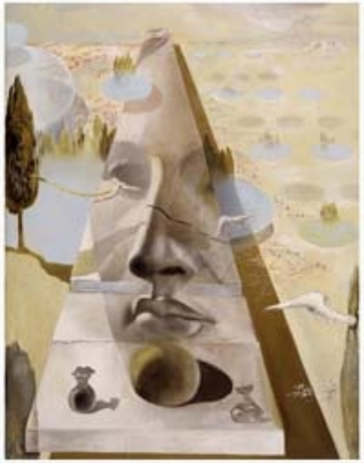


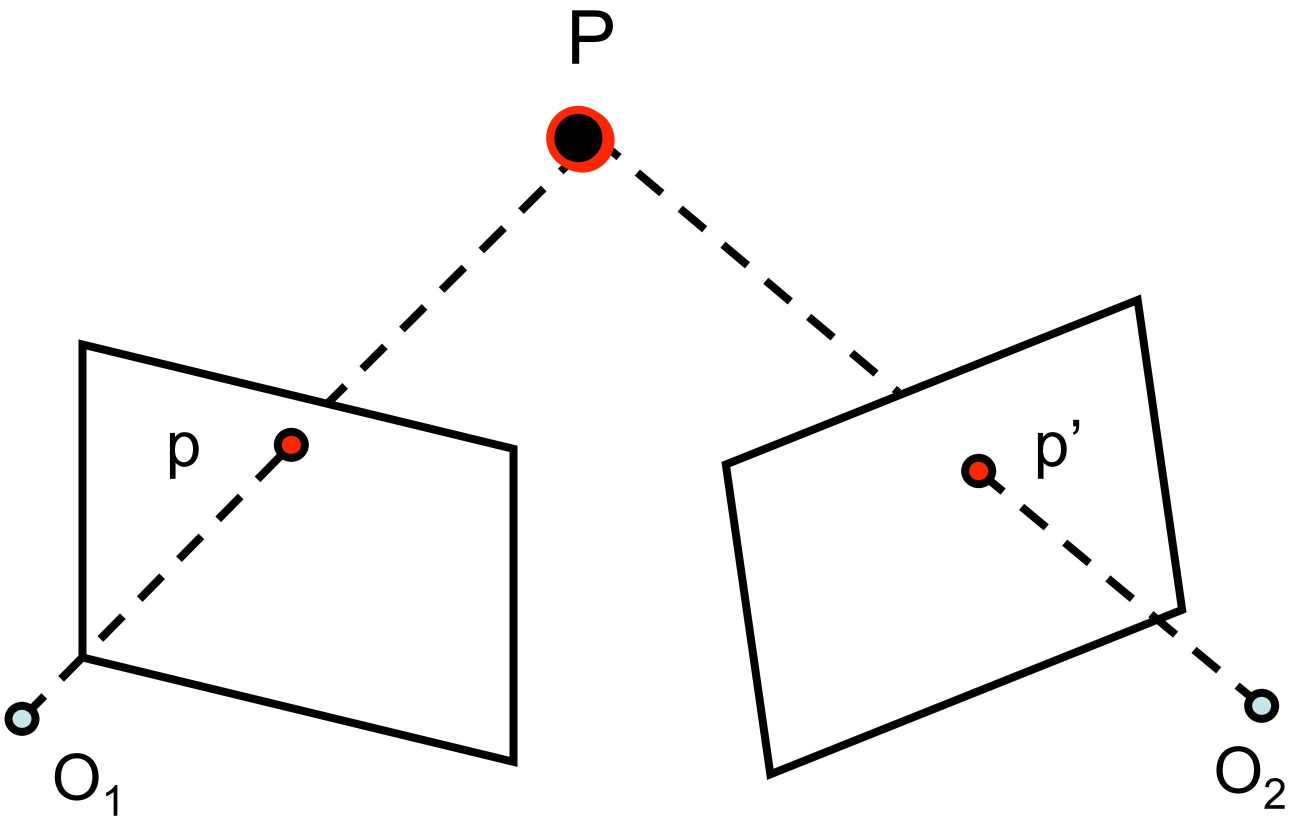
EECS 442 – Computer vision

Stereo systems

- Stereo vision
- Rectification
- Correspondence problem
- Active stereo vision systems

Reading: [HZ] Chapter: 11
[FP] Chapter: 11

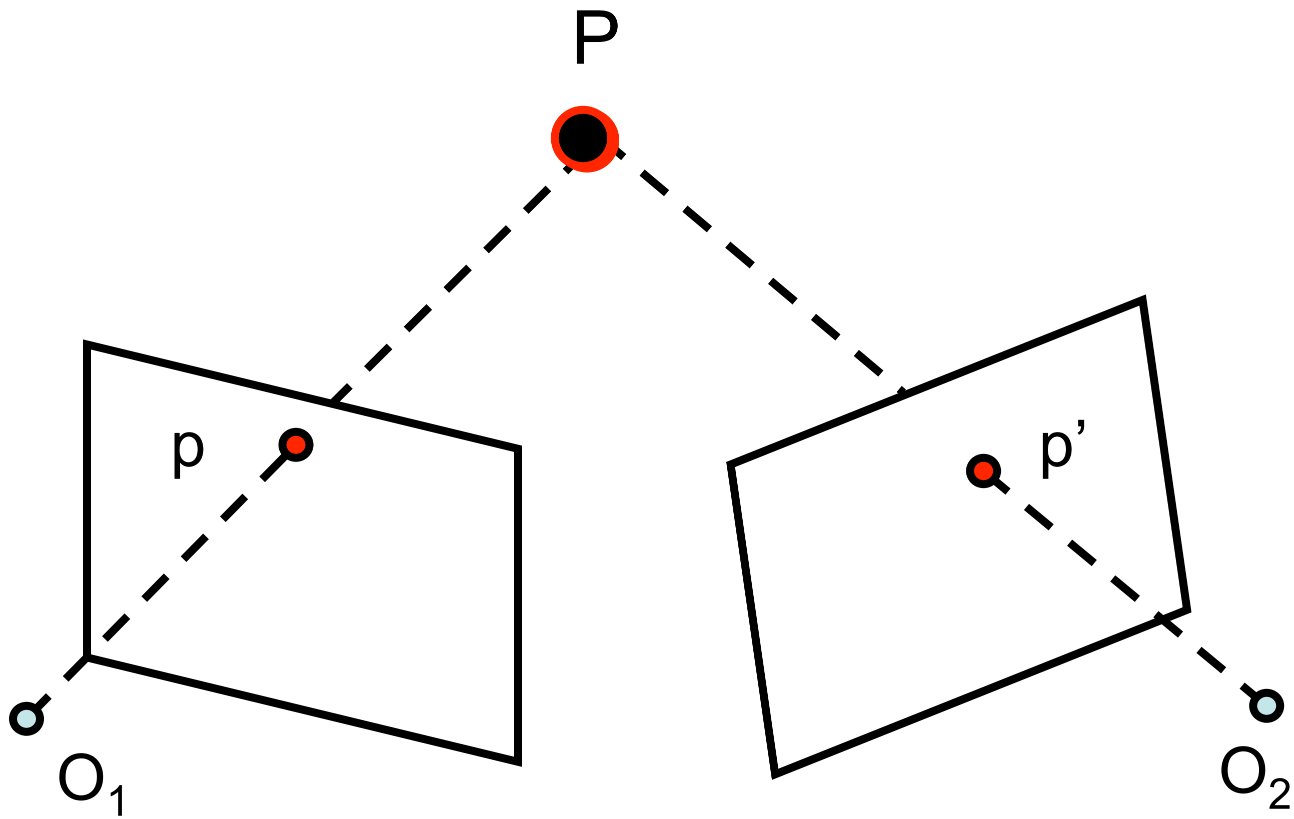
Stereo vision



Goal: estimate the position of P given the observation of P from two view points

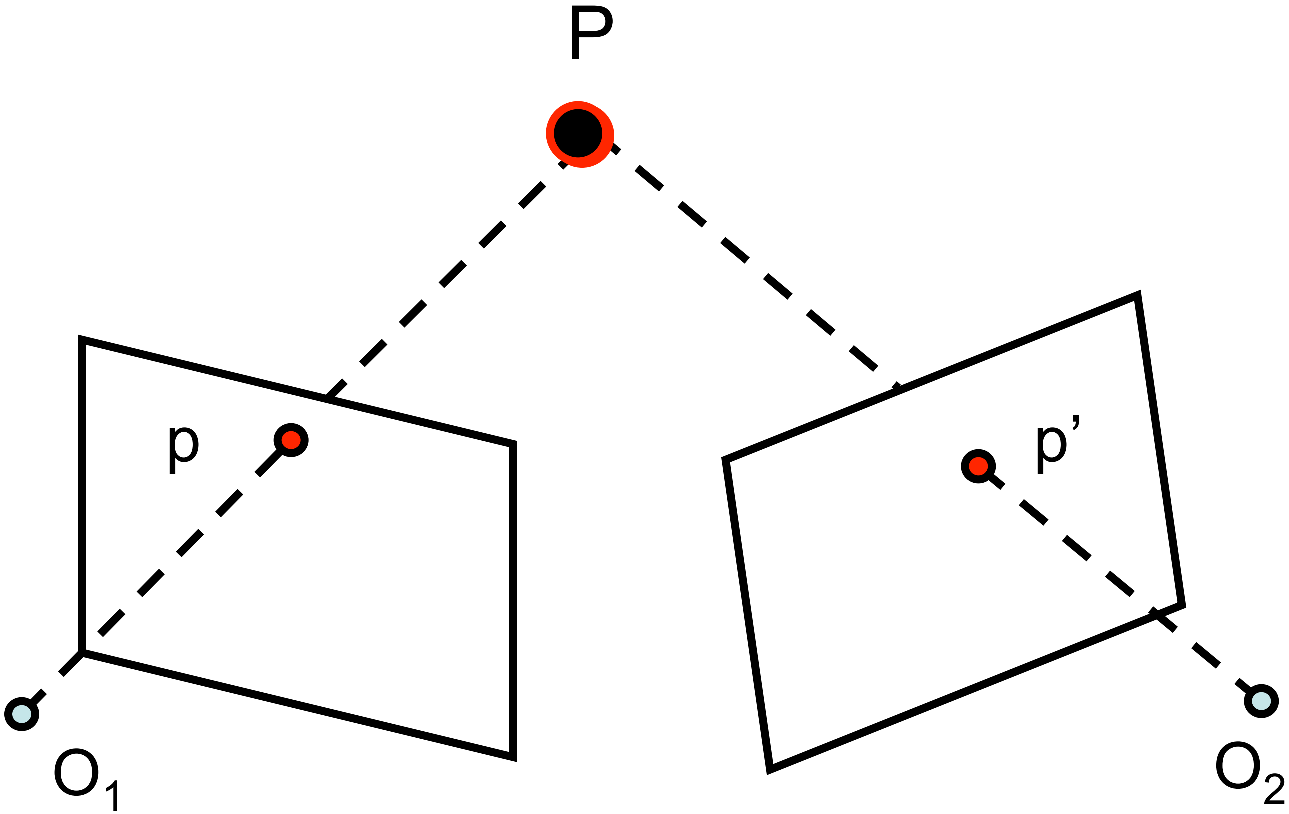
Assumptions: known camera parameters and position (K, R, T)

Stereo vision



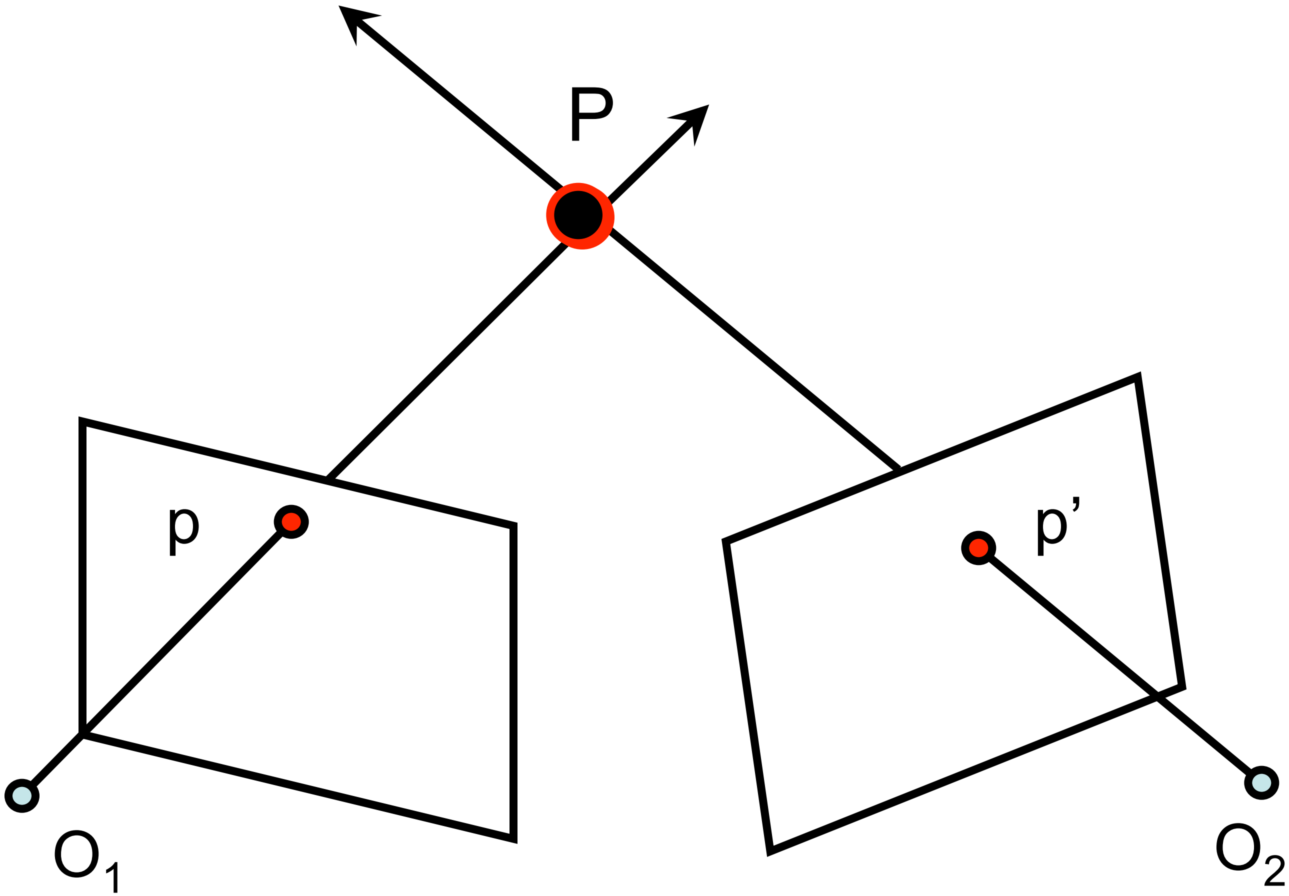
Subgoals:
- Solve the correspondence problem
- Use corresponding observations to triangulate

