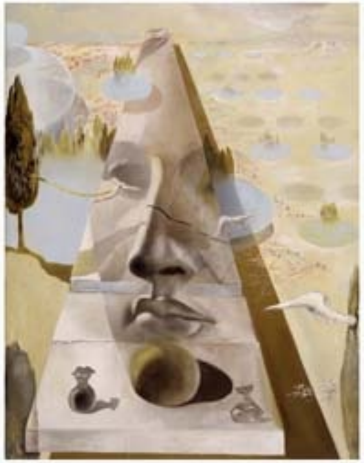


EECS 442 – Computer vision

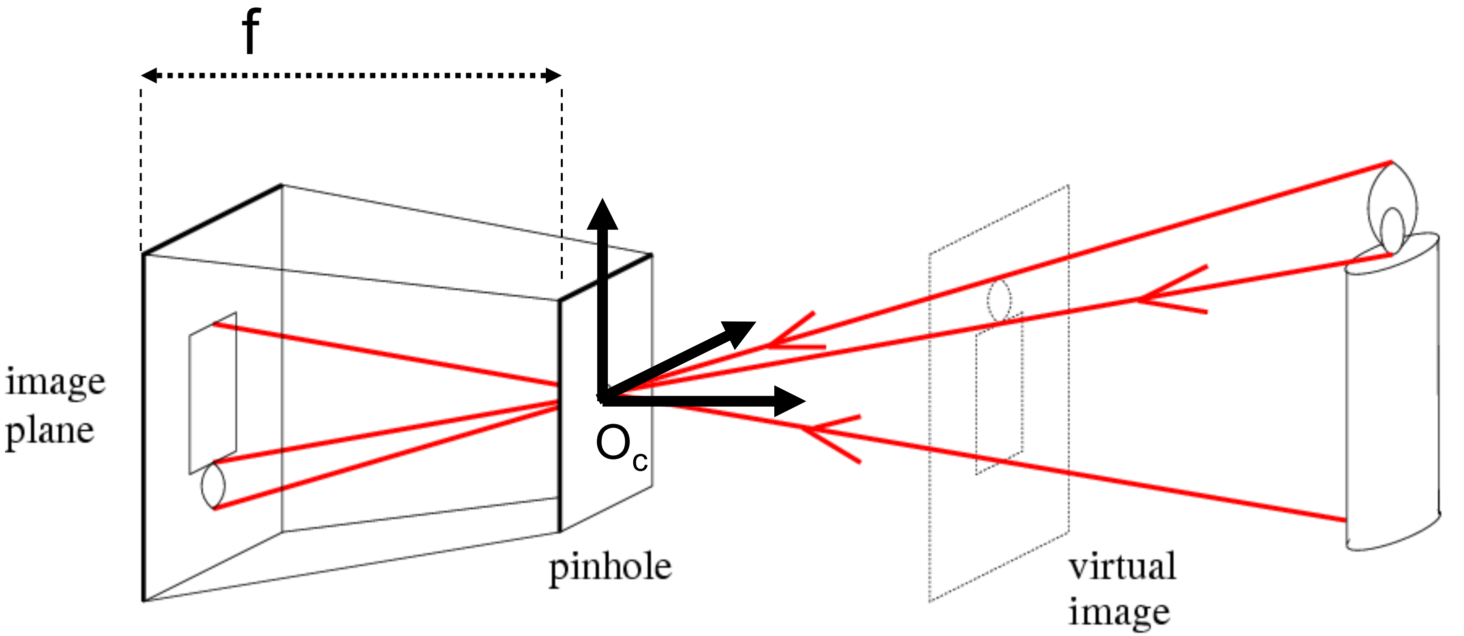
Camera Calibration

- Review camera parameters
- Camera calibration problem
- Example

Reading: [FP] Chapter 3
[HZ] Chapter 7

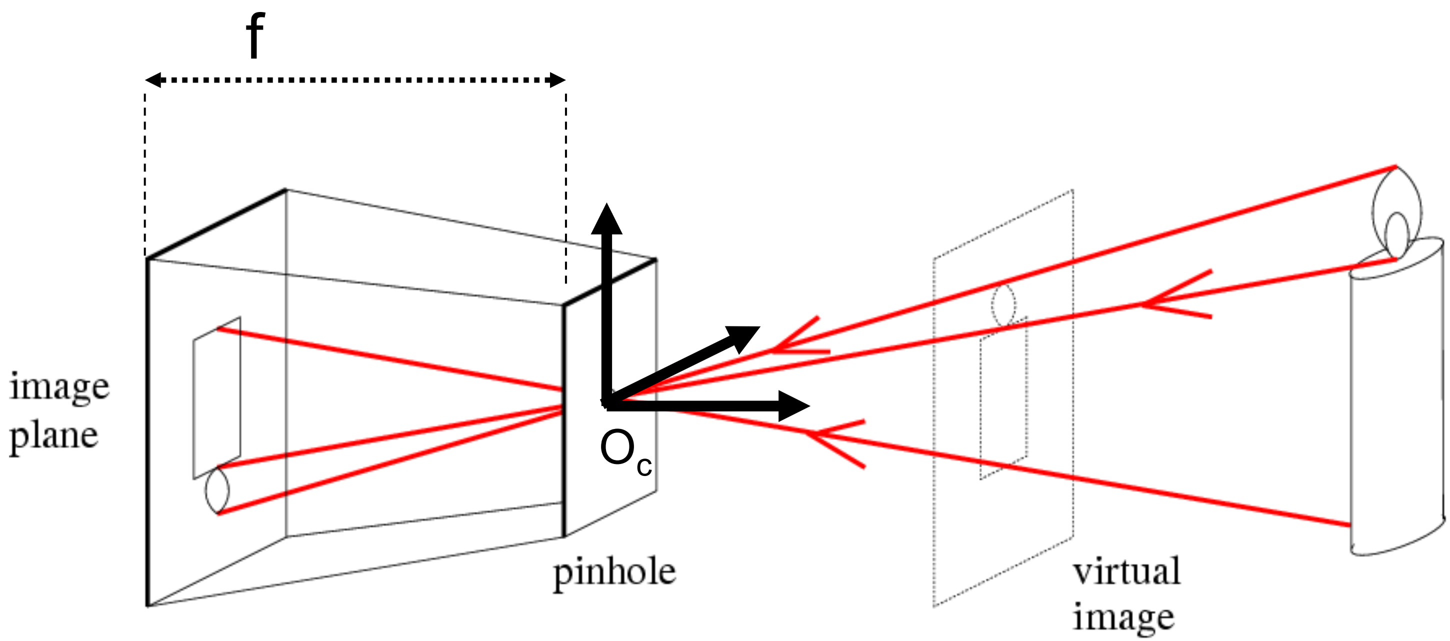
Some slides in this lecture are courtesy to Profs. J. Ponce & F-F Li

