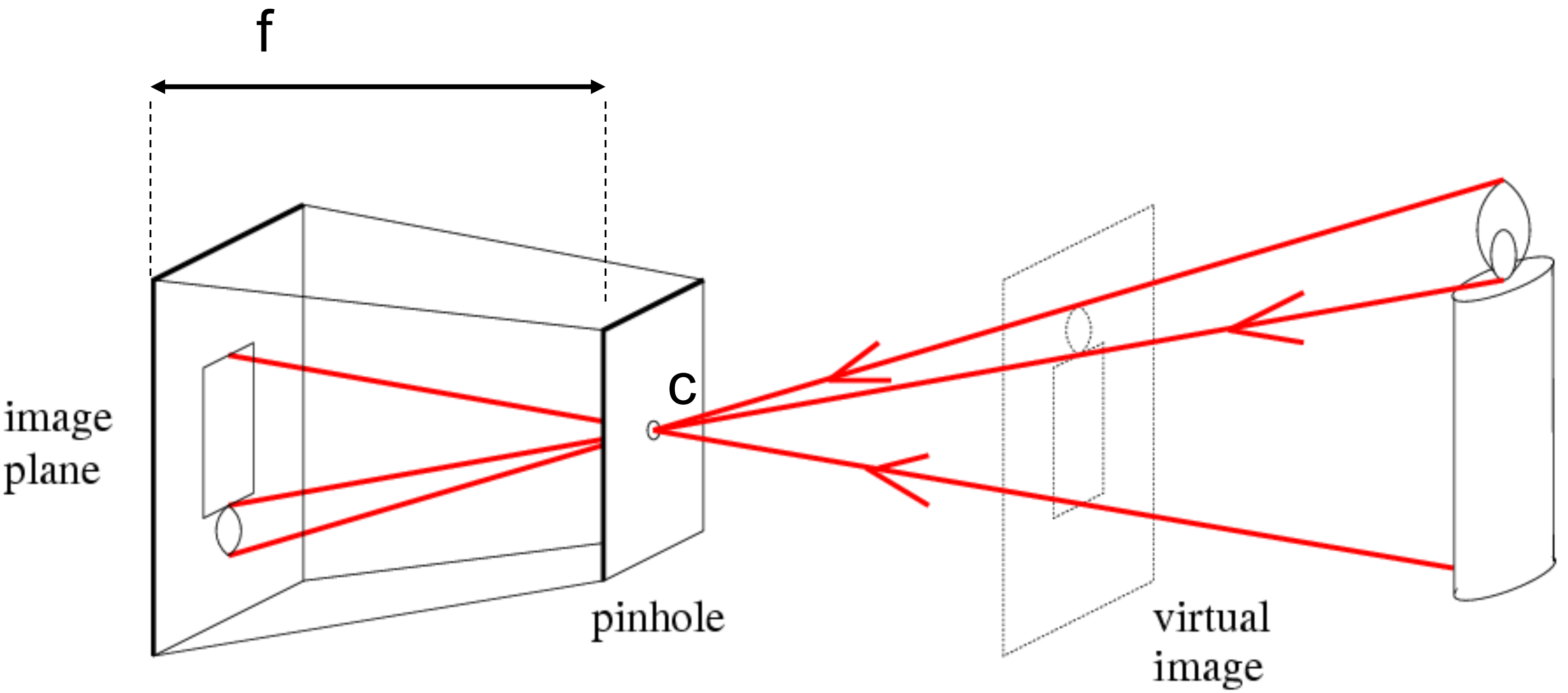


Barrel (fisheye lens)

Cameras

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras
- Other camera models

Pinhole camera

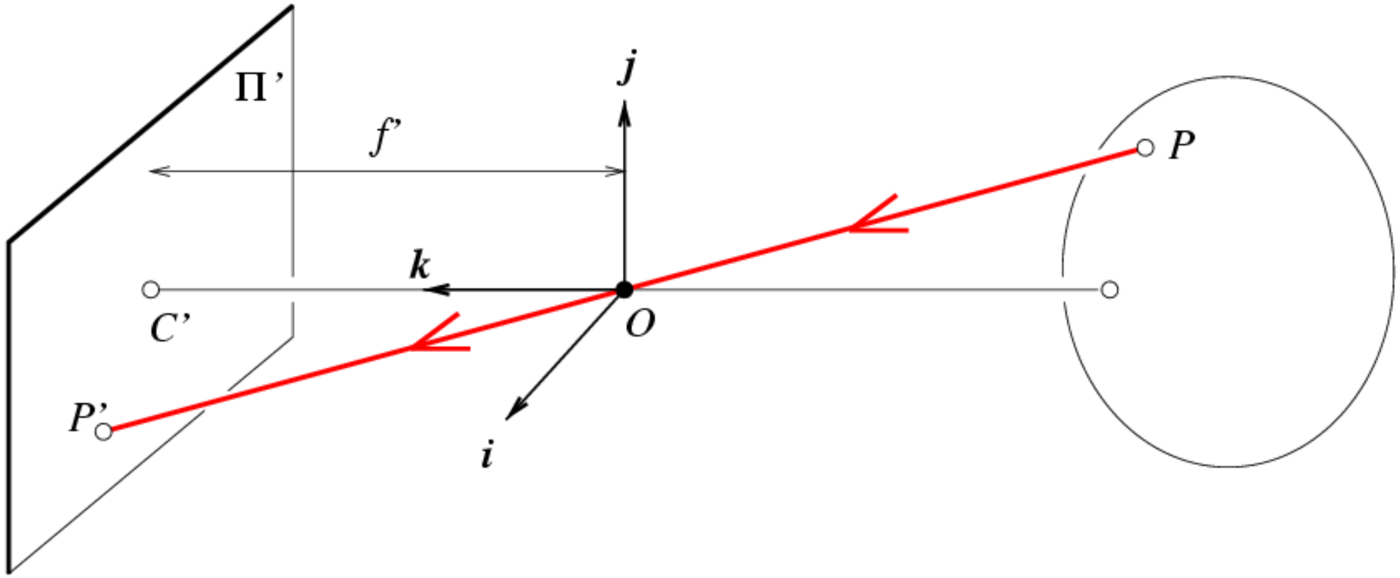


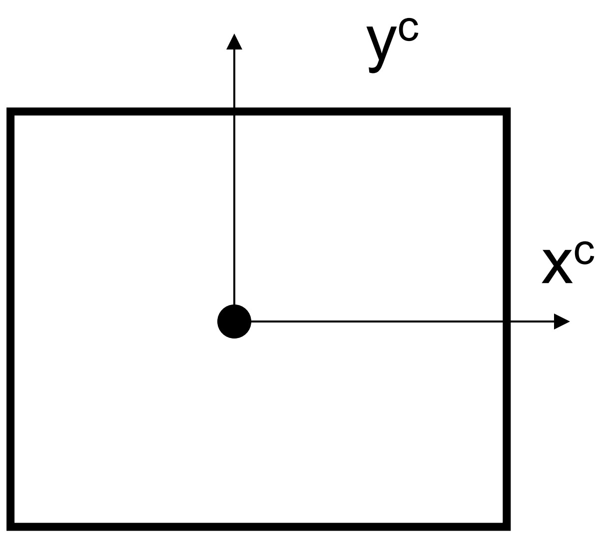
$$(x, y, z) \rightarrow (f\,\frac{x}{z}, f\,\frac{y}{z})$$

$$\Re^3 \xrightarrow{E} \Re^2$$

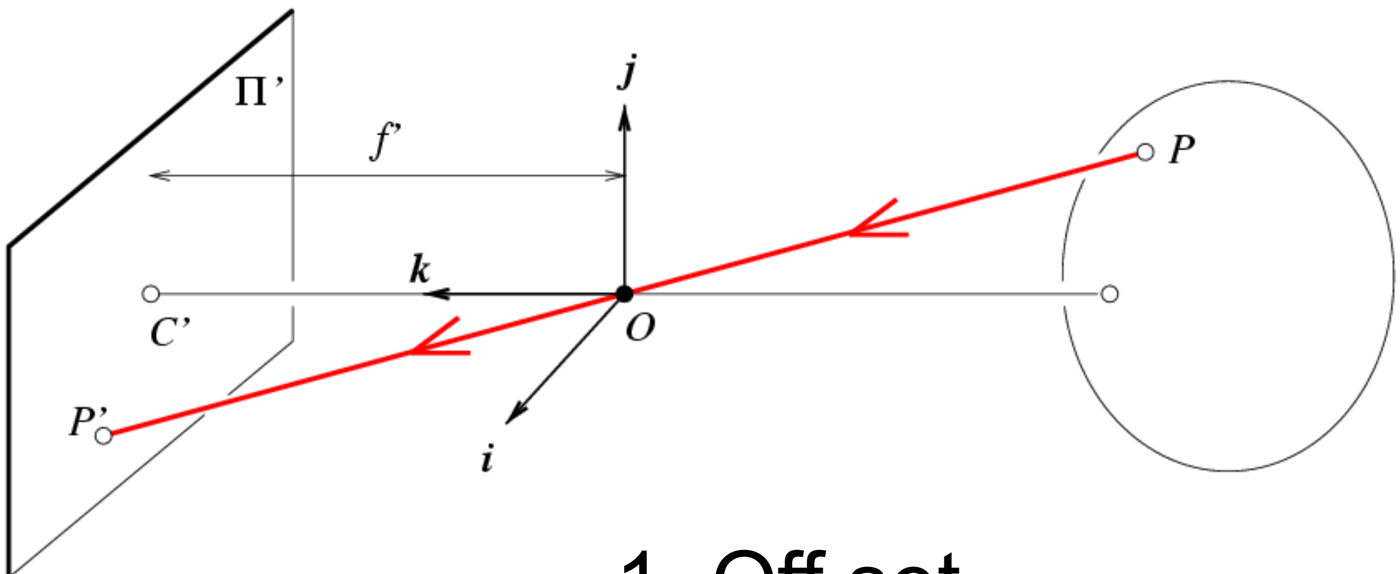
f = focal length
c = center of the camera