Correspondence problem



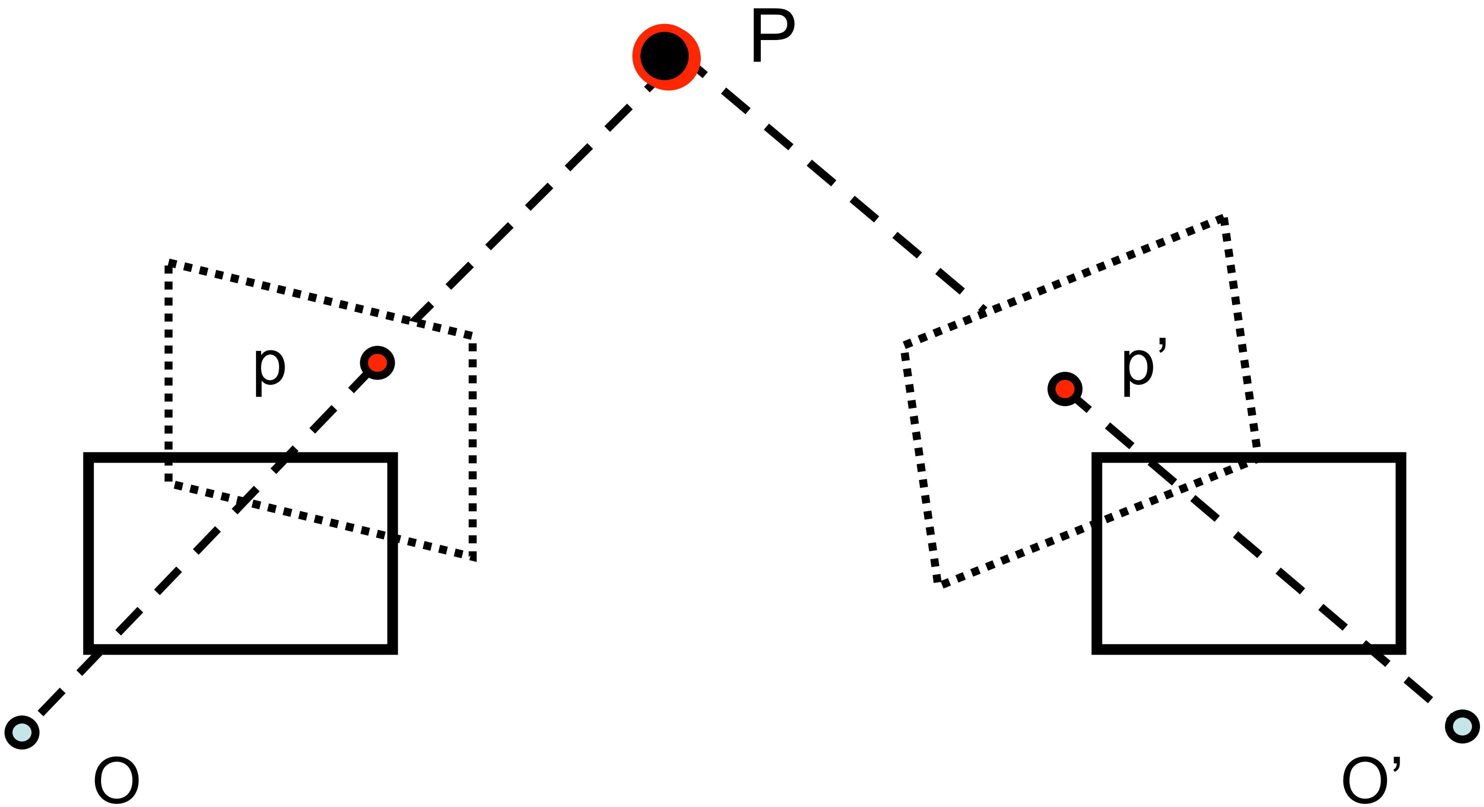
- Given a point in 3d, discover corresponding observations in left and right images

Triangulation



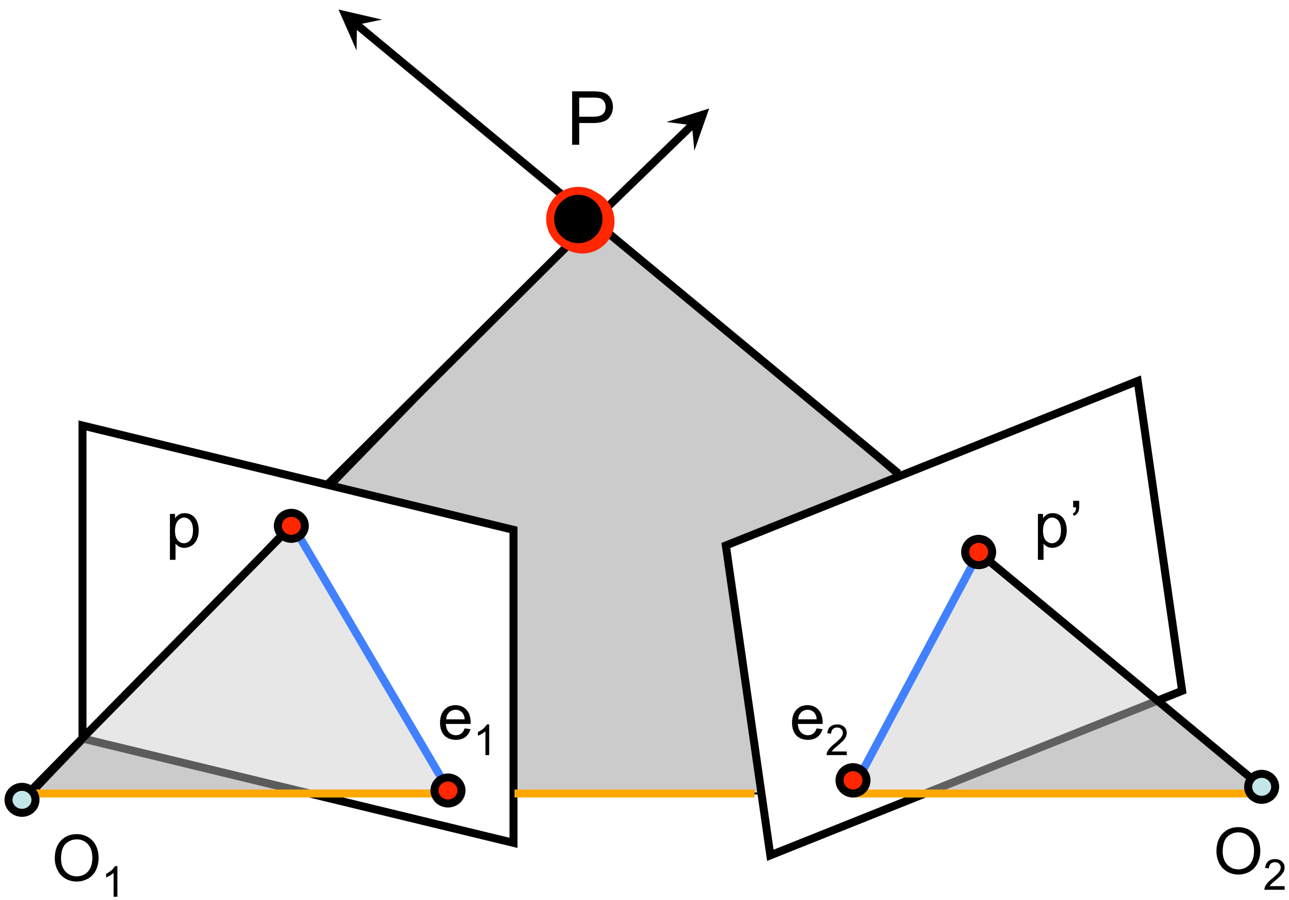
•Intersecting the two lines of sight gives rise to P

Parallel image planes



• When views are **parallel** these two steps becomes much easier!

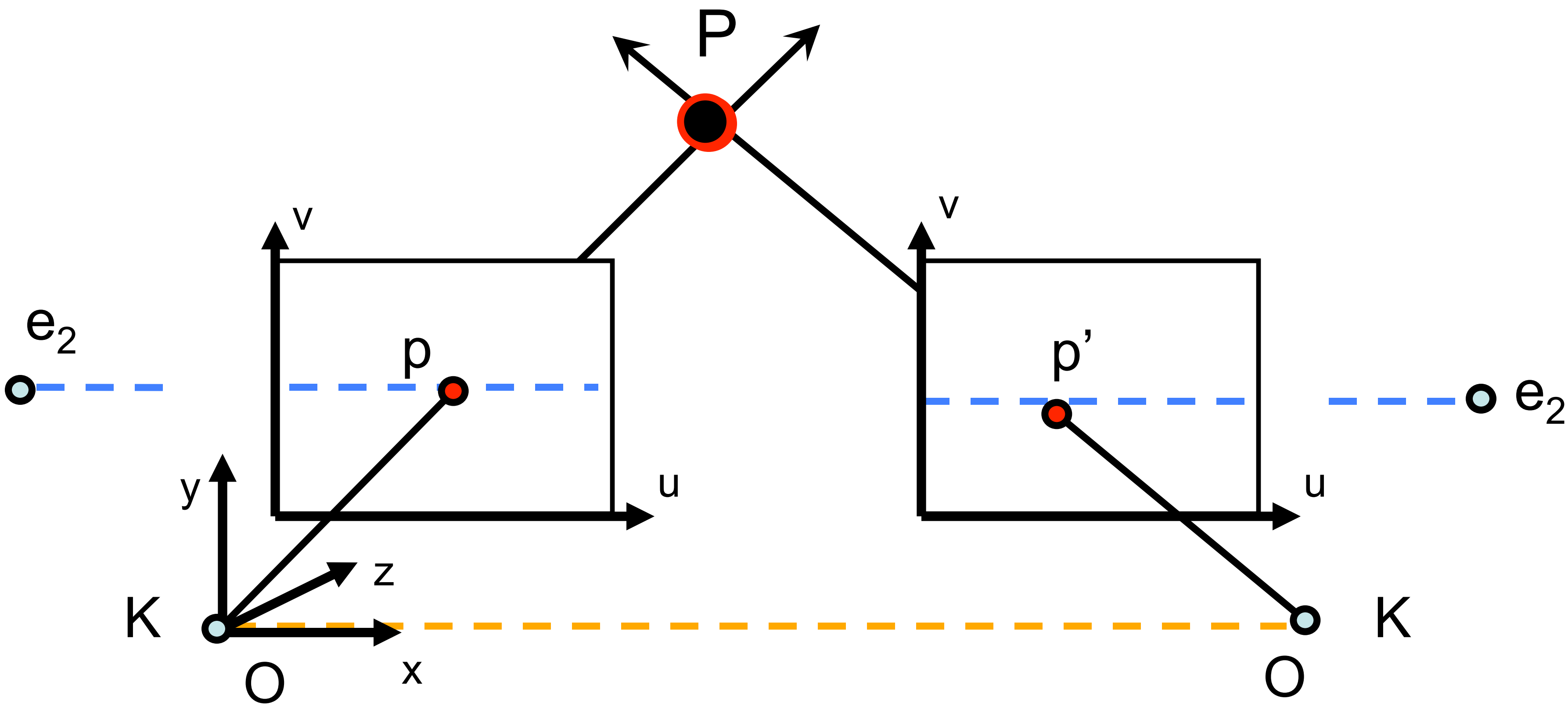
Epipolar geometry



- Epipolar Plane
- Baseline
- Epipolar Lines
- Epipoles e_1, e_2

  = intersections of baseline with image planes
  = projections of the other camera center