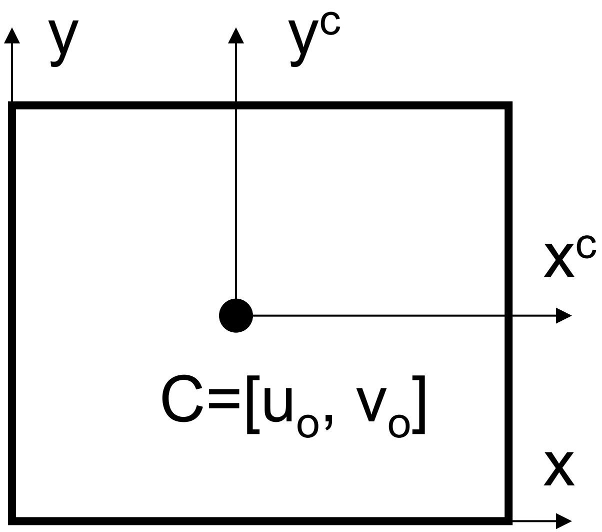
Projective camera



f = focal length

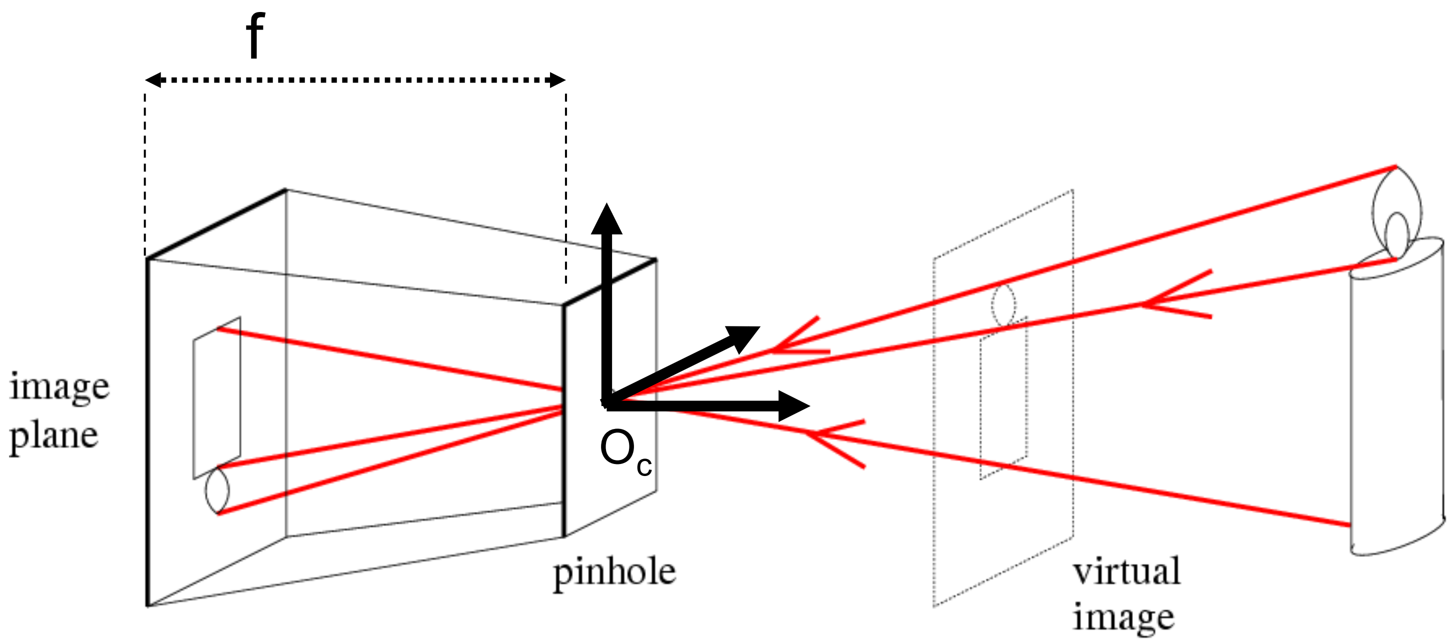
Projective camera





f = focal length
u_o, v_o = offset

Projective camera



Units: k,l [pixel/m]

f [m]

Non-square pixels

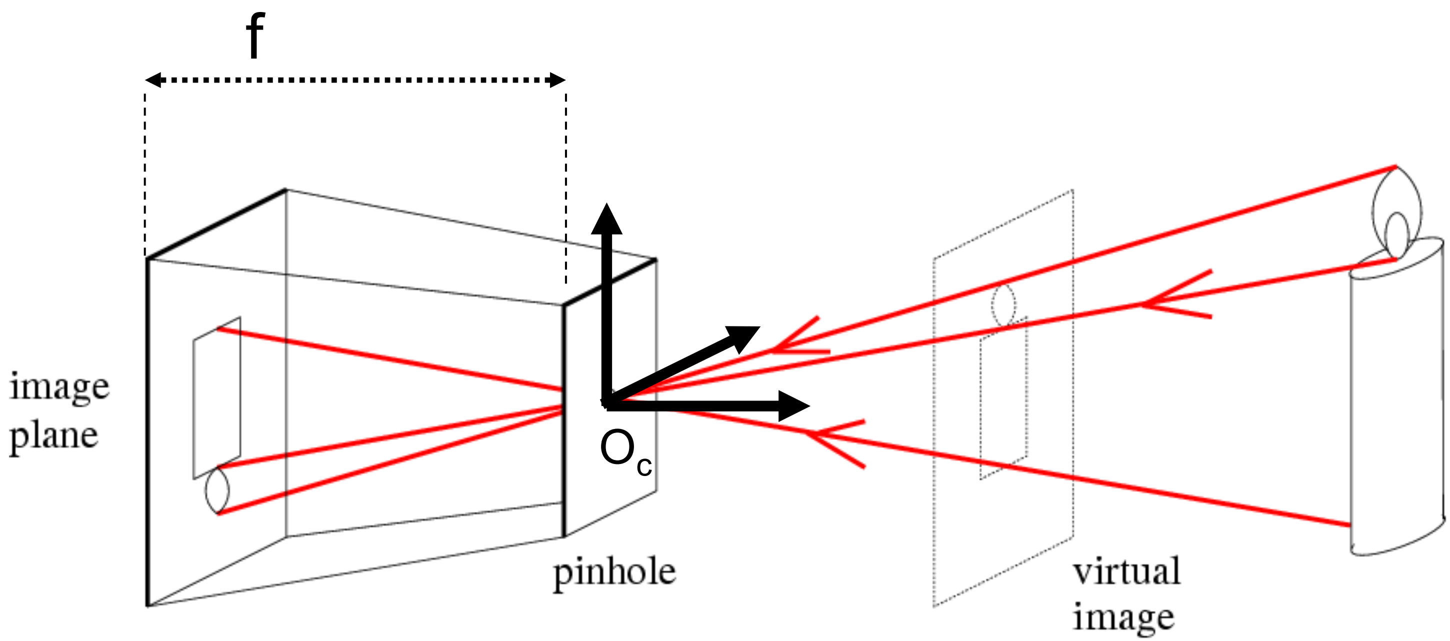
α, β [pixel]

