EECS 442 Computer Vision
Homework 3 Solution

Fall 2016

1 Some Projective Geometry Problems

(a) As mentioned in the problem a group of parallel lines intersect at one point on the image plane as the world coordinate radius from the image plane goes to infinity. Without loss of generality, we consider only one of the lines, denoted by l_0, that goes through the origin of world coordinate system. Using the representation of the line given in the problem we can write the homogeneous coordinate representation of our line as

$$l = \left\{ r \begin{bmatrix} \cos\alpha \\ \cos\beta \\ \cos\gamma \\ 1/r \end{bmatrix} \middle| r \in \mathbb{R} \right\}$$

letting $r \to \infty$ the image of the line $l + 0$ with reach the vanishing point. Therefore using the equations from lecture to convert between world coordinates and image coordinates we can write the vanishing point as

$$\begin{aligned} v &= \lim_{r\to\infty} \begin{bmatrix} K & \mathbf{0} \end{bmatrix} \begin{bmatrix} R & T \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \cos\alpha \\ \cos\beta \\ \cos\gamma \\ 1/r \end{bmatrix} \\ &= \begin{bmatrix} K & \mathbf{0} \end{bmatrix} \begin{bmatrix} R & T \\ \mathbf{0} & \mathbf{1} \end{bmatrix} \begin{bmatrix} \cos\alpha \\ \cos\beta \\ \cos\gamma \\ 0 \end{bmatrix} \\ &= \begin{bmatrix} K & \mathbf{0} \end{bmatrix} \begin{bmatrix} Rd \\ \mathbf{0} \end{bmatrix} \\ &= KRd \end{aligned}$$

where

$$d = \begin{bmatrix} \cos\alpha \\ \cos\beta \\ \cos\gamma \end{bmatrix}$$

(b) In lecture we have already shown K is of the form:

$$K = \begin{bmatrix} \alpha & -\alpha\cot\theta & u_0 \\ 0 & \frac{\beta}{\sin\theta} & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

since $|K| = \alpha\beta/\sin\theta \neq 0$, K^{-1} exists. Here we use the fact that any rotation can be decomposed into three signle rotations around each axis, i.e.

$$R = R_x R_y R_z \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\theta_x & -\sin\theta_x \\ 0 & \sin\theta_x & \cos\theta_x \end{bmatrix} \begin{bmatrix} \cos\theta_x & 0 & -\sin\theta_x \\ 0 & 1 & 0 \\ \sin\theta_x & 0 & \cos\theta_x \end{bmatrix} \begin{bmatrix} \cos\theta_x & -\sin\theta_x & 0 \\ \sin\theta_x & \cos\theta_x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

now considering $|R_i| = \cos^2\theta_i \sin^2\theta_i = 1$ and $|R| = |R_x R_y R_z| = |R_x||R_y||R_z| = 1$, R^{-1} exists. Therefore we can write $d = R^{-1}K^{-1}v$.

(c) From the last problem we have $d = R^{-1}K^{-1}v \Rightarrow Rd = K^{-1}v$. Also we make use of the fact that any rotation matrix is a unitary matrix, i.e. $R^T R = I$.

$$\begin{aligned} d_i^T d_j &= 0 \\ d_i^T (R^T R) d_j &= 0 \\ (Rd_i)^T (Rd_j) &= 0 \\ (K^{-1}v_i)^T (K^{-1}v_j) &= 0 \end{aligned}$$

This holds for $i \neq j$.

2 Affine Camera Calibration

(a) No - you need to calibrate the camera on at least two planar surfaces. If only one planar surface is used, the linear system becomes rank deficient and a unique solution cannot be obtained.

(b) (c) By expanding out the original linear system, we can reformulate the question by placing all the affine camera matrix element unknowns into a single x vector (the A matrix and b vector will be known), then use least squares to find a best fit solution.
This is the original linear system:

$$x = \begin{bmatrix} x_1 & \dots & x_n \\ y_1 & \dots & y_n \end{bmatrix}, P = \begin{bmatrix} p_{11} & p_{12} & p_{13} & p_{14} \\ p_{21} & p_{22} & p_{23} & p_{24} \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} X_1 & \dots & X_n \\ Y_1 & \dots & Y_n \\ Z_1 & \dots & Z_n \\ 1 & \dots & 1 \end{bmatrix}$$

$$x = PX$$

By expanding out the first 3D world to 2D point correspondences, we obtain:

$$x_1 = p_{11}X_1 + p_{12}Y_1 + p_{13}Z_1 + p_{14}$$
$$y_1 = p_{21}Y_2 + p_{22}Y_2 + p_{23}Z_2 + p_{24}$$

A similar expansion can be obtained for all the 3D world to 2D image point correspondences. Since p_{ij} are the unknowns in this case, and everything else is known, we can place the p_{ij} coefficients in an x vector, and write out the corresponding A amtrix and b vector of the linear system. Thus, we obtain

$$Ax = b$$

$$A = \begin{bmatrix} X_1 & Y_1 & Z_1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & X_1 & Y_1 & Z_1 & 1 \\ X_2 & Y_2 & Z_2 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & X_2 & Y_2 & Z_2 & 1 \\ \dots \end{bmatrix}, b = \begin{bmatrix} x_1 \\ y_1 \\ x_2 \\ y_2 \\ \vdots \end{bmatrix}, x = \begin{bmatrix} p_{11} \\ p_{12} \\ p_{13} \\ p_{14} \\ p_{21} \\ p_{22} \\ p_{23} \\ p_{24} \end{bmatrix}$$

Solving via least squares,

$$x = (A^T A)^{-1} A^T b$$