EECS 442
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Prob 1. (a)

$\because$ Camera Matrix has rank of 3.

So, $\exists H$, s.t. $MH = [I \ 0]$

$\because M = [A, b]$

Without loss of generality, let $H_1 = \begin{bmatrix} A^{-1} & -A^{-1}b \\ 0 & 1 \end{bmatrix}$

We have $MH_1 = [I \ 0]_{3\times 4}$

In this case, $M'H_1 = [A', b'] \begin{bmatrix} A^{-1} & -A^{-1}b \\ 0 & 1 \end{bmatrix}$

$$= [A' \cdot A^{-1} + 0, \ A'(-A^{-1}b) + b']$$

$$\Longrightarrow M'H_1 = [A'A^{-1}, \ -A'A^{-1}b + b']$$

Since we know that $e_3^T(-A'A^{-1}b + b') \neq 0$

$\therefore [M'H_1]_{3,4} \neq 0$

let's explicit $M'H_1$ as $\begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \end{bmatrix}$

we can construct a matrix $H_2 = \begin{bmatrix} & I_{3\times 3} & & , 0_{3\times 1} \\ -\frac{x_{31}}{x_{34}}, & -\frac{x_{32}}{x_{34}}, & -\frac{x_{33}}{x_{34}}, & \frac{1}{x_{34}} \end{bmatrix}$

then we have

$$[M'H_1] \cdot H_2 = \begin{bmatrix} x_{11} & x_{12} & x_{13} & x_{14} \\ x_{21} & x_{22} & x_{23} & x_{24} \\ x_{31} & x_{32} & x_{33} & x_{34} \end{bmatrix} \begin{bmatrix} & I_{3\times 3} & & , 0_{3\times 1} \\ -\frac{x_{31}}{x_{34}}, & -\frac{x_{32}}{x_{34}}, & -\frac{x_{33}}{x_{34}}, & \frac{1}{x_{34}} \end{bmatrix}$$

$$= \begin{bmatrix} x_{11} - \frac{x_{14}x_{31}}{x_{34}}, & x_{12} - \frac{x_{14}x_{32}}{x_{34}}, & x_{13} - \frac{x_{14}x_{33}}{x_{34}}, & \frac{x_{14}}{x_{34}} \\ x_{21} - \frac{x_{24}x_{31}}{x_{34}}, & x_{22} - \frac{x_{24}x_{32}}{x_{34}}, & x_{23} - \frac{x_{24}x_{33}}{x_{34}}, & \frac{x_{24}}{x_{34}} \\ 0, & 0, & 0, & 1 \end{bmatrix}$$

In this case, $M'H_1 \cdot H_2$ is in a form a $\hat{M}'$ and $MH_1 \cdot H_2$ is in a form as $\hat{M}$.

Thus $H = H_1 \cdot H_2 = \begin{bmatrix} A^{-1}_{3\times3} & -A^{-1}b_{3\times1} \\ 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} & I_{3\times3} & & , O_{3\times1} \\ -\frac{x_{31}}{x_{34}}, & -\frac{x_{32}}{x_{34}}, & -\frac{x_{33}}{x_{34}} & , \frac{1}{x_{34}} \end{bmatrix}$

is the H that we are looking for.

(b) For the fundametal matrix F, we have

$$x'^T \cdot F \cdot x = 0$$

where $F = M'^{-T} E \cdot M^{-1}$

Thus $x'^T M'^{-T} E M^{-1} x = 0$

$\Rightarrow (x'^T H^T)(M'H)^{-T} E (MH)^{-1} (Hx) = 0$

we can see that the fundamental matrix corresponding to the pairs of matrices (M, M') and $(MH, M'H)$ are the same, where (M, M') corresponding to (x, x') and $(MH, M'H)$ corresponding to (Hx, Hx').

