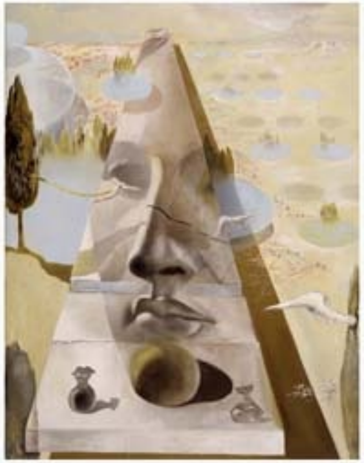


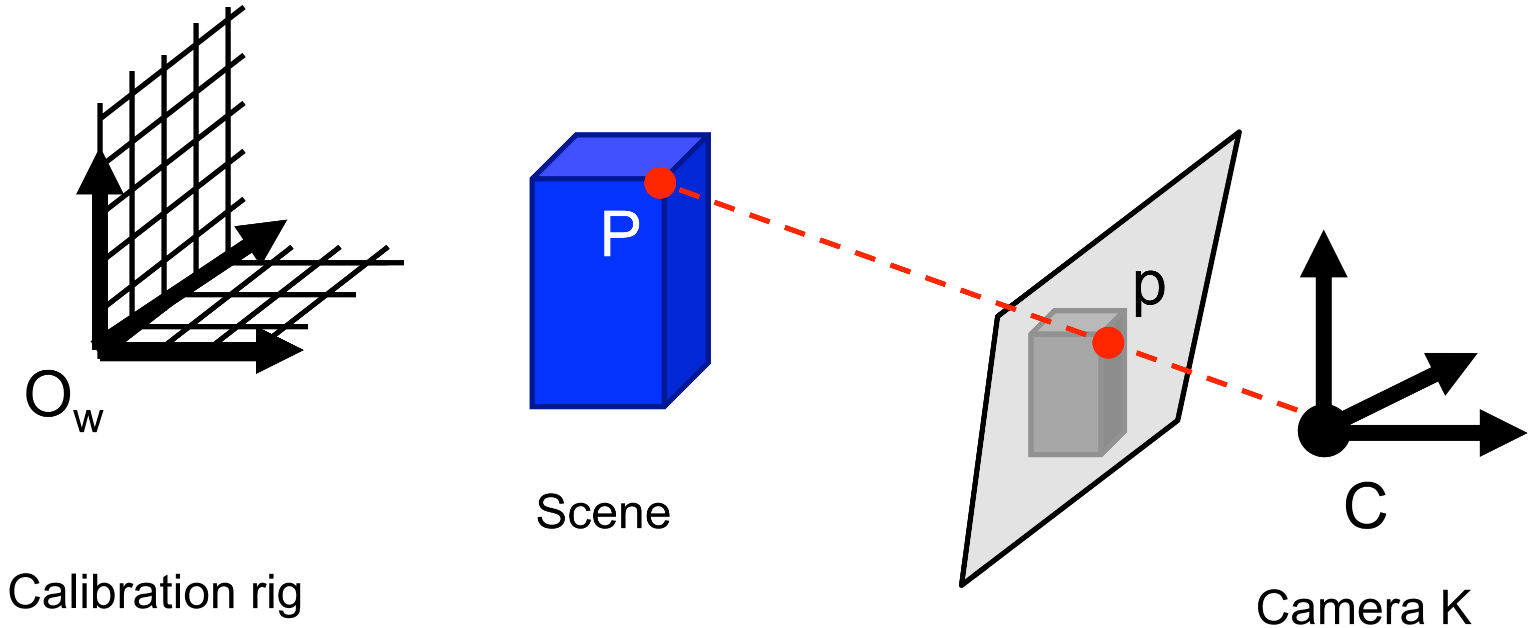
EECS 442 – Computer vision

Epipolar Geometry

- Why is stereo useful?
- Epipolar constraints
- Essential and fundamental matrix
- Estimating F
- Examples

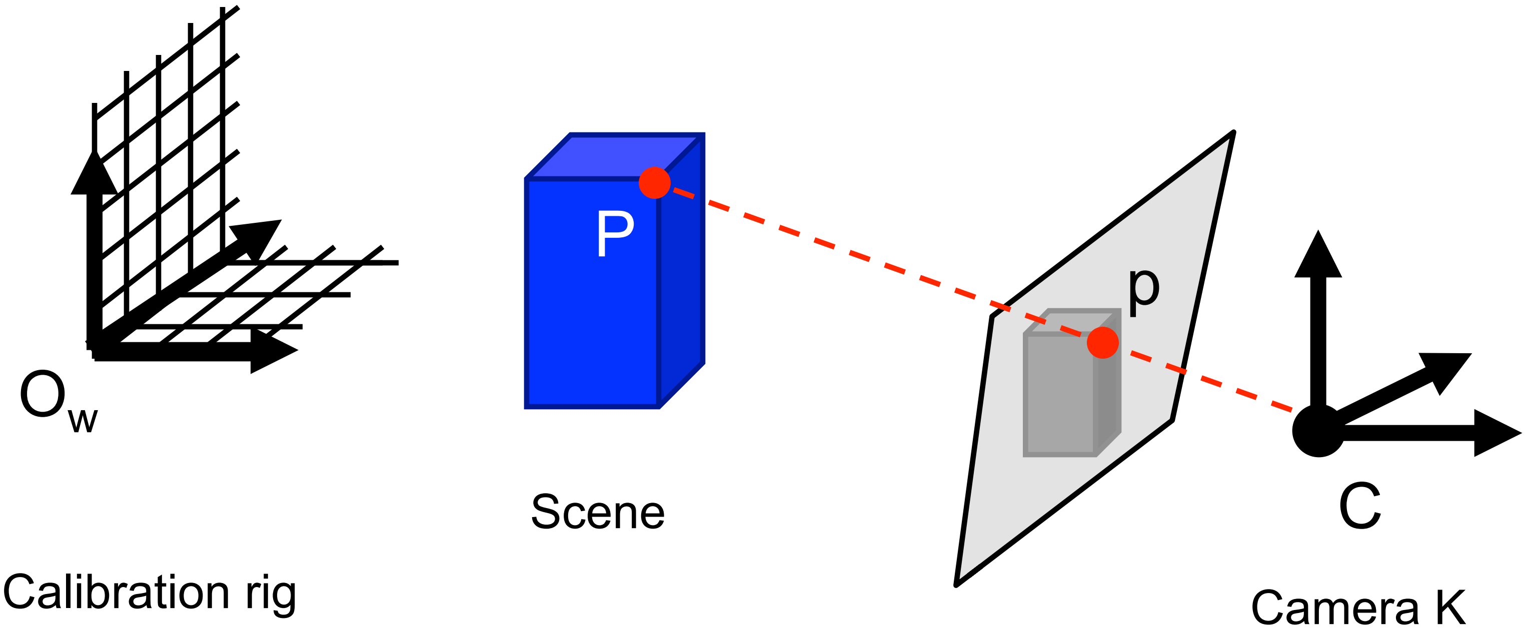
Reading: [AZ] Chapters: 4, 9, 11
[FP] Chapters: 10

Recovering structure from a single view



From calibration rig	→ location/pose of the rig, K
From points and lines at infinity + orthogonal lines and planes	→ structure of the scene, K
Knowledge about scene (point correspondences, geometry of lines & planes, etc…	

Recovering structure from a single view



Why is it so difficult?

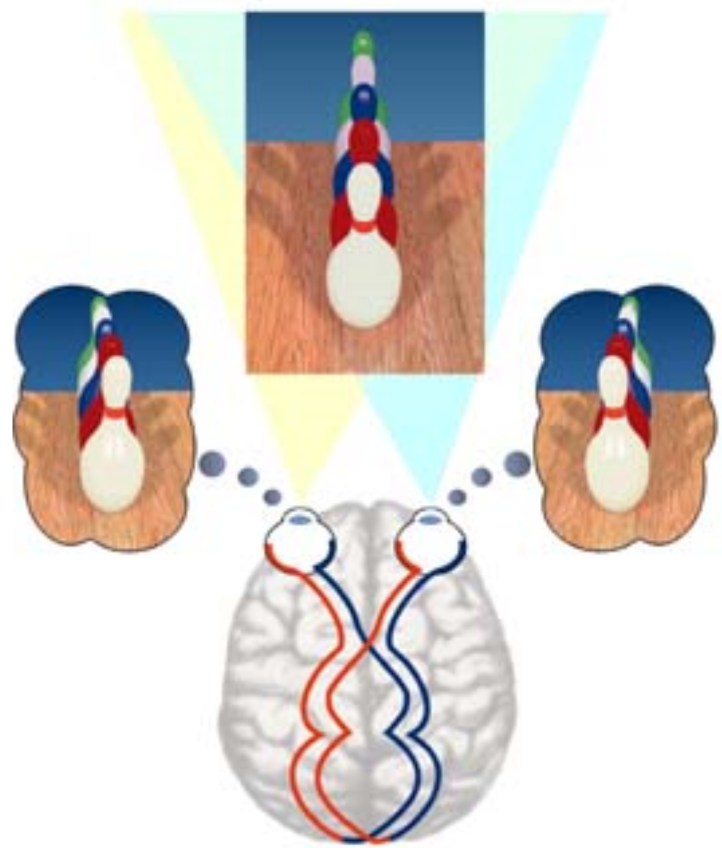
Intrinsic ambiguity of the mapping from 3D to image (2D)

Recovering structure from a single view

Intrinsic ambiguity of the mapping from 3D to image (2D)

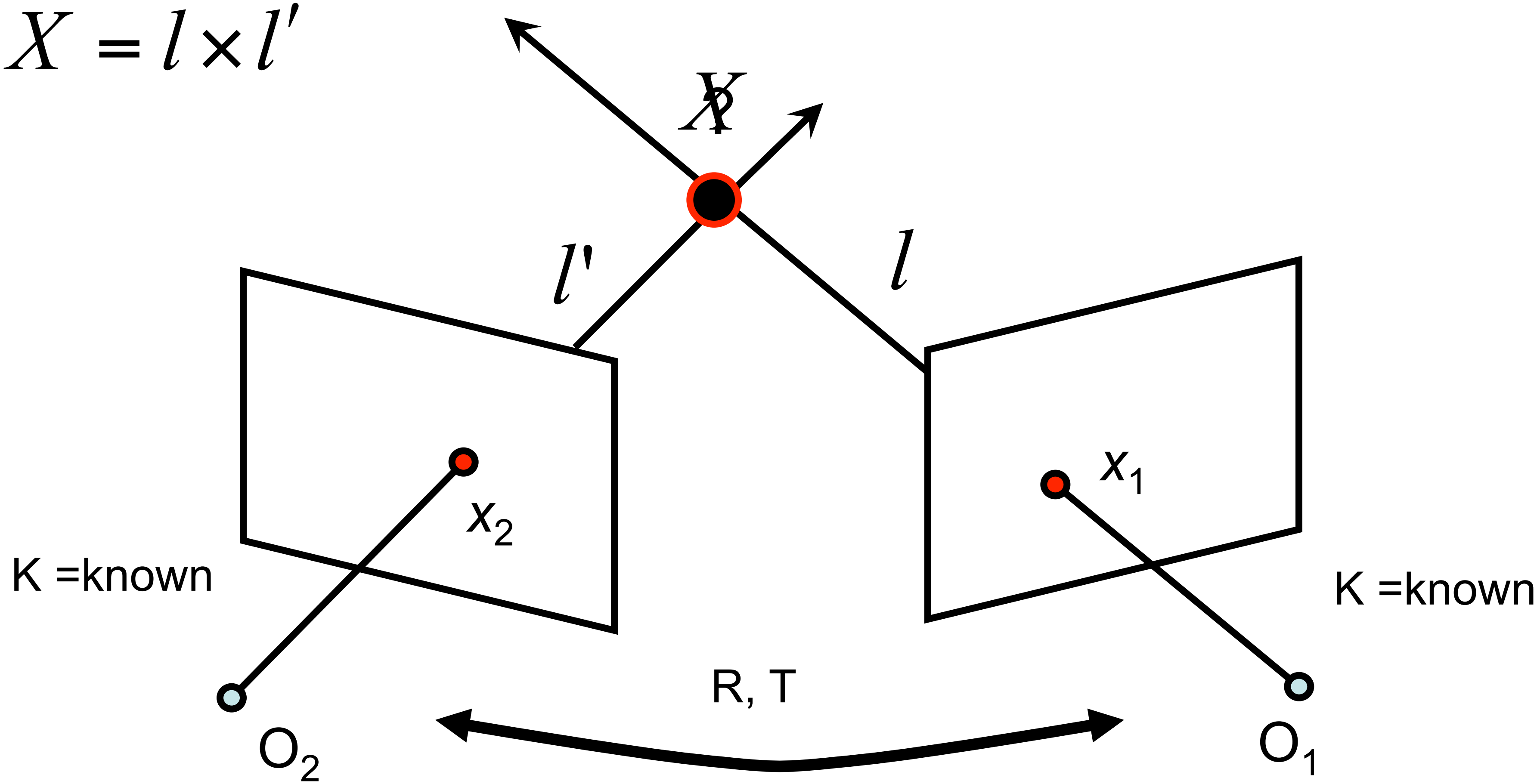
Courtesy slide S. Lazebnik

Two eyes help!



Two eyes help!

$$X = l \times l'$$

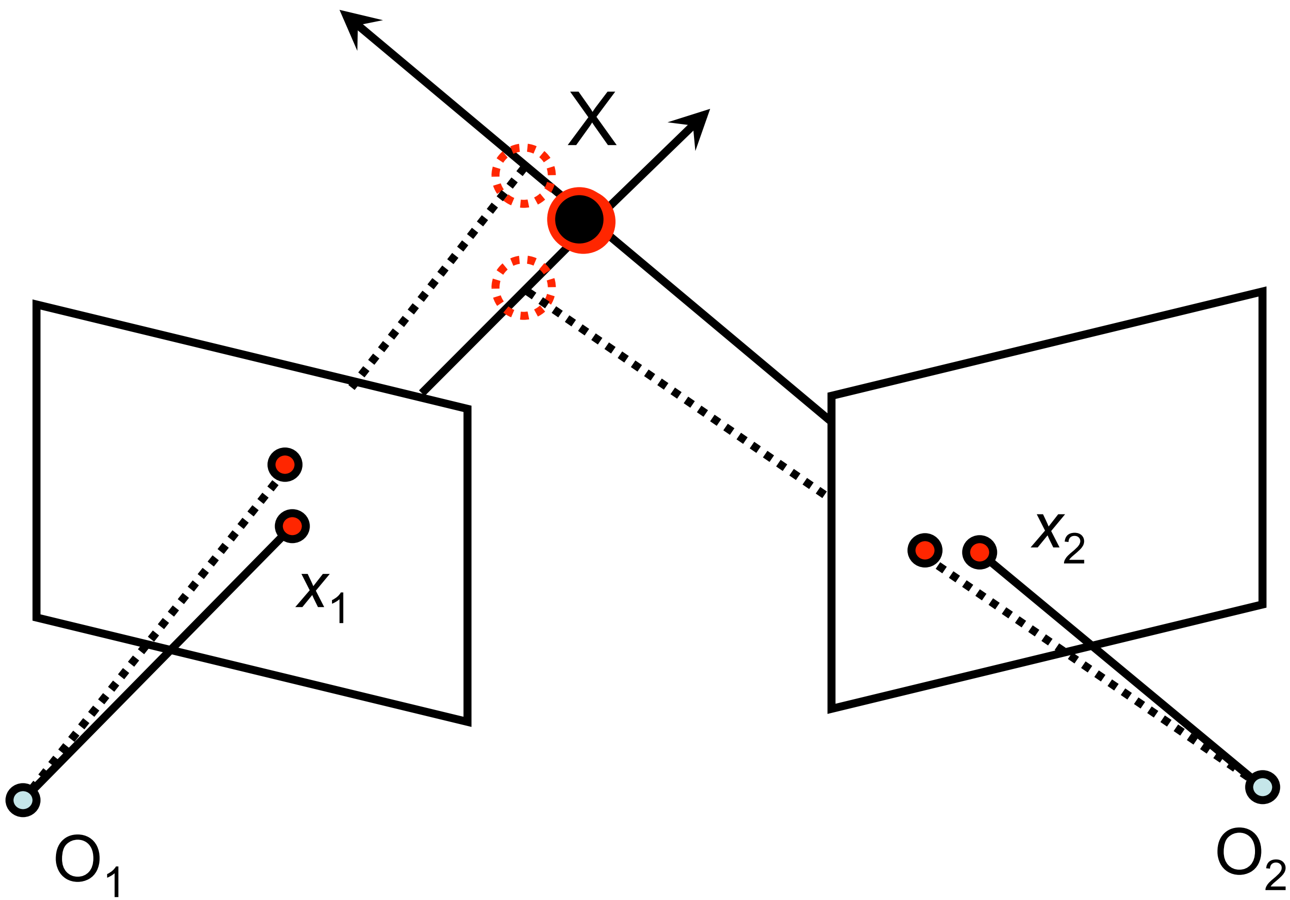


This is called triangulation

Triangulation

- Find X that minimizes

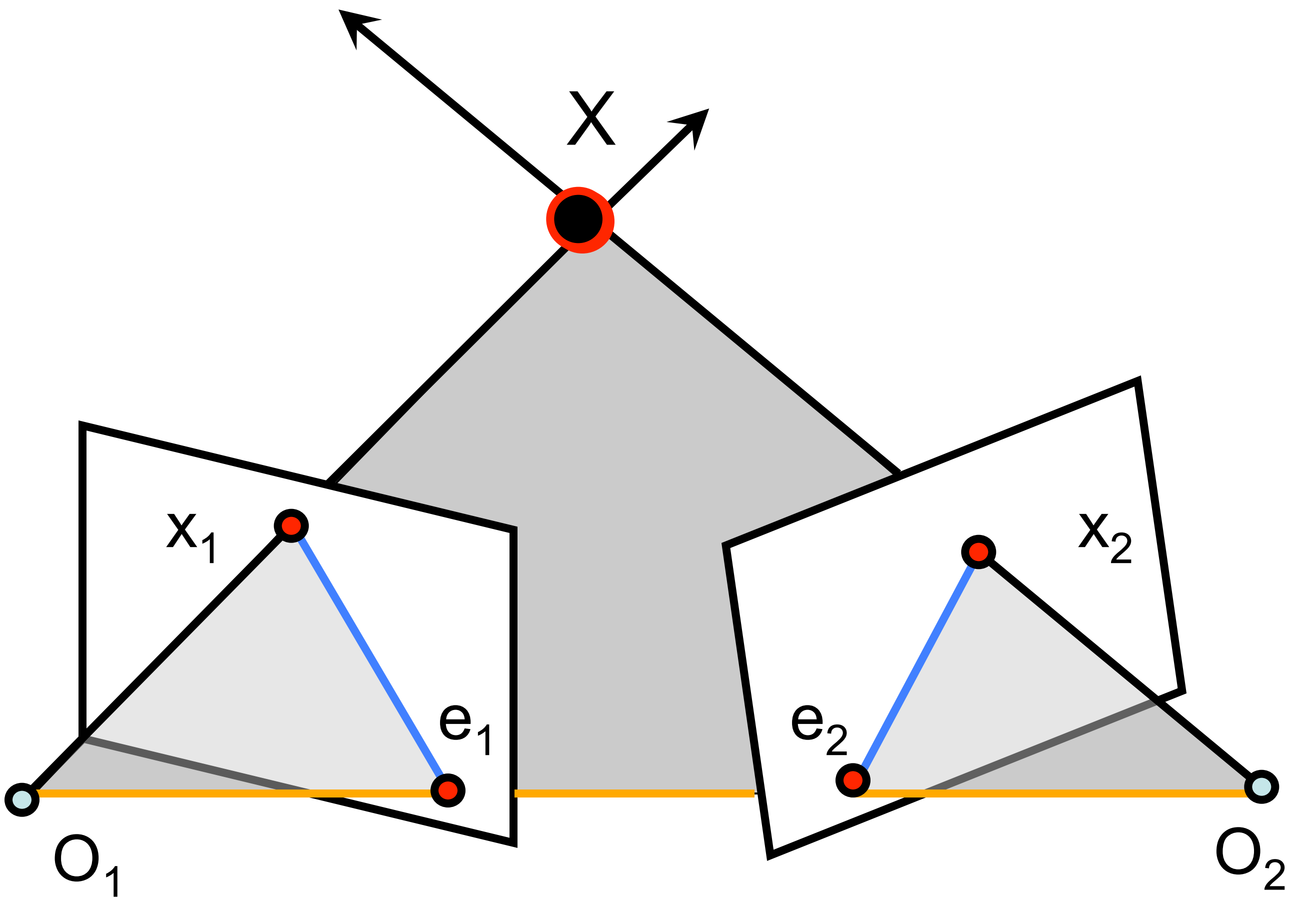
$$d^2(x_1, M_1X) + d^2(x_2, M_2X)$$



Stereo-view geometry

- **Correspondence:** Given a point in one image, how can I find the corresponding point x' in another one ?
- **Camera geometry:** Given corresponding points in two images, find camera matrices, position and pose.
- **Scene geometry:** Find coordinates of 3D point from its projection into 2 or multiple images.