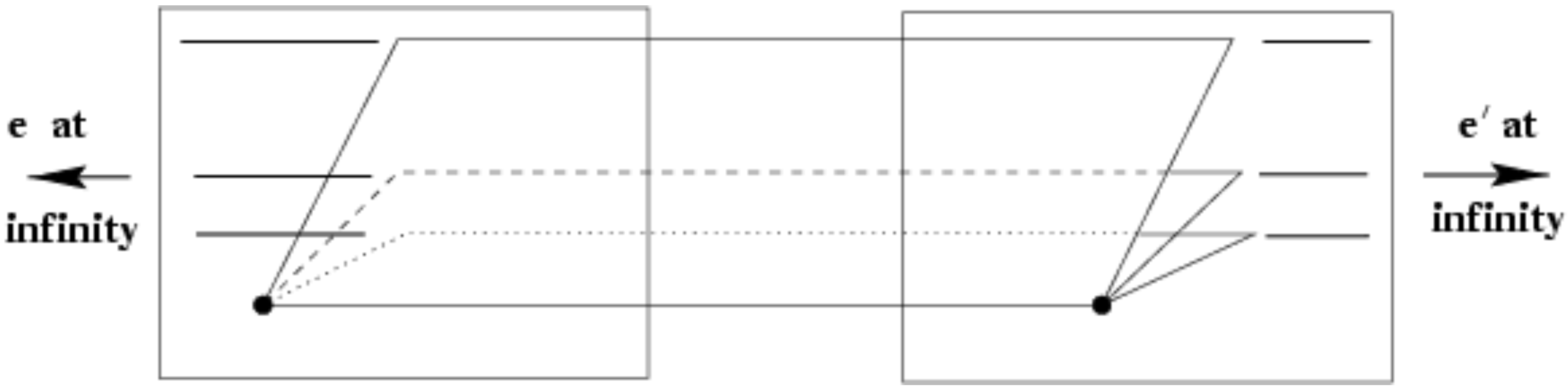
Parallel image planes



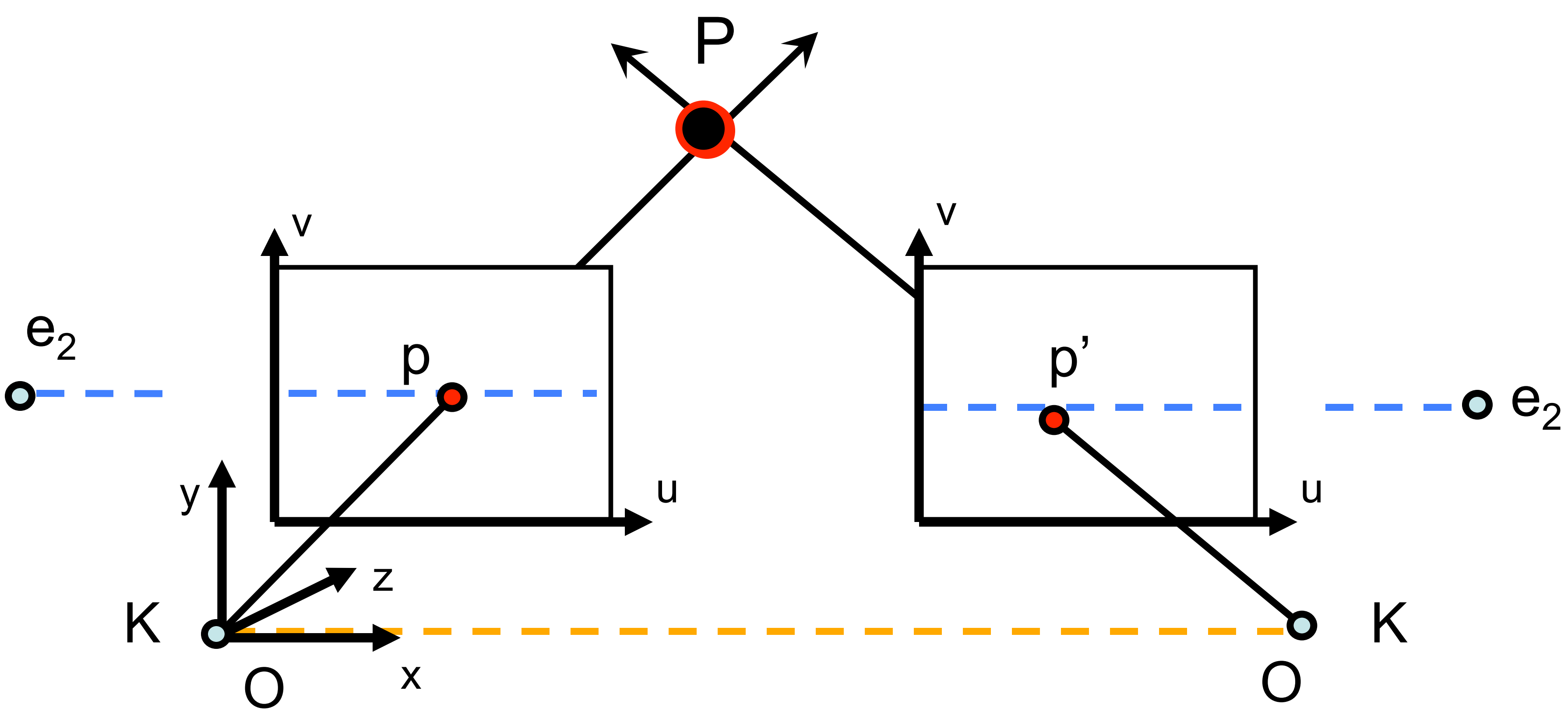
- Parallel epipolar lines
- Epipoles at infinity
- v = v'

Rectification: making two images "parallel"

Example: Parallel image planes



Parallel image planes



K_1=K_2 = known

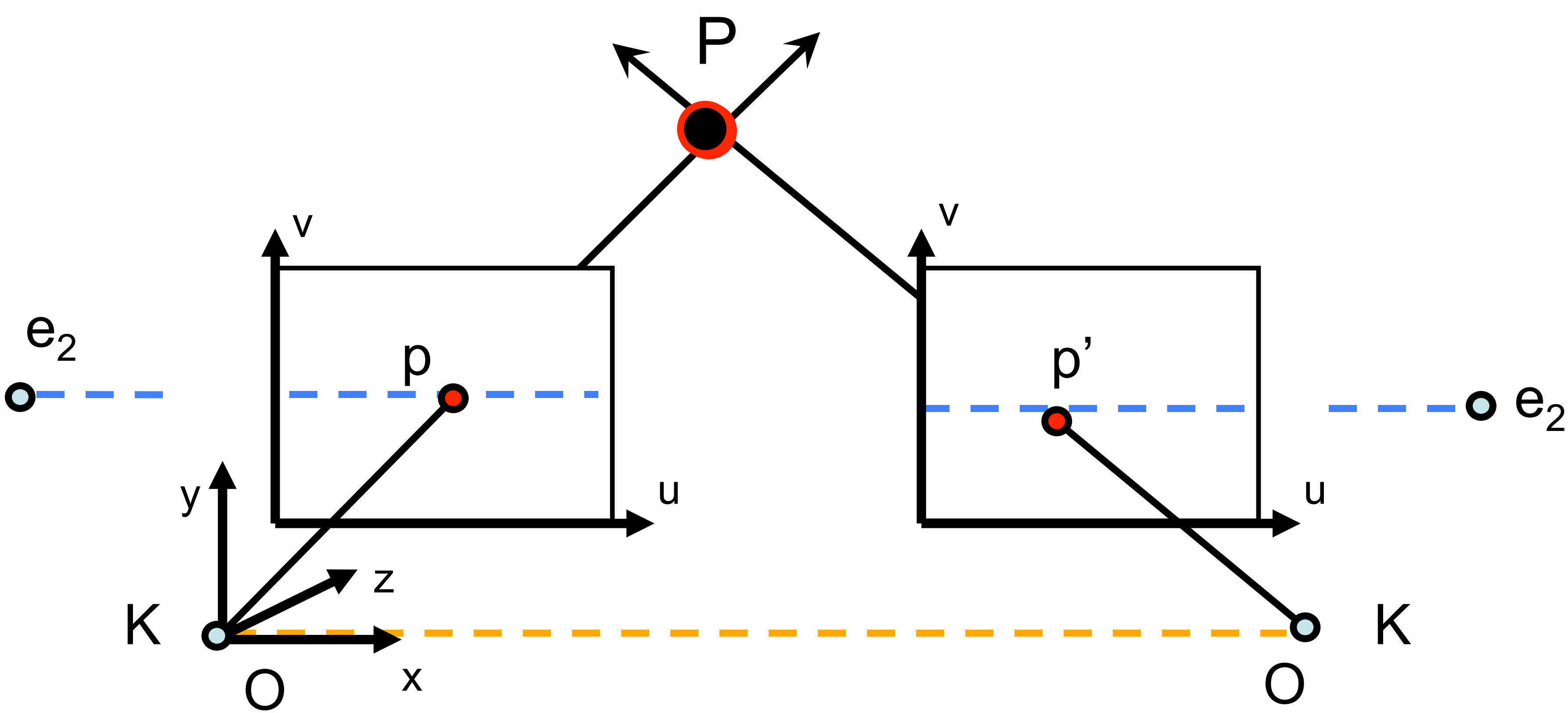
x parallel to O_1O_2

$$E = [t_\times]R$$

Cross product as matrix multiplication

$$\mathbf{a} \times \mathbf{b} = \begin{bmatrix} 0 & -a_z & a_y \\ a_z & 0 & -a_x \\ -a_y & a_x & 0 \end{bmatrix} \begin{bmatrix} b_x \\ b_y \\ b_z \end{bmatrix} = [\mathbf{a}_\times]\mathbf{b}$$

Parallel image planes

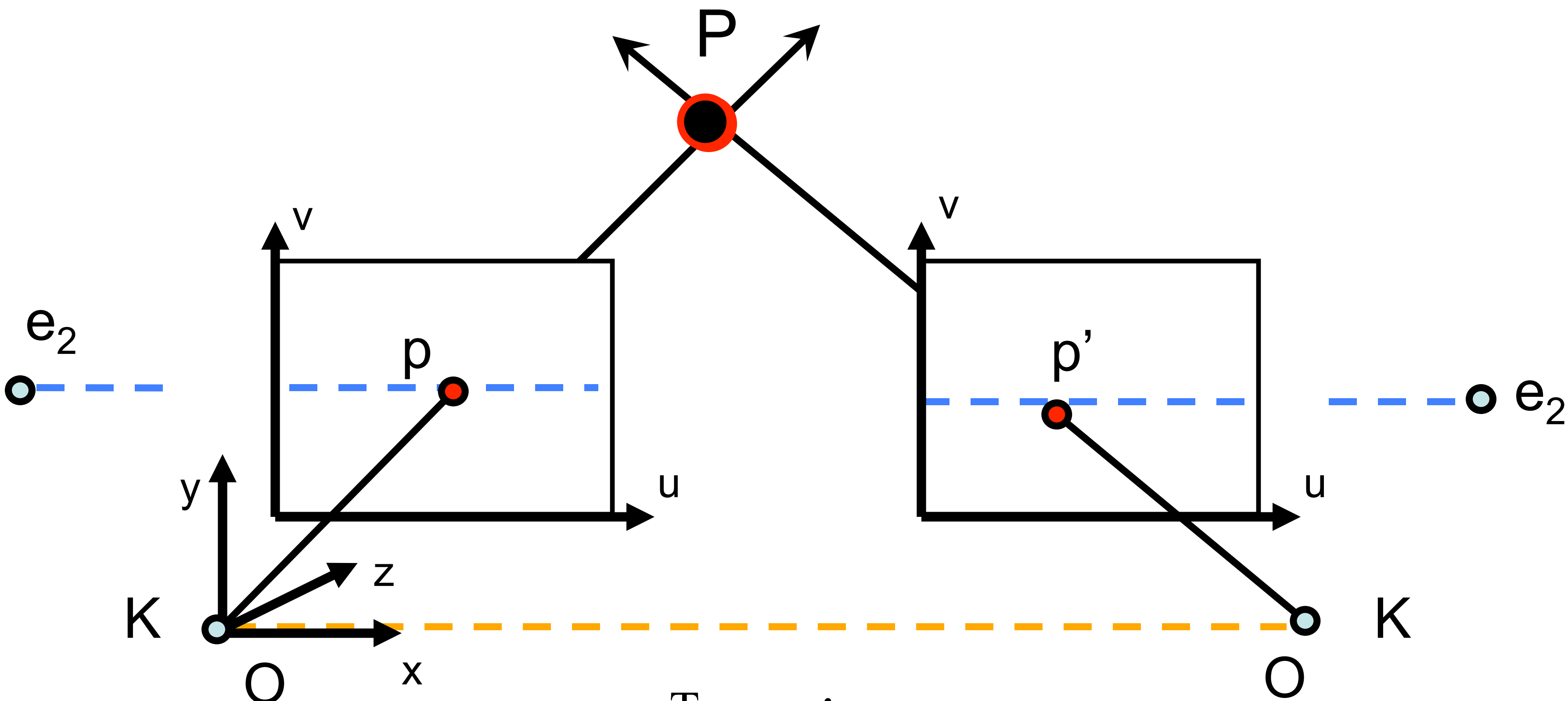


K_1=K_2 = known
x parallel to O_1O_2

$$E = [t_\times]R = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix}$$

→ v = v'?