Coordinate systems

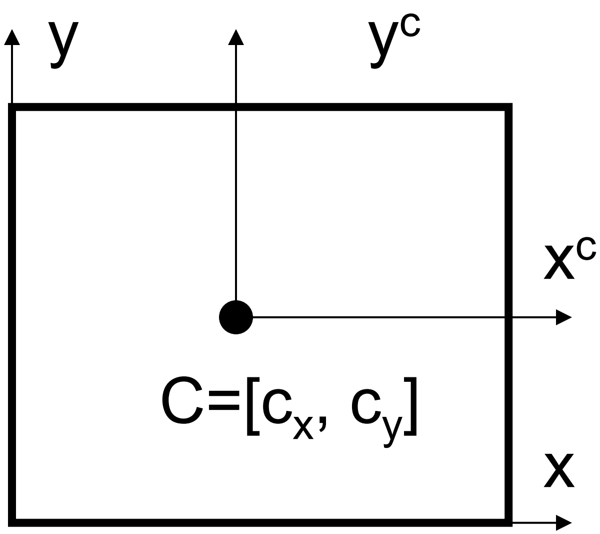




Converting to pixels

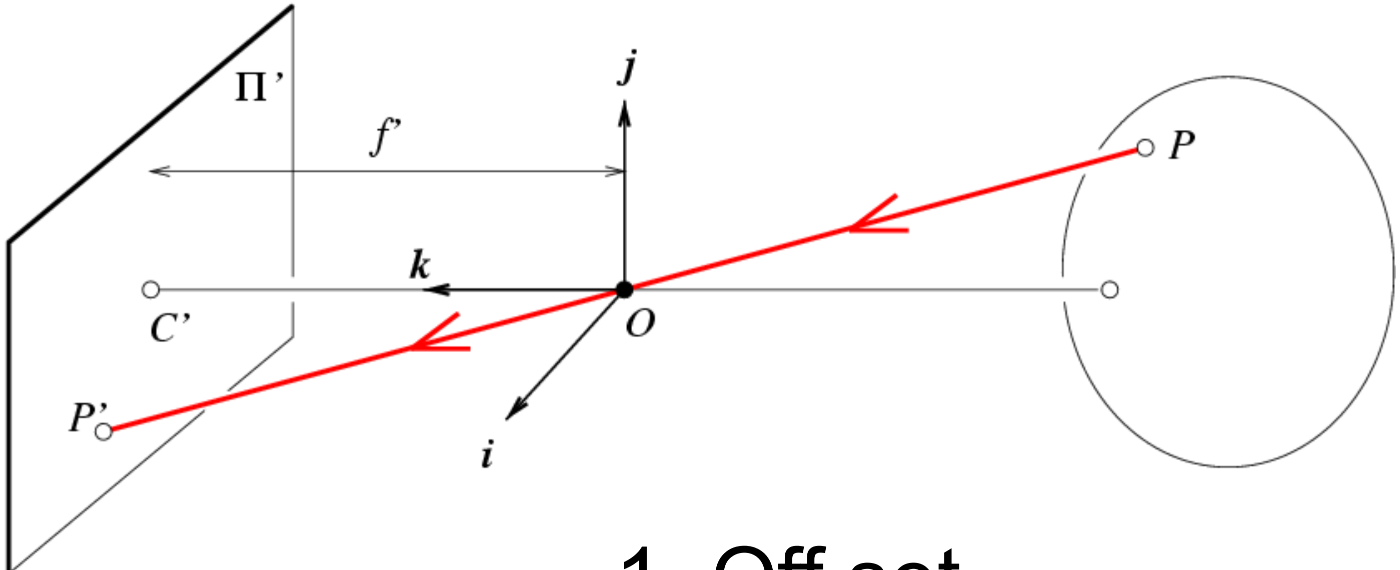


1. Off set

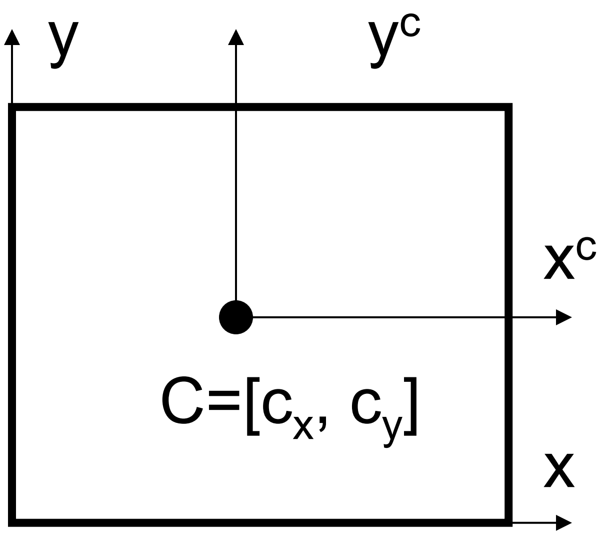


$$(x, y, z) \rightarrow (f\frac{x}{z} + c_x, f\frac{y}{z} + c_y)$$

Converting to pixels



1. Off set
2. From metric to pixels



$$(x,y,z) \rightarrow (\underbrace{\boxed{f\,k}}_{\alpha}\frac{x}{z}+c_x, \underbrace{\boxed{f\,l}}_{\beta}\frac{y}{z}+c_y)$$

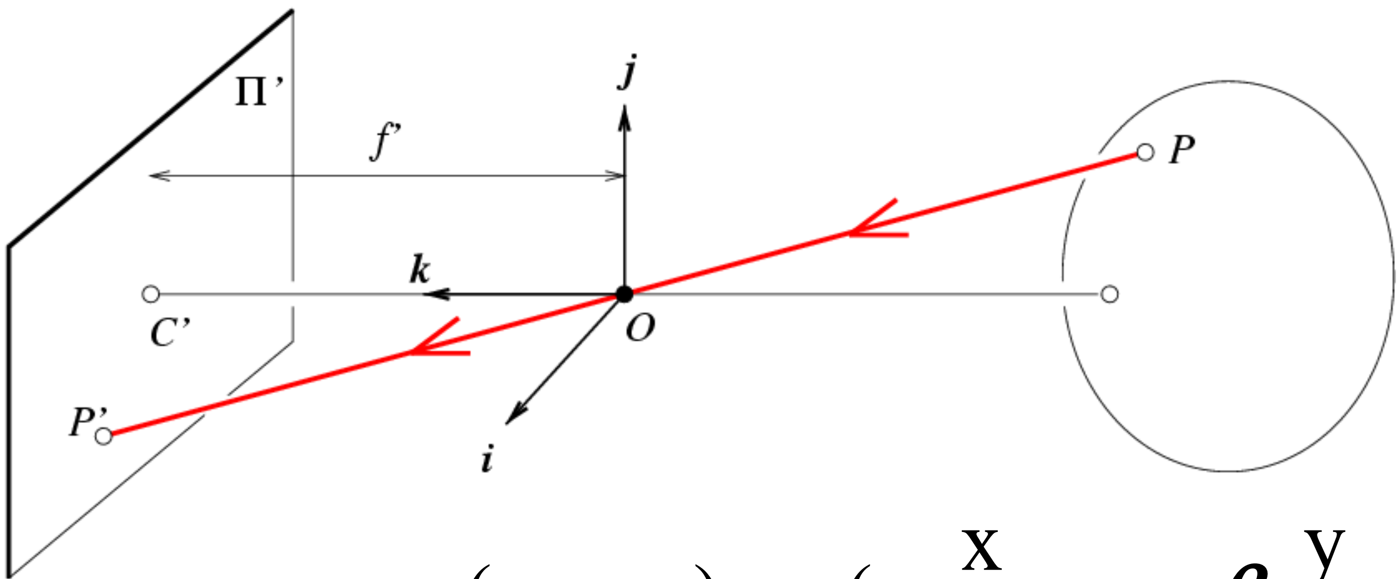
Units: k,l : pixel/m

f : m

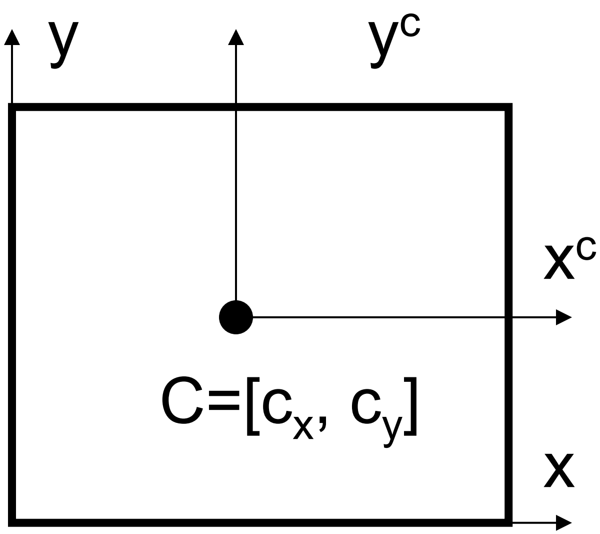
Non-square pixels

α, β : pixel

Converting to pixels



$$(x, y, z) \rightarrow \left(\boldsymbol{\alpha} \frac{x}{z} + c_x, \boldsymbol{\beta} \frac{y}{z} + c_y\right)$$



- Matrix form?

A related question:
- Is this a linear transformation?

$$(x, y, z) \rightarrow (f\frac{x}{z}, f\frac{y}{z})$$

Is this a linear transformation?

No — division by z is nonlinear

How to make it linear?

Homogeneous coordinates

$$(x, y) \Rightarrow \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

homogeneous image coordinates

$$(x, y, z) \Rightarrow \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

homogeneous scene coordinates

- Converting *from* homogeneous coordinates

$$\begin{bmatrix} x \\ y \\ w \end{bmatrix} \Rightarrow (x/w, y/w)$$

$$\begin{bmatrix} x \\ y \\ z \\ w \end{bmatrix} \Rightarrow (x/w, y/w, z/w)$$

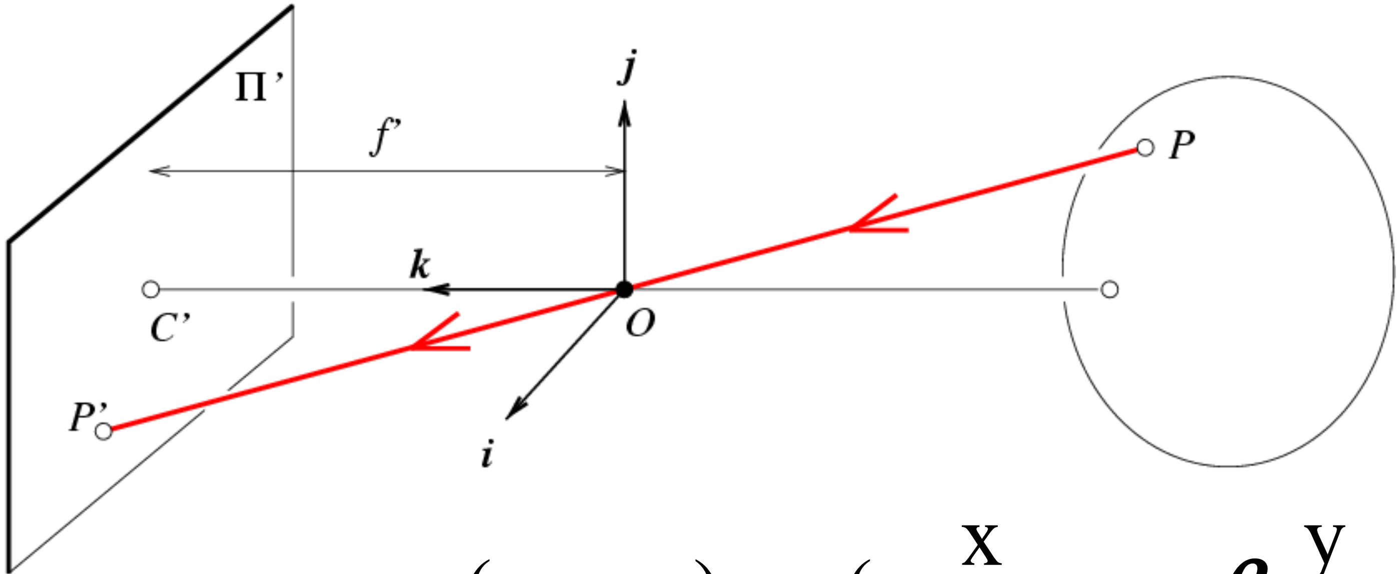
Perspective Projection Transformation

$$X' = \begin{bmatrix} f\,x \\ f\,y \\ z \end{bmatrix} = \underbrace{\begin{bmatrix} f & 0 & 0 & 0 \\ 0 & f & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}_{\text{M}} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

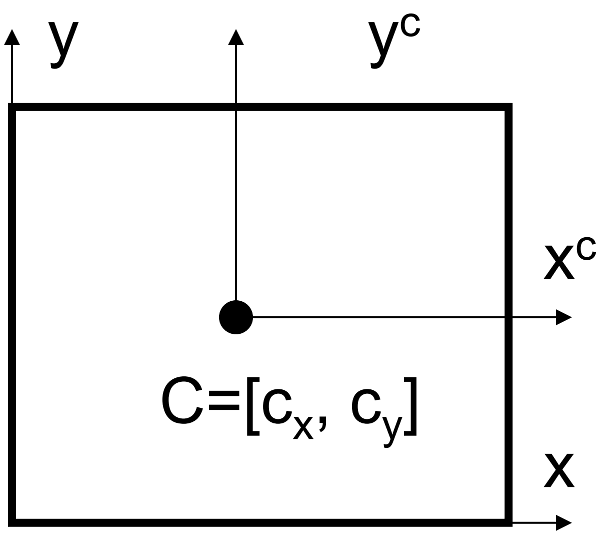
$$X' = M\,X$$

$$\Re^4 \xrightarrow{H} \Re^3$$

Camera Matrix

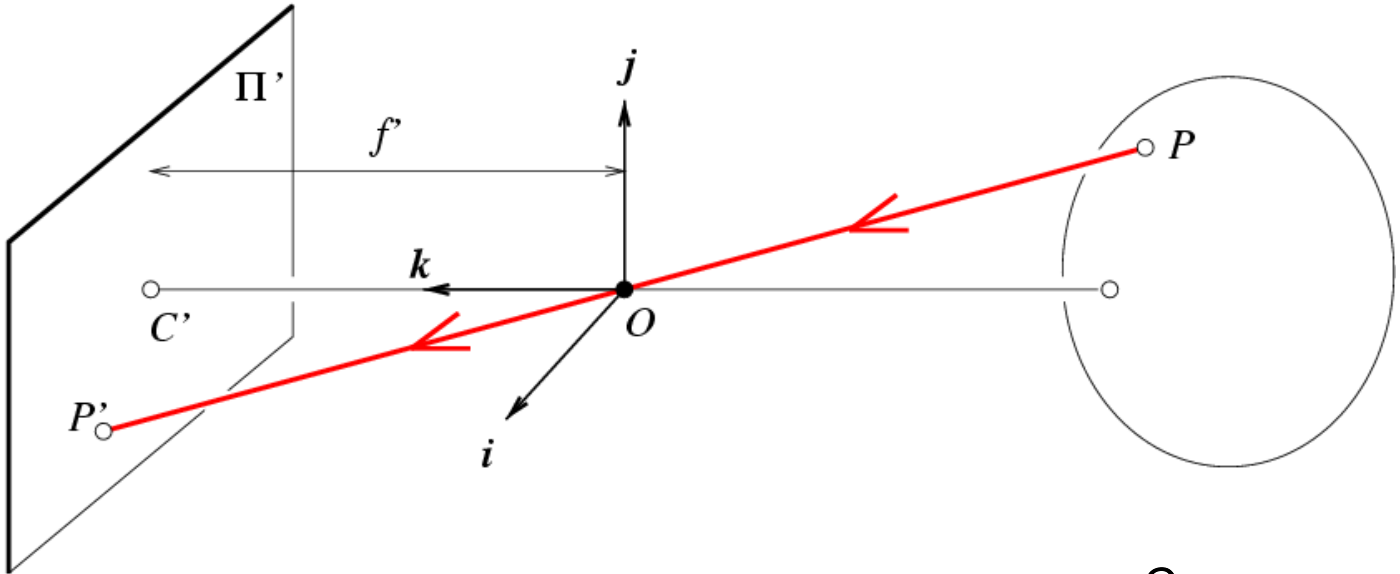


$$(x, y, z) \rightarrow (\alpha \frac{x}{z} + c_x, \beta \frac{y}{z} + c_y)$$



$$X' = \begin{bmatrix} \alpha x + c_x z \\ \beta y + c_y z \\ z \end{bmatrix} = \begin{bmatrix} \alpha & 0 & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Camera Matrix