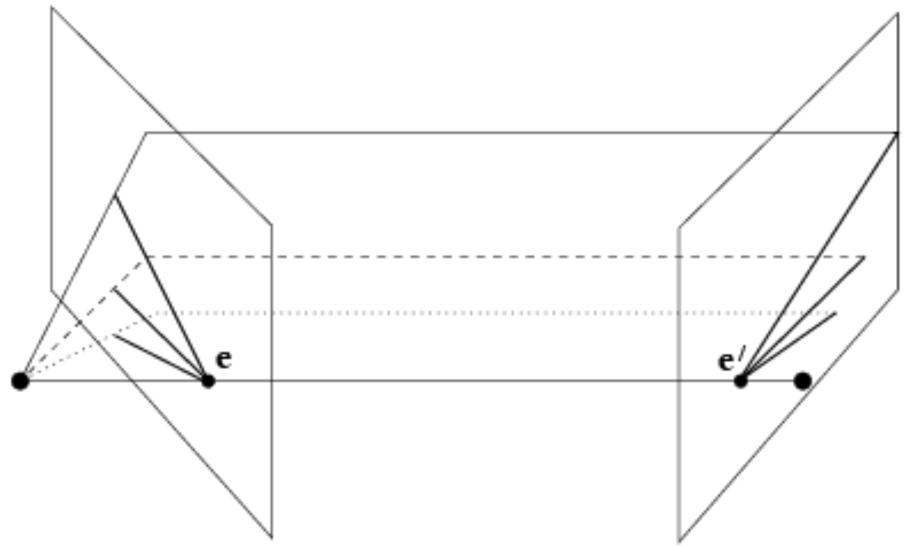
Epipolar geometry

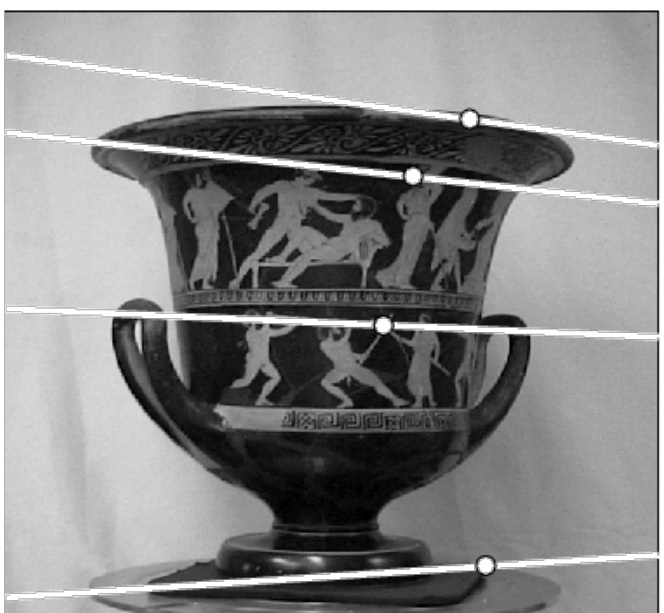


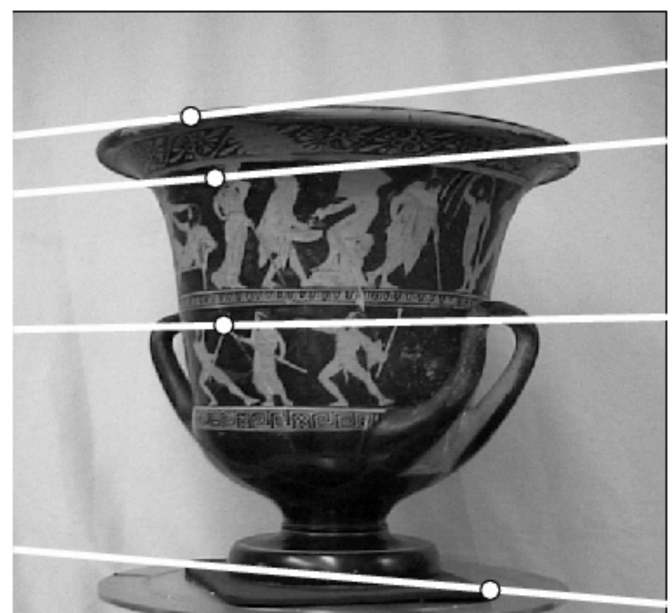
- Epipolar Plane
- Baseline
- Epipolar Lines
- Epipoles e_1, e_2

  = intersections of baseline with image planes
  = projections of the other camera center
  = vanishing points of camera motion direction

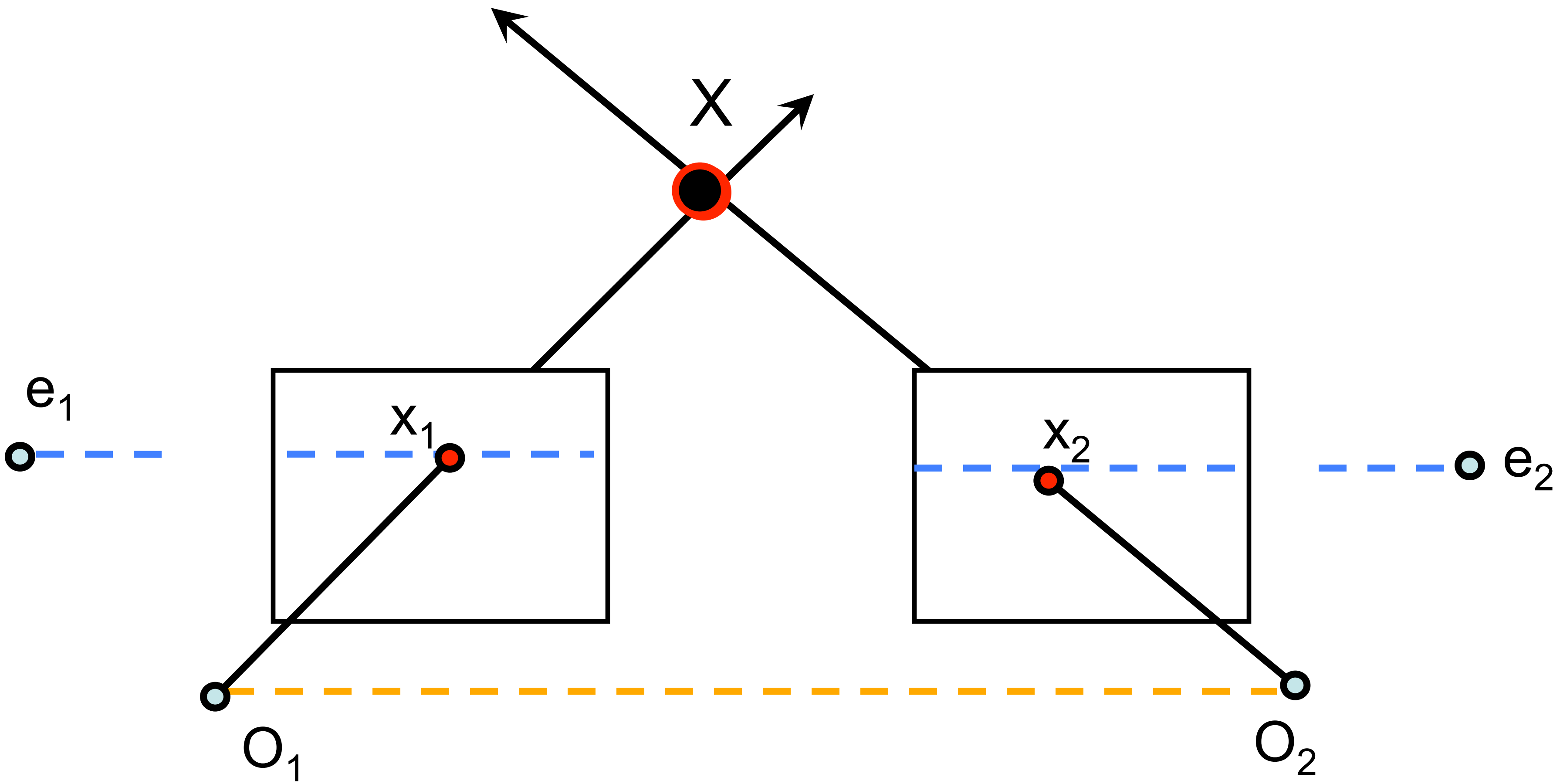
Example: Converging image planes





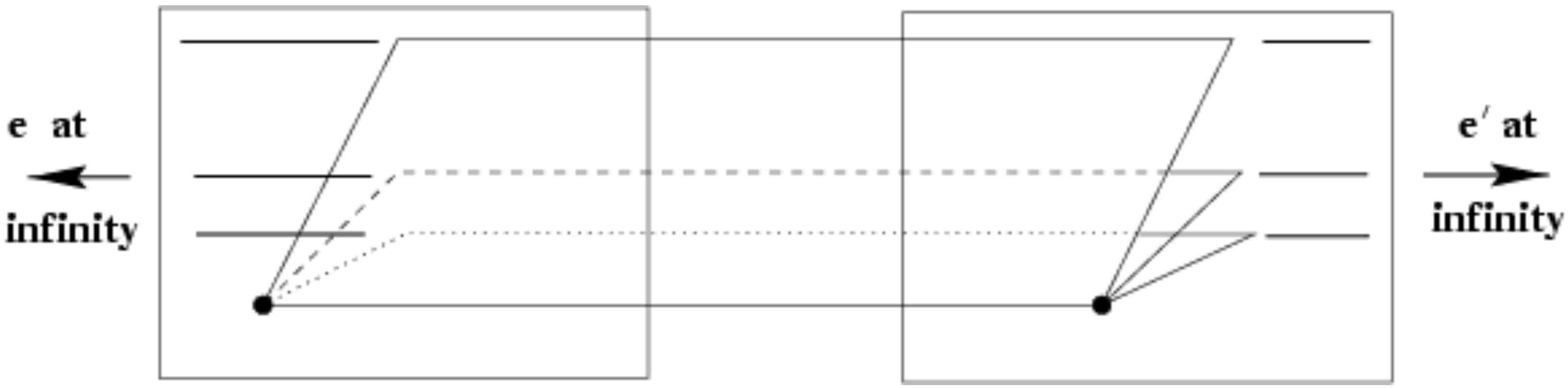


Example: Parallel image planes

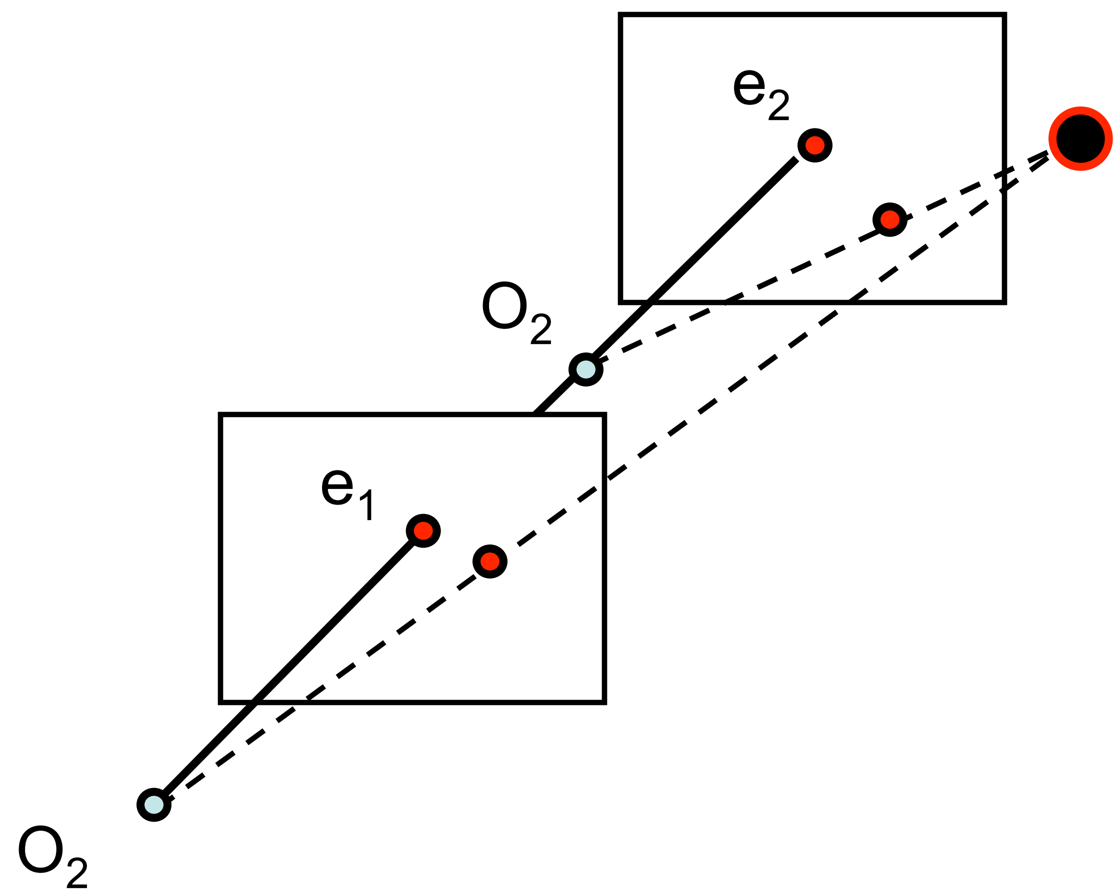


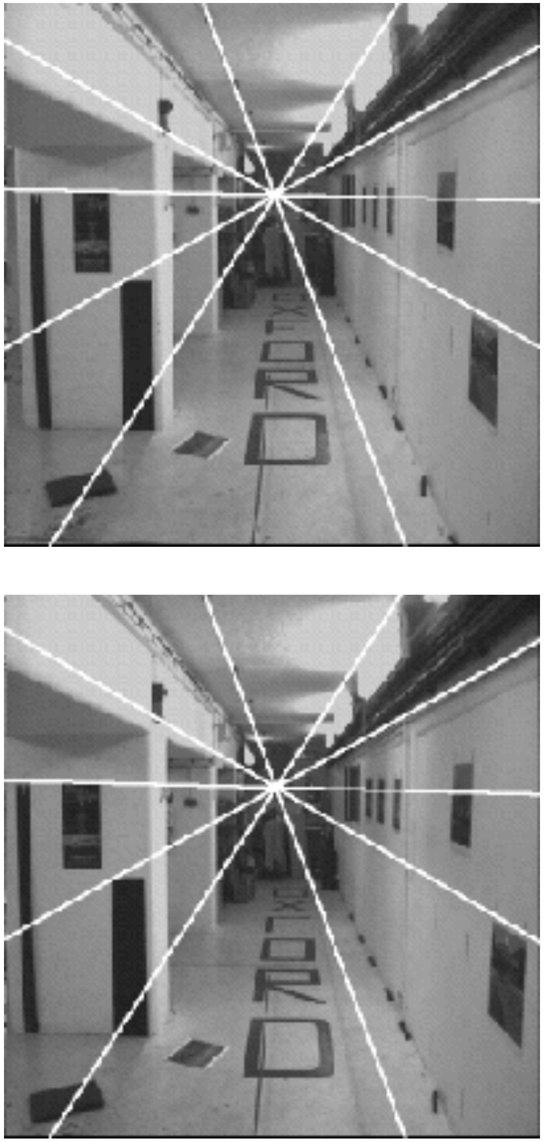
- Baseline intersects the image plane at infinity
- Epipoles are at infinity
- Epipolar lines are parallel to x axis