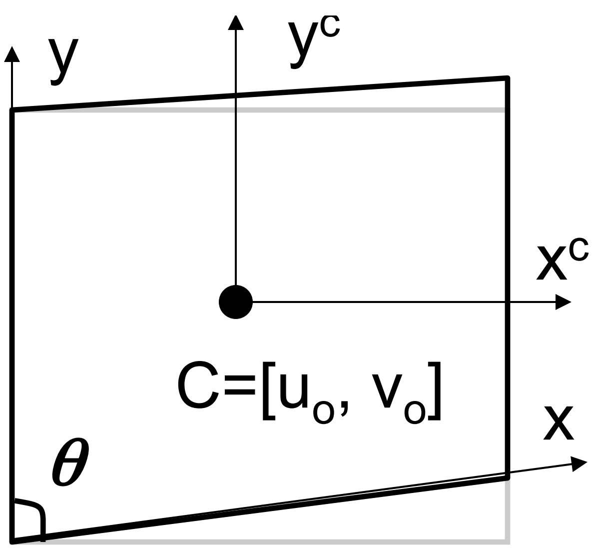
f = focal length

u_o, v_o = offset

$\alpha, \beta \rightarrow$ non-square pixels

Projective camera





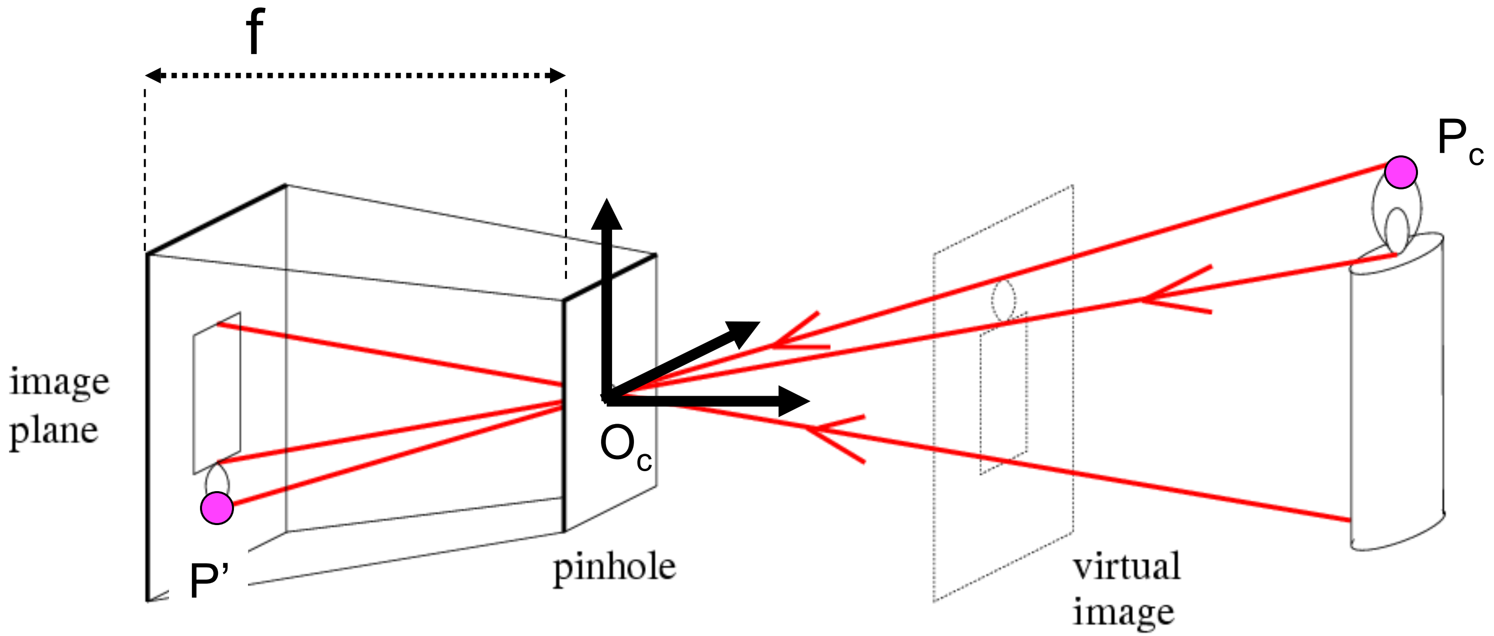
f = focal length

u_o, v_o = offset

$\alpha, \beta \rightarrow$ non-square pixels

θ = skew angle

Projective camera



$$P' = \begin{bmatrix} \alpha & s & u_o & 0 \\ 0 & \beta & v_o & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \\ 1 \end{bmatrix}$$