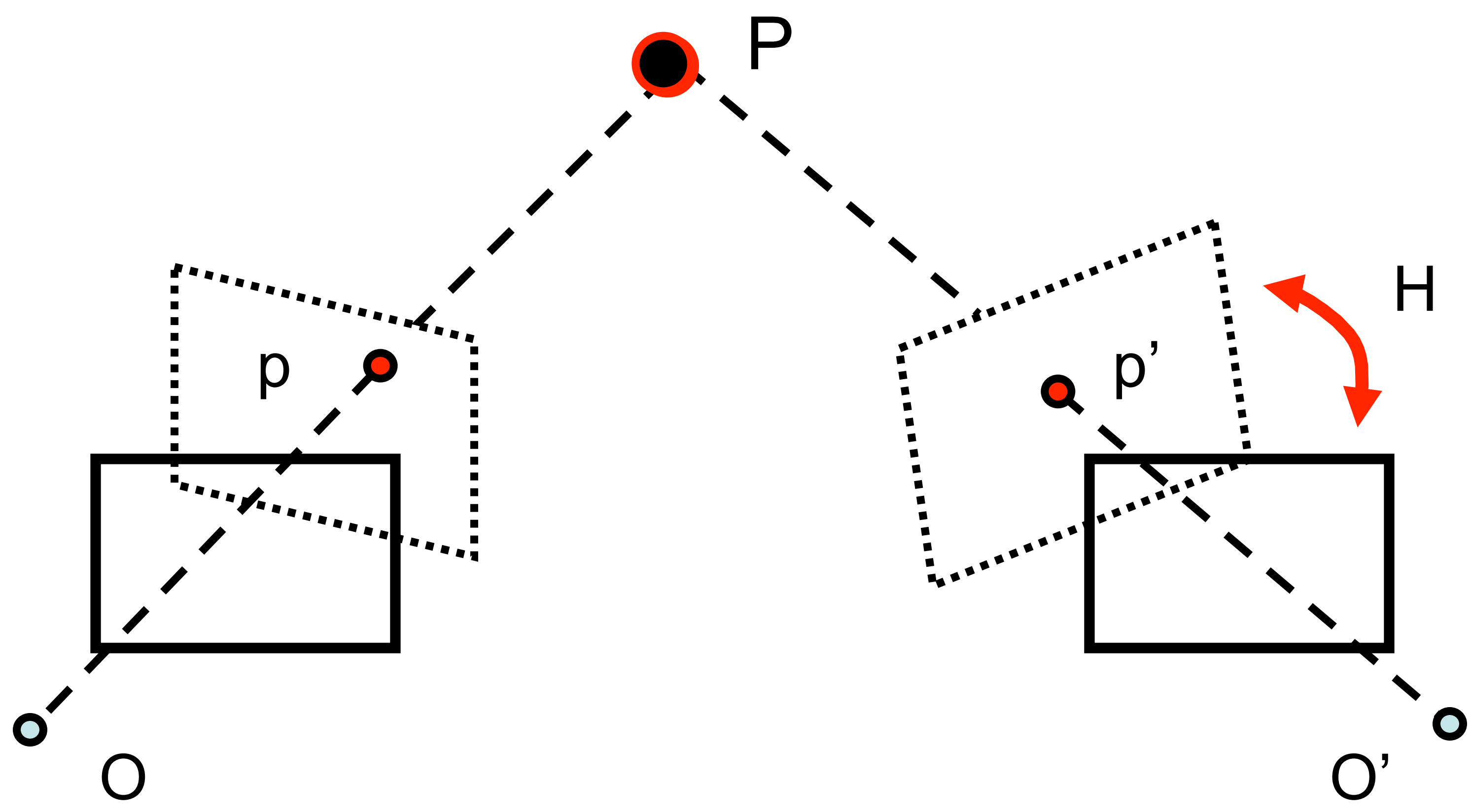
Parallel image planes



$$p^T E\, p' = 0$$

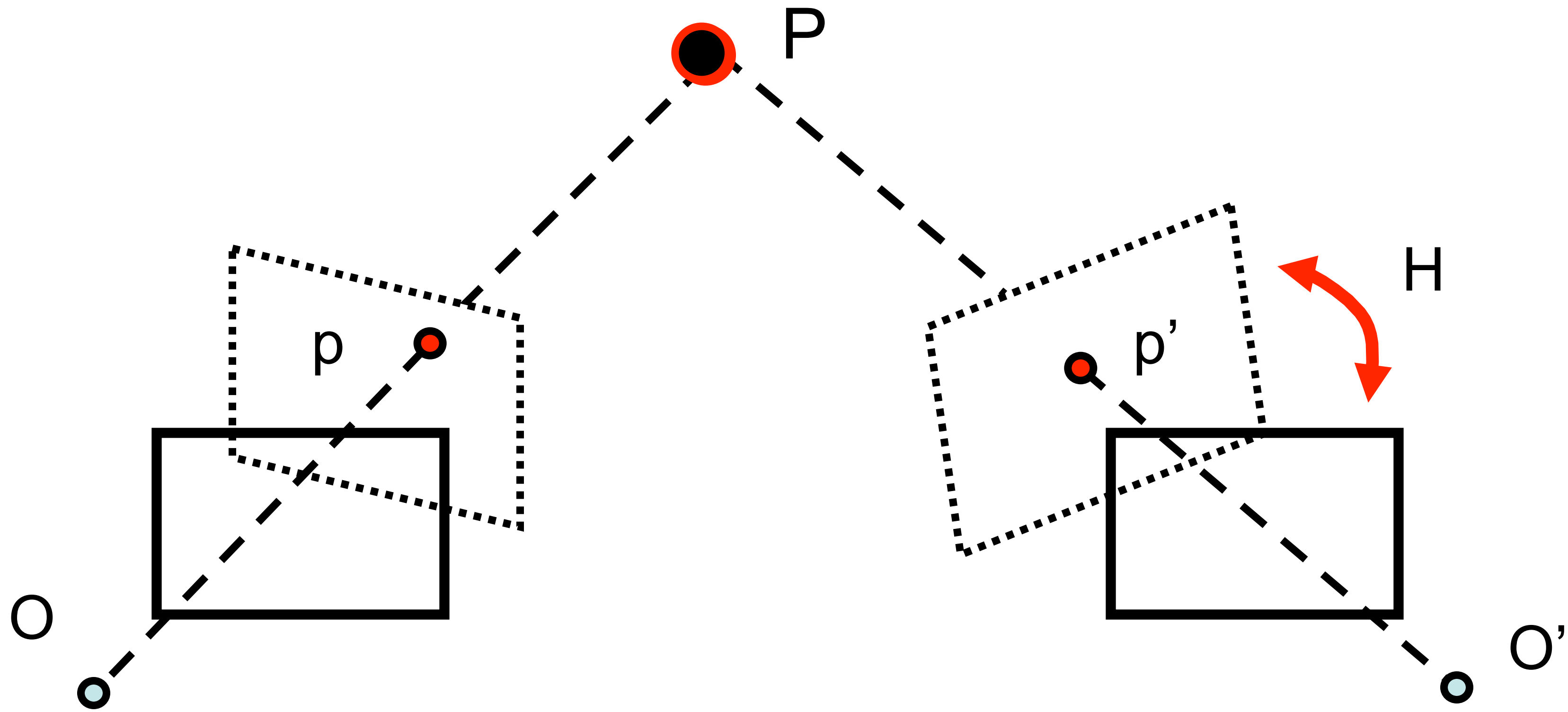
$$\begin{pmatrix} u & v & 1 \end{pmatrix} \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & -T \\ 0 & T & 0 \end{bmatrix} \begin{pmatrix} u' \\ v' \\ 1 \end{pmatrix} = 0 \qquad \begin{pmatrix} u & v & 1 \end{pmatrix} \begin{pmatrix} 0 \\ -T \\ Tv' \end{pmatrix} = 0 \qquad Tv = Tv'$$

Making image planes parallel



GOAL: Estimate the perspective transformation H
that makes the images parallel

Making image planes parallel



GOAL: Estimate the perspective transformation H that makes images parallel

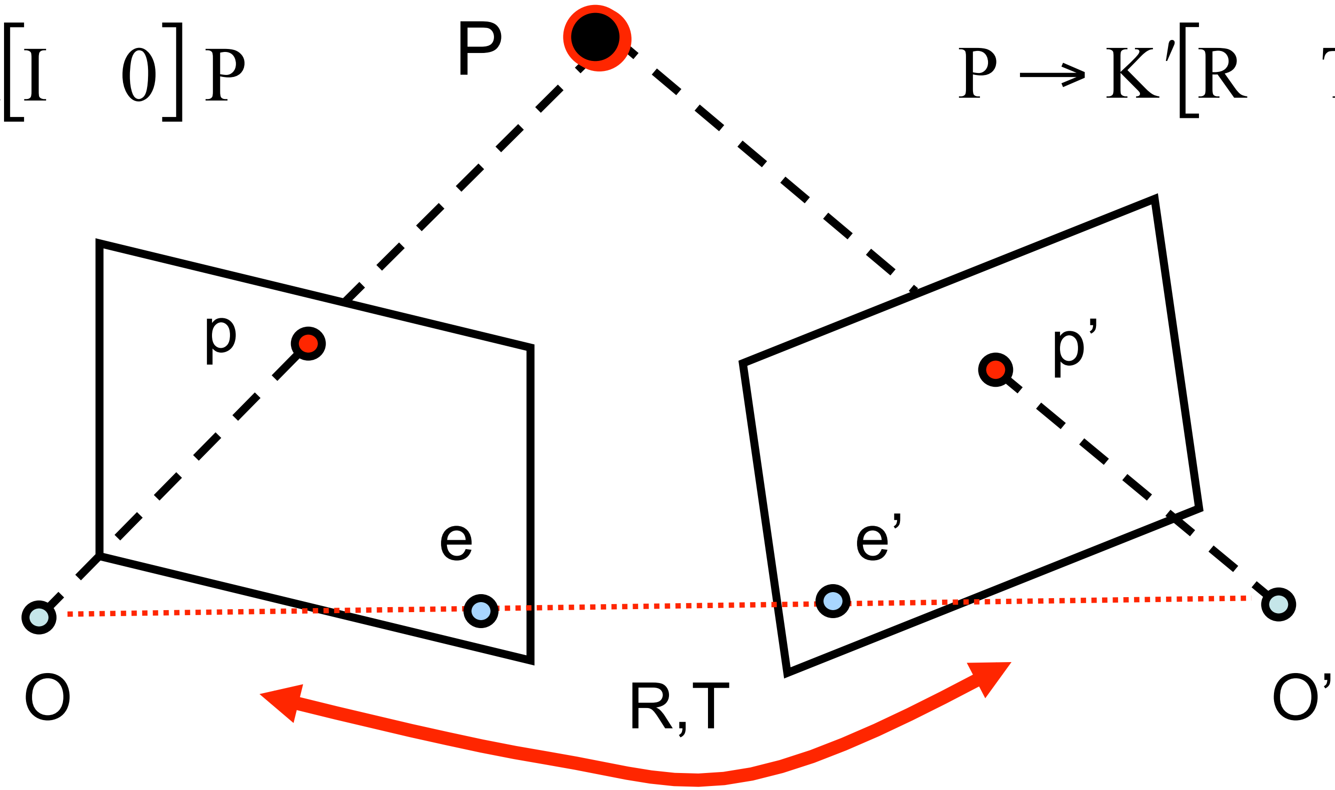
Impose v'=v

- This leaves degrees of freedom for determining H
- If not appropriate H is chosen, severe projective distortions on image take place
- We impose a number of restriction while computing H