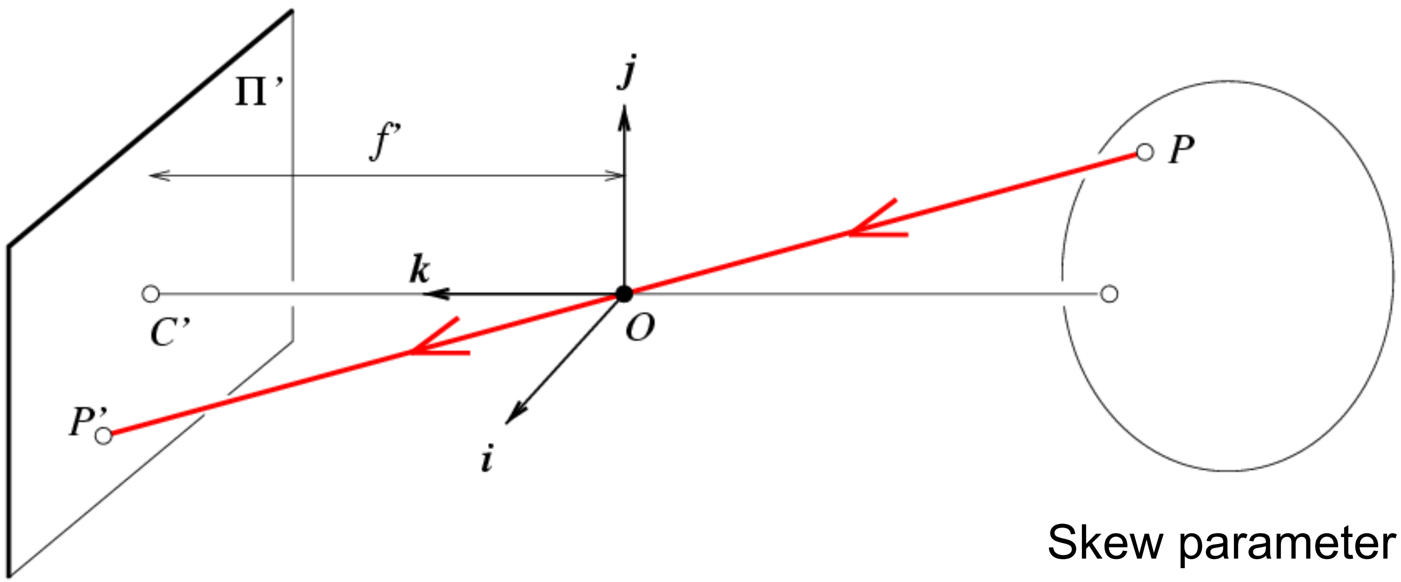
Camera matrix K

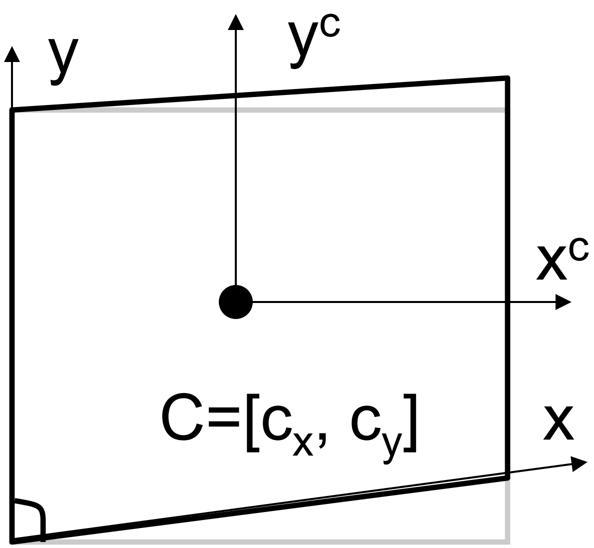
$$X' = M\,X$$

$$= K\begin{bmatrix} I & 0 \end{bmatrix} X$$

$$X' = \begin{bmatrix} \boldsymbol{\alpha} & 0 & c_x & 0 \\ 0 & \boldsymbol{\beta} & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

Finite projective cameras



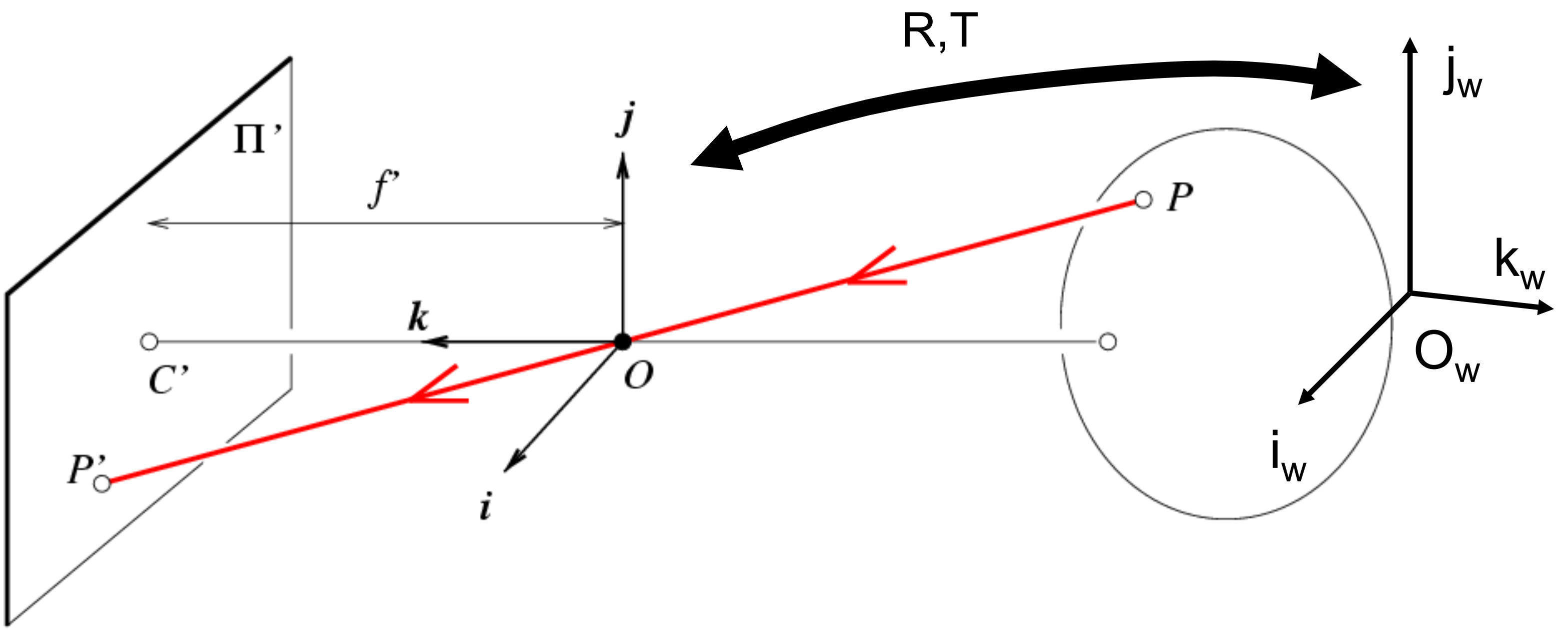


Skew parameter

$$X' = \begin{bmatrix} \alpha & \boxed{s} & c_x & 0 \\ 0 & \beta & c_y & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

K has 5 degrees of freedom!

World reference system

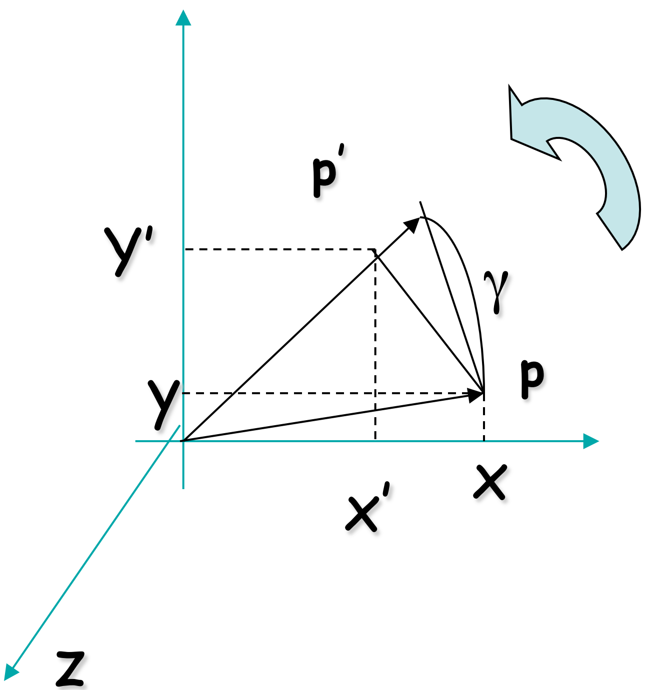


•The mapping so far is defined within the camera reference system

• What if an object is represented in the world reference system

3D Rotation of Points

Rotation around the coordinate axes, counter-clockwise:

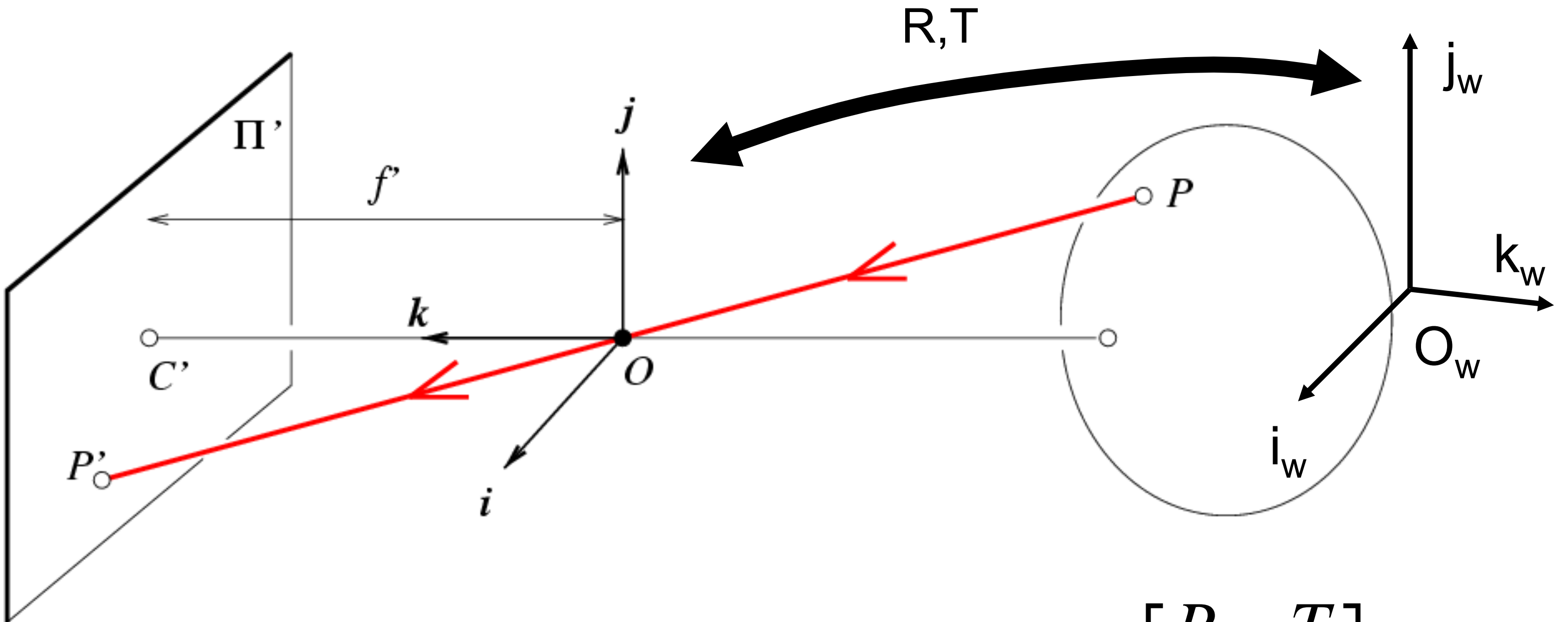


$$R_x(\alpha) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & -\sin\alpha \\ 0 & \sin\alpha & \cos\alpha \end{bmatrix}$$

$$R_y(\beta) = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix}$$

$$R_z(\gamma) = \begin{bmatrix} \cos\gamma & -\sin\gamma & 0 \\ \sin\gamma & \cos\gamma & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

