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Prob 1. (a) The transformation can be devided into two steps:

1, translation from $O_w$ to $O_c$ point

2, Rotation of 135° w.r.t $-C_y$ axis

$\therefore P = R \cdot T$

$$= R_y(\theta) \cdot T\left(\begin{smallmatrix} x-\frac{d}{\sqrt{2}} \\ z-\frac{d}{\sqrt{2}} \end{smallmatrix}\right)$$

$$= R_y(\theta) \cdot \left[\begin{array}{c|c} I & \begin{matrix} -\frac{d}{\sqrt{2}} \\ 0 \\ -\frac{d}{\sqrt{2}} \end{matrix} \\ \hline 0 & 1 \end{array}\right]$$

$$= \left[\begin{array}{ccc|c} \cos 135^\circ & 0 & \sin 135^\circ & \\ 0 & 1 & 0 & 0 \\ -\sin 135^\circ & 0 & \cos 135^\circ & \\ \hline & 0 & & 1 \end{array}\right] \left[\begin{array}{c|c} I & \begin{matrix} -\frac{d}{\sqrt{2}} \\ 0 \\ -\frac{d}{\sqrt{2}} \end{matrix} \\ \hline 0 & 1 \end{array}\right].$$

$$= \begin{bmatrix} -\frac{\sqrt{2}}{2}, & 0, & \frac{\sqrt{2}}{2}, & 0 \\ 0, & 1, & 0, & 0 \\ -\frac{\sqrt{2}}{2}, & 0, & -\frac{\sqrt{2}}{2}, & d \\ 0, & 0, & 0, & 1 \end{bmatrix}$$

where new coordinates $[x', y', z', 1]^T = P \cdot [x, y, z, 1]^T$

$$\therefore \begin{bmatrix} x' \\ y' \\ z' \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} z - \frac{\sqrt{2}}{2} x \\ y \\ d - \frac{\sqrt{2}}{2} x - \frac{\sqrt{2}}{2} z \\ 1 \end{bmatrix}$$

(b) Suppose the coordinates of a, b, c, d are $(0, y, z)$, $(0, y+1, z)$, $(0, y+1, z+1)$, $(0, y, z+1)$

So the original square has an area of 1.

According to the result from question (a).

$$a' = P \cdot a$$

$$\therefore \begin{bmatrix} x_{ac} \\ y_{ac} \\ z_{ac} \\ 1 \end{bmatrix} = \begin{bmatrix} -\frac{\sqrt{2}}{2} & 0 & \frac{\sqrt{2}}{2} & 0 \\ 0 & 1 & 0 & 0 \\ -\frac{\sqrt{2}}{2} & 0 & -\frac{\sqrt{2}}{2} & d \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{aw} \\ y_{aw} \\ z_{aw} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} z \\ y \\ d - \frac{\sqrt{2}}{2} z \\ 1 \end{bmatrix}$$

Same reasoning:

$$\begin{bmatrix} x_{bc} \\ y_{bc} \\ z_{bc} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2} z \\ y+1 \\ d - \frac{\sqrt{2}}{2} z \\ 1 \end{bmatrix}; \quad \begin{bmatrix} x_{cc} \\ y_{cc} \\ z_{cc} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2}(z+1) \\ y+1 \\ d - \frac{\sqrt{2}}{2}(z+1) \\ 1 \end{bmatrix}; \quad \begin{bmatrix} x_{dc} \\ y_{dc} \\ z_{dc} \\ 1 \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{2}}{2}(z+1) \\ y \\ d - \frac{\sqrt{2}}{2}(z+1) \\ 1 \end{bmatrix}$$

$$\therefore \text{new area} = \|\overrightarrow{a_c d_c}\| \cdot \|\overrightarrow{b_c c_c}\|$$

$$= \text{norm}\left(\begin{bmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ -\frac{\sqrt{2}}{2} \end{bmatrix}\right) \cdot \text{norm}\left(\begin{bmatrix} \frac{\sqrt{2}}{2} \\ 0 \\ -\frac{\sqrt{2}}{2} \end{bmatrix}\right)$$

$$= 1 \times 1$$

$$= 1$$

$\therefore$ new area is still an unit area in the camera reference system.

(c) Suppose there is a line in the world frame denoted by two points: $[x_1, y_1, z_1]^T$ & $[x_1+a, y_1+b, z_1+c]^T$

we can define a parallel line denoted by another two points $[x_2, y_2, z_2]^T$ & $[x_2+a, y_2+b, z_2+c]^T$

① After transformation, the coordinates of the two points of the first line would be:

$$P \cdot [x_1, y_1, z_1, 1]^T = \left[\tfrac{\sqrt{2}}{2} z_1 - \tfrac{\sqrt{2}}{2} x_1,\ y_1,\ d - \tfrac{\sqrt{2}}{2} x_1 - \tfrac{\sqrt{2}}{2} z_1,\ 1\right]^T$$

& $$P \cdot [x_1+a, y_1+b, z_1+c, 1]^T = \left[\tfrac{\sqrt{2}}{2}(z_1+c) - \tfrac{\sqrt{2}}{2}(x_1+a),\ y_1+b,\ d - \tfrac{\sqrt{2}}{2}(x_1+a) - \tfrac{\sqrt{2}}{2}(z_1+c),\ 1\right]^T$$

∴ The vector of the first line would be:

$$V_1 = \left[\tfrac{\sqrt{2}}{2} c - \tfrac{\sqrt{2}}{2} a,\ b,\ -\tfrac{\sqrt{2}}{2} a - \tfrac{\sqrt{2}}{2} a\right]^T$$