Example: Parallel image planes



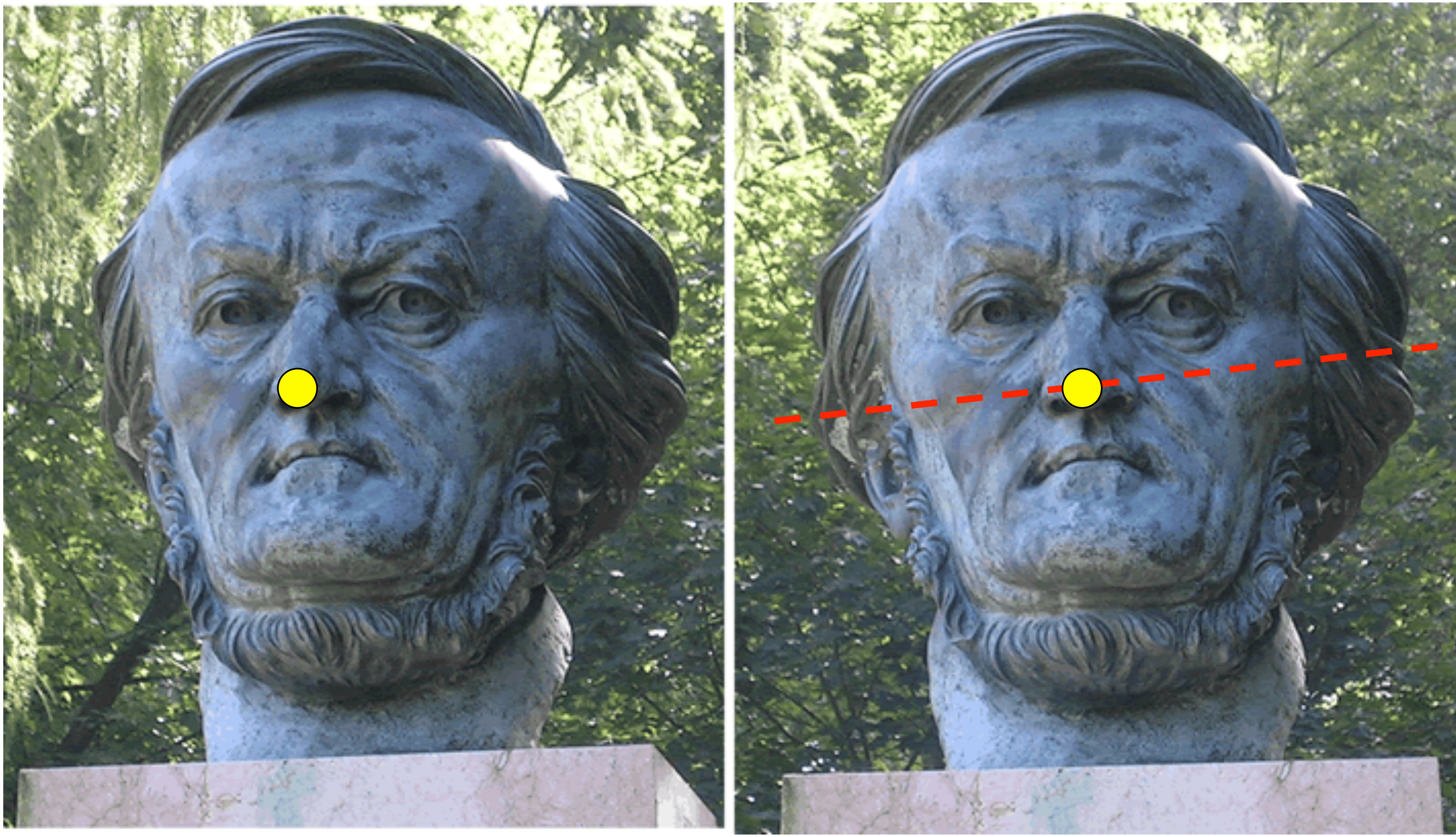
Example: Forward translation





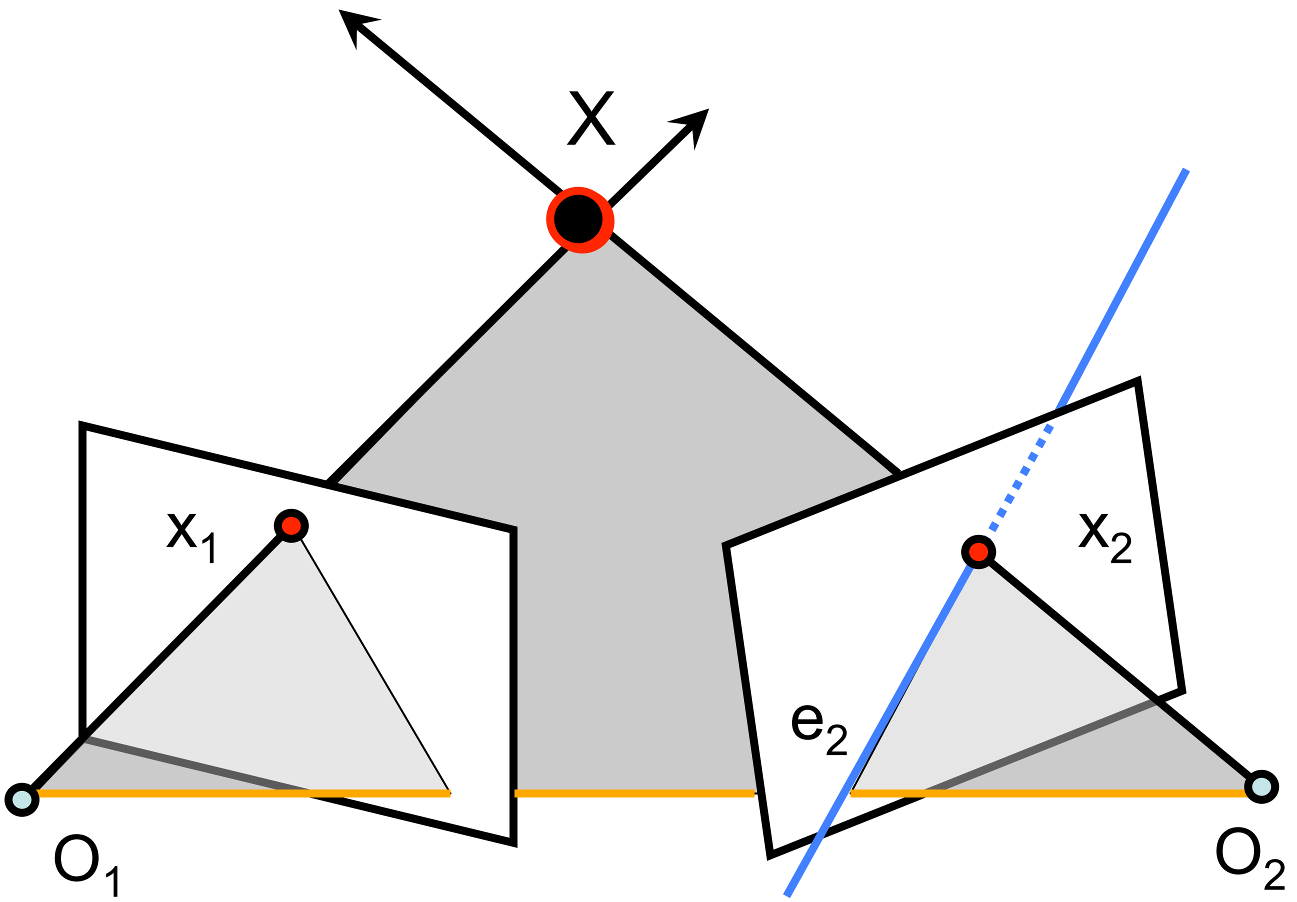
- The epipoles have same position in both images
- Epipole called FOE (focus of expansion)

Epipolar Constraint



- Two views of the same object
- Suppose I know the camera positions and camera matrices
- Given a point on left image, how can I find the corresponding point on right image?

Epipolar geometry

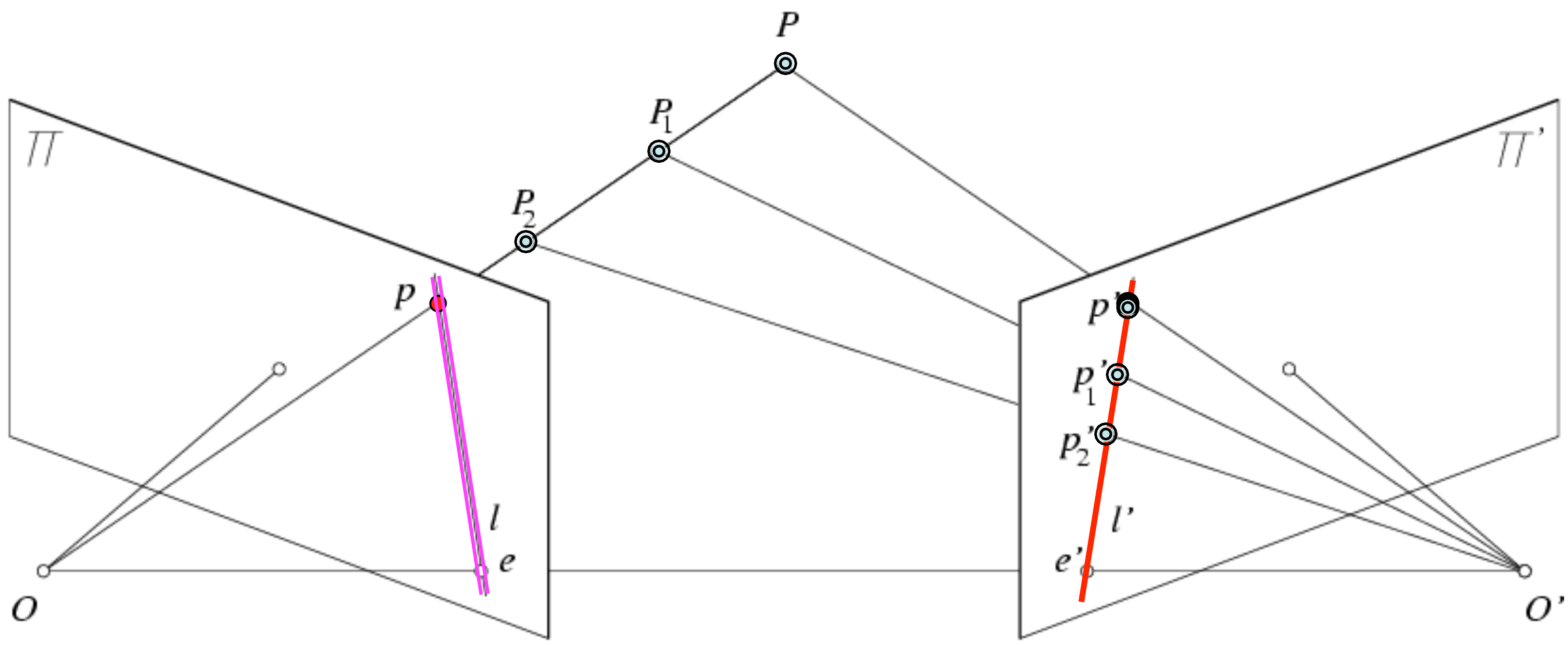


- Epipolar Plane
- Baseline
- Epipolar Lines

- Epipoles e_1, e_2

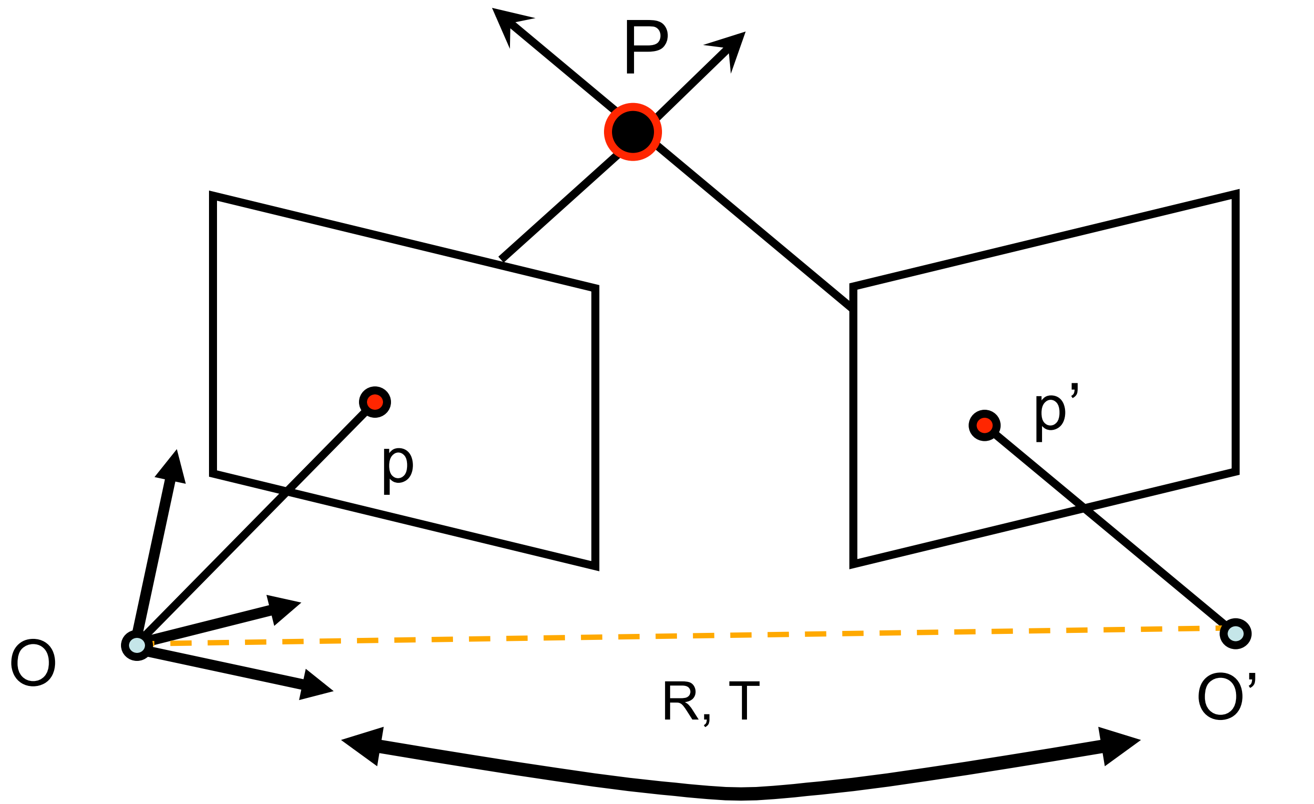
  = intersections of baseline with image planes
  = projections of the other camera center
  = vanishing points of camera motion direction

Epipolar Constraint



- Potential matches for *p* have to lie on the corresponding epipolar line *l'*.
- Potential matches for *p'* have to lie on the corresponding epipolar line *l*.

Epipolar Constraint



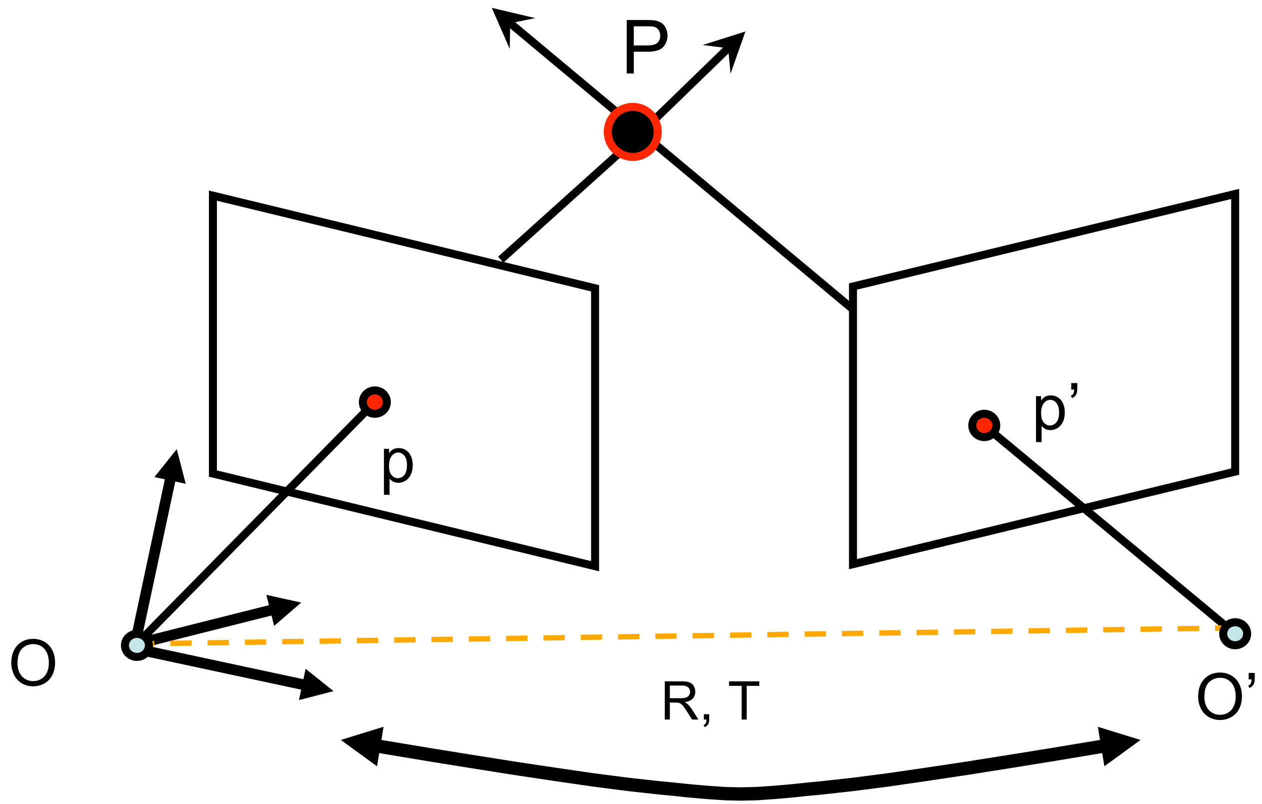
$$M = K\begin{bmatrix} I & 0 \end{bmatrix}$$

$$M' = K\begin{bmatrix} R & T \end{bmatrix}$$

$$P \rightarrow M\,P = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix}$$

$$P \rightarrow M'\,P = \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix}$$

Epipolar Constraint



$$M = K\begin{bmatrix} I & 0 \end{bmatrix}$$

K is known
(canonical cameras)