World reference system



In 4D homogeneous coordinates: $$X = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}_{4\times 4} X_w$$

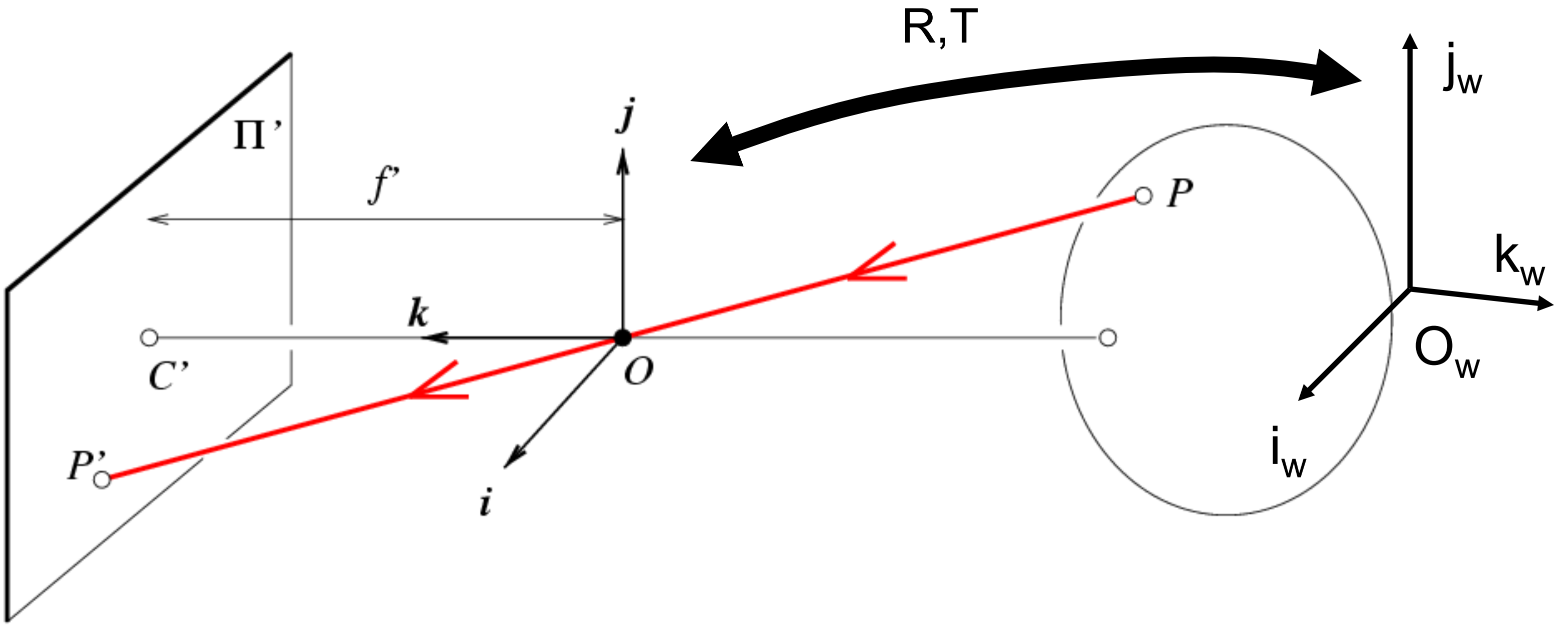
Internal parameters

External parameters

$$X' = K\begin{bmatrix} I & 0 \end{bmatrix} X = K\begin{bmatrix} I & 0 \end{bmatrix}\begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}_{4\times 4} X_w = \boxed{K\begin{bmatrix} R & T \end{bmatrix}} X_w$$

M

Projective cameras

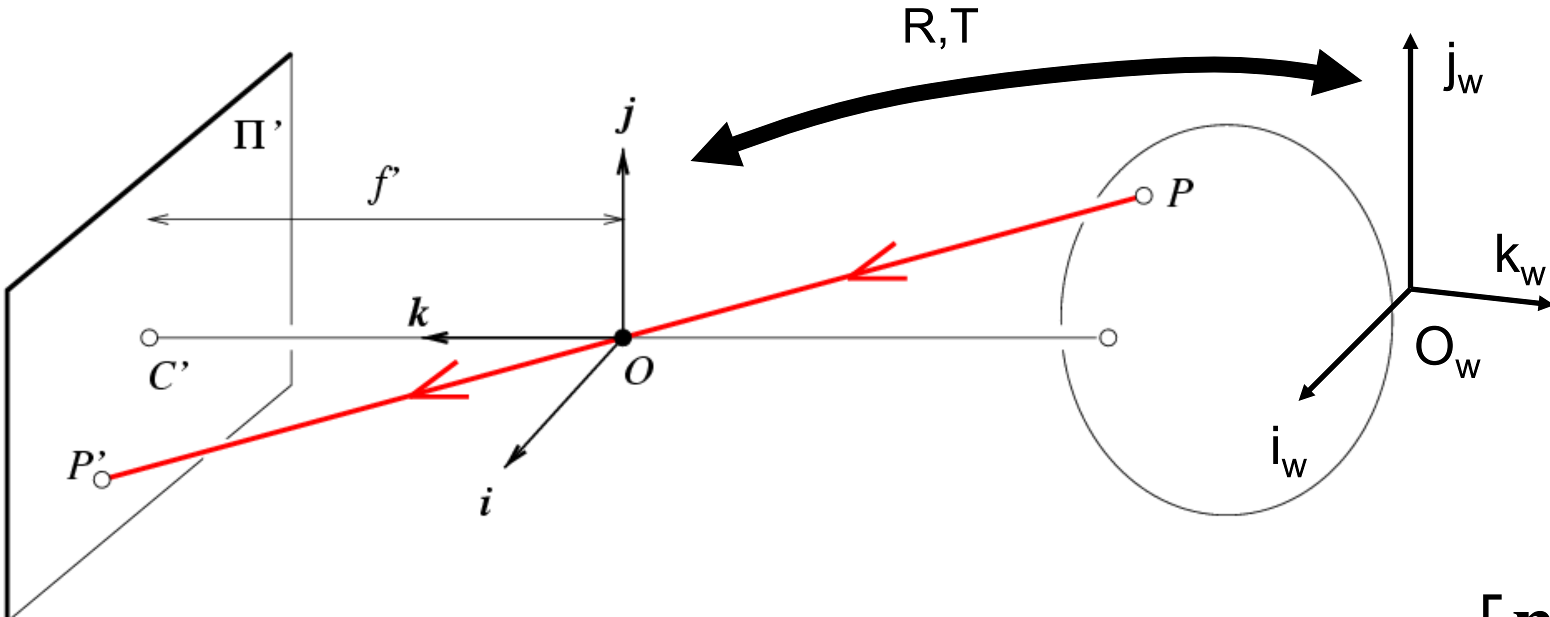


$$X'_{3\times 1} = M_{3x4}\ X_w = \mathrm{K}_{3\times 3}\begin{bmatrix}\mathrm{R} & \mathrm{T}\end{bmatrix}_{3\times 4}\ \mathrm{X}_{\mathrm{w}\,4\times 1} \qquad \mathrm{K} = \begin{bmatrix} \boldsymbol{\alpha} & \mathrm{s} & \mathrm{c_x} \\ 0 & \boldsymbol{\beta} & \mathrm{c_y} \\ 0 & 0 & 1 \end{bmatrix}$$

How many degrees of freedom?

5 + 3 + 3 =11!

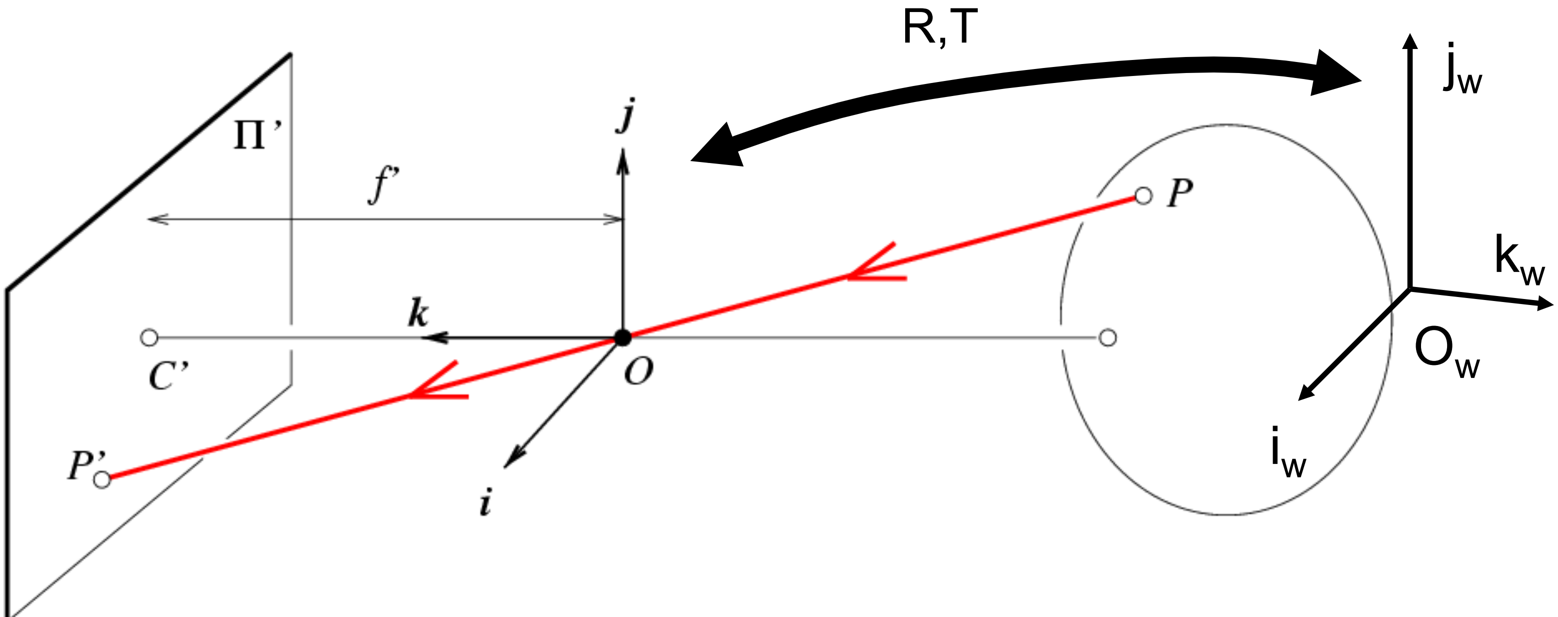
Projective cameras



$$X'_{3\times1} = \mathrm{M}\,X_w = K_{3\times3}\begin{bmatrix} R & T \end{bmatrix}_{3\times4} X_{w\,4\times1} \qquad M = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix}$$

$$= \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix} X_\mathbf{w} = \begin{bmatrix} \mathbf{m}_1 X_\mathbf{w} \\ \mathbf{m}_2 X_\mathbf{w} \\ \mathbf{m}_3 X_\mathbf{w} \end{bmatrix} \xrightarrow{\mathbf{E}} \left(\frac{\mathbf{m}_1 X_w}{\mathbf{m}_3 X_w}, \frac{\mathbf{m}_2 X_w}{\mathbf{m}_3 X_w}\right) = (x, y, z)_w$$

Projective cameras



$$X'_{3\times1} = M\,X_w = K_{3\times3}\begin{bmatrix} R & T \end{bmatrix}_{3\times4} X_{w\,4\times1} \qquad M = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix}$$

$$(x, y, z)_w \rightarrow \left(\frac{\mathbf{m}_1 X_w}{\mathbf{m}_3 X_w}, \frac{\mathbf{m}_2 X_w}{\mathbf{m}_3 X_w}\right)$$

M is defined up to scale!