Making image planes parallel

$$P \rightarrow K\begin{bmatrix} I & 0 \end{bmatrix} P$$

$$P \rightarrow K'\begin{bmatrix} R & T \end{bmatrix} P$$



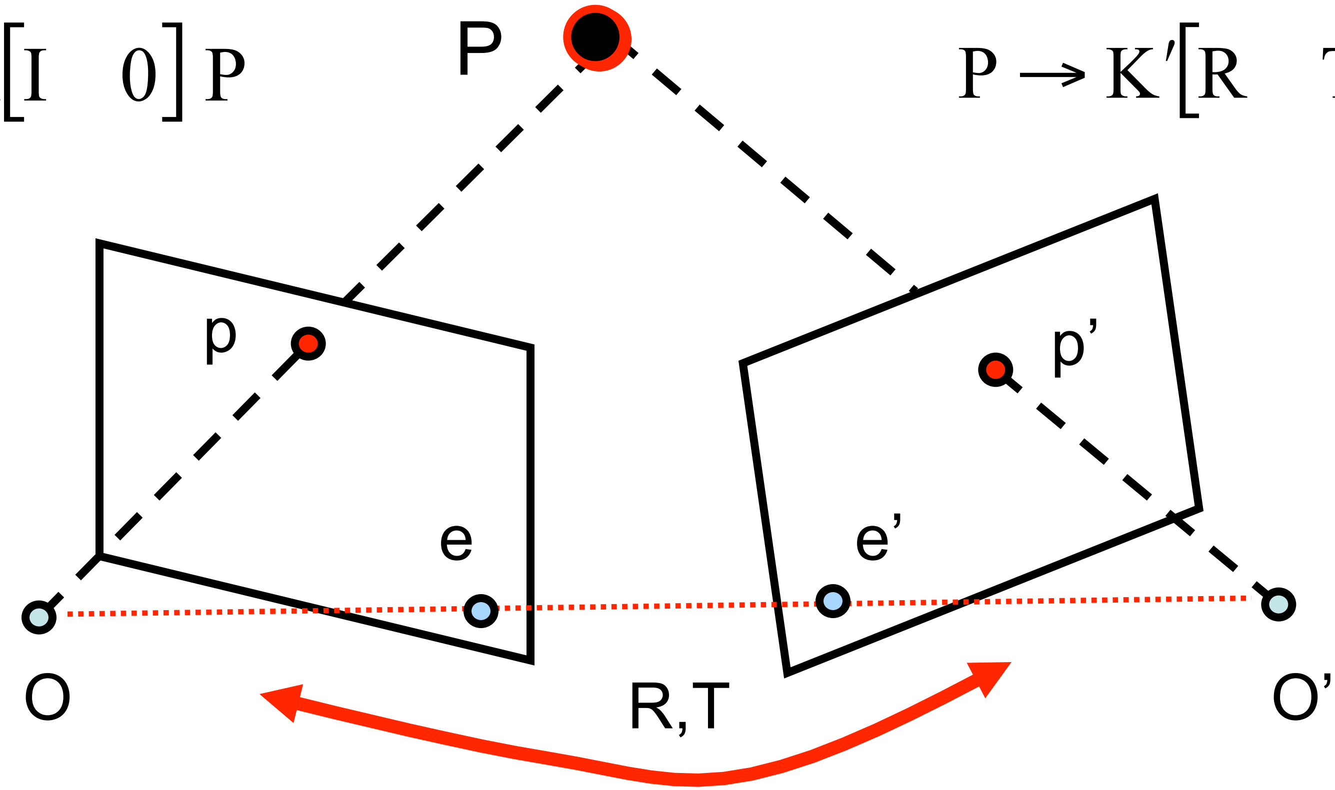
0. Compute epipoles

$$e = [\; e_1 \quad e_2 \quad 1 \;]^T = -K\, R^T\, T$$

$$e' = K'\, T$$

Making image planes parallel

$$P \rightarrow K\begin{bmatrix} I & 0 \end{bmatrix} P \qquad\qquad P \rightarrow K'\begin{bmatrix} R & T \end{bmatrix} P$$

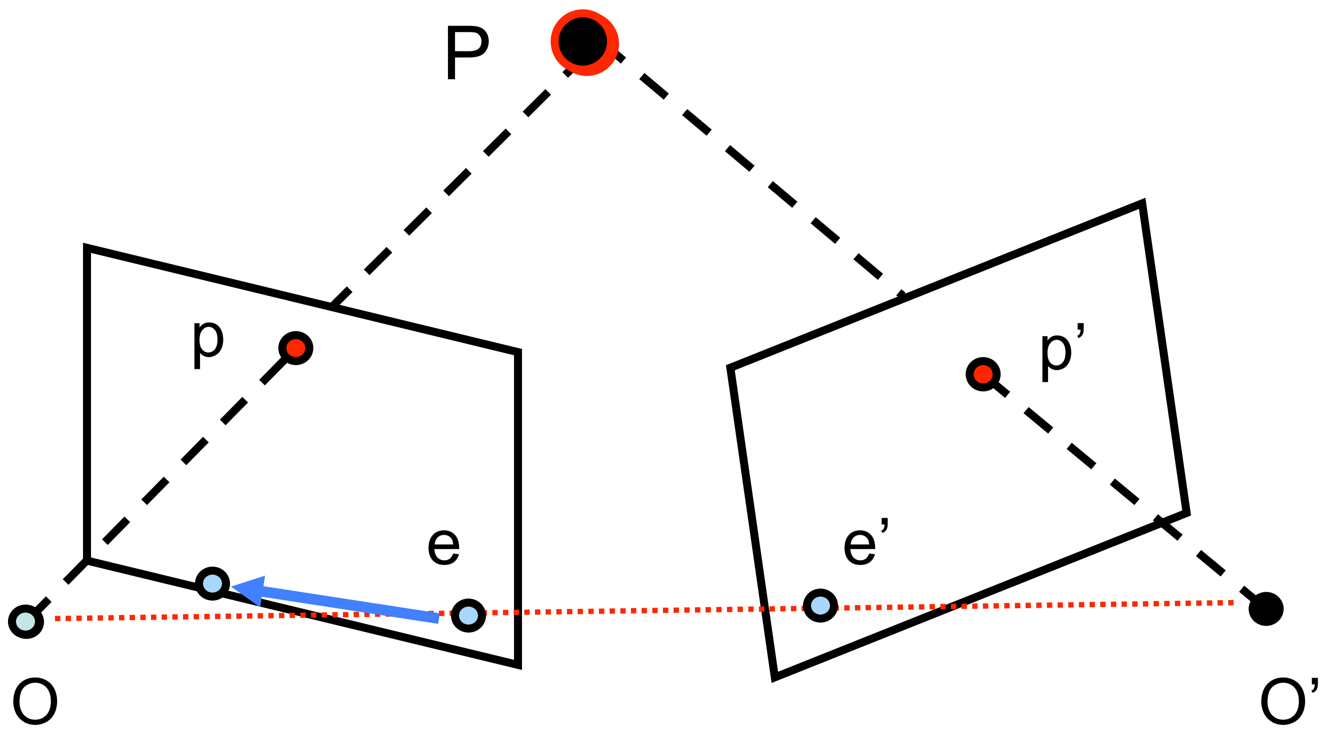


0. Compute epipoles

$$K[\ I \quad 0\ ]\begin{bmatrix} -R^T T \\ 1 \end{bmatrix} = -KR^T T \qquad e' = K'[\ R \quad T\ ]\begin{bmatrix} 0 \\ 1 \end{bmatrix} = K'T$$

Page 244 [HZ]

Making image planes parallel



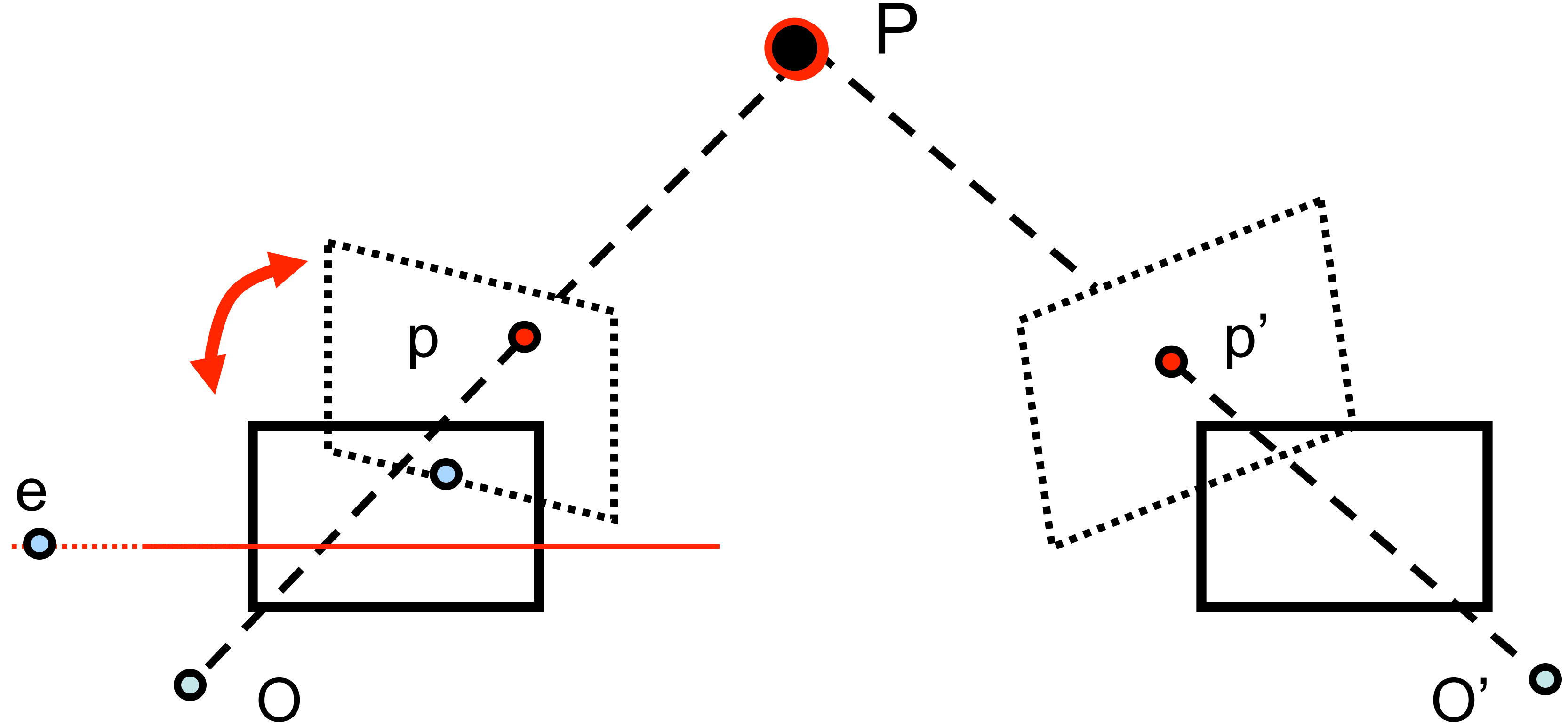
1. Map e to the x-axis at location $[1,0,1]^T$ (normalization)

$$e = [e_1 \quad e_2 \quad 1]^T \rightarrow [1 \quad 0 \quad 1]^T$$

$$\boxed{H_1 = R_H T_H}$$

Page 304 [AZ]

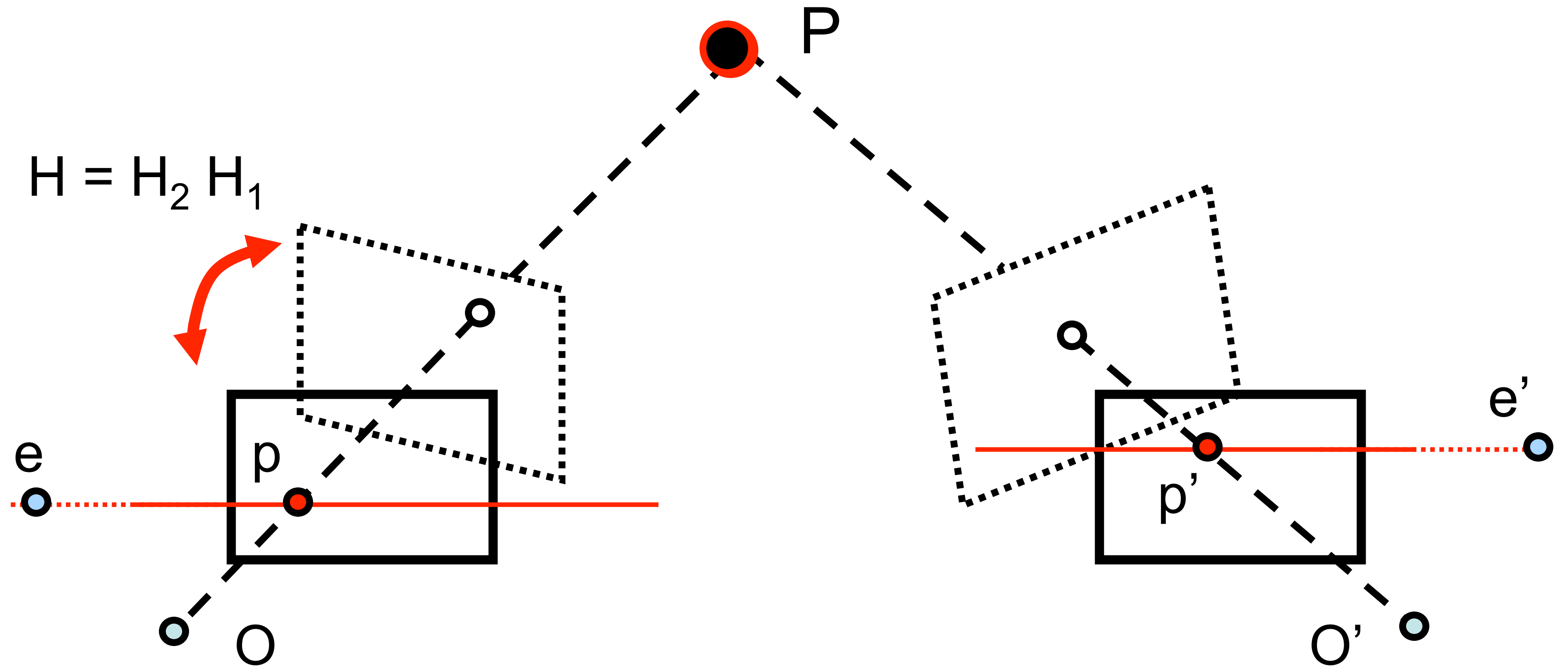
Making image planes parallel



2. Send epipole to infinity: $e = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix}^T \rightarrow \begin{bmatrix} 1 & 0 & 0 \end{bmatrix}^T$