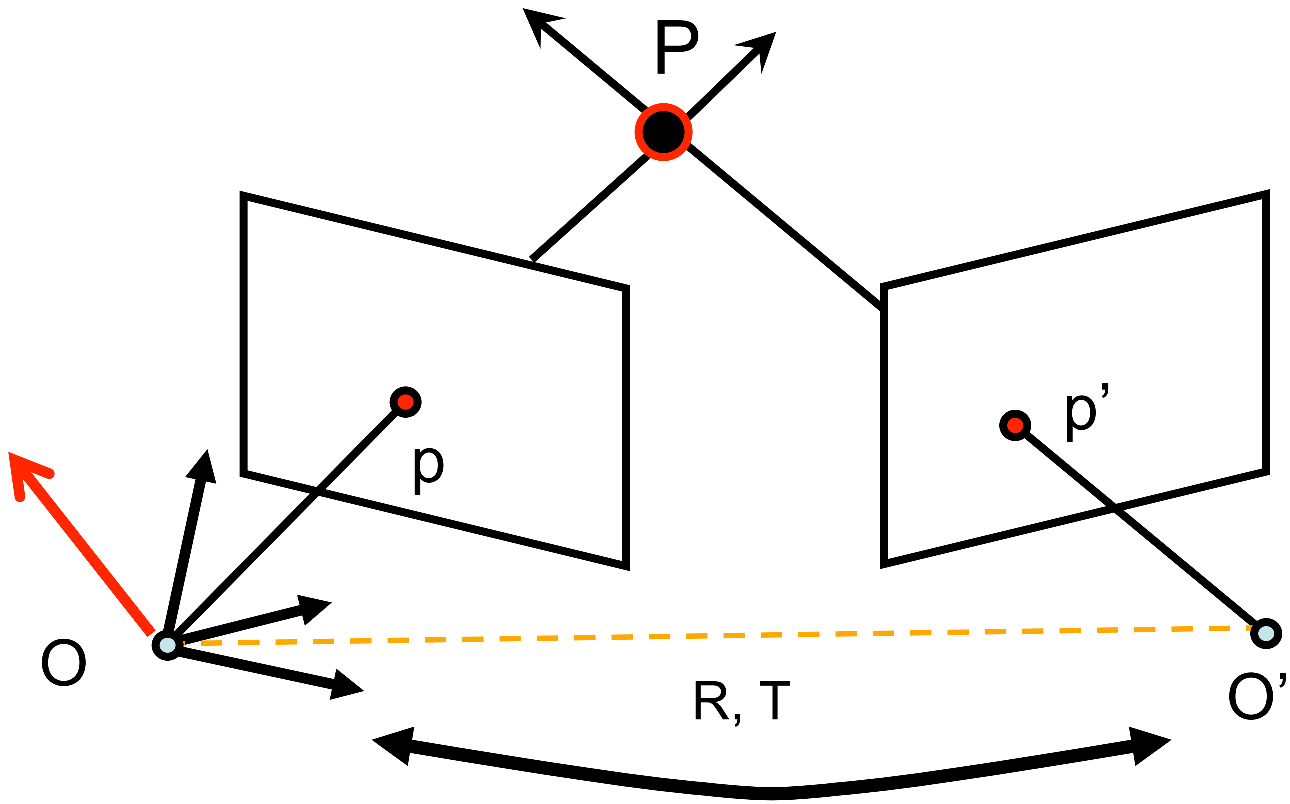
$$M' = K\begin{bmatrix} R & T \end{bmatrix}$$

$$K = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} I & 0 \end{bmatrix}$$

$$M' = \begin{bmatrix} R & T \end{bmatrix}$$

Epipolar Constraint



p' in first camera reference system is $= R\ p'$

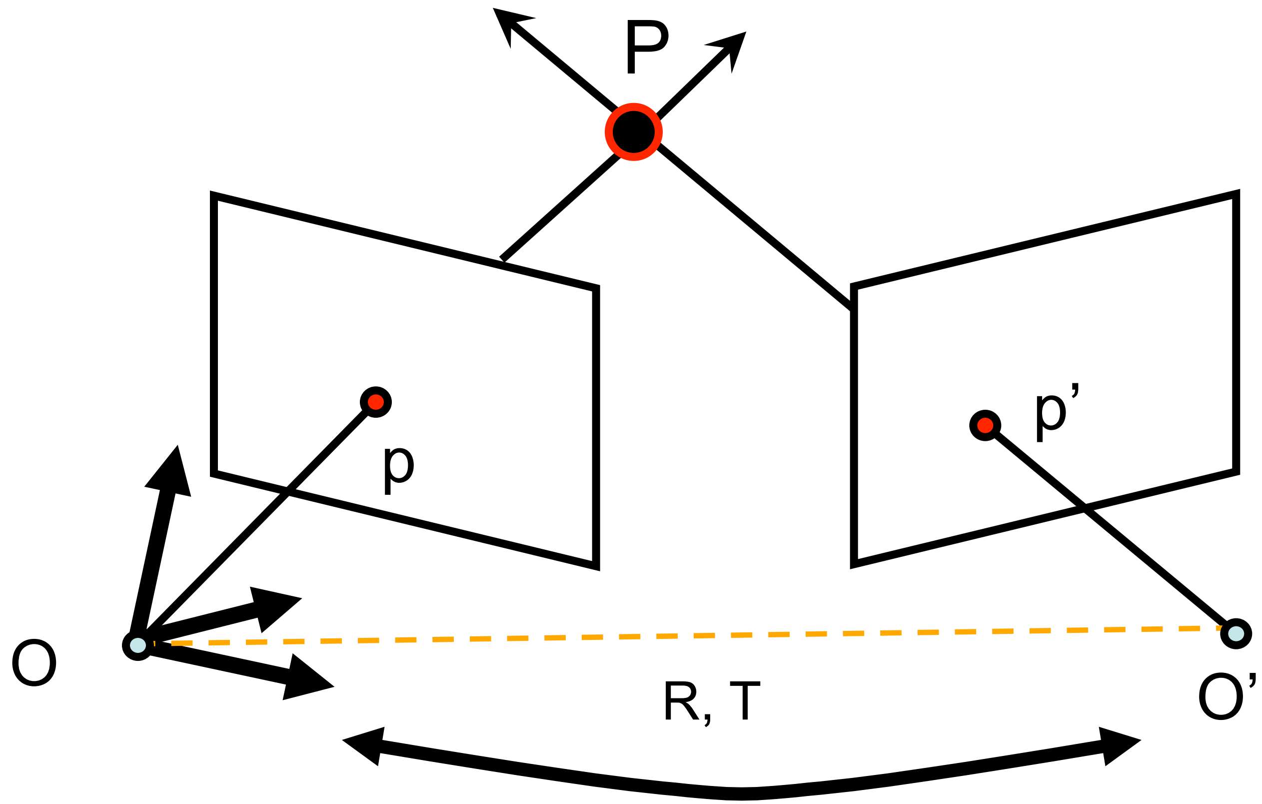
$T \times (R\ p')$ is perpendicular to epipolar plane

$$\rightarrow p^T \cdot \left[T \times (R\ p') \right] = 0$$

Cross product as matrix multiplication

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = [\mathbf{a}_\times]\mathbf{b}$$

Epipolar Constraint



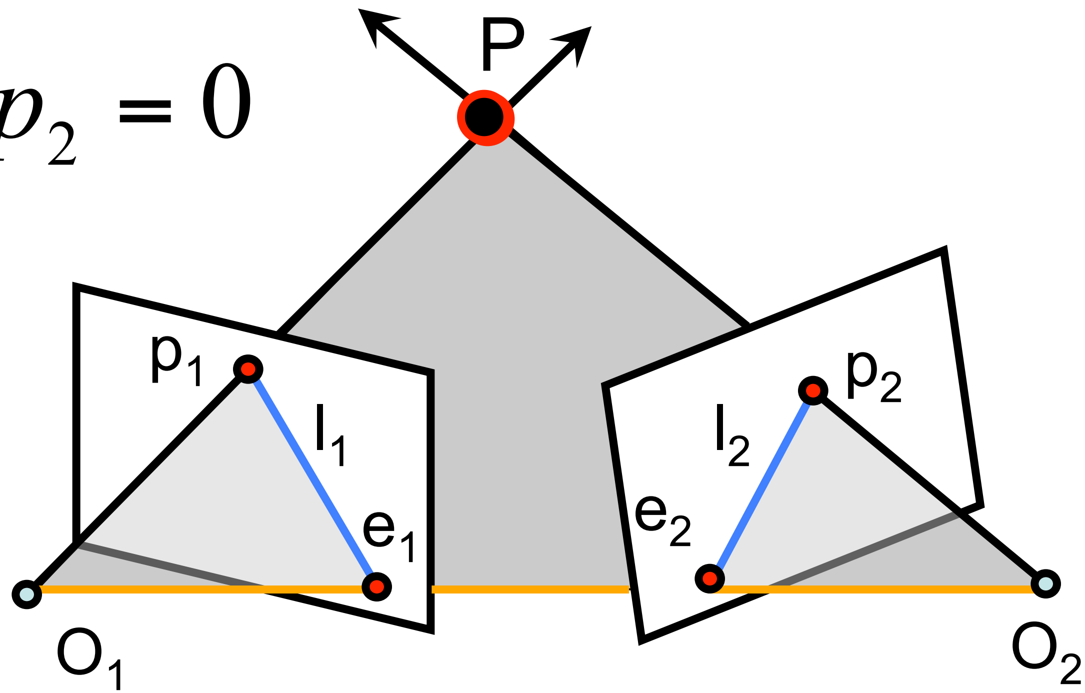
$$p^T \cdot [T \times (R\,p')] = 0 \rightarrow p^T \cdot \boxed{[T_\times] \cdot R}\, p' = 0$$

E = essential matrix

(Longuet-Higgins, 1981)

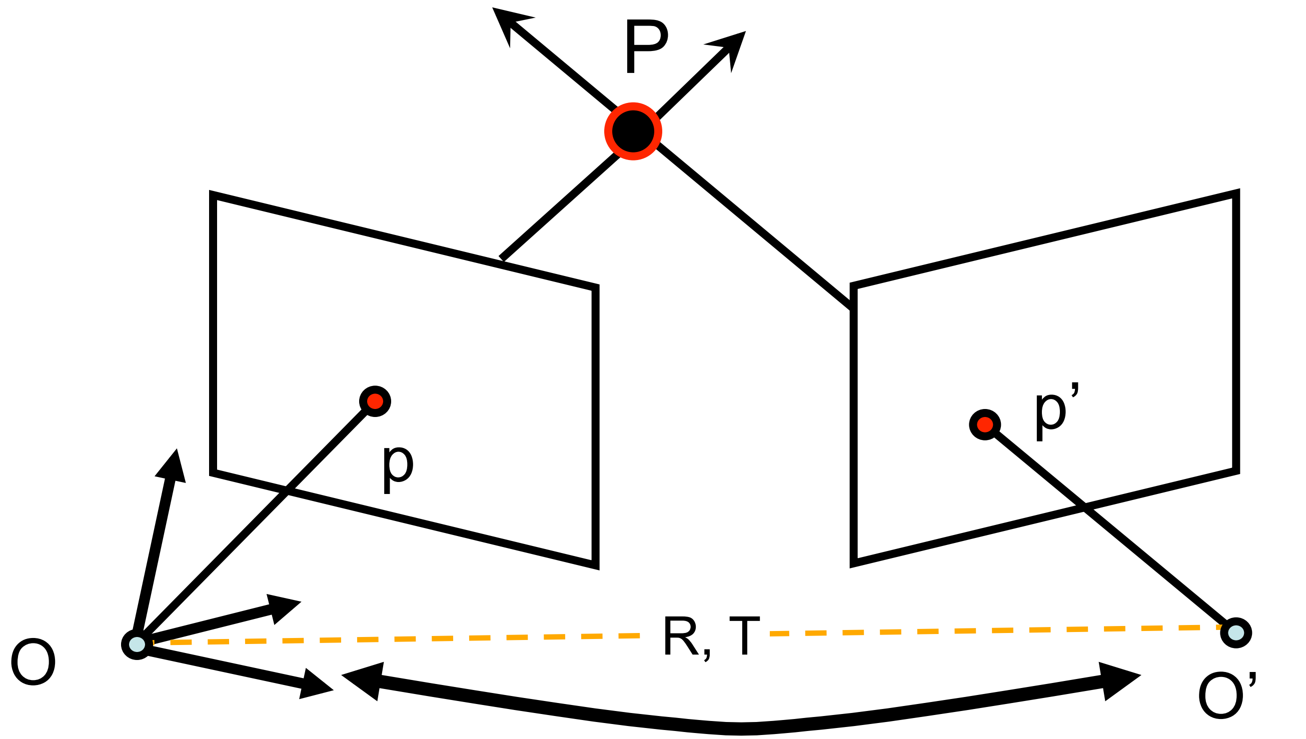
Epipolar Constraint

$$p_1^{\ T} \cdot E\ p_2 = 0$$



- $E\ p_2$ is the epipolar line associated with p_2 ($l_1 = E\ p_2$)
- $E^T p_1$ is the epipolar line associated with p_1 ($l_2 = E^T p_1$)
- $E\ e_2 = 0$ and $E^T e_1 = 0$
- E is 3x3 matrix; 5 DOF
- E is singular (rank two)

Epipolar Constraint



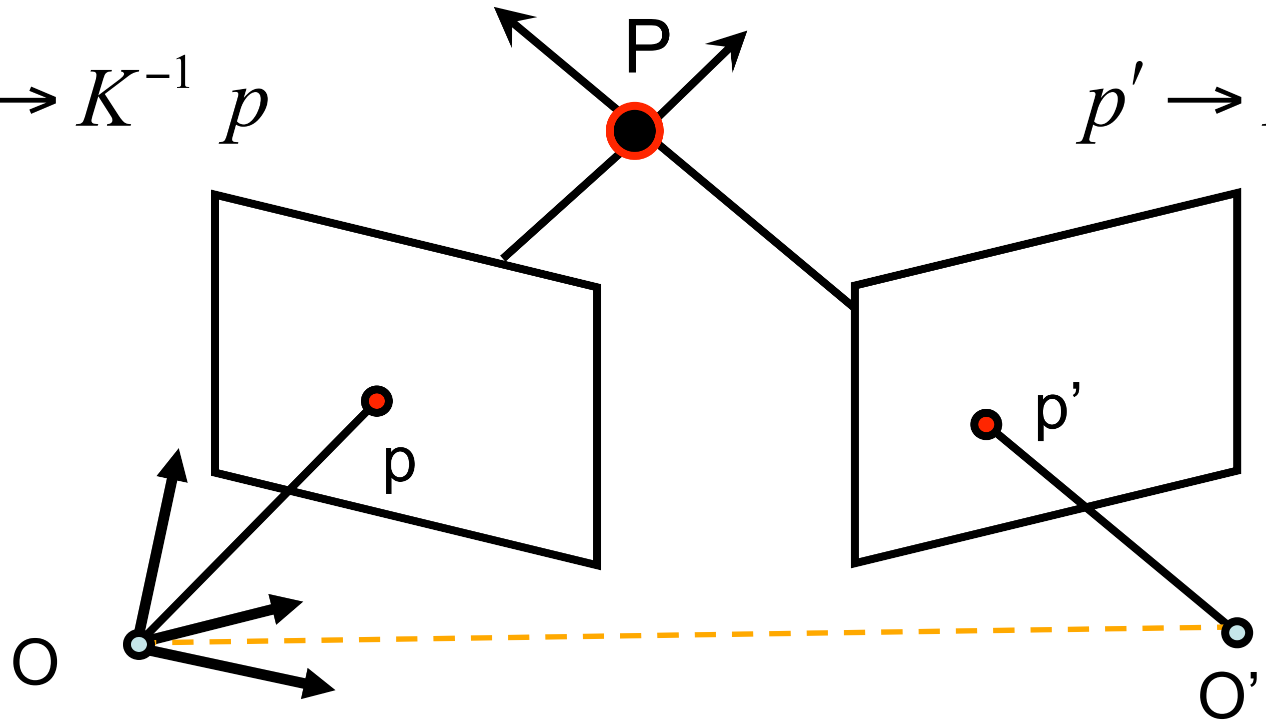
$$M = K\begin{bmatrix} I & 0 \end{bmatrix}$$

K is unknown

$$M' = K\begin{bmatrix} R & T \end{bmatrix}$$

Epipolar Constraint

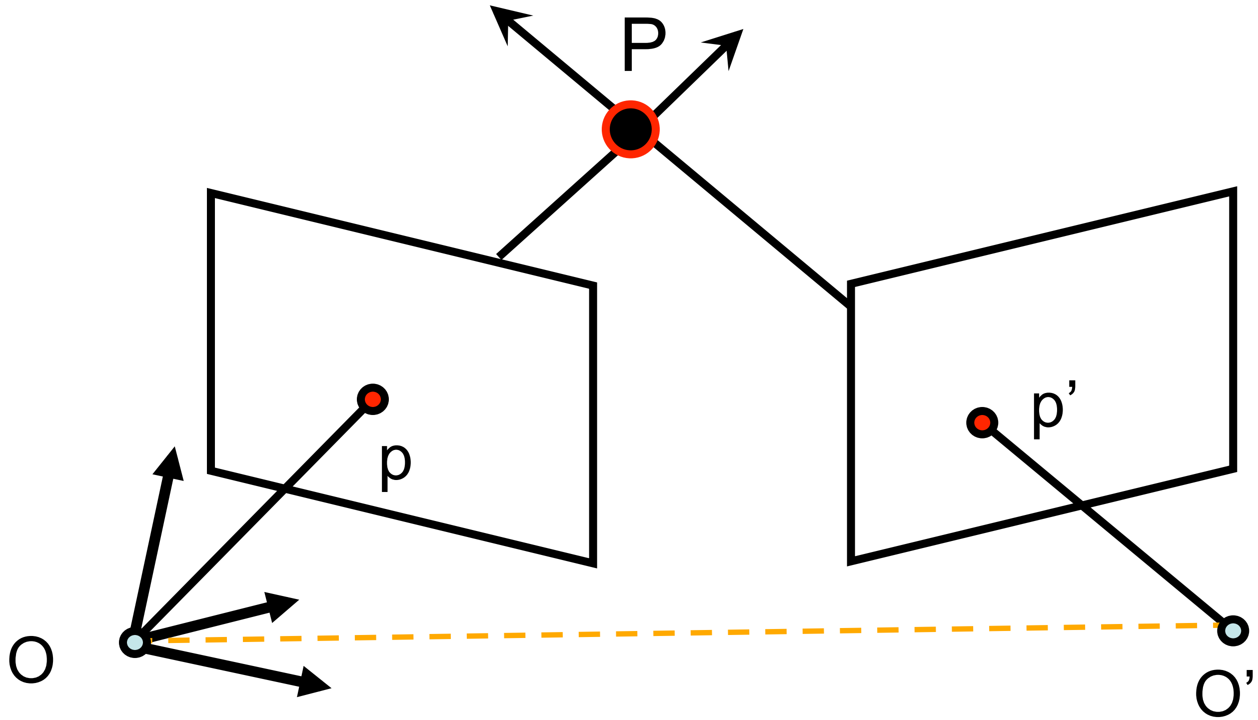
$$p \rightarrow K^{-1}\, p \qquad\qquad p' \rightarrow K'^{-1}\, p'$$



$$\mathrm{p}^{\mathrm{T}} \cdot \left[\mathrm{T}_{\times}\right] \cdot \mathrm{R}\, \mathrm{p}' = 0 \rightarrow (\mathrm{K}^{-1}\, \mathrm{p})^{\mathrm{T}} \cdot \left[\mathrm{T}_{\times}\right] \cdot \mathrm{R}\, \mathrm{K}'^{-1}\, \mathrm{p}' = 0$$

$$\mathrm{p}^{\mathrm{T}} \boxed{\mathrm{K}^{-\mathrm{T}} \cdot \left[\mathrm{T}_{\times}\right] \cdot \mathrm{R}\, \mathrm{K}'^{-1}}\, \mathrm{p}' = 0 \rightarrow \mathrm{p}^{\mathrm{T}} \boxed{\mathrm{F}}\, \mathrm{p}' = 0$$