f = focal length

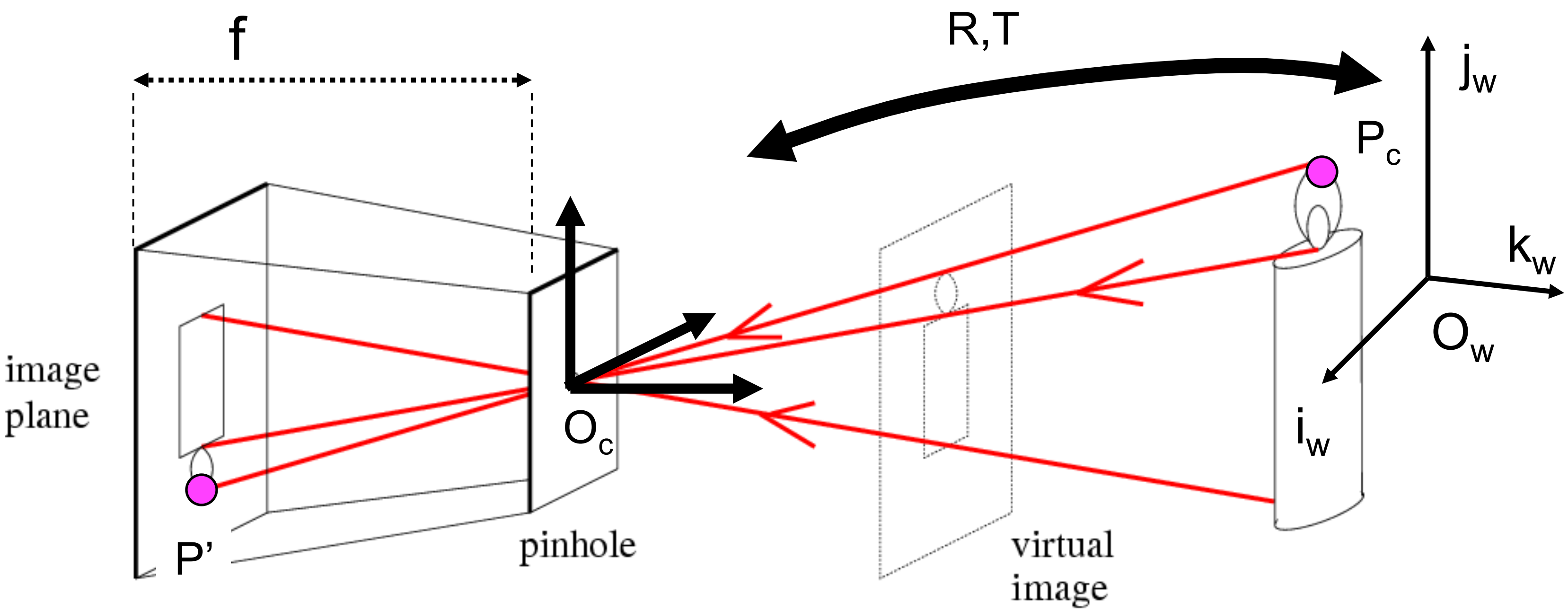
u_o, v_o = offset

$\alpha, \beta \rightarrow$ non-square pixels

θ = skew angle

K has 5 degrees of freedom!

Projective camera

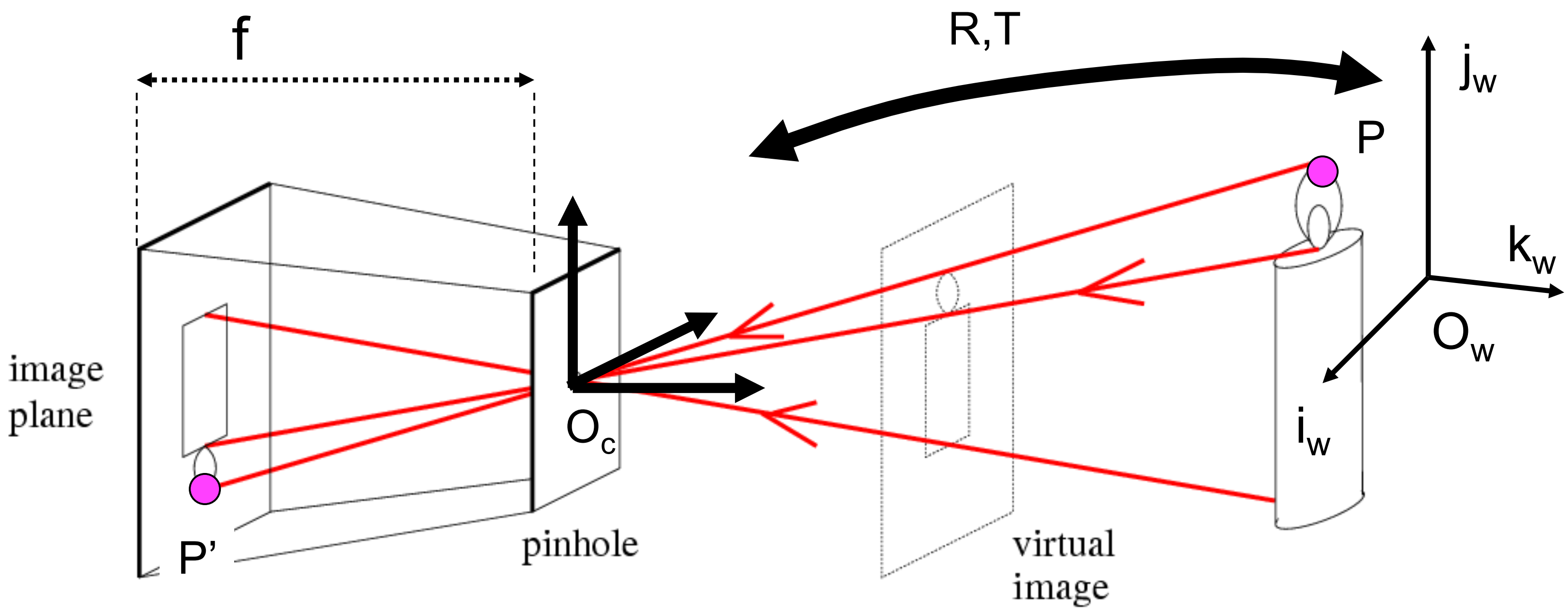


$$P = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}_{4\times4} P_w$$

$$T = -R\,O_c$$

f = focal length
u_o, v_o = offset
$\boldsymbol{\alpha}, \boldsymbol{\beta} \rightarrow$ non-square pixels
θ = skew angle
R,T = rotation, translation

Projective camera



$$P' = M\,P_w$$

$$= K\begin{bmatrix} R & T \end{bmatrix} P_w$$

Internal parameters

External parameters

f = focal length

u_o, v_o = offset

$\alpha, \beta \rightarrow$ non-square pixels

θ = skew angle

R,T = rotation, translation

Projective camera

$$P' = M\,P_w \quad = K\begin{bmatrix} R & T \end{bmatrix} P_w$$

Internal parameters

External parameters

Projective camera

$$P' = M\,P_w = K\begin{bmatrix} R & T \end{bmatrix} P_w$$

$$\boxed{\mathcal{M} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta}\boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix}_{3\times 4}}$$

$$K = \begin{bmatrix} \alpha & -\alpha\cot\theta & u_o \\ 0 & \dfrac{\beta}{\sin\theta} & v_o \\ 0 & 0 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix} \qquad T = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