Multiplying M by a scalar won't change the image

Theorem (Faugeras, 1993)

$$M = K\begin{bmatrix} R & T \end{bmatrix} = \begin{bmatrix} KR & KT \end{bmatrix} = \begin{bmatrix} A & b \end{bmatrix}$$

$$K = \begin{bmatrix} \alpha & s & c_x \\ 0 & \beta & c_y \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{aligned} \boldsymbol{\alpha} &= \mathrm{f\ k}; \\ \boldsymbol{\beta} &= \mathrm{f\ l} \end{aligned} \qquad A = \begin{bmatrix} \mathbf{a}_1 \\ \mathbf{a}_2 \\ \mathbf{a}_3 \end{bmatrix}$$

- A necessary and sufficient condition for $\mathcal{M}$ to be a perspective projection matrix is that $\mathrm{Det}(\mathcal{A}) \neq 0$.
- A necessary and sufficient condition for $\mathcal{M}$ to be a zero-skew perspective projection matrix is that $\mathrm{Det}(\mathcal{A}) \neq 0$ and

$$(\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3) = 0.$$

- A necessary and sufficient condition for $\mathcal{M}$ to be a perspective projection matrix with zero skew and unit aspect-ratio is that $\mathrm{Det}(\mathcal{A}) \neq 0$ and

$$\begin{cases} (\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3) = 0, \\ (\boldsymbol{a}_1 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_1 \times \boldsymbol{a}_3) = (\boldsymbol{a}_2 \times \boldsymbol{a}_3) \cdot (\boldsymbol{a}_2 \times \boldsymbol{a}_3). \end{cases}$$

Properties of Projection

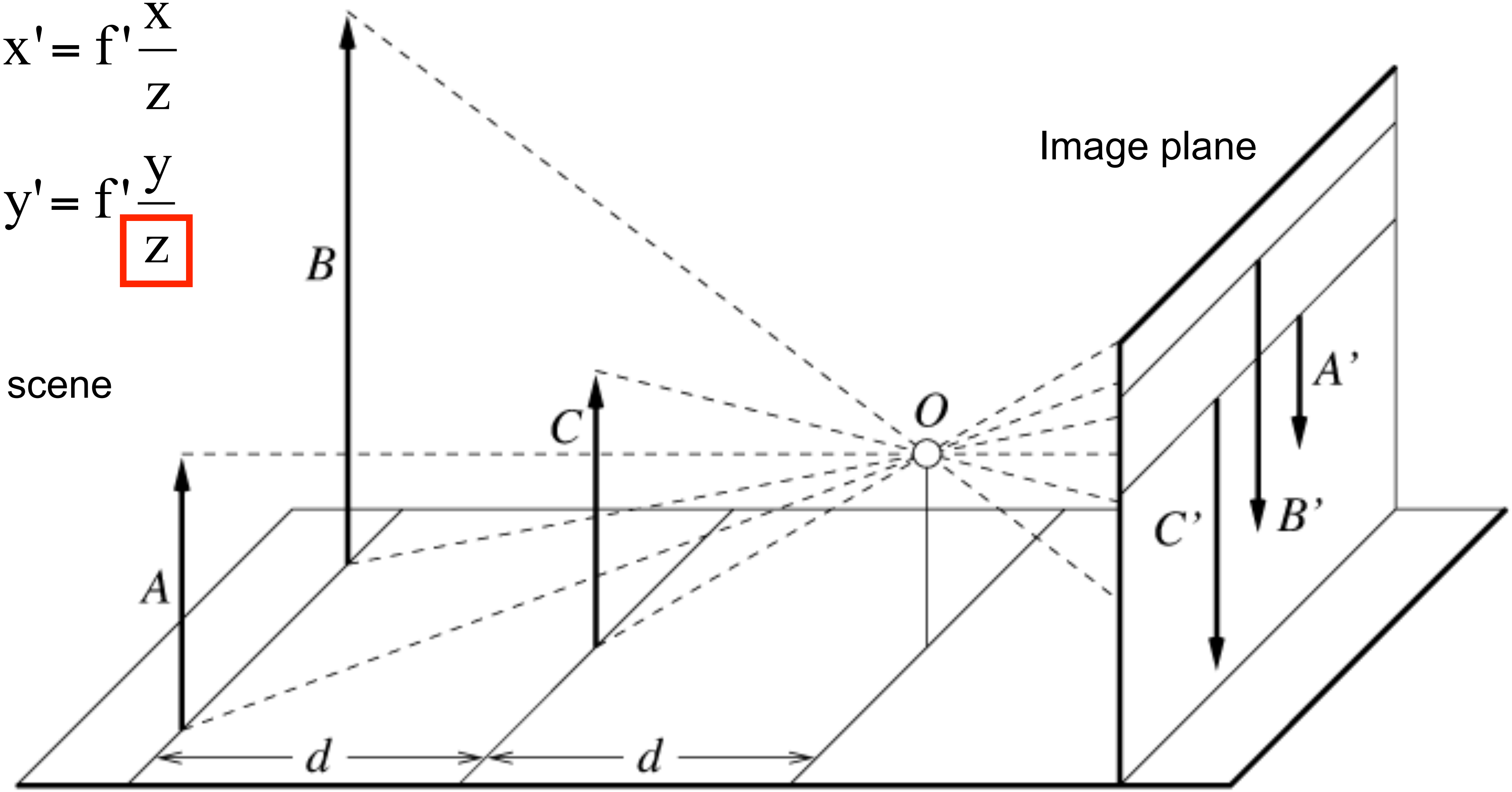
- Points project to points
- Lines project to lines
- Distant objects look smaller

Properties of Projection

•Distant objects look smaller

$$\begin{cases} x' = f'\dfrac{x}{z} \\ y' = f'\dfrac{y}{\boxed{z}} \end{cases}$$

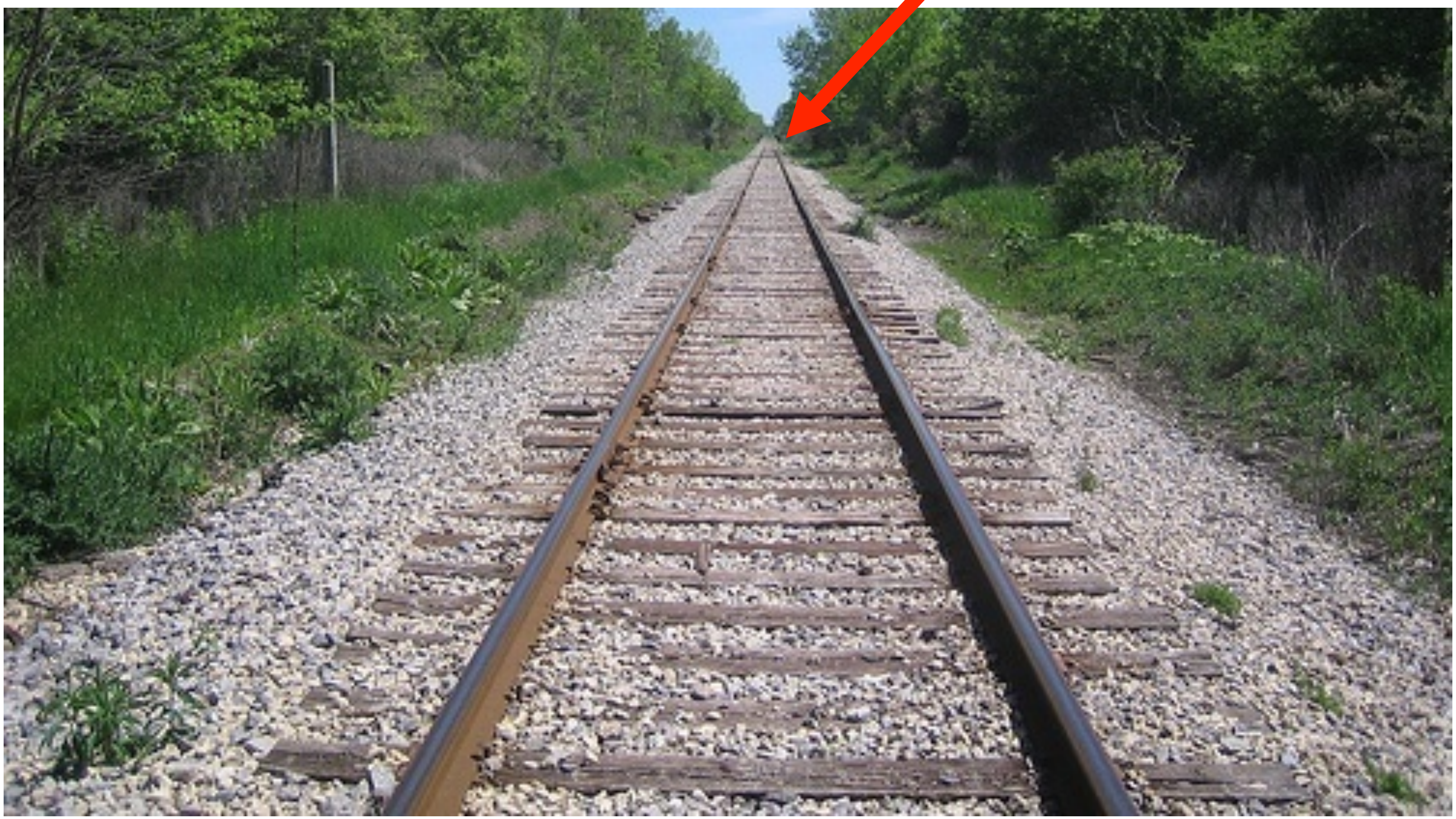
scene



Properties of Projection

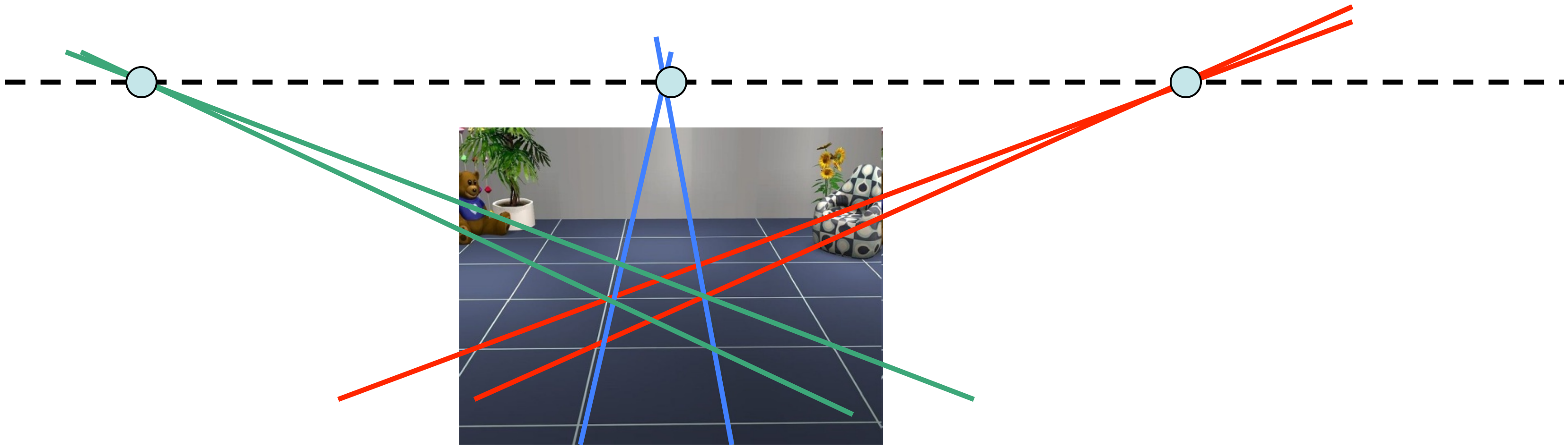
- Angles are not preserved
- Parallel lines meet!