Epipolar Constraint



$$\boxed{p'^{T} F\, p = 0}$$

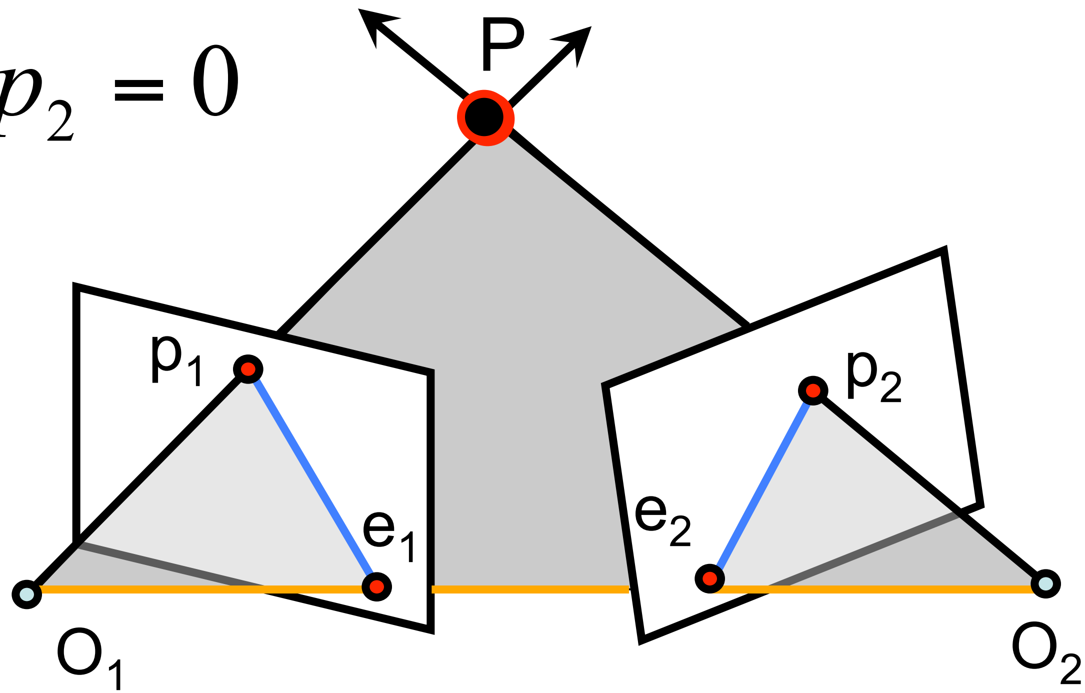
$$F = K'^{-T} \cdot \left[T_{\times}\right] \cdot R\, K^{-1}$$

F = Fundamental Matrix

(Faugeras and Luong, 1992)

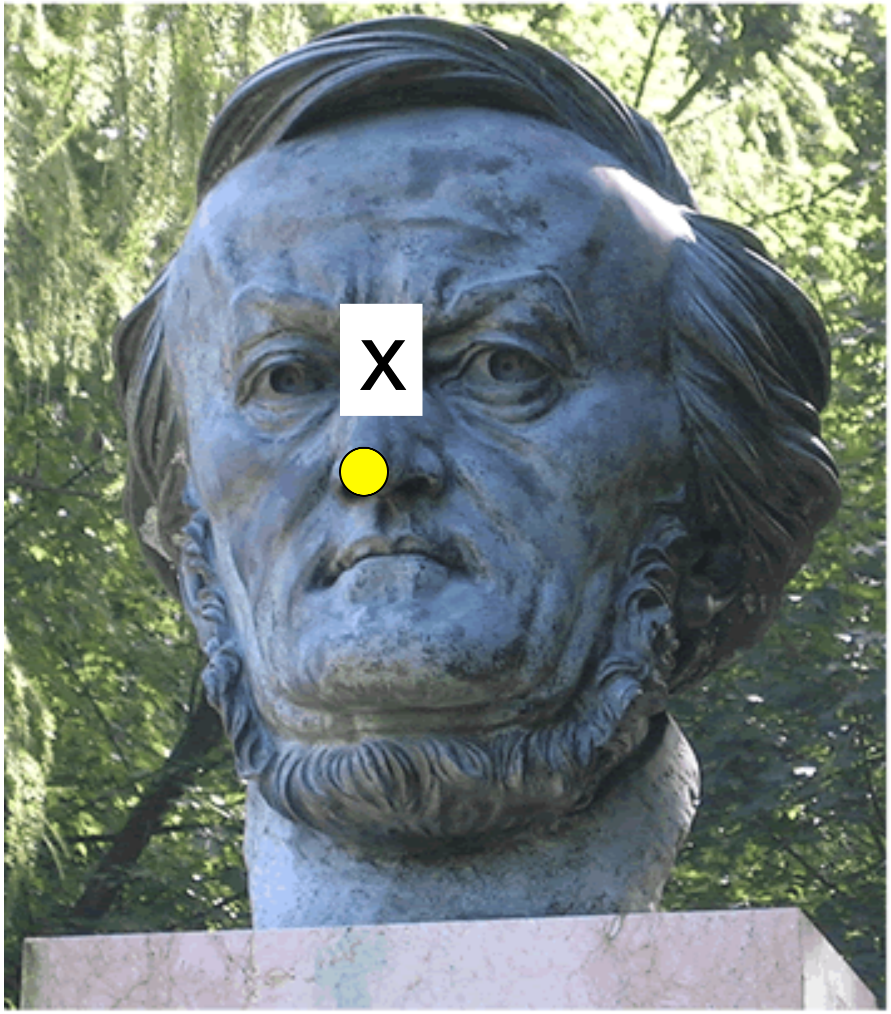
Epipolar Constraint

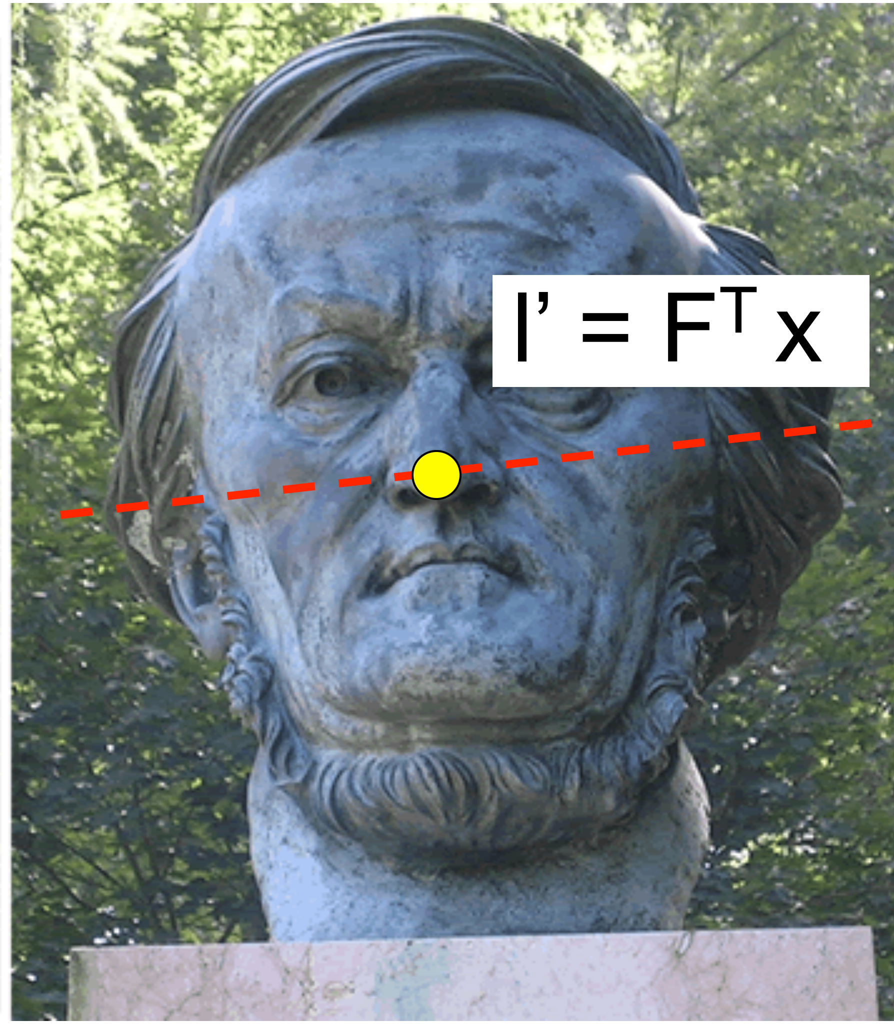
$$p_1^{\ T} \cdot F\ p_2 = 0$$



- $F\ p_2$ is the epipolar line associated with p_2 ($l_1 = F\ p_2$)
- $F^T p_1$ is the epipolar line associated with x_1 ($l_2 = F^T p_1$)
- $F\ e_2 = 0$ and $F^T e_1 = 0$
- F is 3x3 matrix; 7 DOF
- F is singular (rank two)

Why F is useful?





- Suppose F is known
- No additional information about the scene and camera is given
- Given a point on left image, how can I find the corresponding point on right image?

Why F is useful?

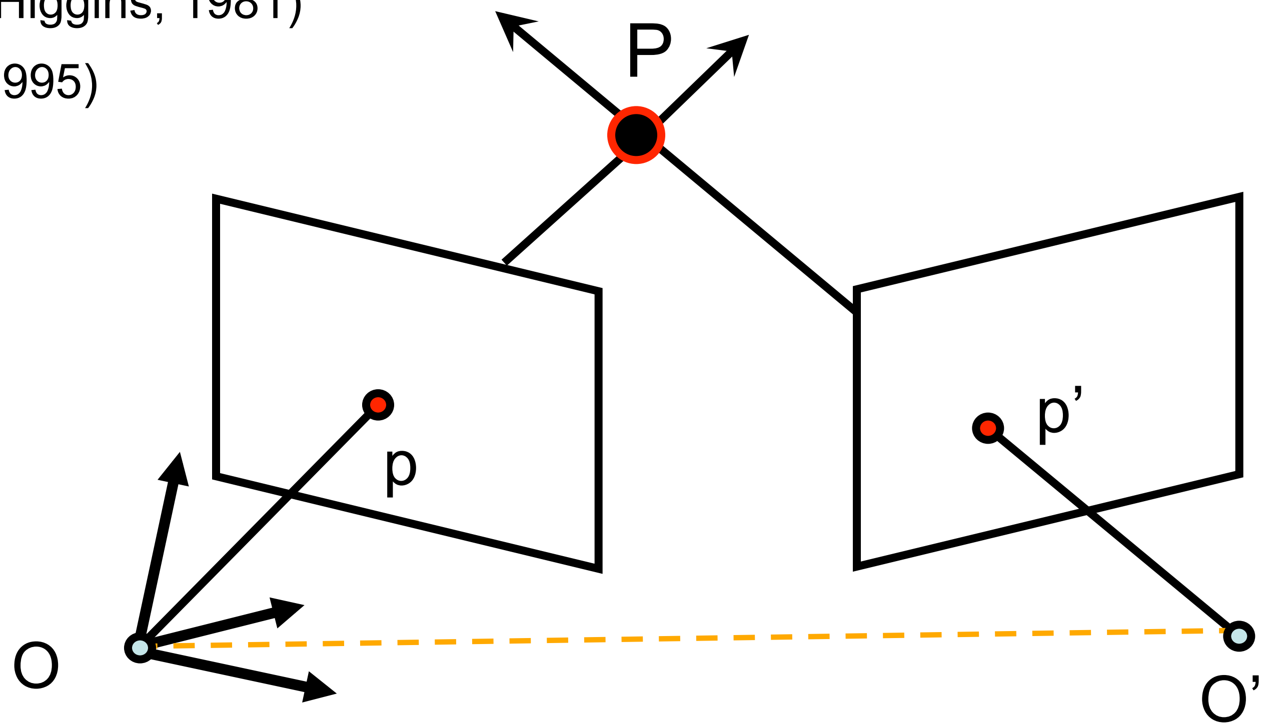
- F captures information about the epipolar geometry of 2 views + camera parameters
- MORE IMPORTANTLY: F gives constraints on how the scene changes under view point transformation (without reconstructing the scene!)
- Powerful tool in:
    - 3D reconstruction
    - Multi-view object/scene matching

Estimating F

The Eight-Point Algorithm

(Longuet-Higgins, 1981)

(Hartley, 1995)



$$P \rightarrow p = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \qquad P \rightarrow p' = \begin{bmatrix} u' \\ v' \\ 1 \end{bmatrix} \qquad p'^{T} F\, p = 0$$

Estimating F

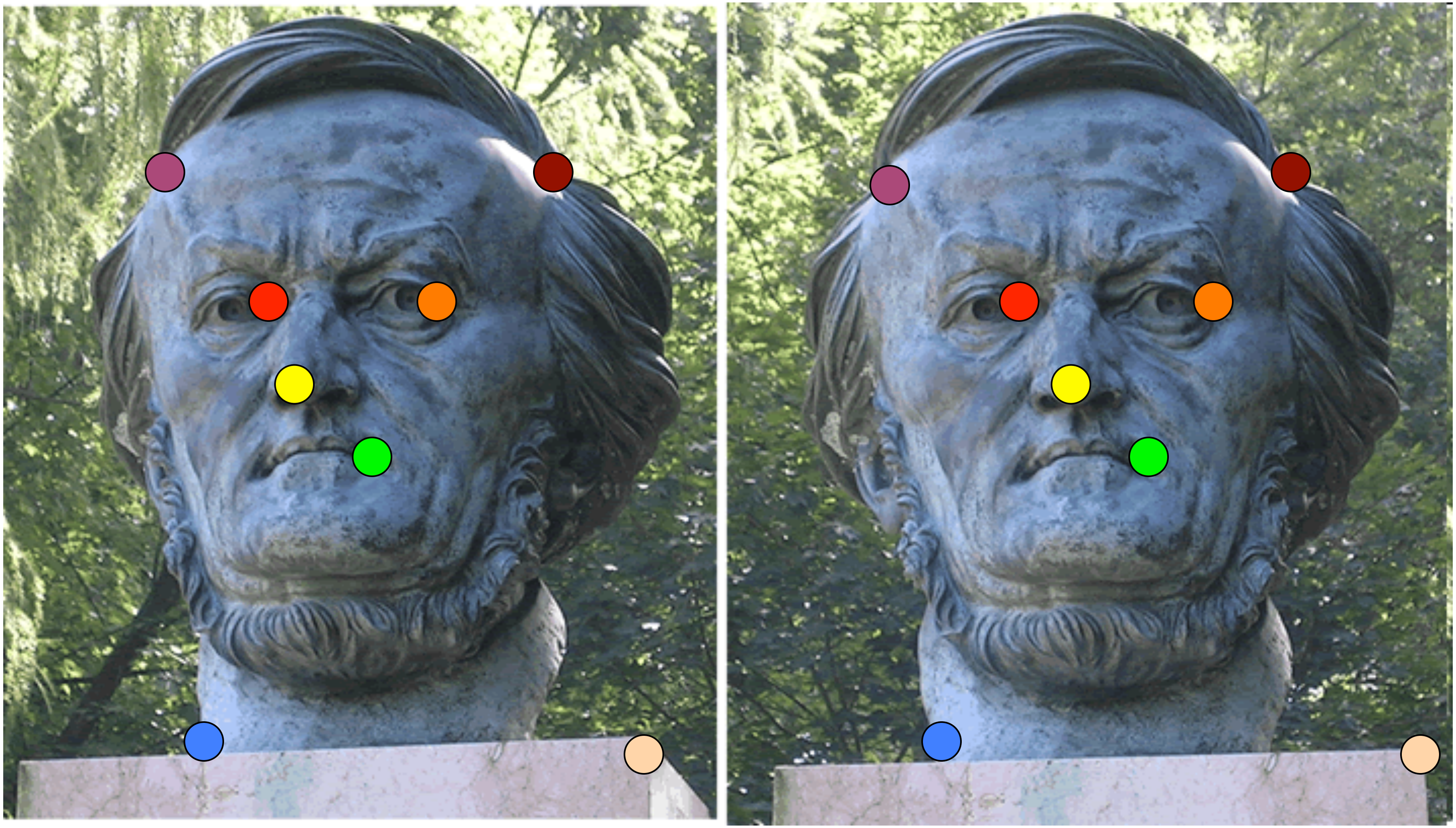
$$p'^{T} Fp = 0 \quad \Rightarrow$$

$$(u, v, 1) \begin{pmatrix} F_{11} & F_{12} & F_{13} \\ F_{21} & F_{22} & F_{23} \\ F_{31} & F_{32} & F_{33} \end{pmatrix} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0$$

$$\Rightarrow \quad (uu', uv', u, vu', vv', v, u', v', 1) \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix} = 0$$

Let's take 8 corresponding points

Estimating F



Estimating F

$$\underbrace{\begin{pmatrix} u_1u_1' & u_1v_1' & u_1 & v_1u_1' & v_1v_1' & v_1 & u_1' & v_1' & 1 \\ u_2u_2' & u_2v_2' & u_2 & v_2u_2' & v_2v_2' & v_2 & u_2' & v_2' & 1 \\ u_3u_3' & u_3v_3' & u_3 & v_3u_3' & v_3v_3' & v_3 & u_3' & v_3' & 1 \\ u_4u_4' & u_4v_4' & u_4 & v_4u_4' & v_4v_4' & v_4 & u_4' & v_4' & 1 \\ u_5u_5' & u_5v_5' & u_5 & v_5u_5' & v_5v_5' & v_5 & u_5' & v_5' & 1 \\ u_6u_6' & u_6v_6' & u_6 & v_6u_6' & v_6v_6' & v_6 & u_6' & v_6' & 1 \\ u_7u_7' & u_7v_7' & u_7 & v_7u_7' & v_7v_7' & v_7 & u_7' & v_7' & 1 \\ u_8u_8' & u_8v_8' & u_8 & v_8u_8' & v_8v_8' & v_8 & u_8' & v_8' & 1 \end{pmatrix}}_{\mathbf{W}} \underbrace{\begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix}}_{\mathbf{f}} = 0$$

- Homogeneous system $\mathbf{W}\,\mathbf{f} = 0$
- Rank 8 $\longrightarrow$ A non-zero solution exists (unique)
- If N>8 $\longrightarrow$ Lsq. solution by SVD! $\longrightarrow \hat{\mathrm{F}}$

$\|\mathbf{f}\| = 1$

$\hat{F}$ satisfies: $p'^T \hat{F} p = 0$

and estimated $\hat{F}$ may have full rank ($\det(\hat{F}) \neq 0$)