Goal of calibration

Estimate intrinsic and extrinsic parameters from 1 or multiple images

$$P' = M\ P_w \quad = K\begin{bmatrix} R & T \end{bmatrix} P_w$$

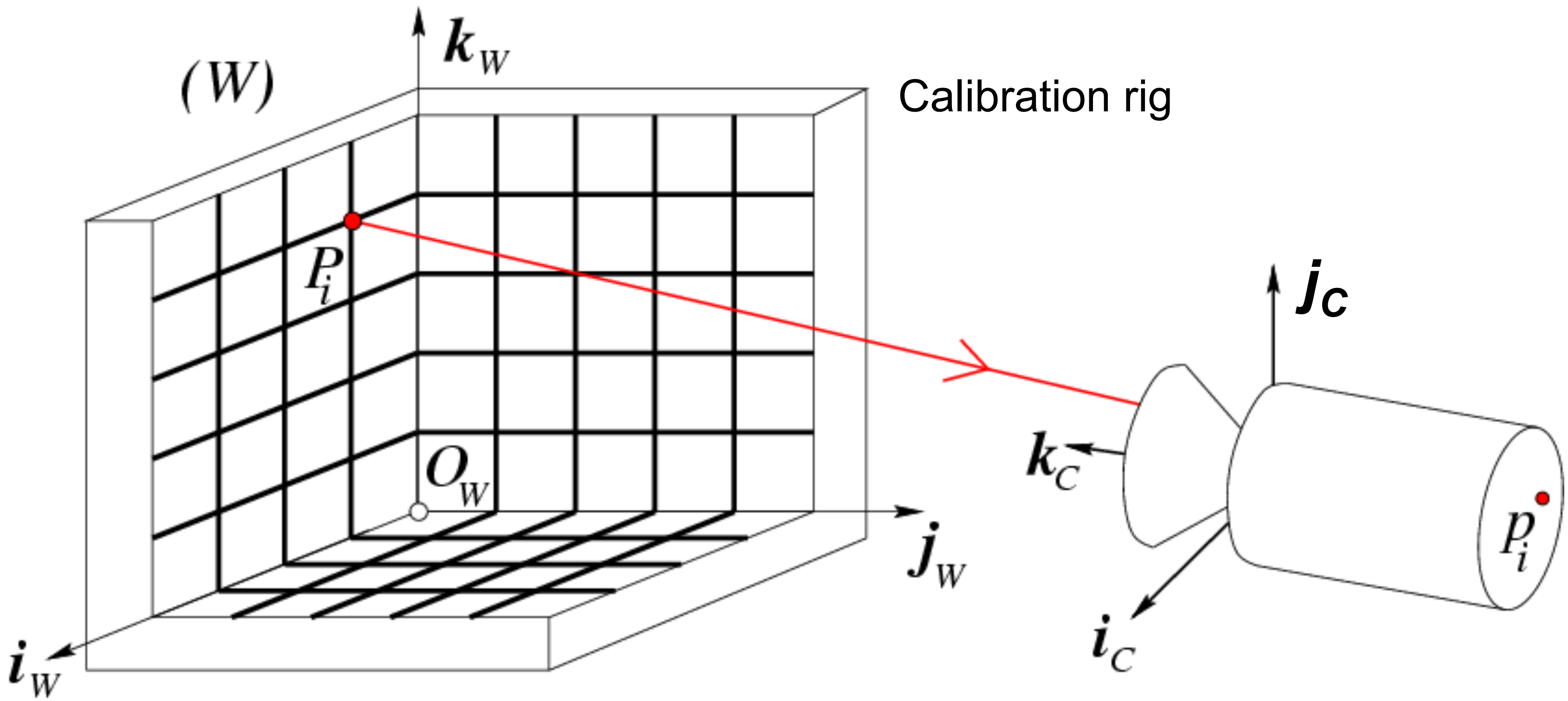
$$\boxed{\mathcal{M} = \begin{pmatrix} \alpha \boldsymbol{r}_1^T - \alpha \cot\theta \boldsymbol{r}_2^T + u_0 \boldsymbol{r}_3^T & \alpha t_x - \alpha \cot\theta t_y + u_0 t_z \\ \dfrac{\beta}{\sin\theta}\boldsymbol{r}_2^T + v_0 \boldsymbol{r}_3^T & \dfrac{\beta}{\sin\theta} t_y + v_0 t_z \\ \boldsymbol{r}_3^T & t_z \end{pmatrix}_{3\times 4}}$$

$$K = \begin{bmatrix} \alpha & -\alpha\cot\theta & u_o \\ 0 & \dfrac{\beta}{\sin\theta} & v_o \\ 0 & 0 & 1 \end{bmatrix} \qquad R = \begin{bmatrix} \mathbf{r}_1^T \\ \mathbf{r}_2^T \\ \mathbf{r}_3^T \end{bmatrix} \qquad T = \begin{bmatrix} t_x \\ t_y \\ t_z \end{bmatrix}$$