(c) According to Hint, the fundemental matrix corresponding to a pair of camera matrices $M = [I \mid 0]$ and $M' = [A \mid b]$ is equal to $[b]_\times A$

Thus, applying result from question (a), we have

$$F = [b]_\times A,$$

where $b = \begin{bmatrix} b_1 \\ b_2 \\ 1 \end{bmatrix}$ $\quad A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 0 \end{bmatrix}$.

$$\therefore F = [b]_\times \cdot A = \begin{bmatrix} 0 & -1 & b_2 \\ 1 & 0 & -b_1 \\ -b_2 & b_1 & 0 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} -a_{21} & -a_{22} & -a_{23} \\ a_{11} & a_{12} & a_{13} \\ -a_{11}b_2 + a_{21}b_1 & -a_{12}b_2 + a_{22}b_1 & -a_{13}b_2 + a_{23}b_1 \end{bmatrix}$$

Thus, we can multiply any scale factor to make one element as 1.

For example, we may multiply $\frac{1}{a_{11}}$

$$\text{Then, } F = \begin{bmatrix} -\frac{a_{21}}{a_{11}} & -\frac{a_{22}}{a_{11}} & \frac{-a_{23}}{a_{11}} \\ 1 & \frac{a_{12}}{a_{11}} & \frac{a_{1}'}{a_{11}} \\ -b_2 + \frac{a_{21}b_1}{a_{11}} & \frac{-a_{12}b_2 + a_{22}b_1}{a_{11}} & \frac{-a_{13}b_2 + a_{23}b_1}{a_{11}} \end{bmatrix}$$

which is expressed by seven parameters.

Prob 2. Let k pass through x but not epipole e.

Then x can be expressed as the cross-mutiply of k and l, i.e.

$$x = [k]_\times l \quad \text{------} \quad ①$$

Since we know that fundamental matrix F has the property $F \cdot x = l'$ ------ ②

put ① into ②, we have.

$$l' = F \cdot x = F \cdot [k]_\times l$$

i.e. $l' = F \cdot [k]_\times \cdot l$ $\blacksquare$

Prob 3. 3.1 Fundamental Matrix.

① Linear Least Square

For each pair of point, $p' \cdot p^T$ will generate a 3×3 matrix
$$\begin{bmatrix} x_1' x_1 & y_1' x_1 & x_1 \\ x_1' y_1 & y_1' y_1 & y_1 \\ x_1' & y_1' & 1 \end{bmatrix}$$

Same reasoning, for all the N pairs of points, we can write a matrix A, such that
$$A = \begin{bmatrix} x_1' x_1 & x_1' y_1 & x_1' & y_1' x_1 & y_1' y_1 & y_1' & x_1 & y_1 & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ x_n' x_n & x_n' y_n & x_n & y_n' x_n & y_n' y_n & y_n' & x_n & y_n & 1 \end{bmatrix}$$

Theoretically. $\text{rank}(A) = 8$.

So we do SVD for matrix A.

$[u \ s \ v] = svd(A)$

Then pick the column of V that corresponds to the minimum singular value $V(:,9) = [v_1 \ v_2 \ \cdots \ v_9]^T$

$$\text{Let } F = \begin{bmatrix} v_1 & v_2 & v_3 \\ v_4 & v_5 & v_6 \\ v_7 & v_8 & v_9 \end{bmatrix}$$

Since we know that $\text{rank}(F) = 2$, so this F is not the final Fundamental Matrix. Instead, we shall set its rank into 2.

Thus. $[u \ s \ v] = svd(F)$

Then $F \leftarrow F - u(:,3) * S(3,3) * [V(:,3)]^T$

② Normalized Version.

Before actually compute the Fundamental Matrix, in order to reduce the error by the uncentered origin, we shall center our data into a circle by multiplying a matrix T, where

$$T = \begin{bmatrix} 1/d & 0 & -\bar{x}/d \\ 0 & 1/d & -\bar{y}/d \\ 0 & 0 & 1 \end{bmatrix}$$

in which, $$\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n}$$

$$\bar{y} = \sum_{i=1}^{n} \frac{y_i}{n}$$

$$d = \sum_{i=1}^{n} \frac{\sqrt{(x_i - \bar{x})^2 + (y_i - \bar{y})^2}}{n\sqrt{2}}$$

By this way, for data x1 and x2, we can produce two matrix T1 and T2.

Then we put $xx1 = T1 \cdot X1$; $xx2 = T2 \cdot X2$ into the Linear least square step, it will result in a Scaled Fundamental Matrix F_s.