$$\boxed{H_2 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}}$$

Making image planes parallel

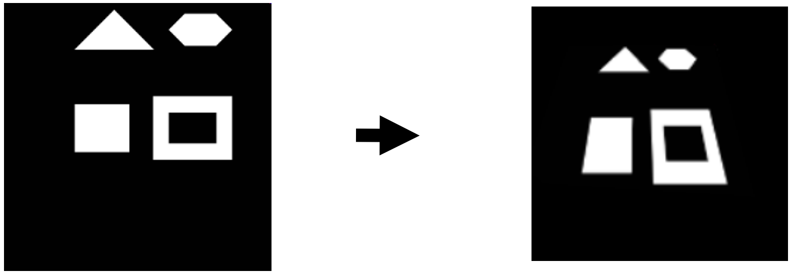


3. Define: $H = H_2 H_1$

4. Align epipolar lines

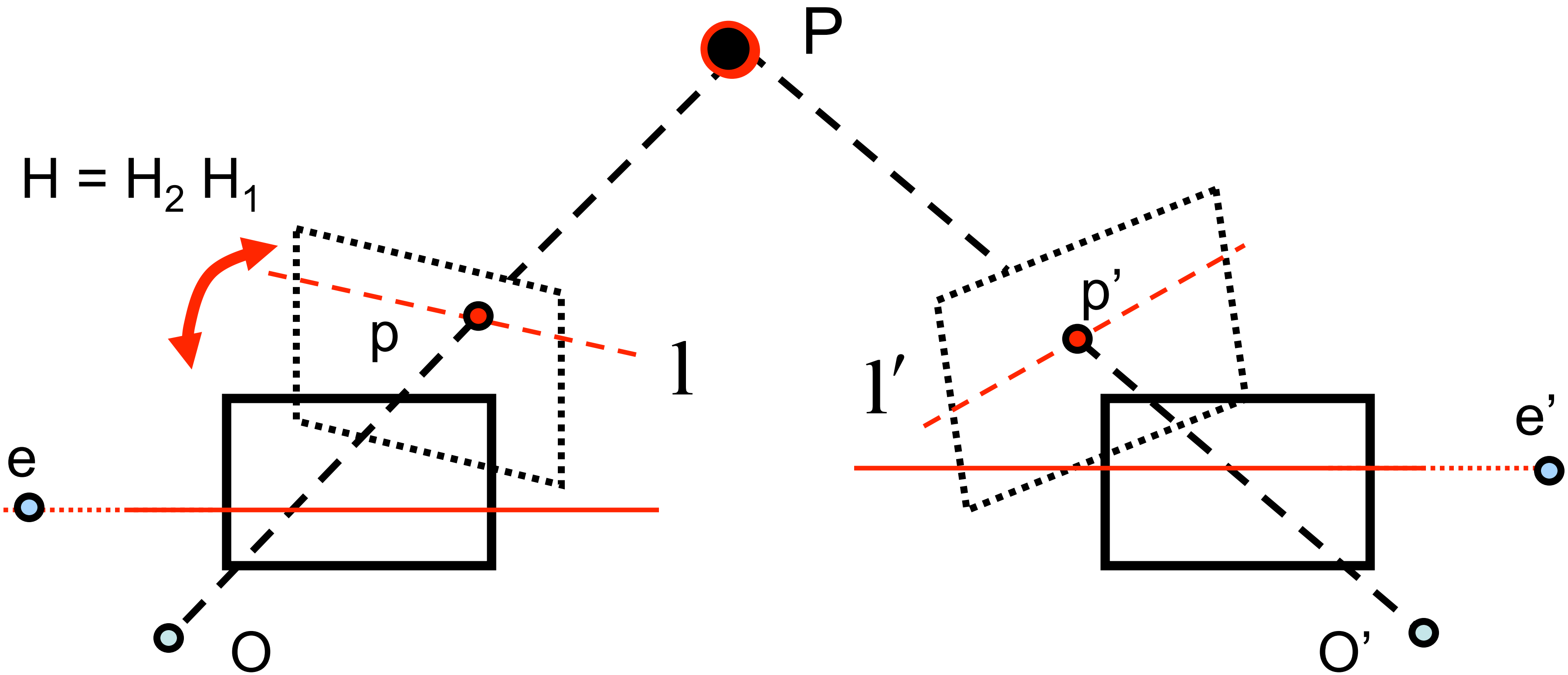
Projective transformation of a line (in 2D)

$$H = \begin{bmatrix} A & t \\ v & b \end{bmatrix}$$



$$l' \rightarrow H^{-T} l$$

Making image planes parallel



3. Define: $H = H_2 H_1$

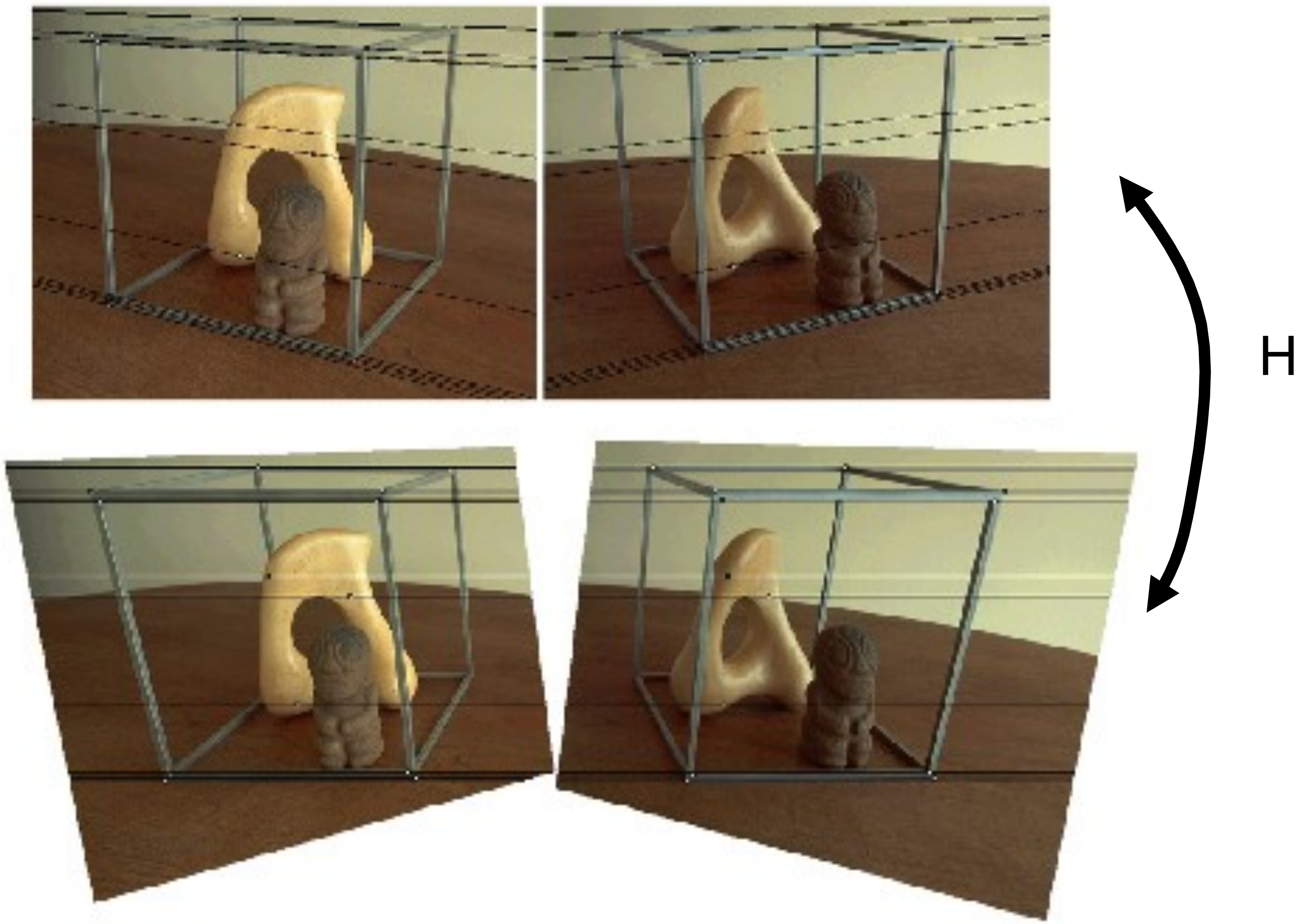
4. Align epipolar lines

[HZ] Chapters: 11 (sec. 11.12)

$$\overline{H'}^{-T} l' = \overline{H}^{-T} l$$

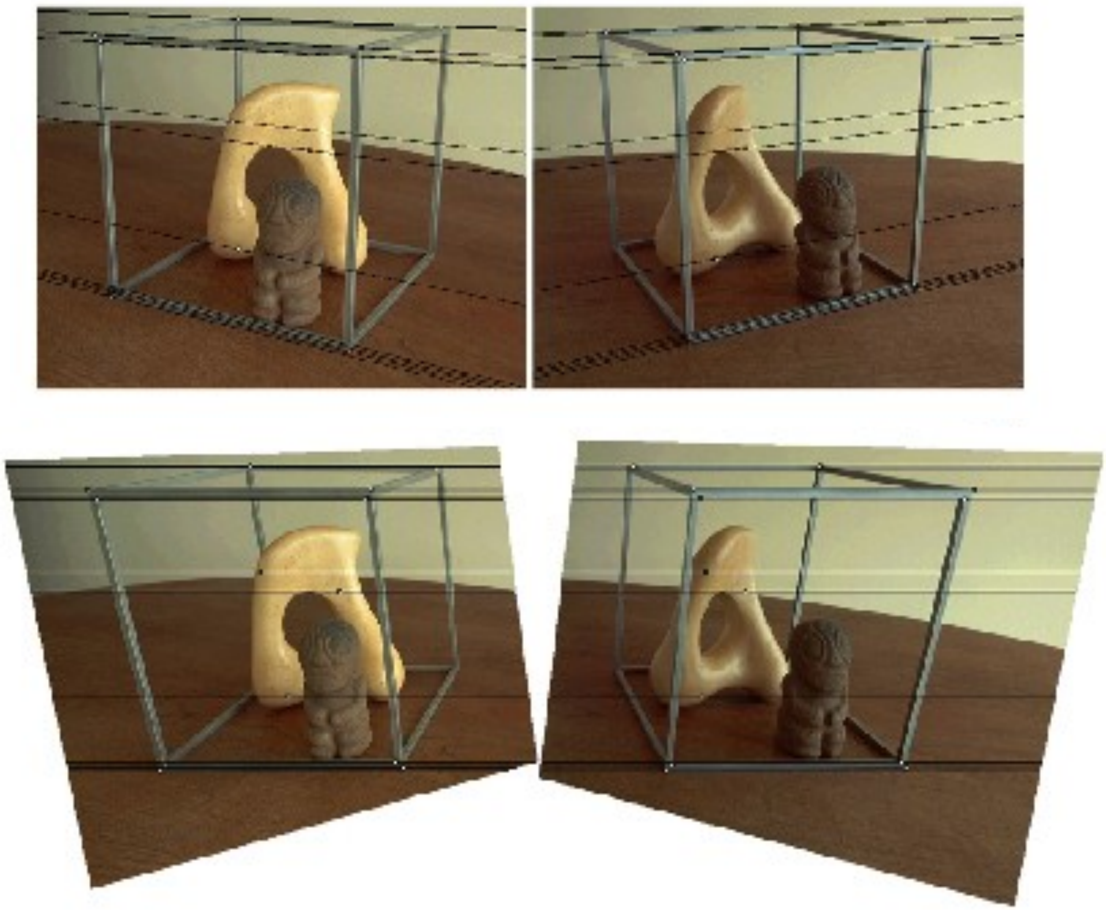
These are called matched pair of transformation

Making image planes parallel



Courtesy figure S. Lazebnik

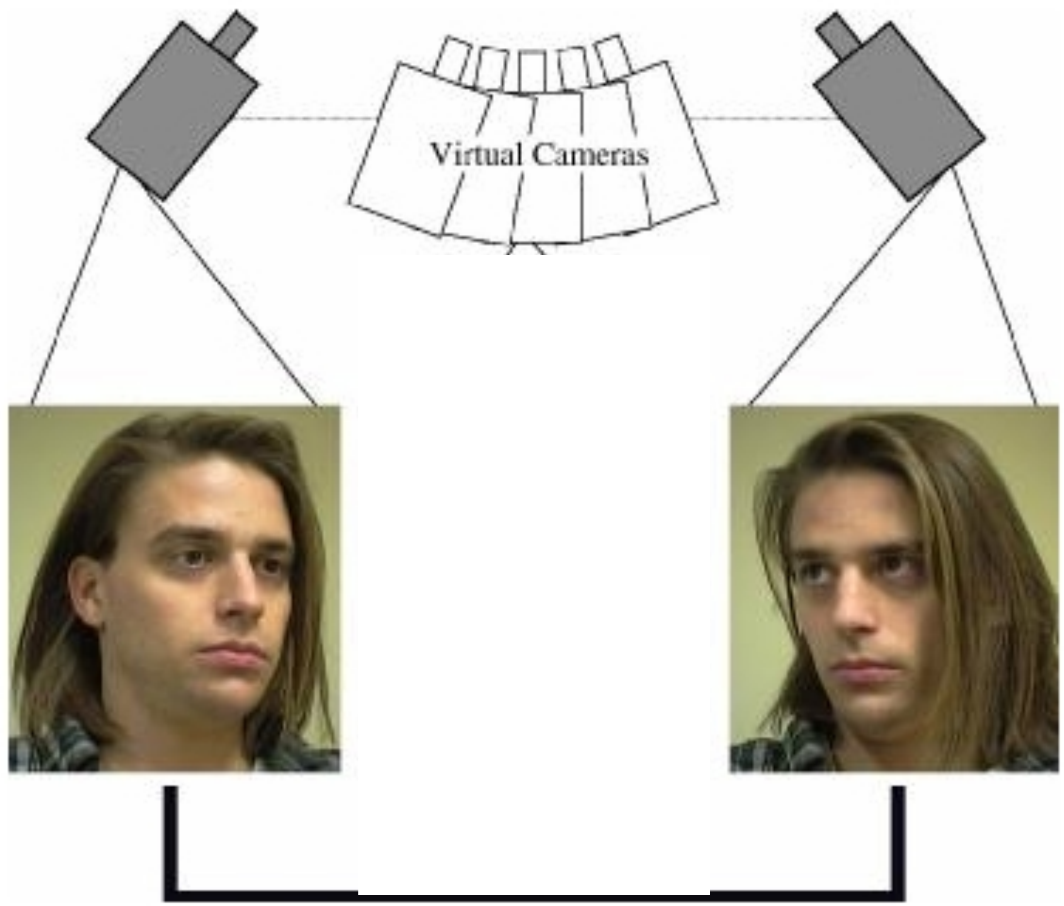
Why rectification is useful?