Change notation:
$P = P_w$

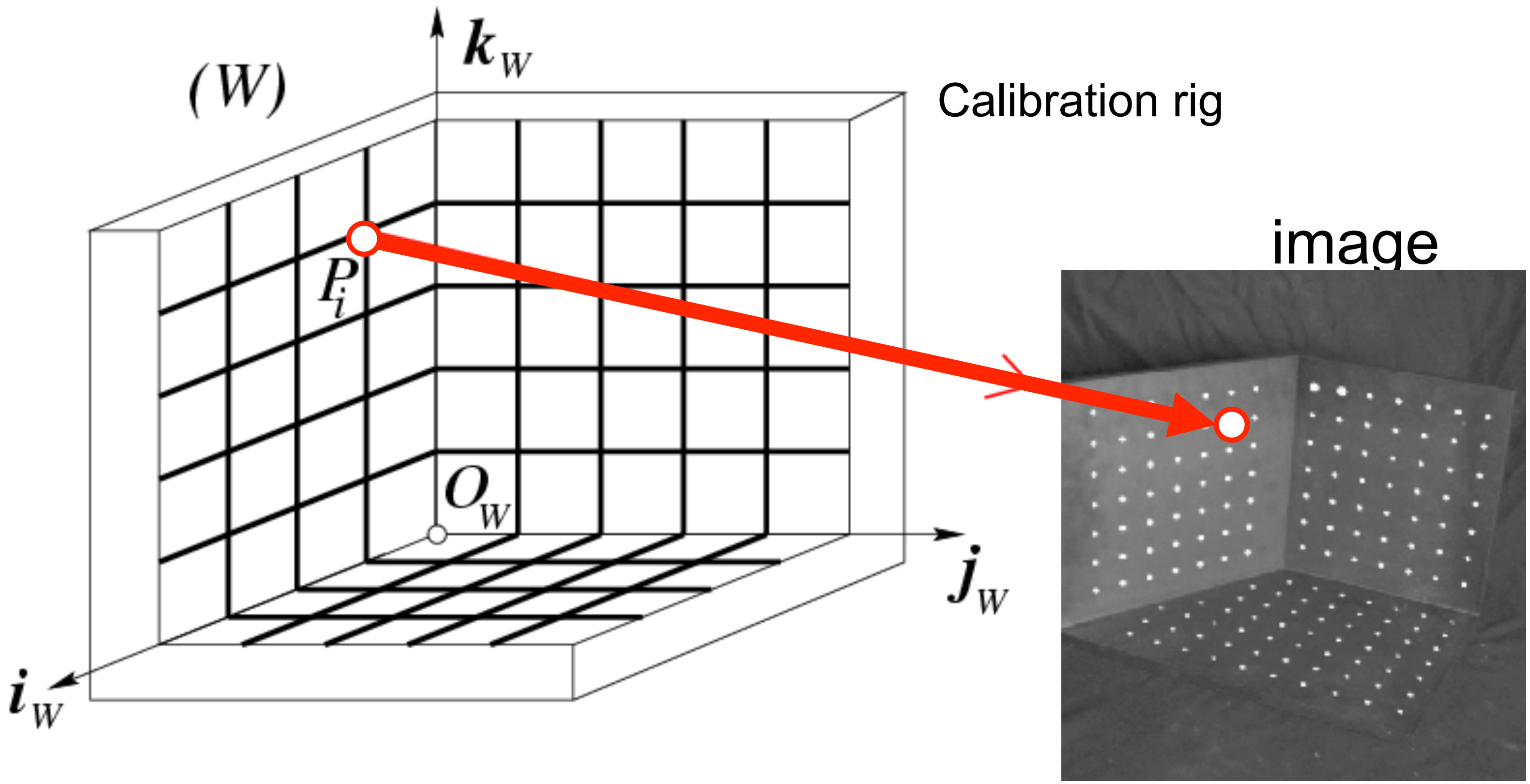
$p = P'$

Calibration Problem



•$P_1 \ldots P_n$ with known positions in $[O_w, i_w, j_w, k_w]$
•$p_1, \ldots p_n$ known positions in the image
Goal: compute intrinsic and extrinsic parameters

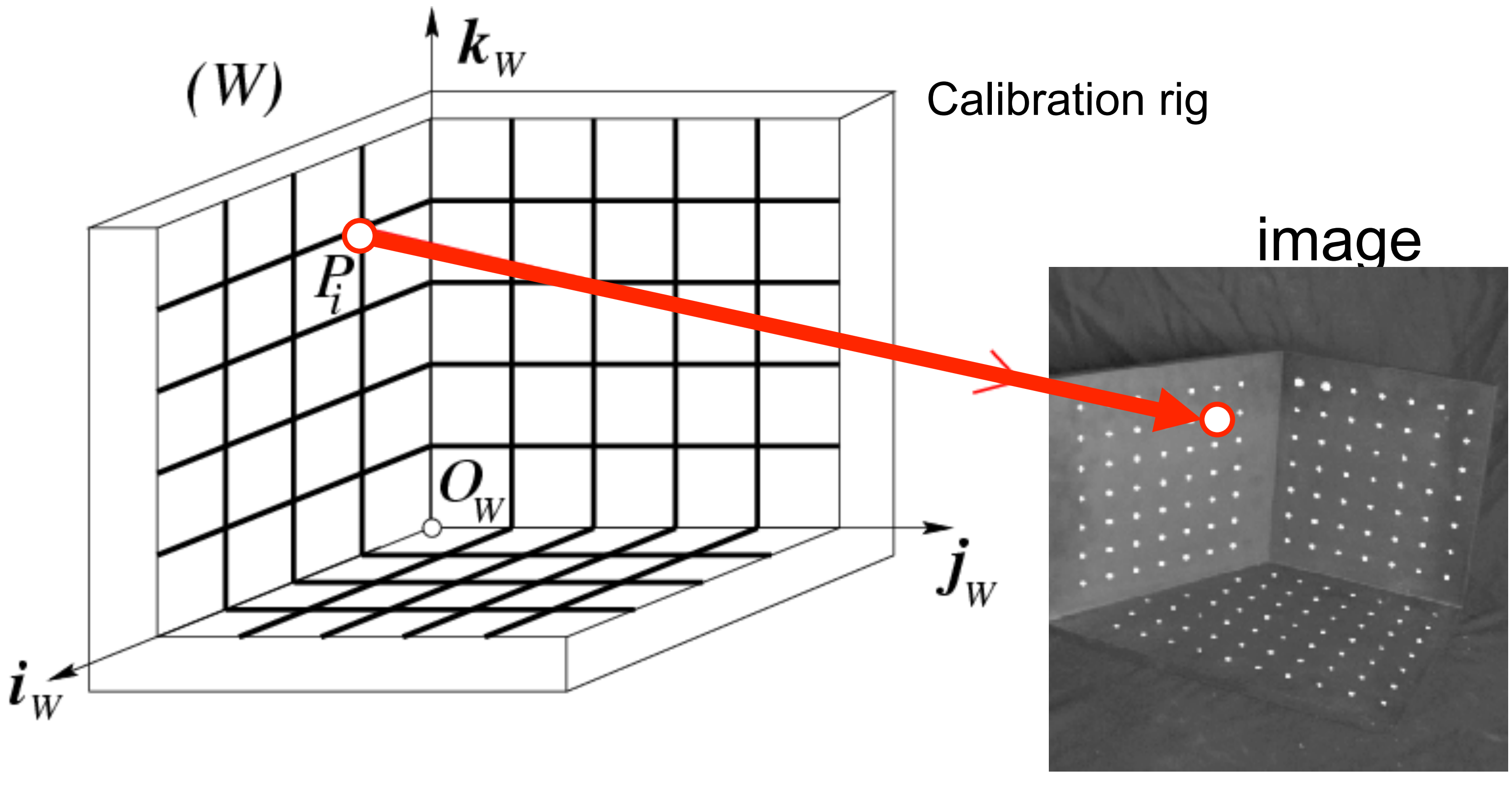
Calibration Problem



- $P_1 \ldots P_n$ with known positions in $[O_w, i_w, j_w, k_w]$
- $p_1, \ldots p_n$ known positions in the image