To get the final Fundamental Matrix

$$F = T_1^T \cdot F_s \cdot T_2$$

☆ Matlab Code is attached here and uploaded in CANVAS.

☆ Image result is also attached here or can be generated by my code. Errors are shown in images.

3.2 Stereo Rectification

After we calculate the Fundamental Matrix F, we know that : $\text{rank}(F) = 2$. $\quad p_2^T \cdot F \cdot p_1 = 0$

For epipole e_1 and e_2 in Image J_1 and J_2, we have:

$$F \cdot e_1 = 0 \qquad F^T \cdot e_2 = 0$$

$$\Rightarrow \quad e_1 \in \mathcal{N}(F) \qquad e_2 \in \mathcal{N}(F^T)$$

First, we shall find a matrix H_1 for J_1, H_2 for J_2.

In order to translate the epipole to infinty, and make sure we have less distortion. We shall first translate the origin of the picture to its center

$$\bar{e} = T \cdot e \quad \text{where} \quad T = \begin{bmatrix} 1 & 0 & -\frac{width}{2} \\ 0 & 1 & -\frac{length}{2} \\ 0 & 0 & 1 \end{bmatrix}$$

The set the epipoler line horizontal, let $\phi = \angle \bar{e}$

$$R = \begin{bmatrix} \cos\phi & -\sin\phi & 0 \\ \sin\phi & \cos\phi & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad \hat{e} = R\bar{e} = \begin{bmatrix} \hat{e}_1 \\ 0 \\ 1 \end{bmatrix}$$

Then

$$G = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1/\hat{e}_1 & 0 & 1 \end{bmatrix}$$

By this way, $H = G \cdot R \cdot T$, we can get H_1 and H_2

Now we transformed picture J_1 and J_2 into $\tilde{J}_1$ and $\tilde{J}_2$

$$J_1 \xrightarrow{H_1} \tilde{J}_1$$

$$J_2 \xrightarrow{H_2} \tilde{J}_2$$

However, this is NOT the final result. We should set $\tilde{J}_2$ as a standard and correct $\tilde{J}_1$ to the right position

$$\begin{array}{ccc} J_1 & & J_2 \\ \downarrow{\scriptstyle H_1} & & \downarrow{\scriptstyle H_2} \\ \tilde{J}_1 & & \tilde{J}_2 \\ \downarrow{\scriptstyle H_0} & & \\ \hat{J}_1 & & \end{array}$$

So that

Then $\hat{J}_1$ and $\tilde{J}_2$ is our final result. that minimizes $d(H_0 \tilde{p}_1, \tilde{p}_2)$.

To calculate H_0, let $H_0 = \begin{bmatrix} s_1 & s_3 & d_1 \\ 0 & s_2 & d_2 \\ 0 & 0 & 1 \end{bmatrix}$

we Know that

$$\begin{bmatrix} \tilde{p}_{1,x_1} & \tilde{p}_{1,y_1} & 1 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ \tilde{p}_{1,x_n} & \tilde{p}_{1,y_n} & 1 & 0 & 0 \\ 0 & 0 & 0 & \tilde{p}_{1,y_1} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & \tilde{p}_{1,y_n} & 1 \end{bmatrix}_{2n\times 5} \cdot \begin{bmatrix} s_1 \\ s_3 \\ d_1 \\ s_2 \\ d_2 \end{bmatrix}_{5\times 1} = \begin{bmatrix} \tilde{p}_{2,x_1} \\ \vdots \\ \tilde{p}_{2,x_n} \\ \tilde{p}_{2,y_1} \\ \vdots \\ \tilde{p}_{2,y_n} \end{bmatrix}$$

Denote as $A \cdot x = b$

$\therefore x = A \backslash b$

Thus we can calculate H_0,

Then $\begin{cases} H_1 \Leftarrow H_0 \cdot H_1 \\ H_2 \Leftarrow H_2 \end{cases} \Rightarrow \begin{cases} \hat{J}_1 = H_0 H_1 J_1 \\ \tilde{J}_2 = H_2 J_2 \end{cases}$

☆ Matlab code is attached below and uploaded.

☆ H transform Matrix H_1 and H_2 are attach below.

☆ Images and Errors are also attached and shown below.