Vanishing point



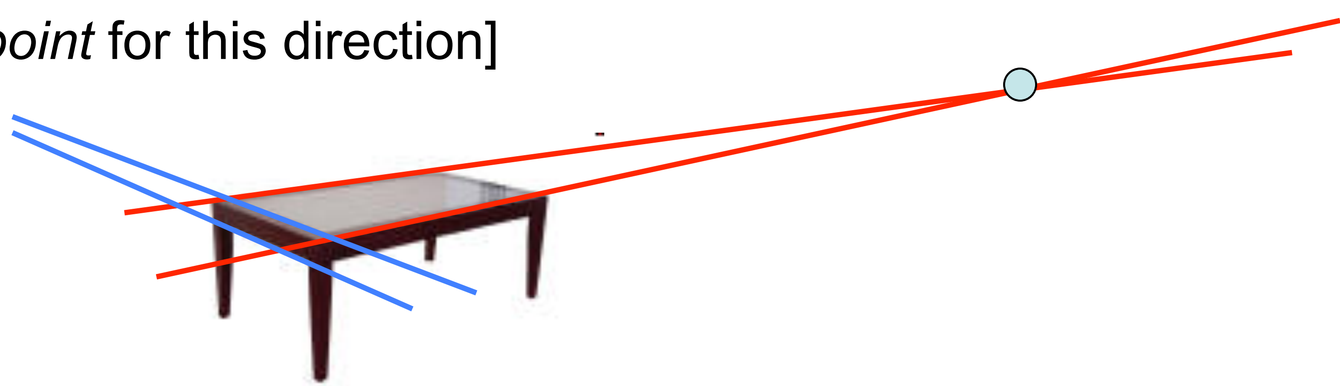
Vanishing points

•Sets of parallel lines on the same plane lead to *collinear* vanishing points [The line is called the *horizon* for that plane]

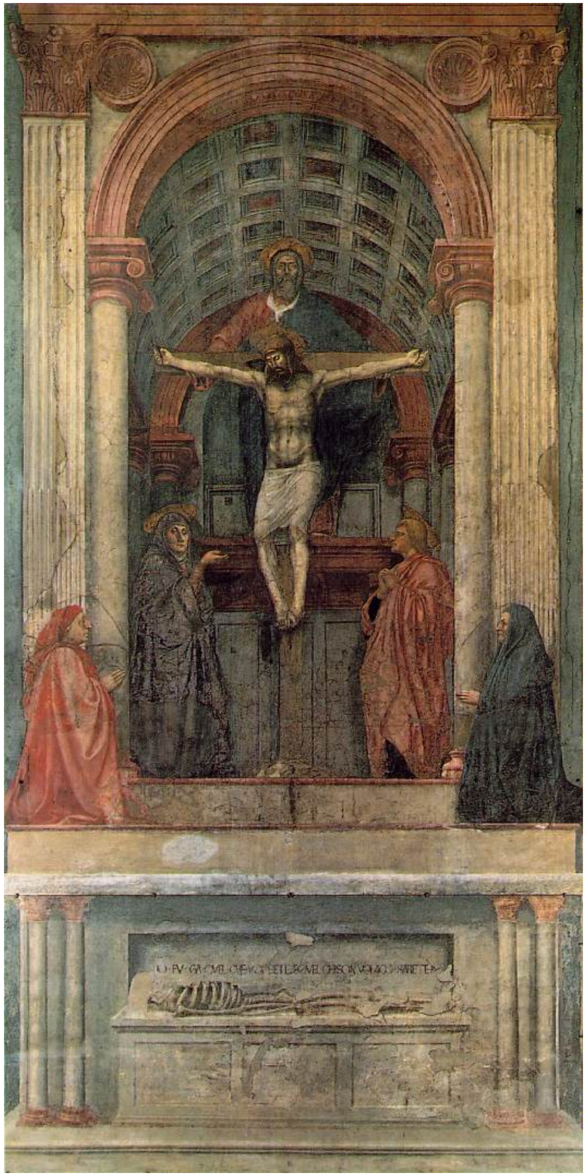


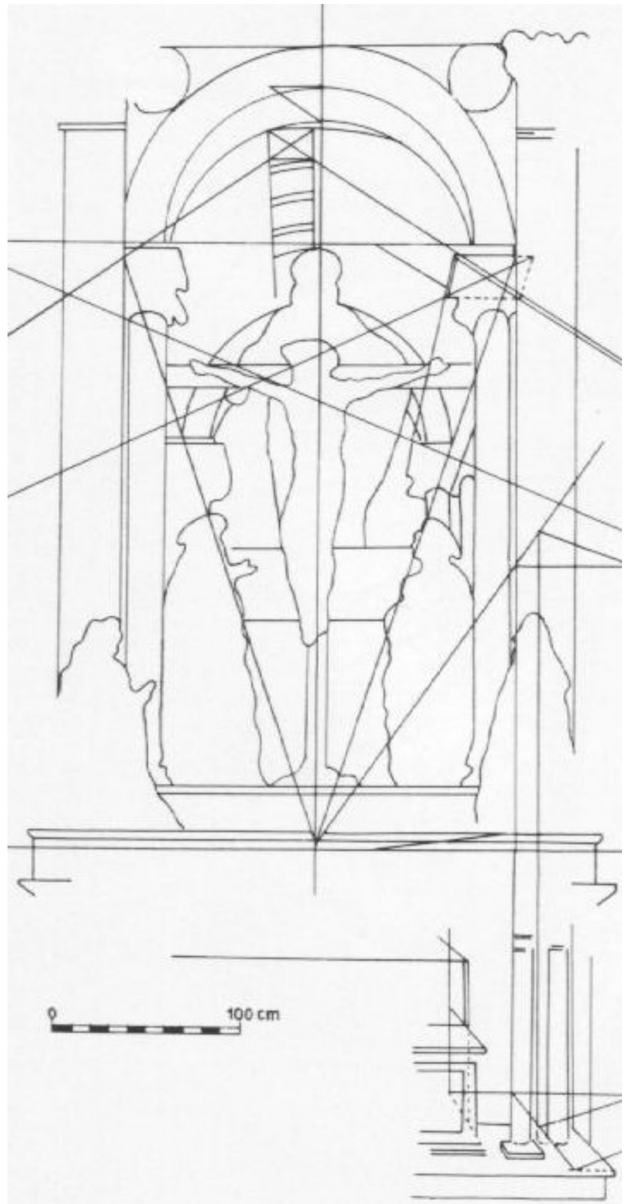
• Each set of parallel lines meets at a different point

[The *vanishing point* for this direction]