But remember: fundamental matrix is Rank2

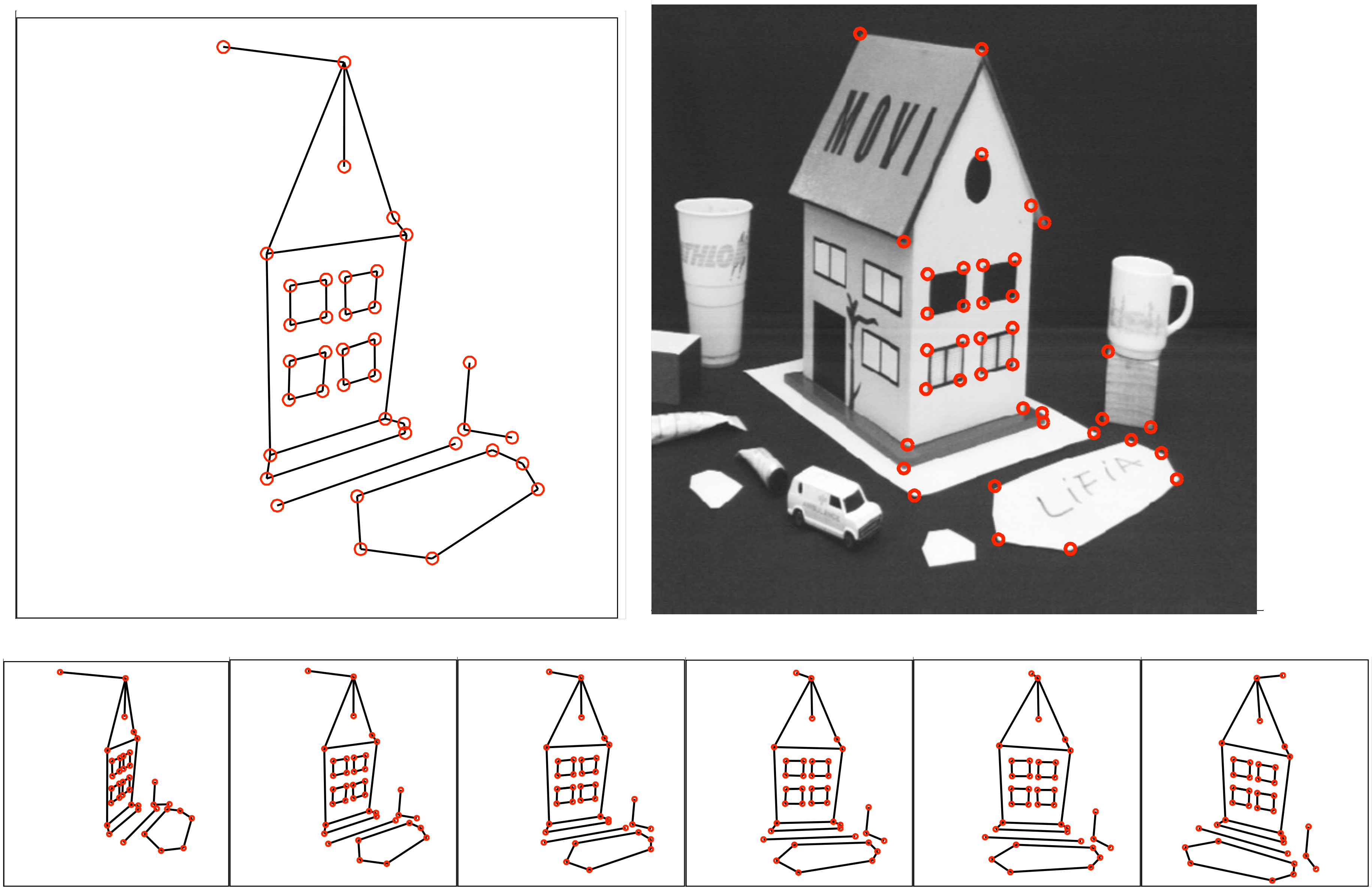
Find F that minimizes $\left\| \mathrm{F} - \hat{\mathrm{F}} \right\| = 0$

Frobenius norm (*)

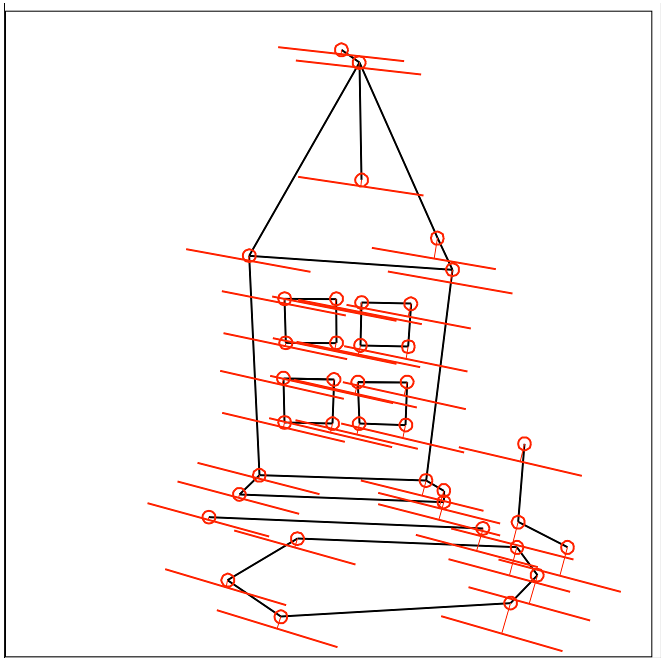
Subject to det(F)=0

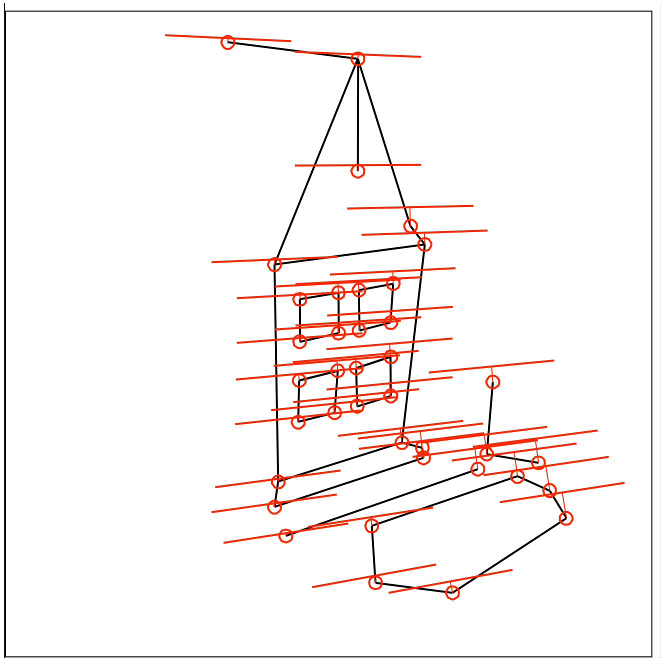
SVD (again!) can be used to solve this problem

(*) Sqrt root of the sum of squares of all entries



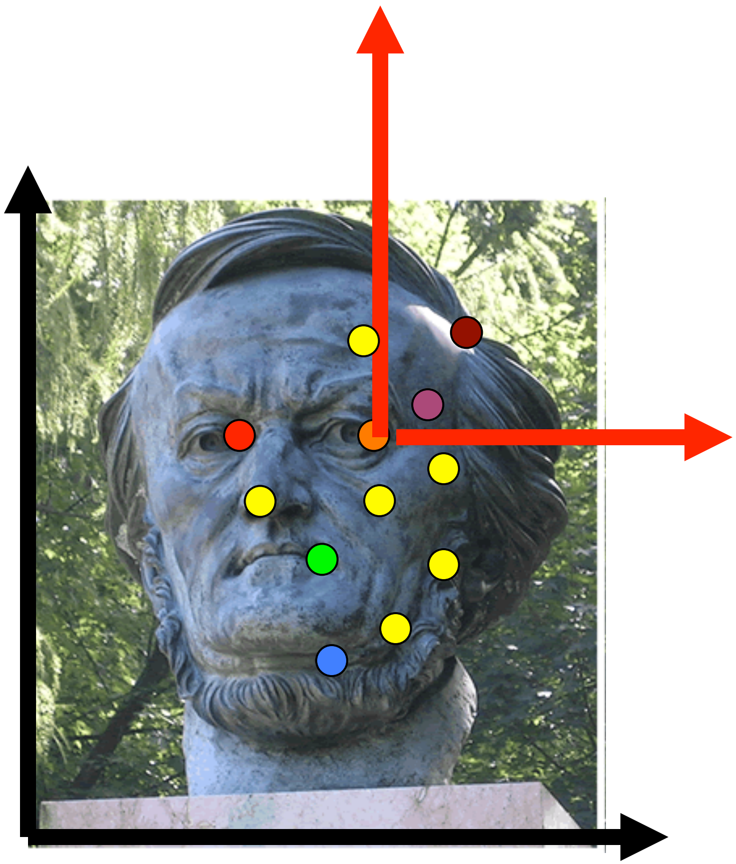
Data courtesy of R. Mohr and B. Boufama.





Mean errors:
10.0pixel
9.1pixel

Problems with the 8-Point Algorithm



$$\mathbf{W}\,\mathbf{f} = 0, \quad \|\mathbf{f}\| = 1 \xrightarrow{\text{Lsq solution by SVD}} \mathrm{F}$$

- Recall the structure of W:
  - do we see any potential (numerical) issue?

Problems with the 8-Point Algorithm

$$\mathbf{W}\,\mathbf{f} = 0$$

$$\mathbf{W}\begin{pmatrix} u_1u'_1 & u_1v'_1 & u_1 & v_1u'_1 & v_1v'_1 & v_1 & u'_1 & v'_1 & 1 \\ u_2u'_2 & u_2v'_2 & u_2 & v_2u'_2 & v_2v'_2 & v_2 & u'_2 & v'_2 & 1 \\ u_3u'_3 & u_3v'_3 & u_3 & v_3u'_3 & v_3v'_3 & v_3 & u'_3 & v'_3 & 1 \\ u_4u'_4 & u_4v'_4 & u_4 & v_4u'_4 & v_4v'_4 & v_4 & u'_4 & v'_4 & 1 \\ u_5u'_5 & u_5v'_5 & u_5 & v_5u'_5 & v_5v'_5 & v_5 & u'_5 & v'_5 & 1 \\ u_6u'_6 & u_6v'_6 & u_6 & v_6u'_6 & v_6v'_6 & v_6 & u'_6 & v'_6 & 1 \\ u_7u'_7 & u_7v'_7 & u_7 & v_7u'_7 & v_7v'_7 & v_7 & u'_7 & v'_7 & 1 \\ u_8u'_8 & u_8v'_8 & u_8 & v_8u'_8 & v_8v'_8 & v_8 & u'_8 & v'_8 & 1 \end{pmatrix} \begin{pmatrix} F_{11} \\ F_{12} \\ F_{13} \\ F_{21} \\ F_{22} \\ F_{23} \\ F_{31} \\ F_{32} \\ F_{33} \end{pmatrix}\mathbf{f} = 0$$

- Highly un-balanced (not well conditioned)
- Values of W must have similar magnitude
- This creates problems during the SVD decomposition

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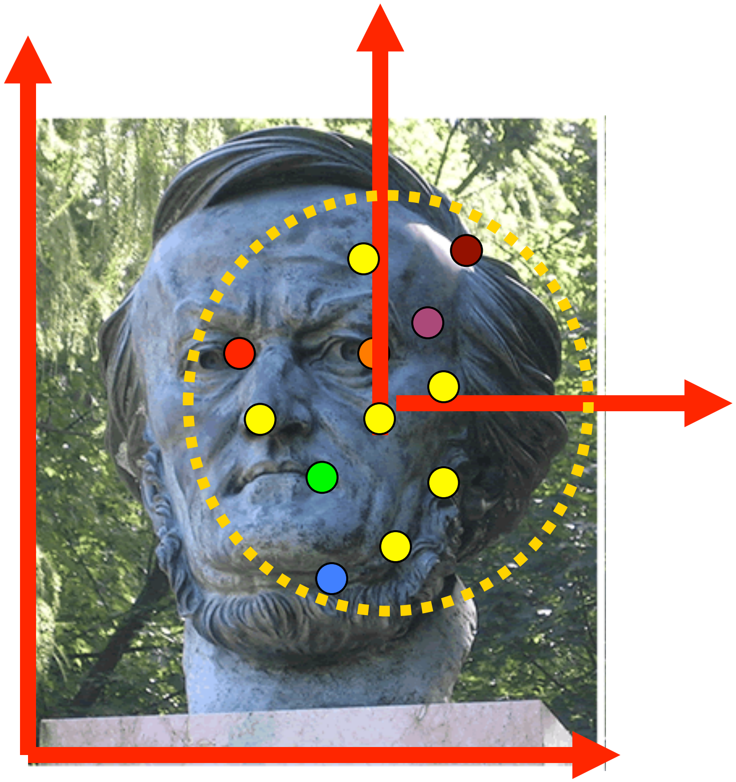
Normalization



IDEA: Transform image coordinate such that the matrix **W** become better conditioned

Apply following transformation T: (translation and scaling)

- Origin = centroid of image points
- Mean square distance of the data points from origin is 2 pixels

$$q_i = T_i \, p_i \qquad q'_i = T'_i \, p'_i \qquad \text{(normalization)}$$

The Normalized Eight-Point Algorithm

0. Compute T_i and T_i'

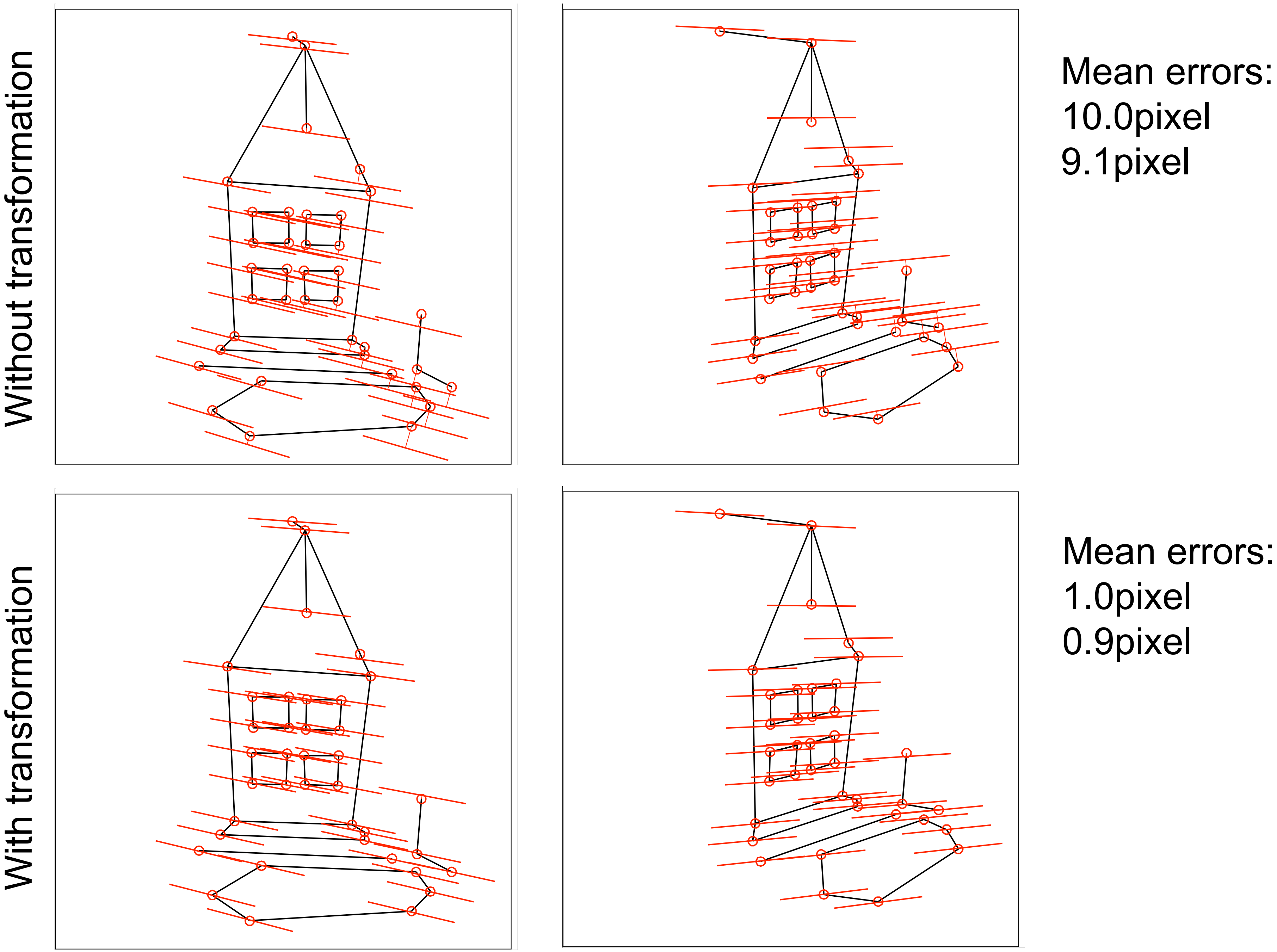
1. Normalize coordinates:

$$q_i = T_i \, p_i \qquad\qquad q_i' = T_i' \, p_i'$$

2. Use the eight-point algorithm to compute F'$_q$ from the points q_i and q' $_i$.

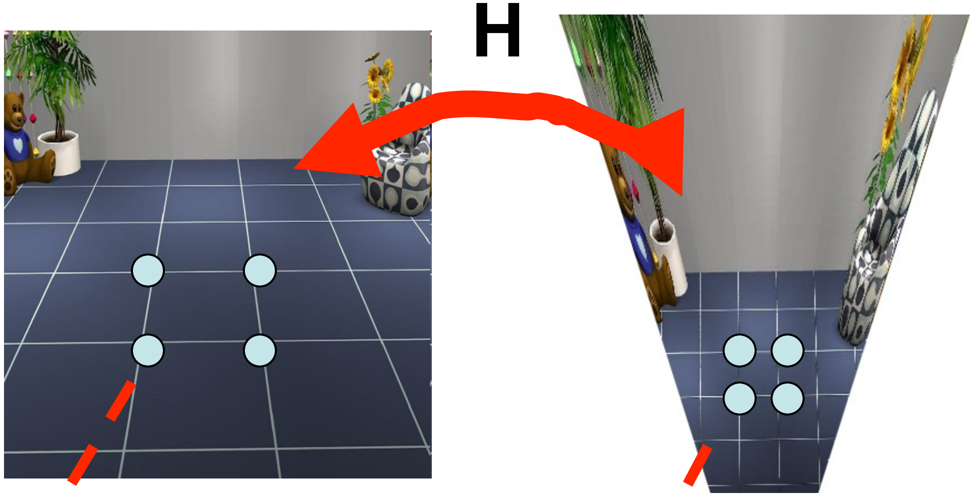
1. Enforce the rank-2 constraint. $\longrightarrow F_q$ $\quad \begin{cases} q'^T \; F_q q = 0 \\ \det(F_q) = 0 \end{cases}$