Goal: compute intrinsic and extrinsic parameters

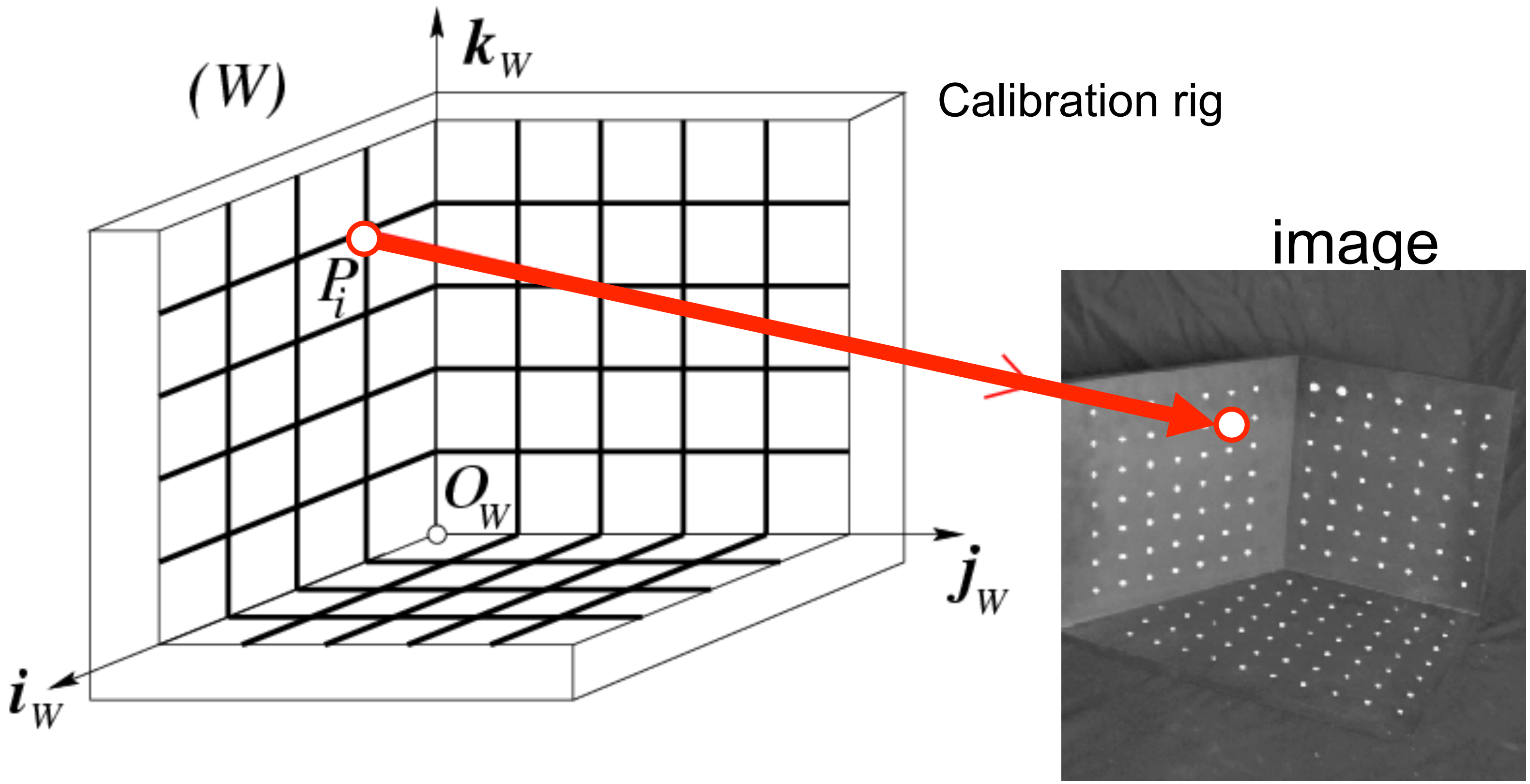
Calibration Problem



How many correspondences do we need?

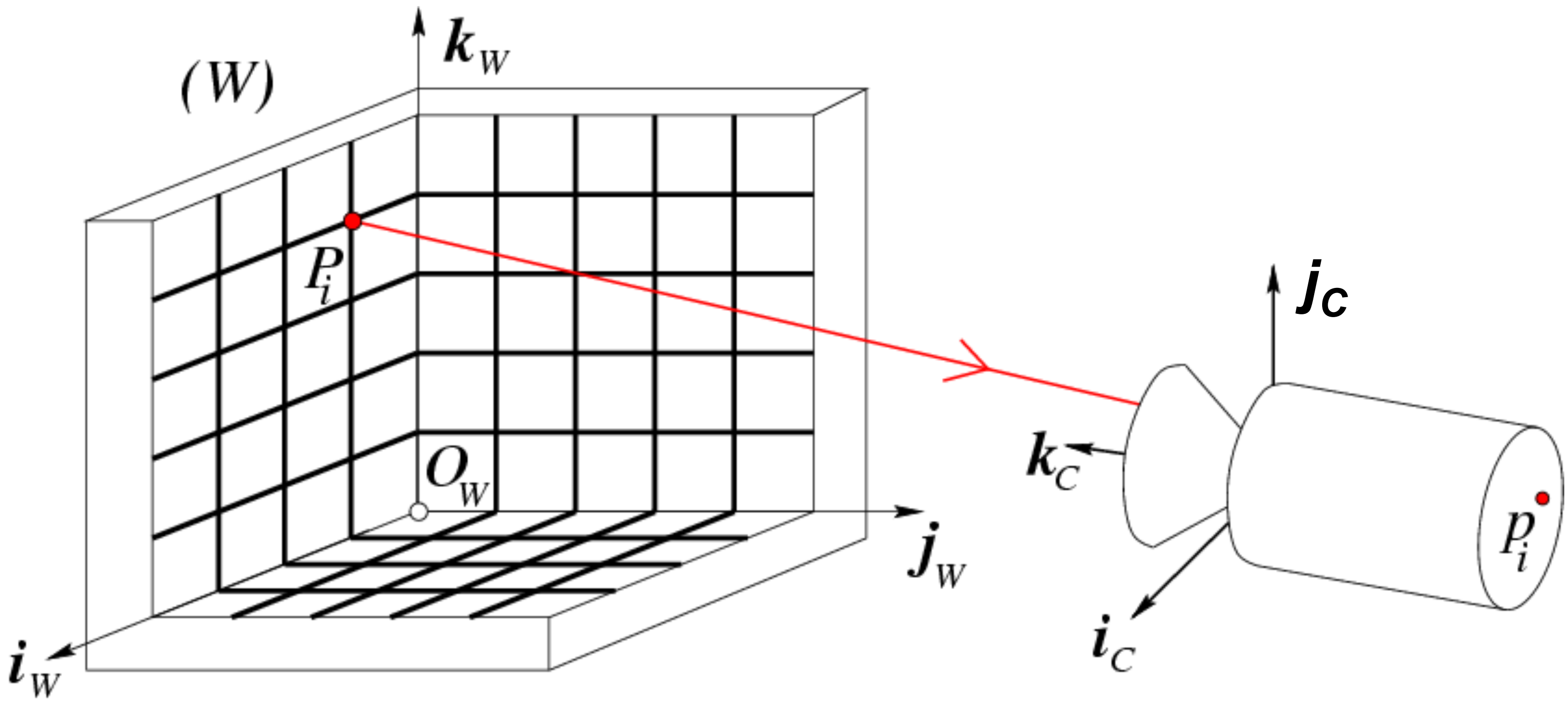
- M has 11 unknown
- We need 11 equations
- 6 correspondences would do it