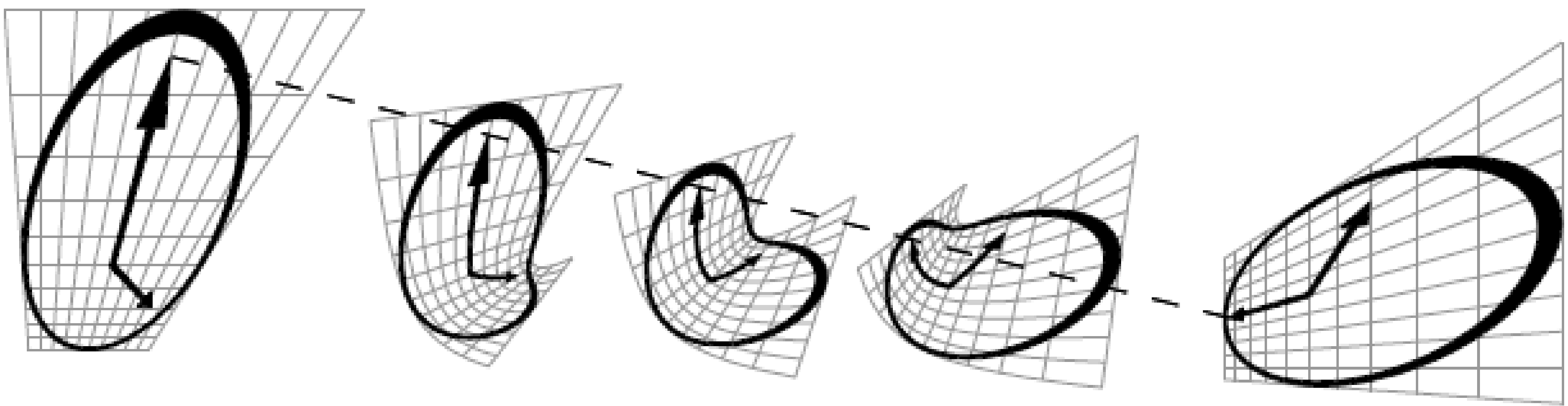
- Makes the correspondence problem easier
- Makes triangulation easy

Application: view morphing

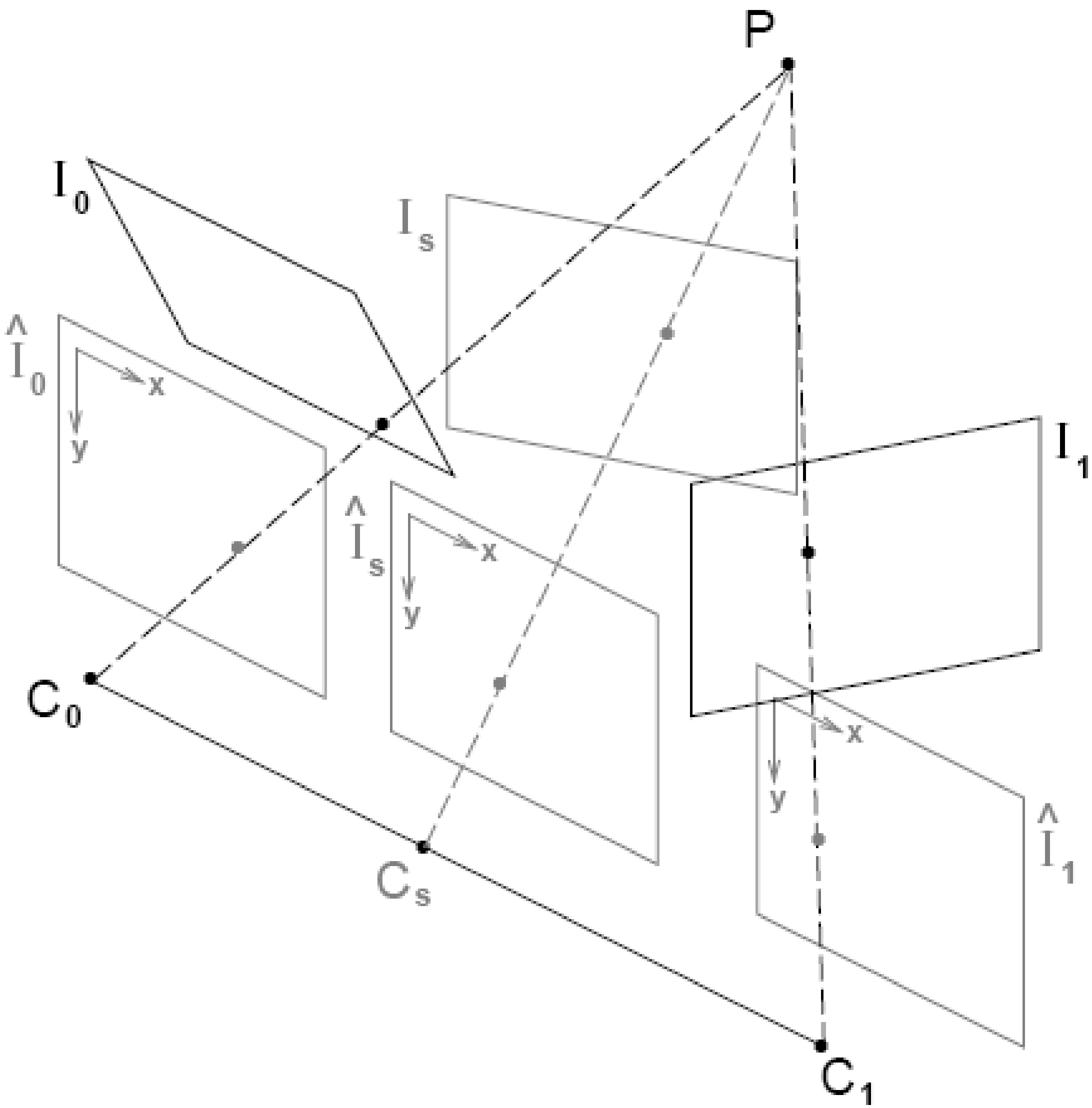
S. M. Seitz and C. R. Dyer, *Proc. SIGGRAPH 96*, 1996, 21-30