② Same reasoning, after the transformation, the coordinates of the two points in the second line would be:

$$P \cdot [x_2, y_2, z_2, 1]^T = \left[\tfrac{\sqrt{2}}{2} z_2 - \tfrac{\sqrt{2}}{2} x_2,\ y_2,\ d - \tfrac{\sqrt{2}}{2} x_2 - \tfrac{\sqrt{2}}{2} z_2,\ 1\right]^T$$

& $$P \cdot [x_2+a, y_2+b, z_2+c, 1]^T = \left[\tfrac{\sqrt{2}}{2}(z_2+c) - \tfrac{\sqrt{2}}{2}(x_2+a),\ y_2+b,\ d - \tfrac{\sqrt{2}}{2}(x_2+a) - \tfrac{\sqrt{2}}{2}(z_2+c),\ 1\right]^T$$

∴ The vector of the second line is:

$$V_2 = \left[\tfrac{\sqrt{2}}{2} c - \tfrac{\sqrt{2}}{2} a,\ b,\ -\tfrac{\sqrt{2}}{2} a - \tfrac{\sqrt{2}}{2} a\right]^T$$

According to ① & ②, we know that $V_1 = V_2$. which means that line 1 and line 2 are still parallel to each other in the camera reference system.

(d) Same as question (b)

suppose $a_{[w]} = [0, y, z]^T$ & $b_{[w]} = [0, y+1, z]^T$

then, in world frame, $V_{ab[w]} = [0, 1, 0]^T$

Aftere transformation

$$a_{[c]} = \left[\frac{\sqrt{2}}{2}z, y, d - \frac{\sqrt{2}}{2}z\right]^T$$

$$b_{[c]} = \left[\frac{\sqrt{2}}{2}z, y+1, d - \frac{\sqrt{2}}{2}z\right]^T$$

$\therefore$ In camara frame: $V_{ab[c]} = [0, 1, 0]^T$

Comparing $V_{ab[w]}$ and $V_{ab[c]}$, we know that the vector defined by a and b have the same orientation in both reference system.

Problem 2.

$$P' = \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} -f \cdot k \cdot \frac{x}{z} \\ -f \cdot l \cdot \frac{y}{z} \end{bmatrix} = \begin{bmatrix} -\frac{f' x}{z} \\ -\frac{f' y}{z} \end{bmatrix}$$

∴ for a certain point $Q = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$ $(-\infty \leq t \leq -1)$

$$Q = \begin{bmatrix} 1 \\ 1 \\ t \end{bmatrix} = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

We have $P' = \begin{bmatrix} -\frac{f' x}{z} \\ -\frac{f' y}{z} \end{bmatrix} = \begin{bmatrix} -\frac{f' \times 1}{t} \\ -\frac{f' \times 1}{t} \end{bmatrix}$ $(-\infty \leq t \leq -1)$

∴ The two end points are:

$P_1' \xrightarrow{t=-\infty} \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ ; $P_2' \xrightarrow{t=-1} \begin{bmatrix} f' \\ f' \end{bmatrix}$

Prob 3. (a) $x_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ $x_2 = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

let line $l$ be defined as : $y = ax + b$

then $\begin{cases} 3 = a + b \\ 1 = 3a + b \end{cases} \Longrightarrow \begin{cases} a = -1 \\ b = 4 \end{cases}$

$\therefore$ line $l$ can be represent as : $y = -x + 4$

(b) $\begin{bmatrix} x_1' \\ 1 \end{bmatrix} = H \cdot \begin{bmatrix} x_1 \\ 1 \end{bmatrix} = \begin{bmatrix} 1.520 & -1.902 & 1 \\ 3.3 & 23.49 & 3 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix} = \begin{bmatrix} -3.186 \\ 76.77 \\ 11 \end{bmatrix}$

$\Longrightarrow x_1' = \begin{bmatrix} -3.186 \\ 76.77 \end{bmatrix} \div 11 = \begin{bmatrix} -0.2896 \\ 6.9791 \end{bmatrix}$

Same reasoning

$\begin{bmatrix} x_2' \\ 1 \end{bmatrix} = H \cdot \begin{bmatrix} x_2 \\ 1 \end{bmatrix} \overset{\text{Matlab}}{=\!=} \begin{bmatrix} 3.658 \\ 36.39 \\ 7 \end{bmatrix} \Longrightarrow x_2' = \begin{bmatrix} 0.5226 \\ 5.1986 \end{bmatrix}$

let $l'$ be defined as: $y = a'x + b'$

then, put $x_1'$, $x_2'$ into the function:

$\begin{cases} 6.9791 = a'(-0.2896) + b' \\ 5.1986 = a'(0.5226) + b' \end{cases} \Longrightarrow \begin{cases} a' = -2.1922 \\ b' = 6.3442 \end{cases}$

$\therefore$ $l'$ can be represent as : $y = -2.1922x + 6.3442$

(c) For any two given points $x_1$ & $x_2$.

$$l = x_1 \times x_2$$

After transformed by a plan projective transformation $H$,

$$l' = x_1' \times x_2'$$
$$= (Hx_1) \times (Hx_2)$$
$$= \det(H) \cdot H^{-T} \cdot (x_1 \times x_2)$$
$$= \det(H) \cdot H^{-T} \cdot l$$

$$\therefore H' = \det(H) \cdot H^{-T}$$

$$= 9.0054 \cdot \begin{bmatrix} 1.6090 & -0.0333 & -1.5091 \\ 0.5443 & 0.0577 & -0.7176 \\ -3.2421 & -0.1399 & 4.6618 \end{bmatrix}$$

$$= \begin{bmatrix} 14.4900 & -0.3000 & -13.5900 \\ 4.9020 & 0.5200 & -6.4620 \\ -29.1960 & -1.2600 & 41.9814 \end{bmatrix}$$