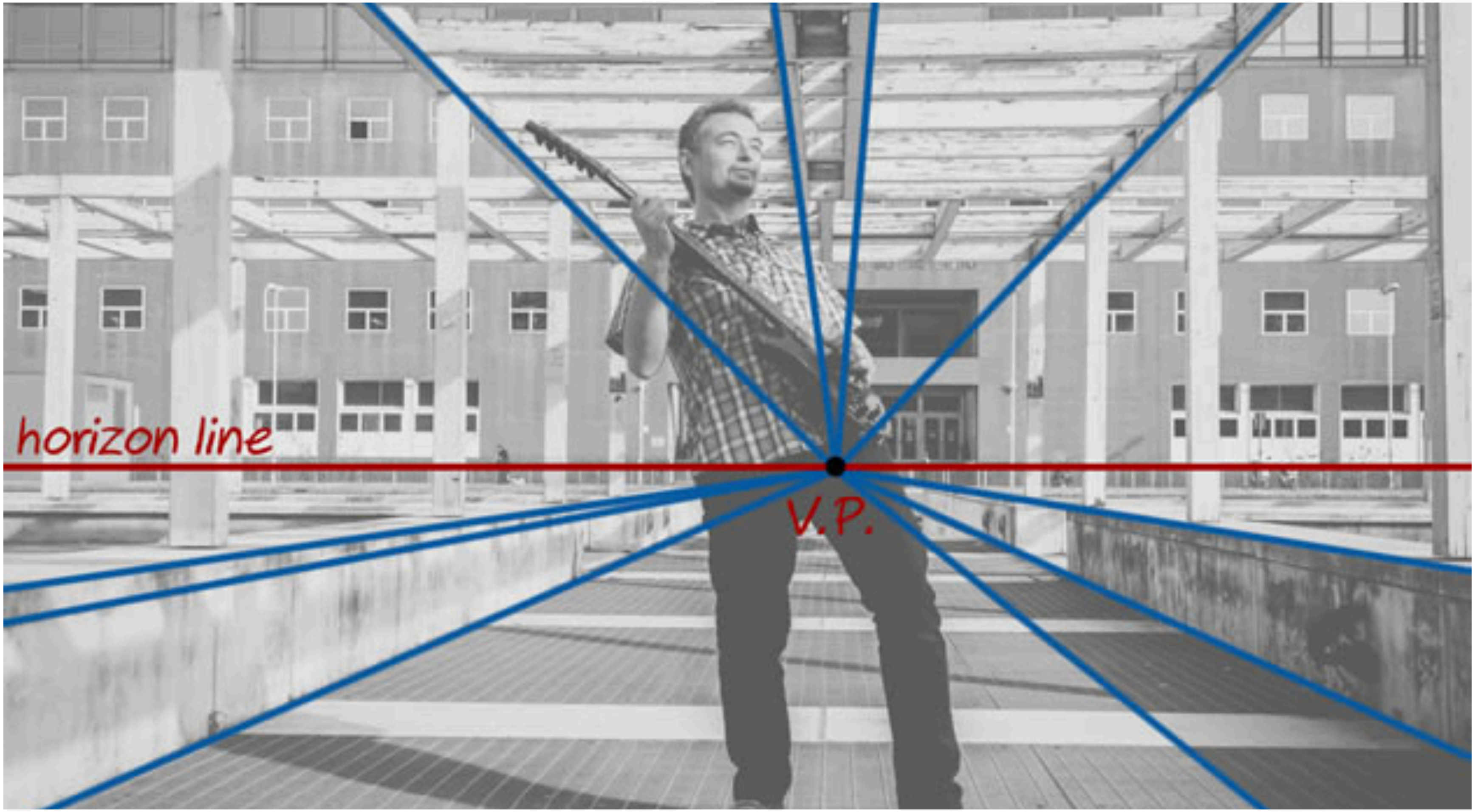
One-point perspective

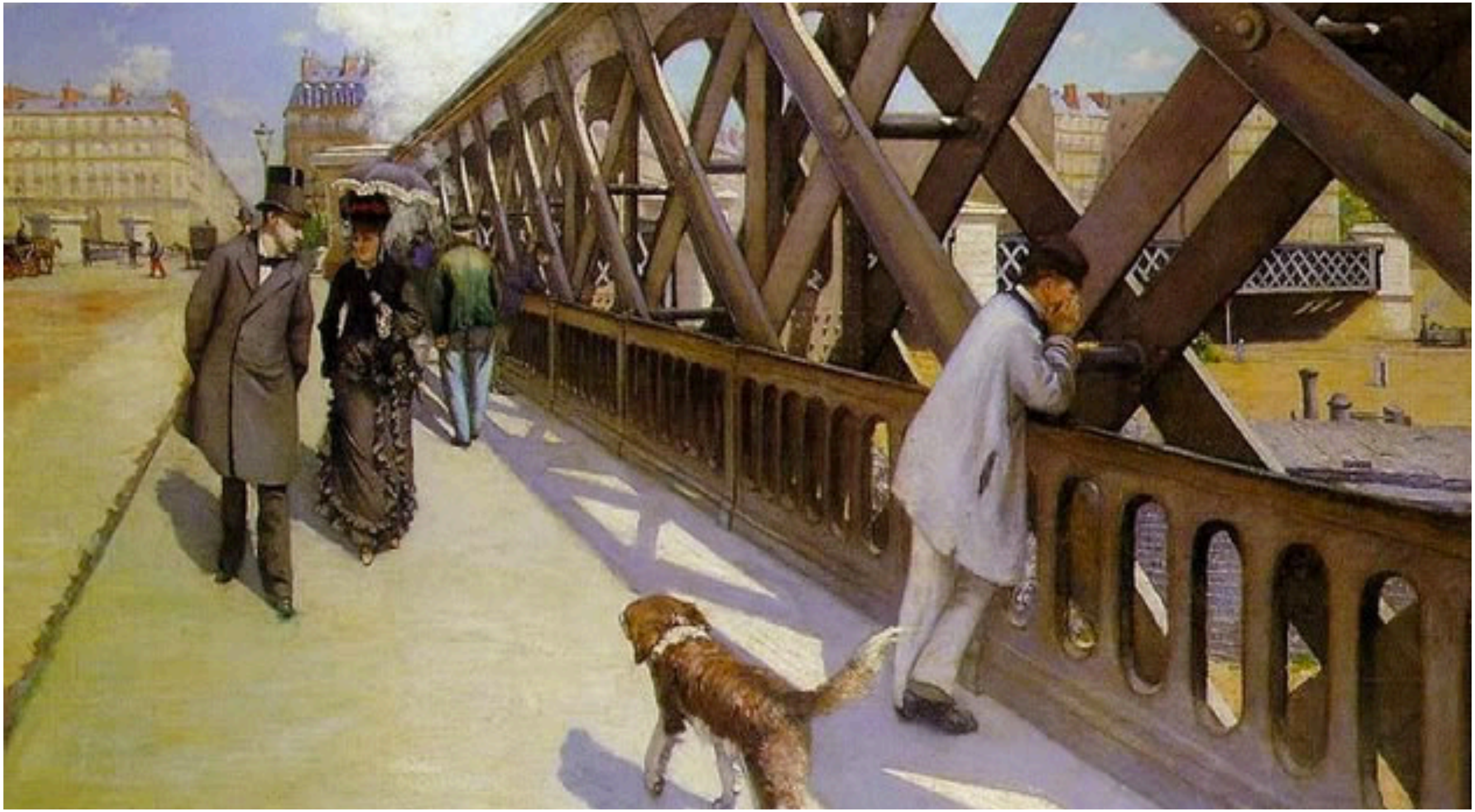


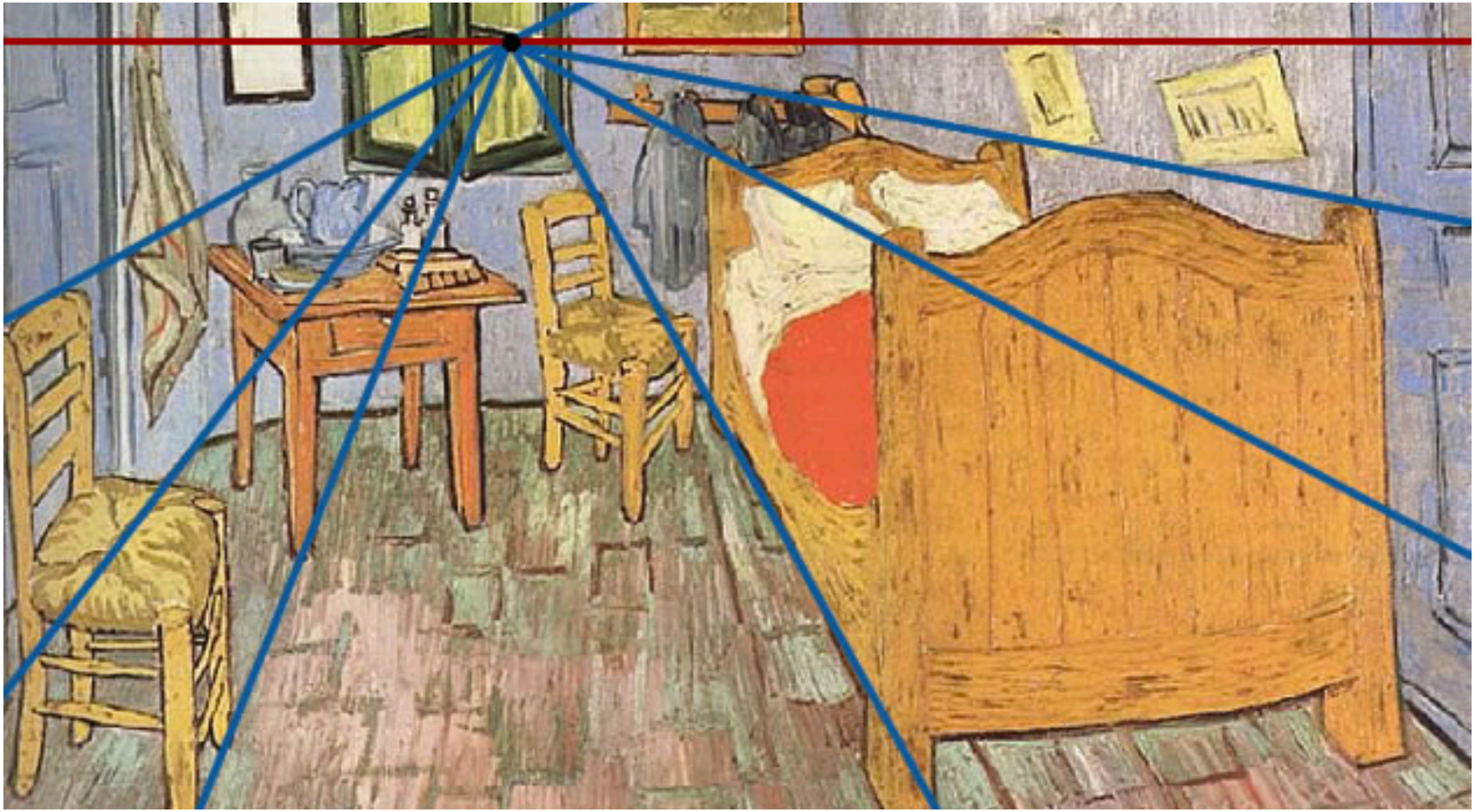


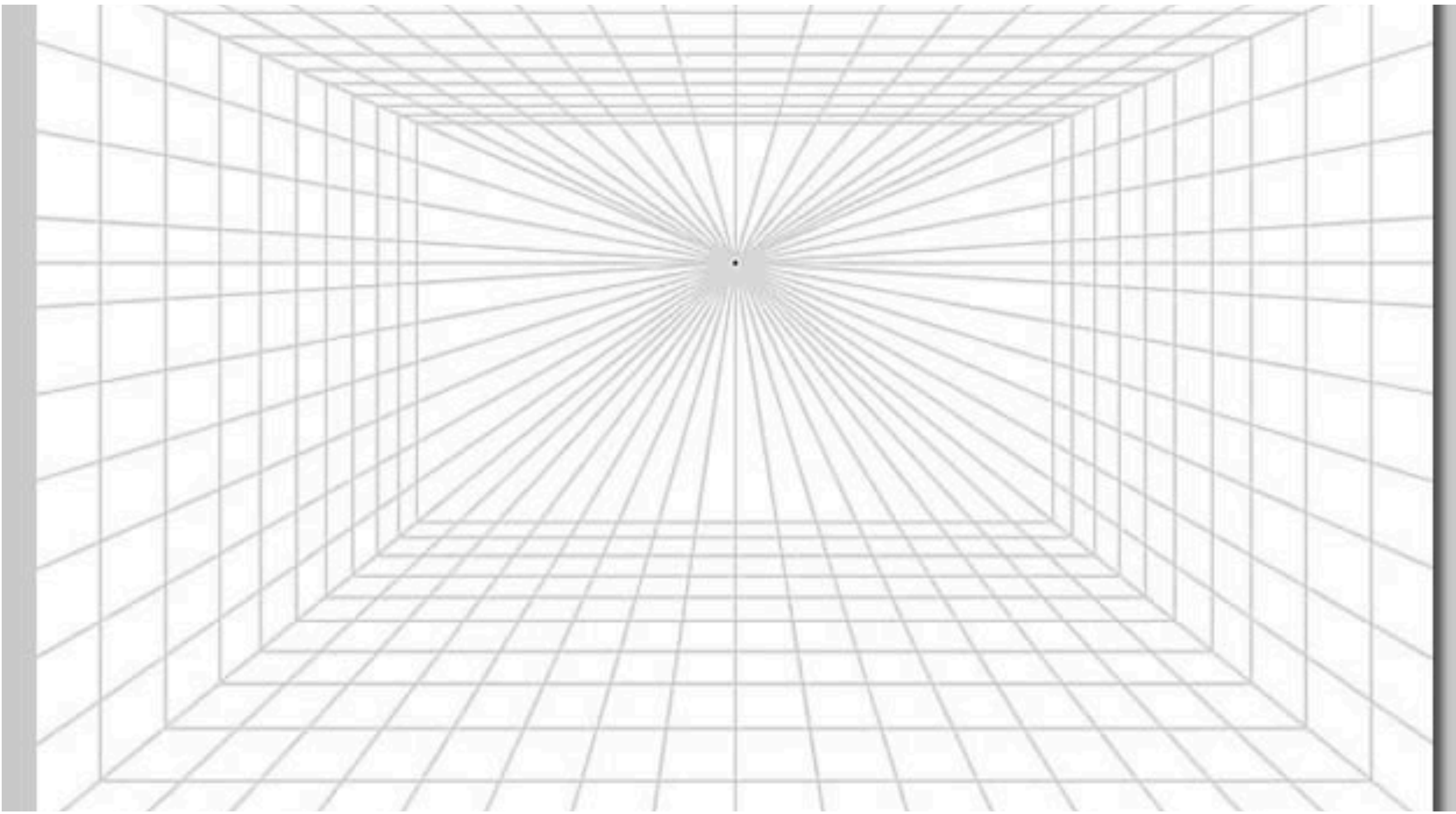
- Masaccio, *Trinity*, Santa Maria Novella, Florence, 1425-28

Credit slide S. Lazebnik


horizon line
V.P.



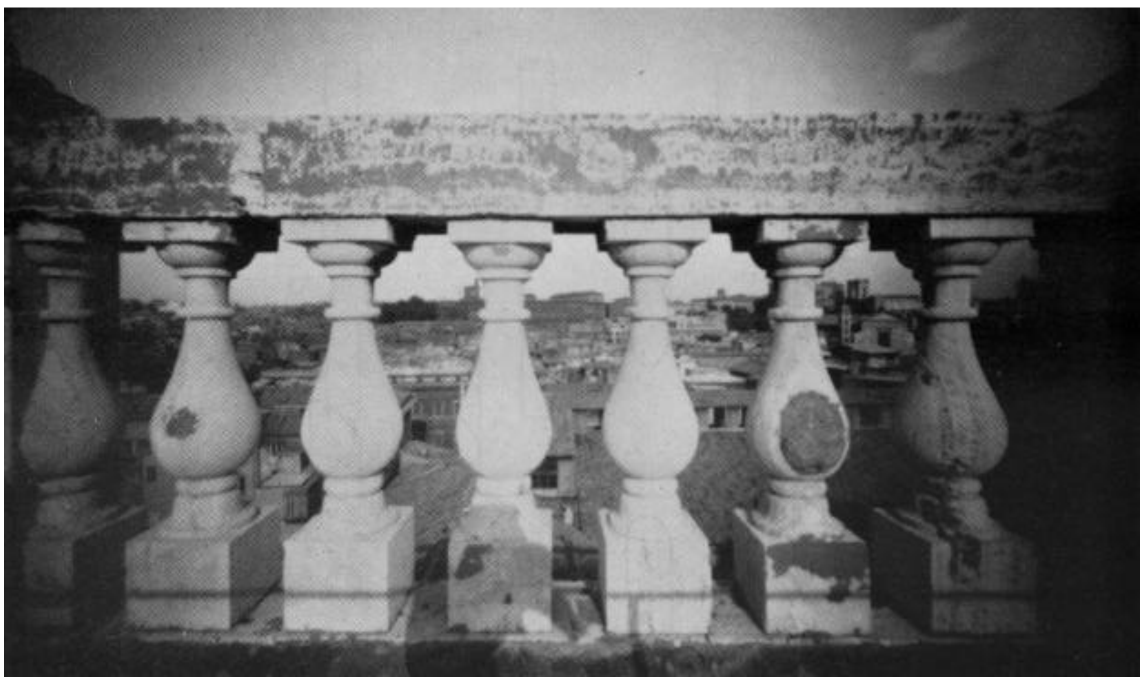


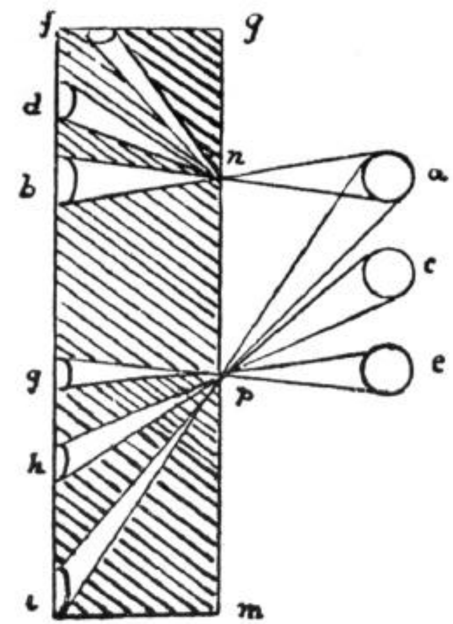


Properties of Projection

Objects on the periphery are expanded

- The exterior columns appear bigger
- The distortion is not due to lens flaws
- Problem pointed out by DaVinci