2. De-normalize F_q: $\quad F = T'^T F_q T$



Without transformation
With transformation
Mean errors:
10.0pixel
9.1pixel
Mean errors:
1.0pixel
0.9pixel

Same issue for the DLT algorithm

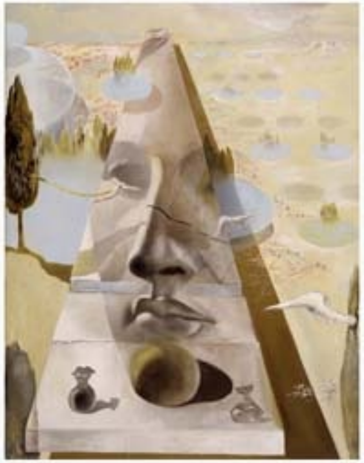


x_i

x_i'

$$\mathrm{x}_i' = \mathrm{H}\,\mathrm{x}_i$$

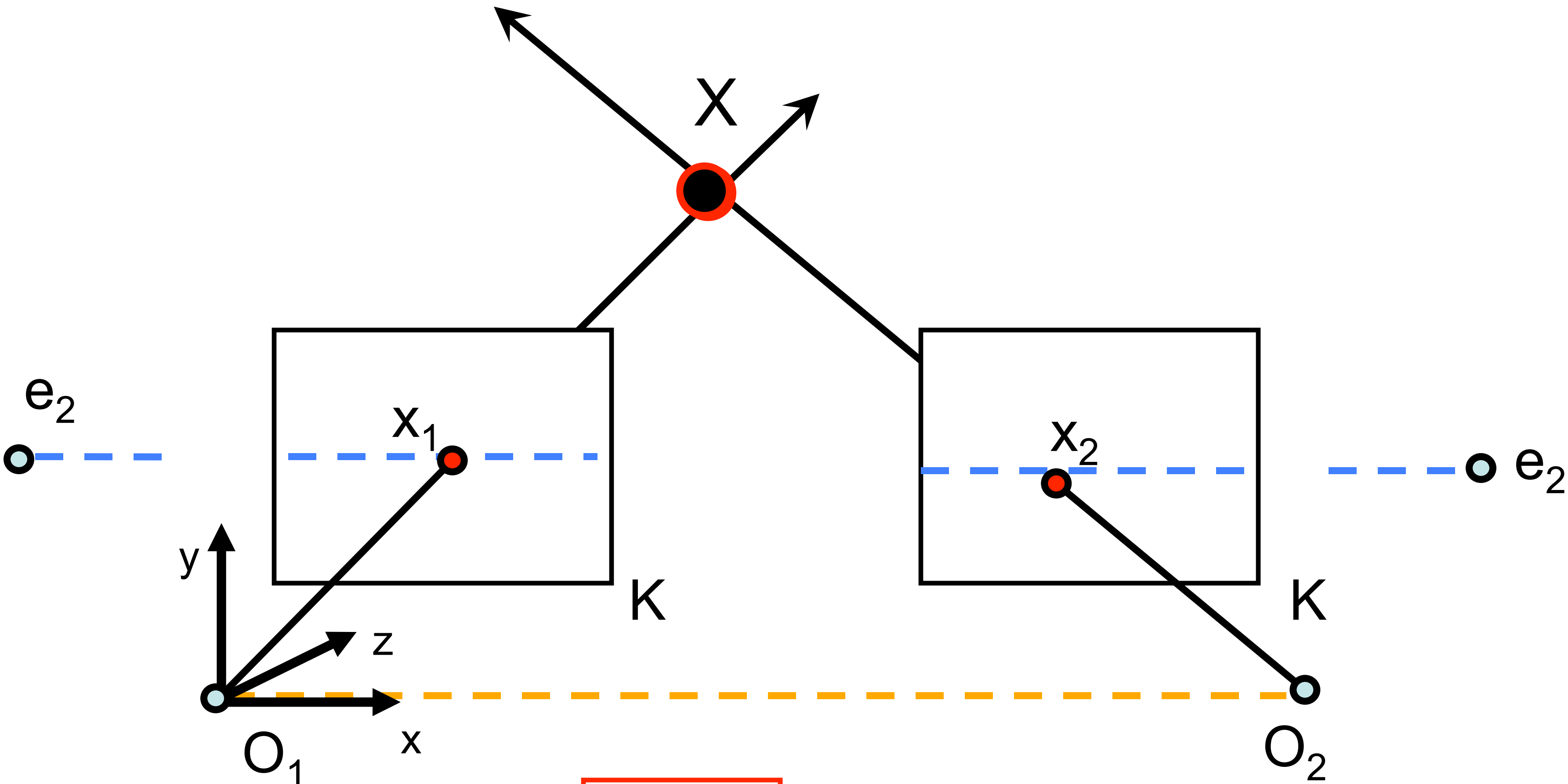
[Section 4.4 in AZ]



Epipolar Geometry

- Why is stereo useful?
- Epipolar constraints
- Essential and fundamental matrix
- Estimating F
- Examples: parallel planes

Example: Parallel image planes



$$\boxed{E=?}$$

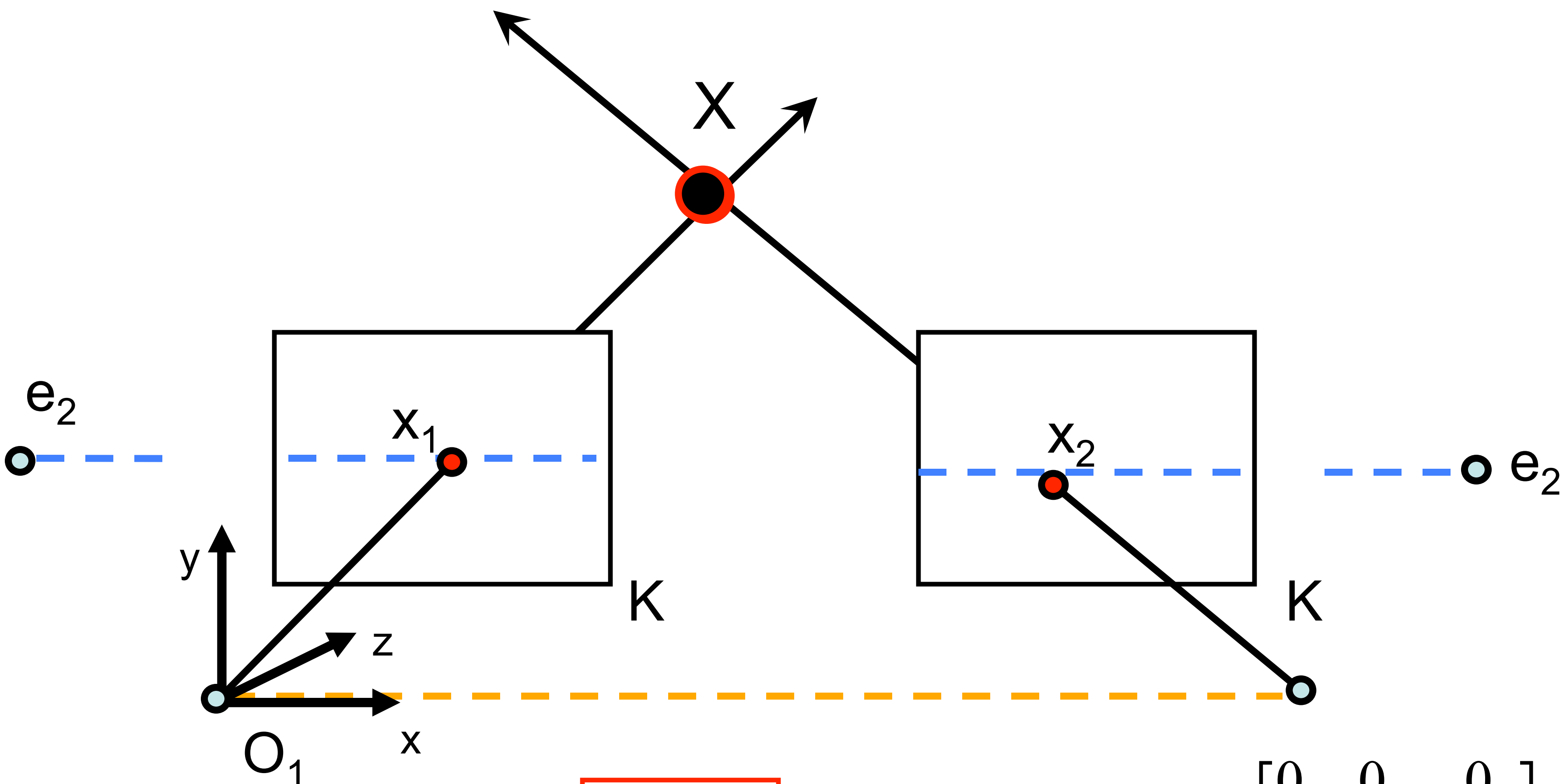
$K_1=K_2$ = known

x parallel to O_1O_2

Hint :

$R = I \qquad t = (T, 0, 0)$

Example: Parallel image planes



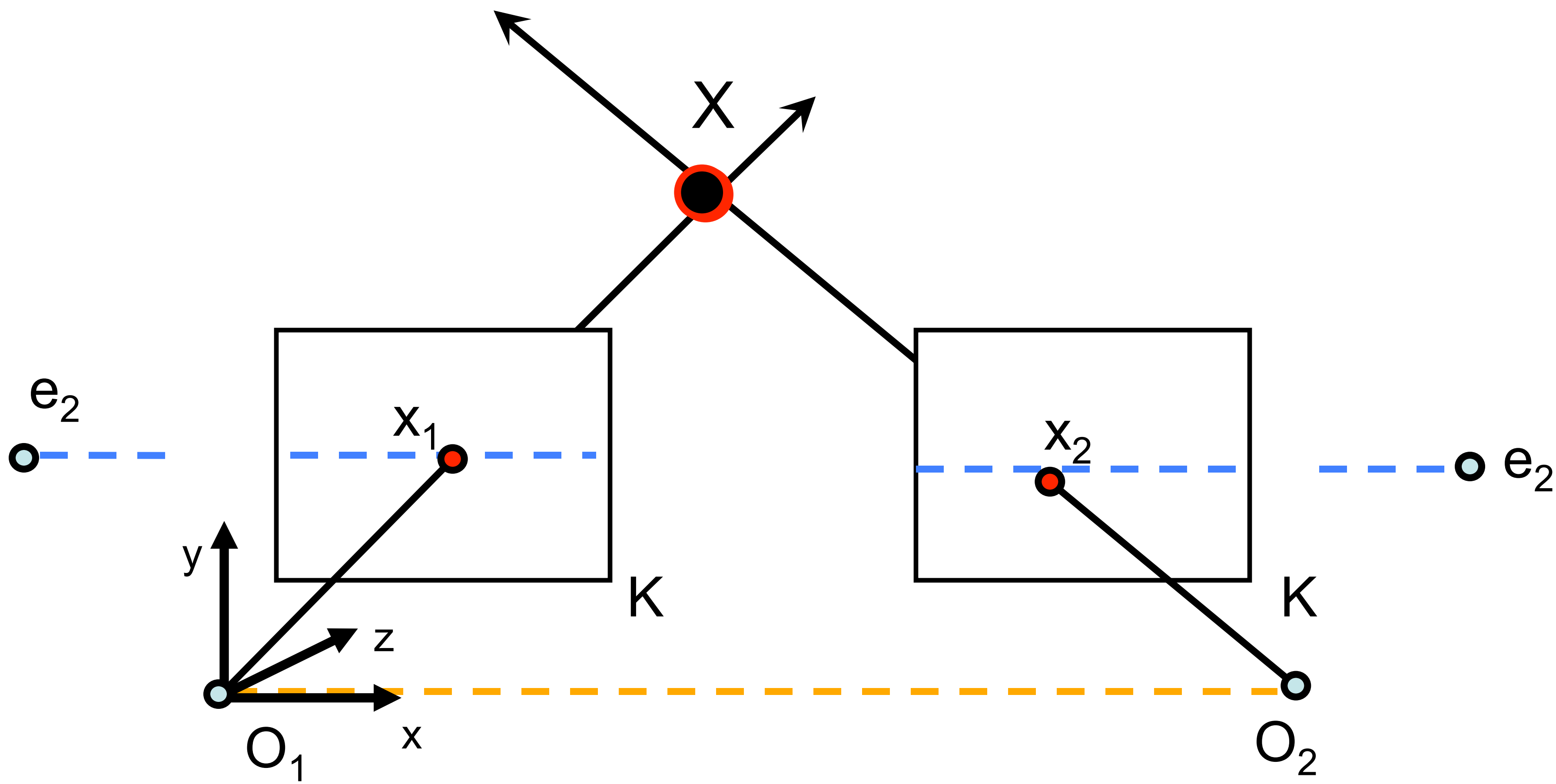
$\boxed{E=?}$

$$E = [t_\times]R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix}$$

$K_1 = K_2$ = known

x parallel to O_1O_2

Example: Parallel image planes



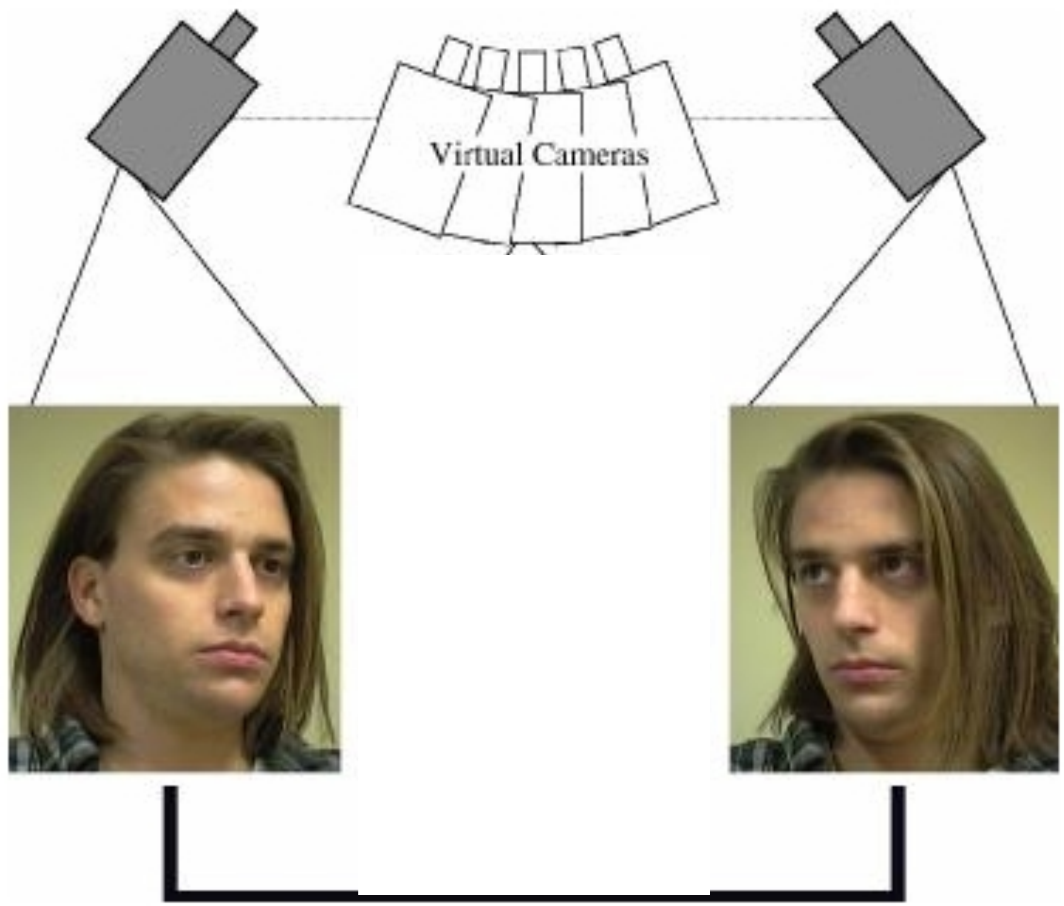
Rectification: making two images "parallel"

Why it is useful?

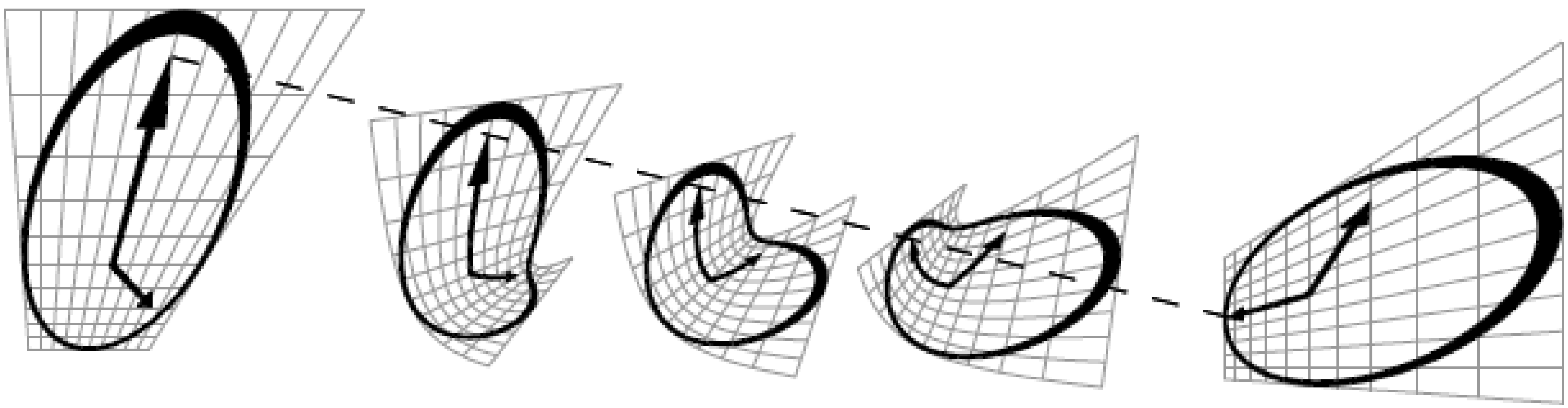
- Epipolar constraint → y = y'
- New views can be synthesized by linear interpolation