Calibration Problem



In practice, using more than 6 correspondences enables more robust results

Calibration Problem



$$P_i \rightarrow M\,P_i \rightarrow p_i = \begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{m}_1\,P_i}{\mathbf{m}_3\,P_i} \\ \dfrac{\mathbf{m}_2\,P_i}{\mathbf{m}_3\,P_i} \end{bmatrix} \qquad M = \begin{bmatrix} \mathbf{m}_1 \\ \mathbf{m}_2 \\ \mathbf{m}_3 \end{bmatrix}$$

in pixels

Calibration Problem

$$\begin{bmatrix} u_i \\ v_i \end{bmatrix} = \begin{bmatrix} \dfrac{\mathbf{m}_1 \, P_i}{\mathbf{m}_3 \, P_i} \\ \dfrac{\mathbf{m}_2 \, P_i}{\mathbf{m}_3 \, P_i} \end{bmatrix}$$

$$u_i = \frac{\mathbf{m}_1 \, P_i}{\mathbf{m}_3 \, P_i} \rightarrow u_i(\mathbf{m}_3 \, P_i) = \mathbf{m}_1 \, P_i \rightarrow u_i(\mathbf{m}_3 \, P_i) - \mathbf{m}_1 \, P_i = 0$$

$$v_i = \frac{\mathbf{m}_2 \, P_i}{\mathbf{m}_3 \, P_i} \rightarrow v_i(\mathbf{m}_3 \, P_i) = \mathbf{m}_2 \, P_i \rightarrow v_i(\mathbf{m}_3 \, P_i) - \mathbf{m}_2 \, P_i = 0$$

Calibration Problem

$$\left\{\begin{array}{l} u_1(\mathbf{m}_3\ P_1) - \mathbf{m}_1\ P_1 = 0 \\ v_1(\mathbf{m}_3\ P_1) - \mathbf{m}_2\ P_1 = 0 \\ \quad\vdots \\ u_i(\mathbf{m}_3\ P_i) - \mathbf{m}_1\ P_i = 0 \\ v_i(\mathbf{m}_3\ P_i) - \mathbf{m}_2\ P_i = 0 \\ \quad\vdots \\ u_n(\mathbf{m}_3\ P_n) - \mathbf{m}_1\ P_n = 0 \\ v_n(\mathbf{m}_3\ P_n) - \mathbf{m}_2\ P_n = 0 \end{array}\right.$$

Block Matrix Multiplication

$$\mathrm{A} = \begin{bmatrix} \mathrm{A}_{11} & \mathrm{A}_{12} \\ \mathrm{A}_{21} & \mathrm{A}_{22} \end{bmatrix} \qquad \mathrm{B} = \begin{bmatrix} \mathrm{B}_{11} & \mathrm{B}_{12} \\ \mathrm{B}_{21} & \mathrm{B}_{22} \end{bmatrix}$$

What is *AB* ?

$$AB = \begin{bmatrix} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{bmatrix}$$

Calibration Problem

$$
\begin{cases}
-u_1(\mathbf{m}_3\ P_1) + \mathbf{m}_1\ P_1 = 0 \\
-v_1(\mathbf{m}_3\ P_1) + \mathbf{m}_2\ P_1 = 0 \\
\vdots \\
-u_n(\mathbf{m}_3\ P_n) + \mathbf{m}_1\ P_n = 0 \\
-v_n(\mathbf{m}_3\ P_n) + \mathbf{m}_2\ P_n = 0
\end{cases}
\longrightarrow
\boxed{\underset{\text{known}}{\boxed{\mathcal{P}}}\ \underset{\text{unknown}}{\boxed{\boldsymbol{m}}} = 0}
$$

Homogenous linear system

$$
\mathcal{P} \stackrel{\text{def}}{=} \begin{pmatrix}
\boldsymbol{P}_1^T & \mathbf{0}^T & -u_1 \underset{1\text{x}4}{\boxed{\boldsymbol{P}_1^T}} \\
\mathbf{0}^T & \boldsymbol{P}_1^T & -v_1 \boldsymbol{P}_1^T \\
\cdots & \cdots & \cdots \\
\boldsymbol{P}_n^T & \mathbf{0}^T & -u_n \boldsymbol{P}_n^T \\
\mathbf{0}^T & \boldsymbol{P}_n^T & -v_n \boldsymbol{P}_n^T
\end{pmatrix}_{2n\ \text{x}\ 12}
\qquad
\boldsymbol{m} \stackrel{\text{def}}{=} \begin{pmatrix}
\underset{4\text{x}1}{\boxed{\mathbf{m}_1^{\mathrm{T}}}} \\
\mathbf{m}_2^{\mathrm{T}} \\
\mathbf{m}_3^{\mathrm{T}}
\end{pmatrix}_{12\text{x}1}
$$

Homogeneous M x N Linear Systems

M=number of equations = 2n
N=number of unknown = 11

$$\boldsymbol{P}\;\boldsymbol{m} = \boldsymbol{0}$$

Rectangular system (M>N)

- 0 is always a solution
- To find non-zero solution

  Minimize $|\mathbf{P}\ \mathbf{m}|^2$

  under the constraint $|\mathbf{m}|^2 = 1$

Calibration Problem

$$\mathcal{P}\boldsymbol{m} = 0$$

- How do we solve this homogenous linear system?
- Using DLT (Direct Linear Transformation) algorithm via SVD decomposition

Next lecture

- How do we solve this system