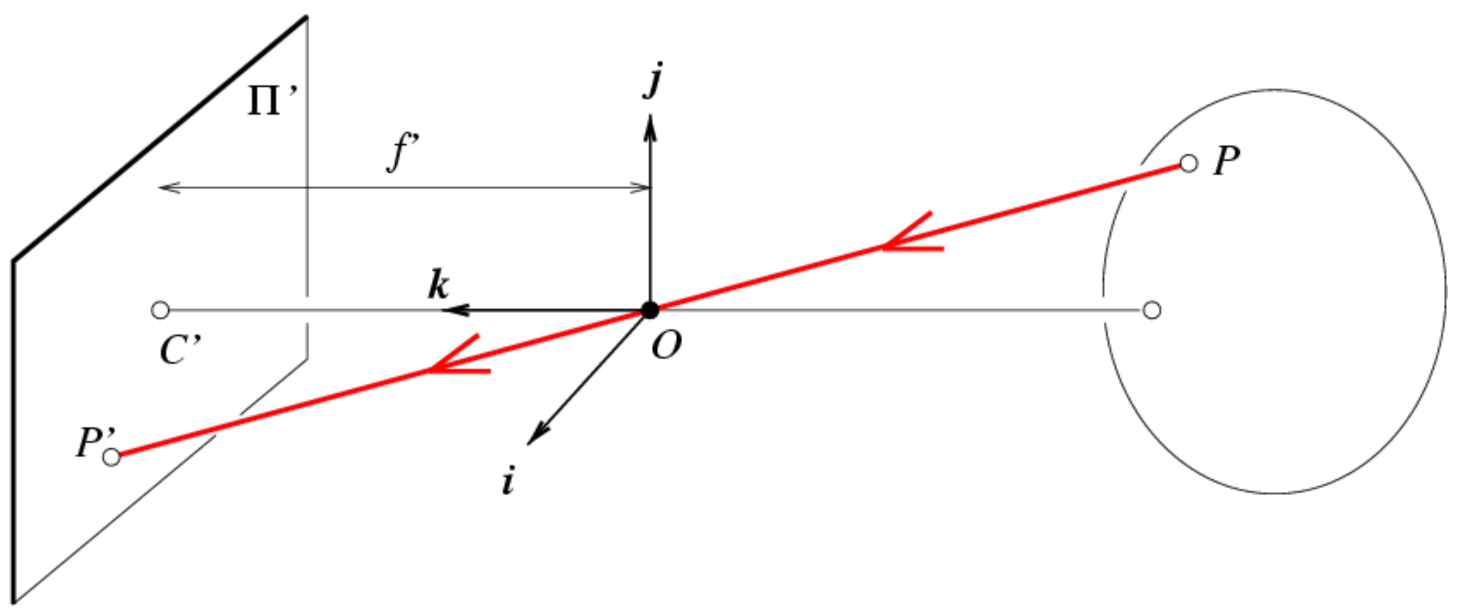


Slide by F. Durand

Credit slide S. Lazebnik

Properties of Projection

- Degenerate cases
  - Line through focal point projects to a point.
  - Plane through focal point projects to line
  - Plane perpendicular to image plane projects to part of the image (with horizon).

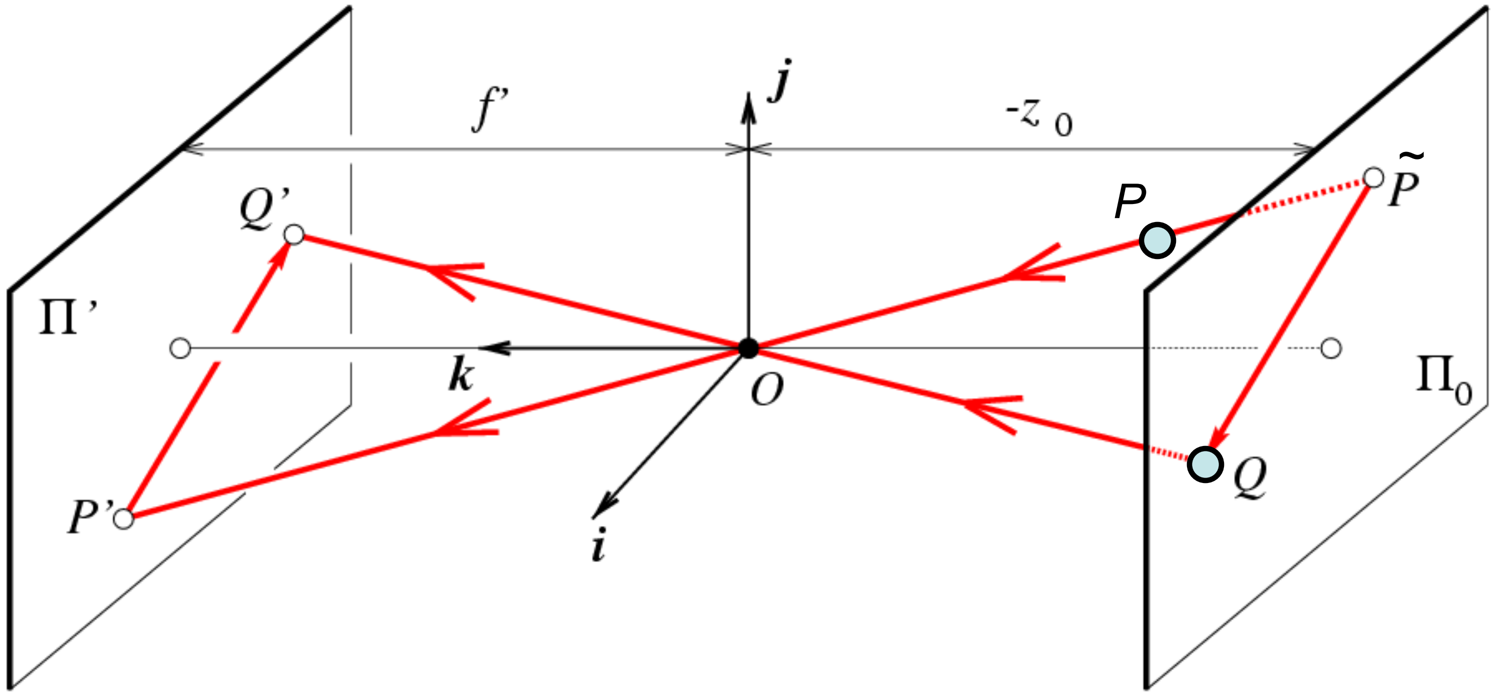


Cameras

- Pinhole cameras
- Cameras & lenses
- The geometry of pinhole cameras
- Other camera models

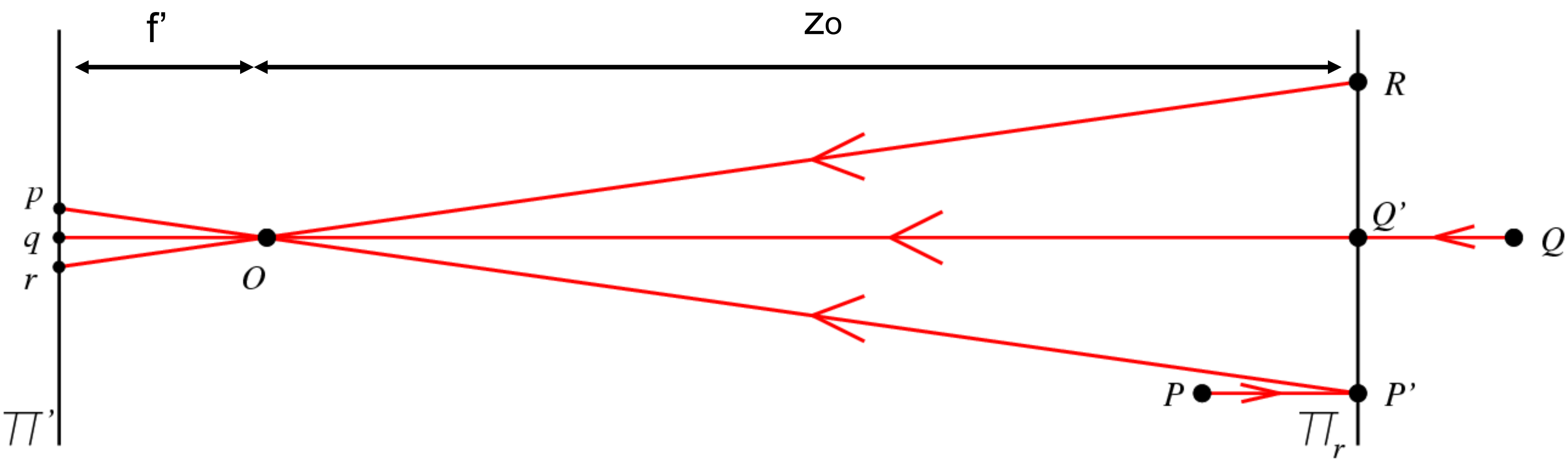
Weak perspective projection

When the relative scene depth is small compared to its distance from the camera



Weak perspective projection

When the relative scene depth is small compared to its distance from the camera

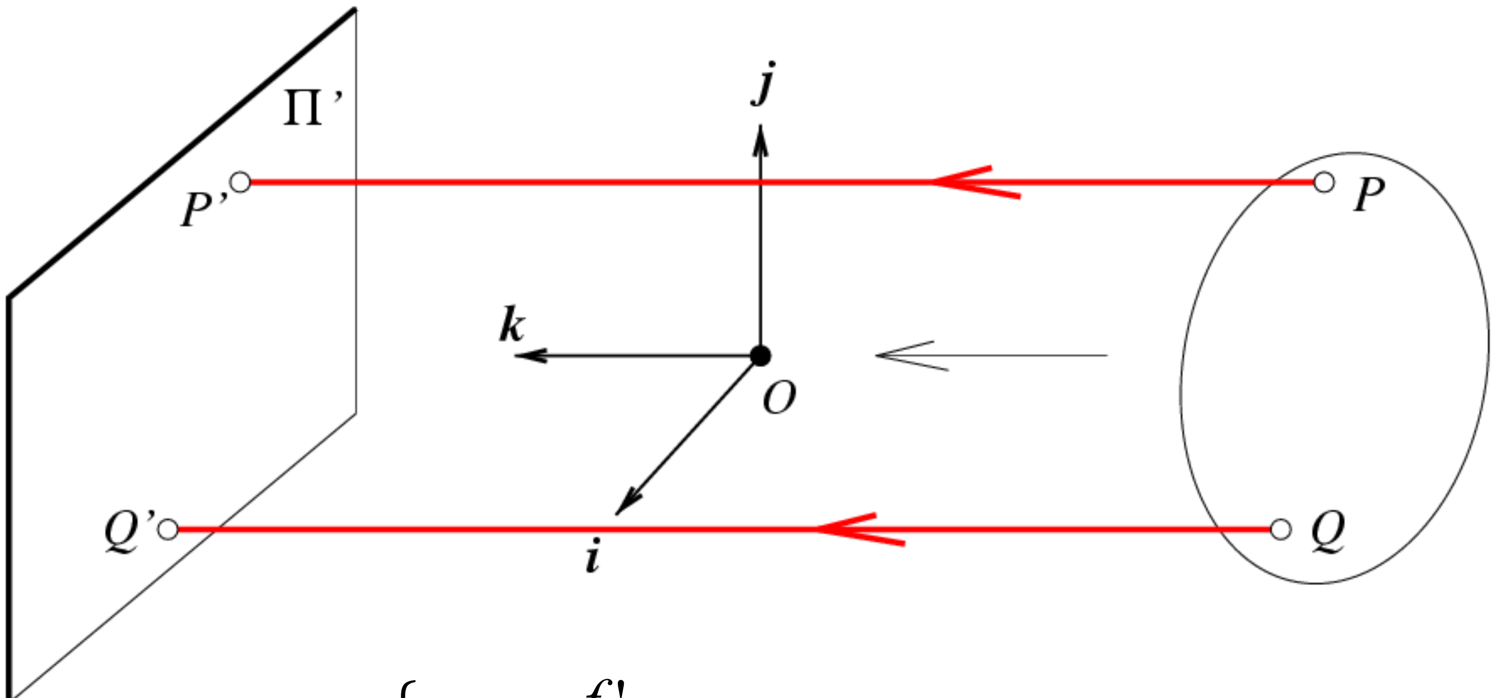


$$\begin{cases} x' = -\dfrac{f'}{z}x \\ \\ y' = -\dfrac{f'}{z}y \end{cases} \rightarrow \begin{cases} x' = -\boxed{\dfrac{f'}{z_0}}x \\ y' = -\dfrac{f'}{z_0}y \end{cases}$$

Magnification m

Orthographic (affine) projection

Distance from center of projection to image plane is infinite



$$\begin{cases} x' = -\dfrac{f'}{z}x \\ \\ y' = -\dfrac{f'}{z}y \end{cases} \rightarrow \begin{cases} x' = -x \\ y' = -y \end{cases}$$

Pros and Cons of These Models

- Weak perspective much simpler math.
  - Accurate when object is small and distant.
  - Most useful for recognition.
- Pinhole perspective much more accurate for scenes.
  - Used in structure from motion.

Weak perspective projection

Qingming Festival by the Riverside Zhang Zeduan ~900 AD

The Kangxi Emperor's Southern Inspection Tour (1691-1698) By Wang Hui

Next lecture

- How to calibrate a camera?