Application: view morphing

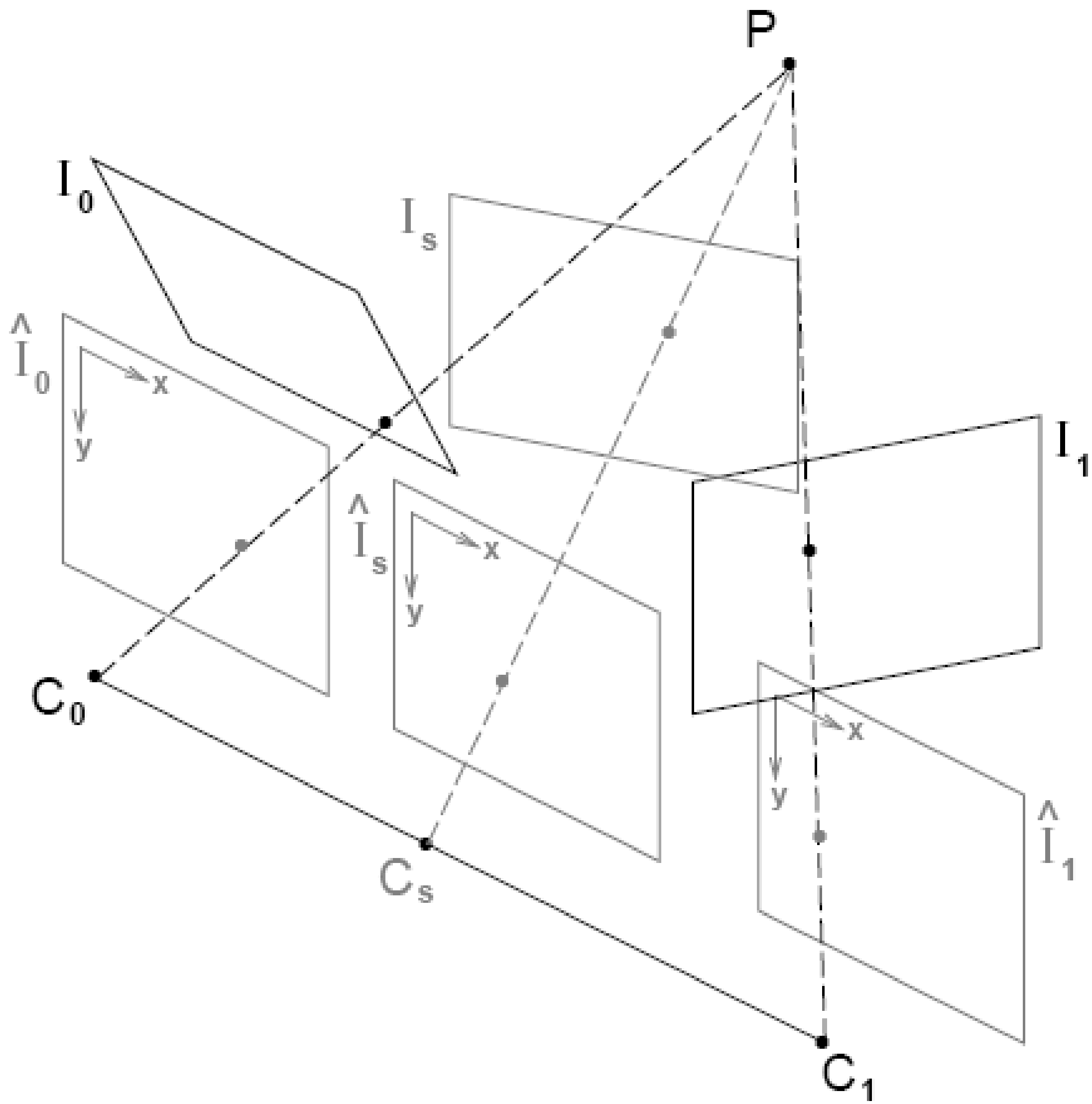
S. M. Seitz and C. R. Dyer, *Proc. SIGGRAPH 96*, 1996, 21-30



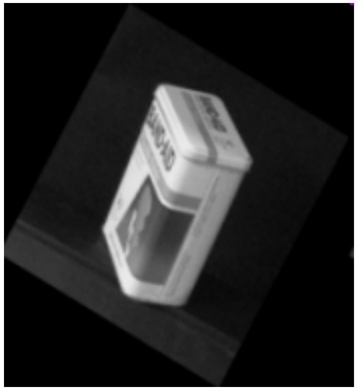
Morphing without using geometry

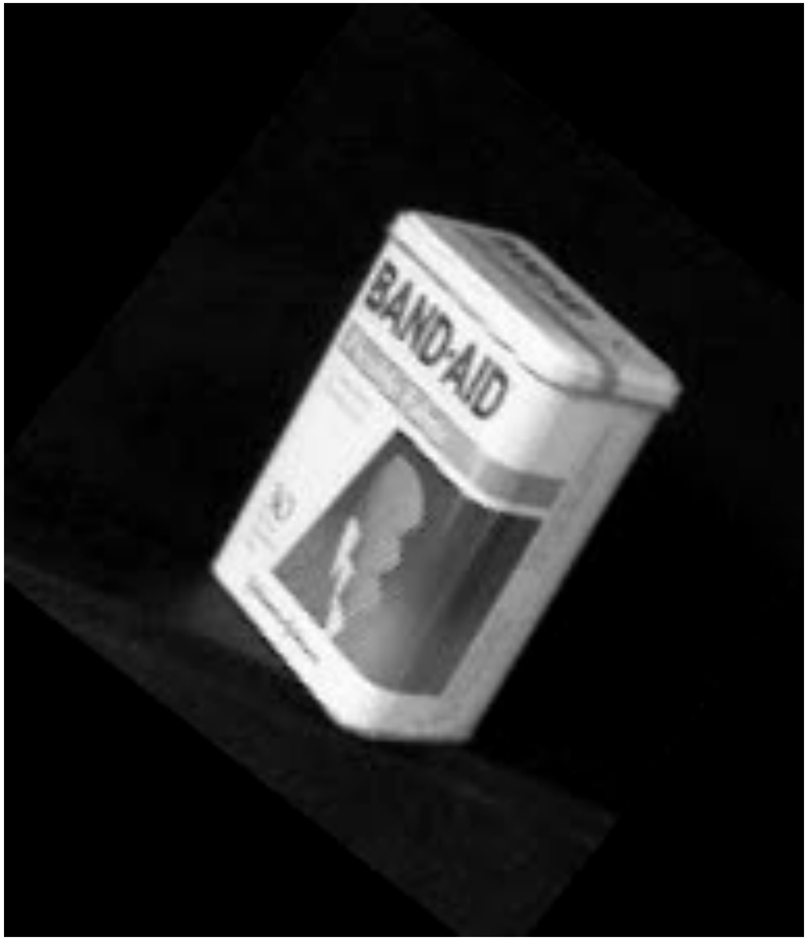


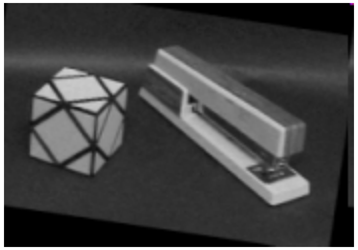
Rectification

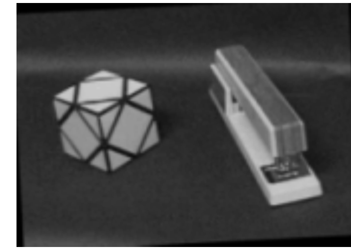


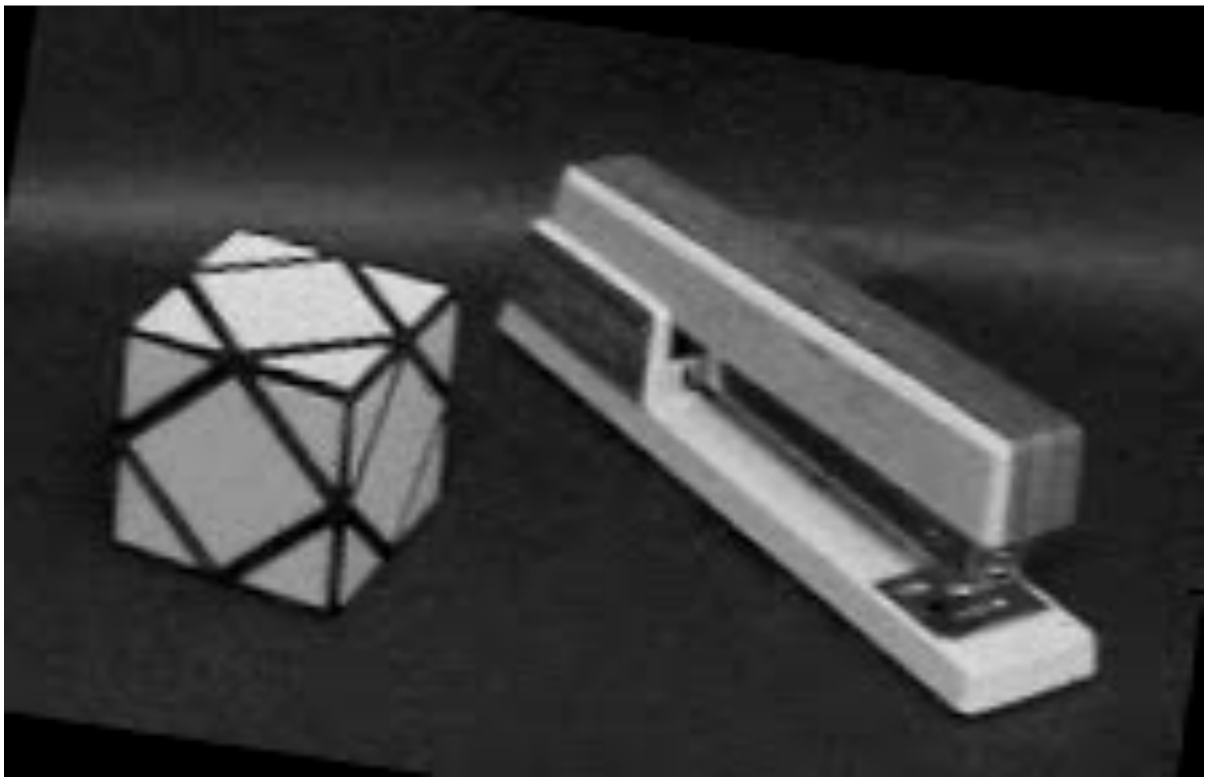

BAND-AID

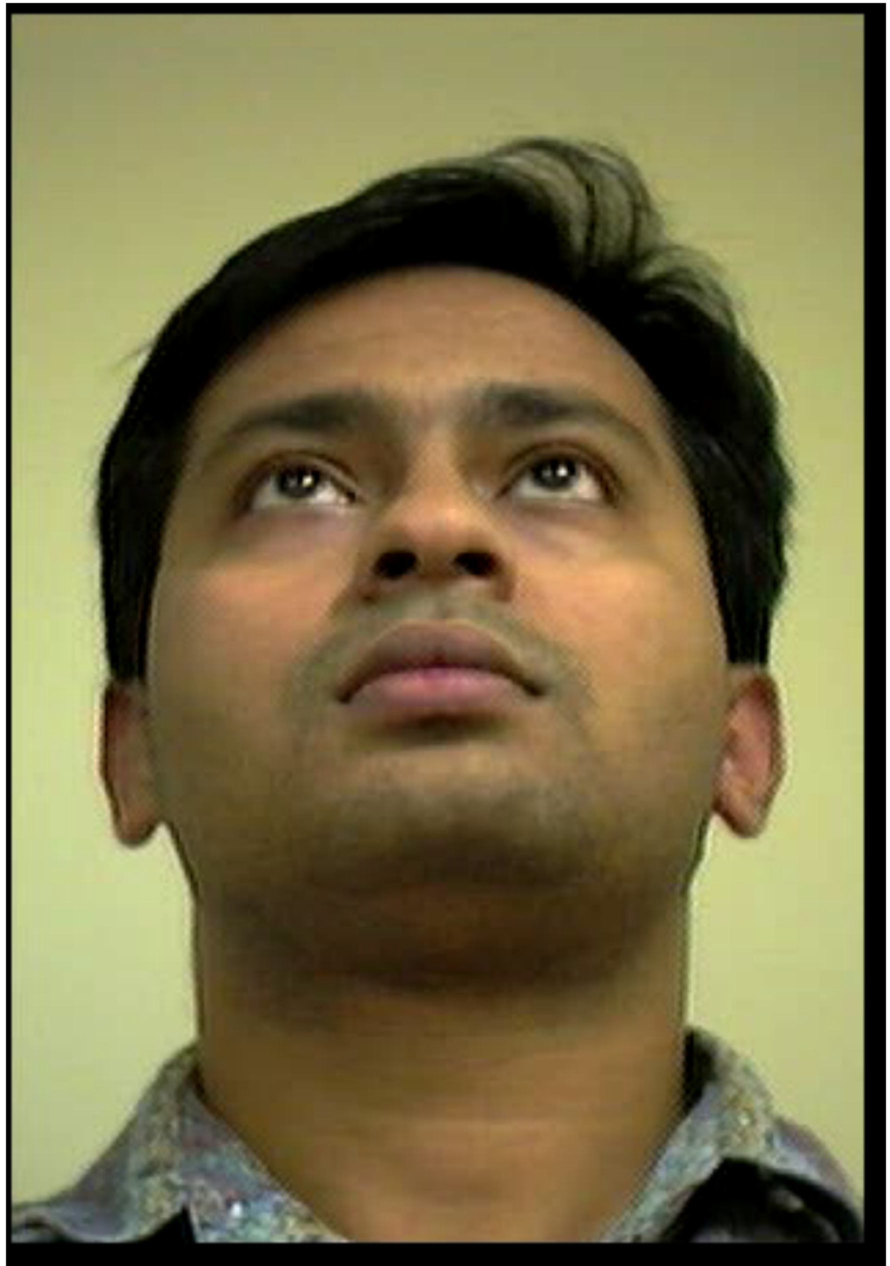



BAND-AID

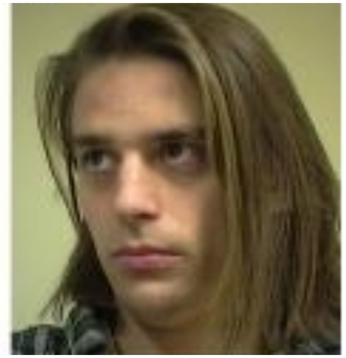
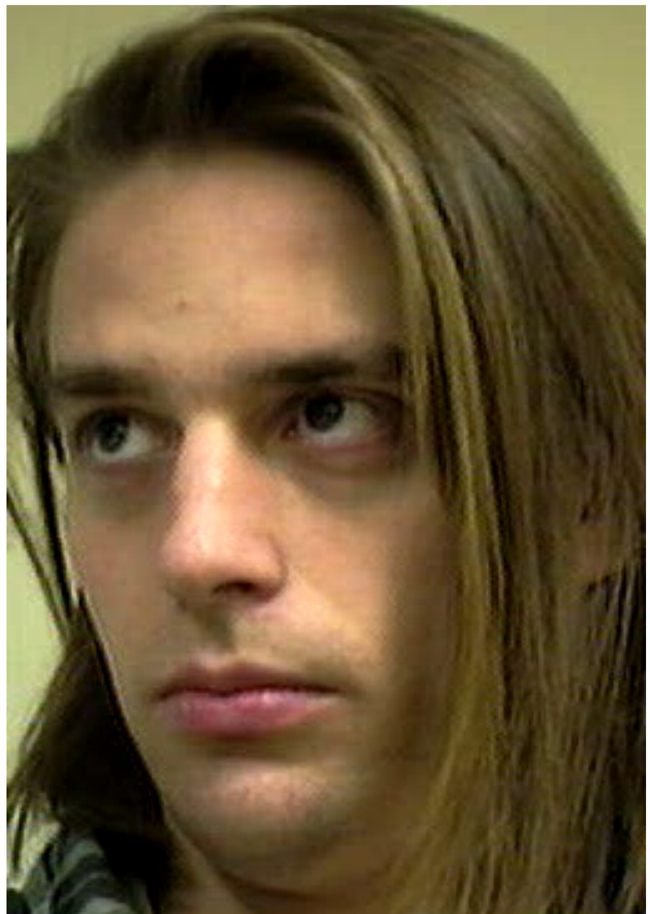


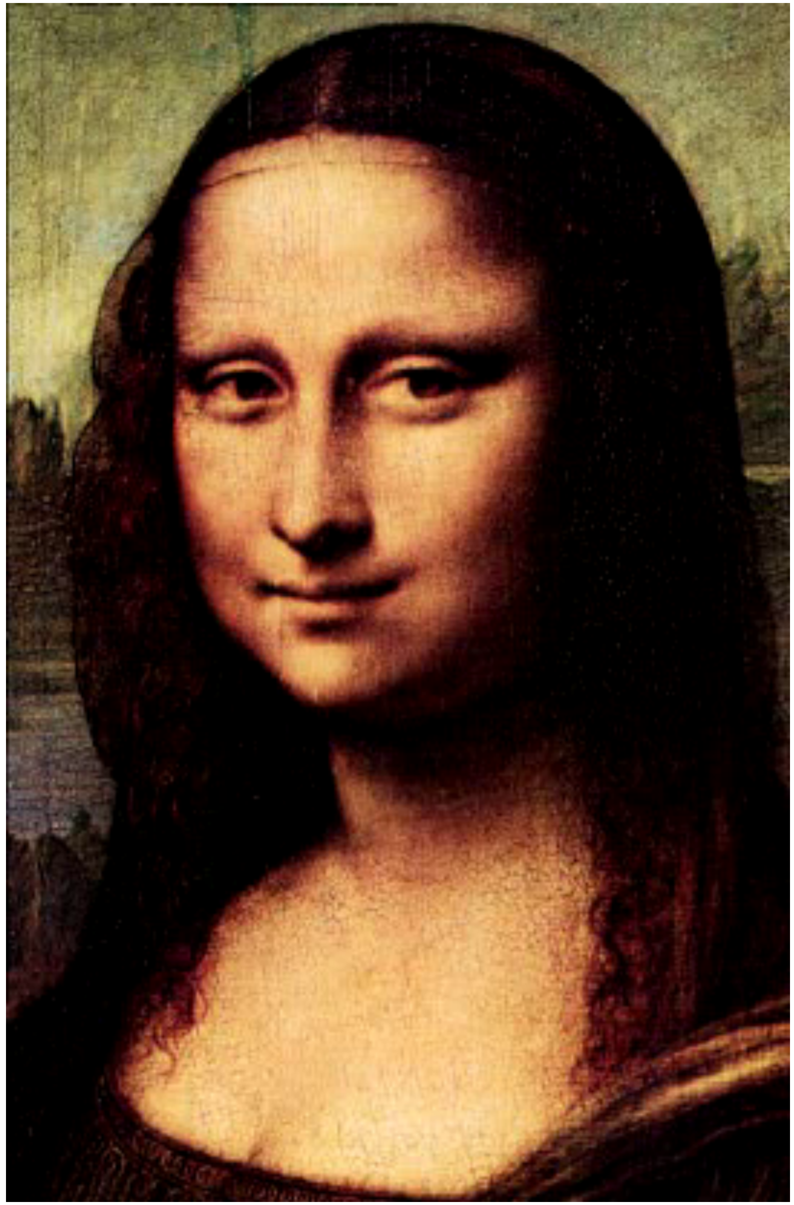








From its reflection!

The Fundamental Matrix Song

http://dar fmatrix/