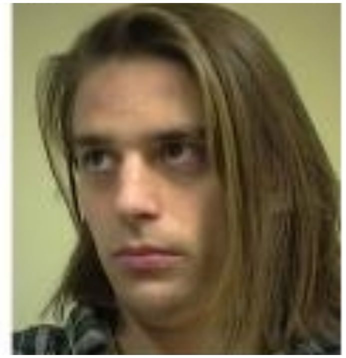
Morphing without using geometry

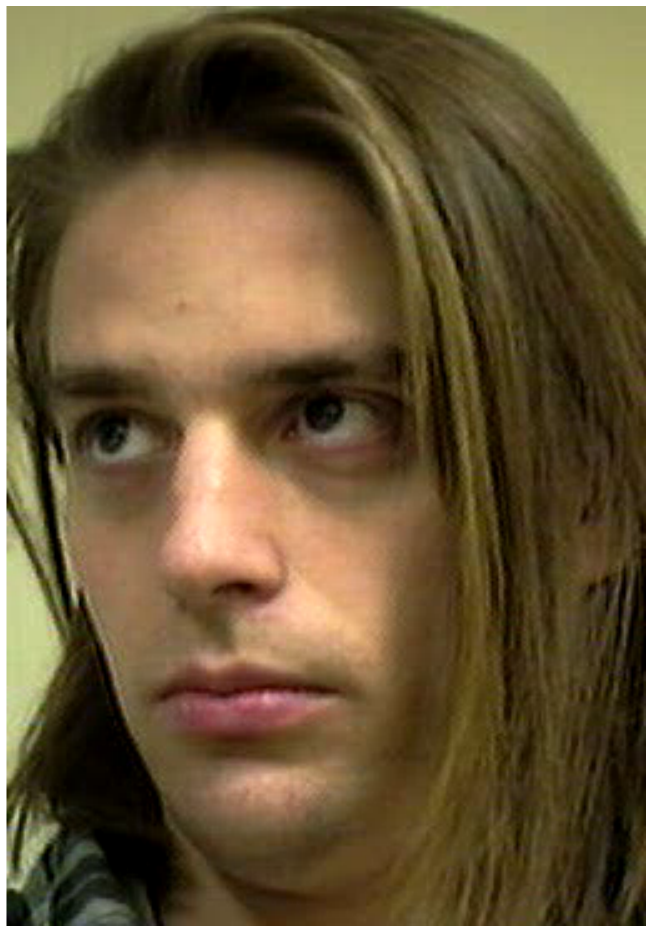


Rectification

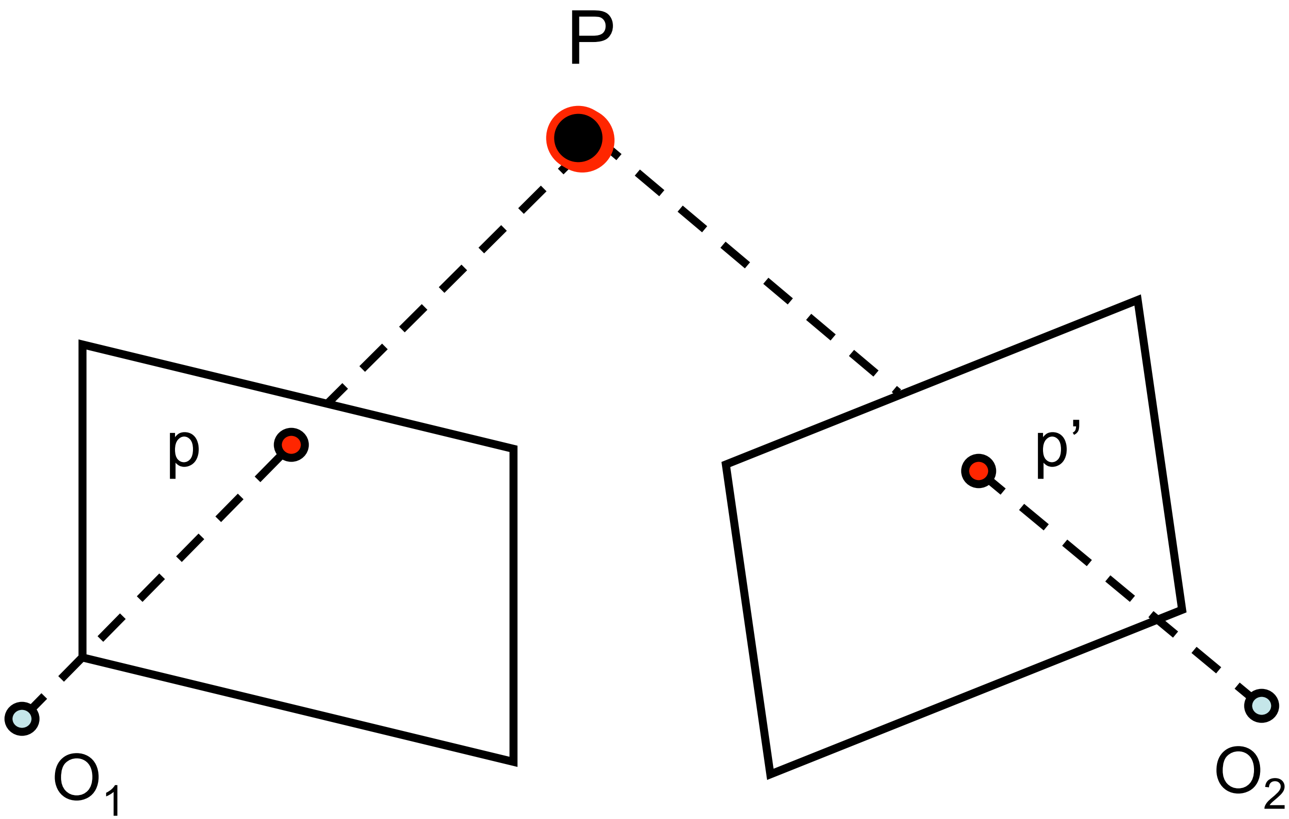








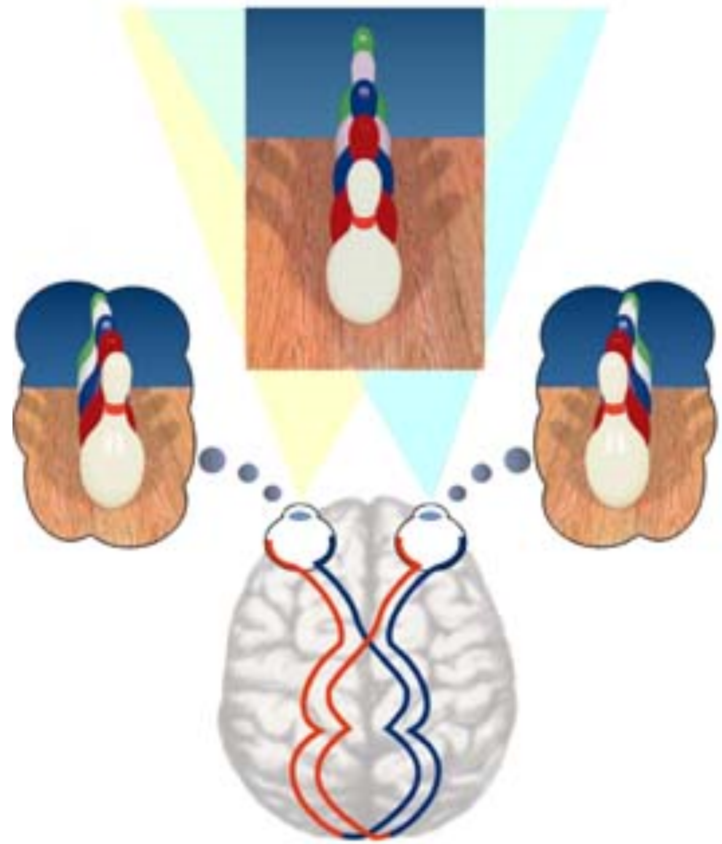
Stereo vision



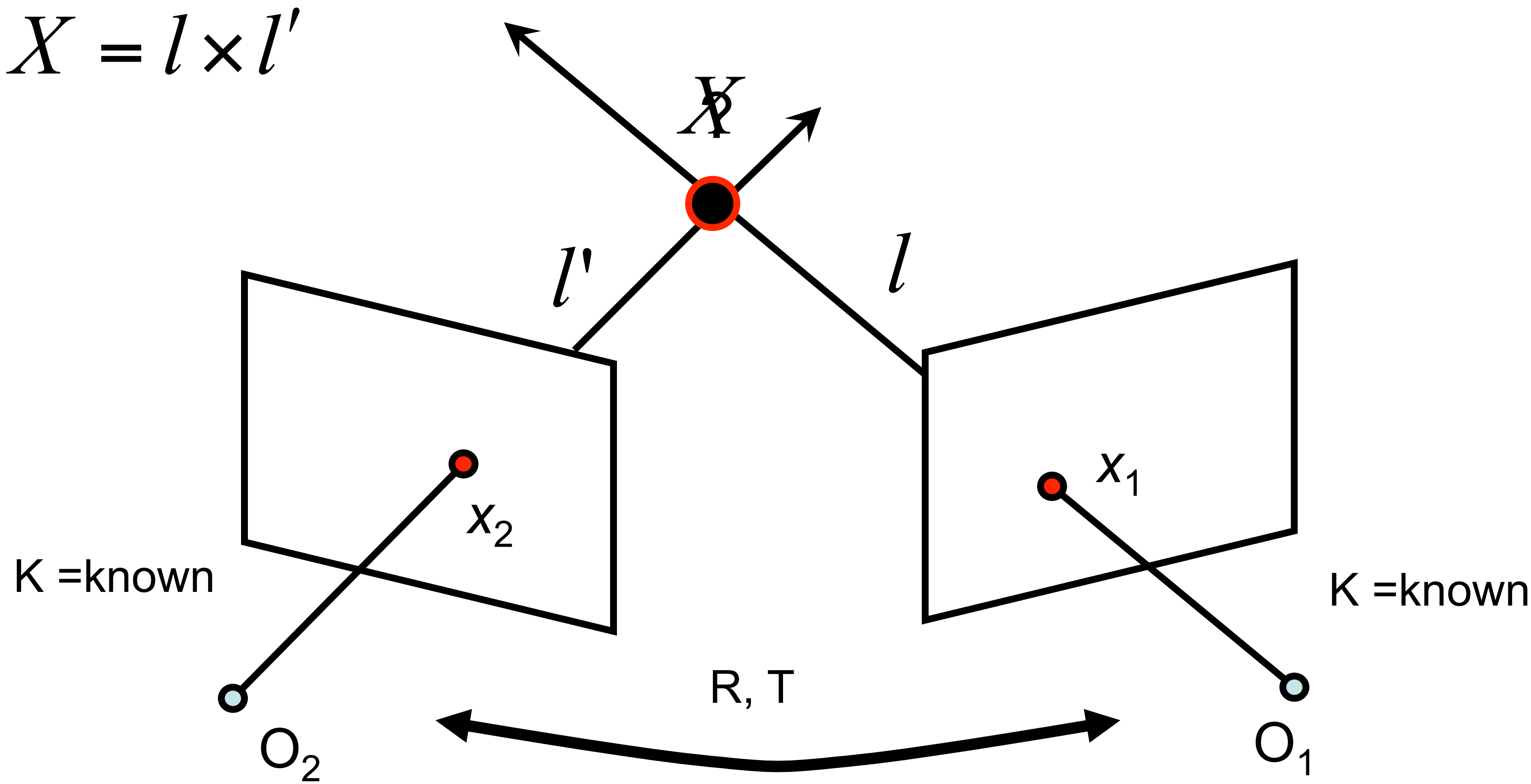
Subgoals:

- Solve the correspondence problem
- Use corresponding observations to triangulate

Two eyes help!



Two eyes help!

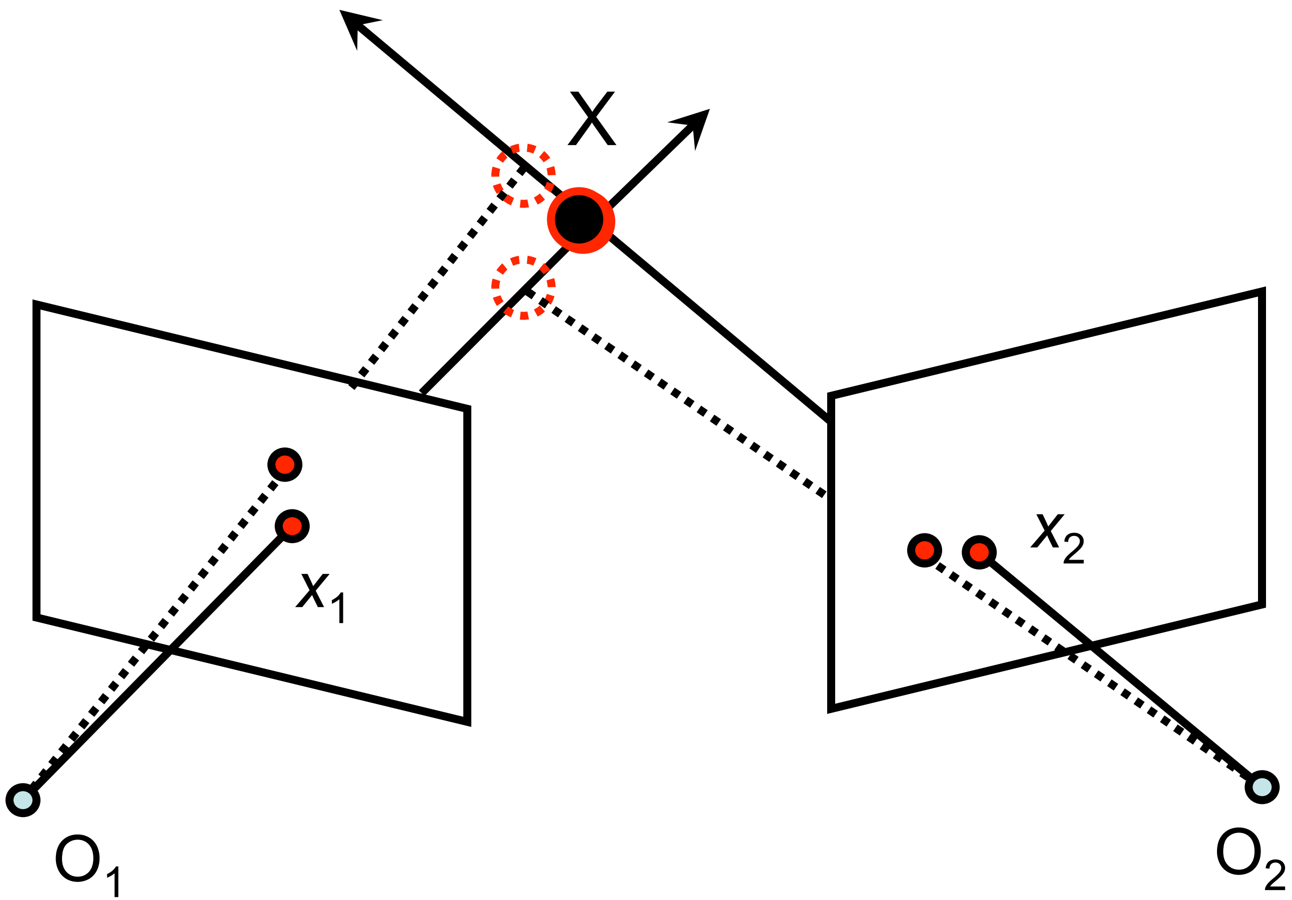


This is called triangulation

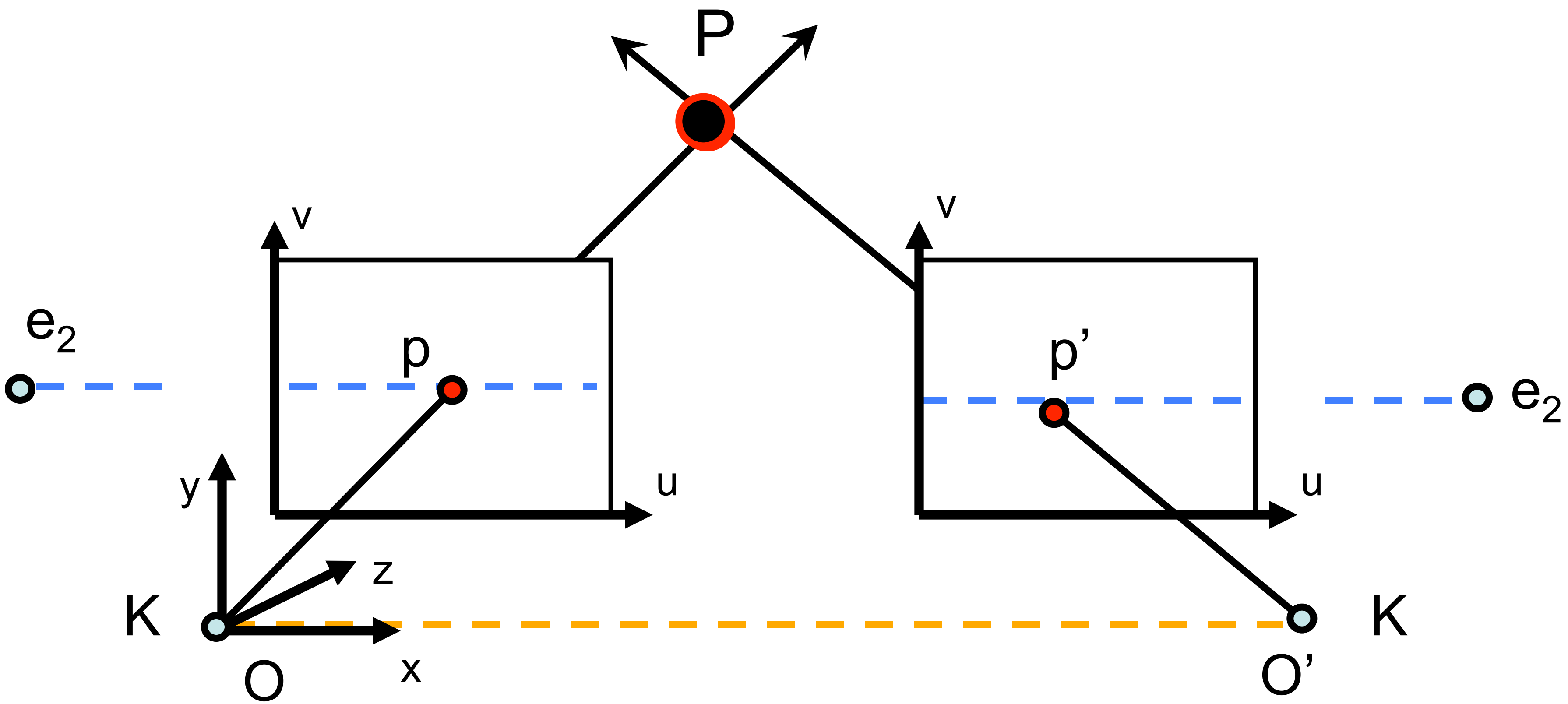
Triangulation

- Find X that minimizes

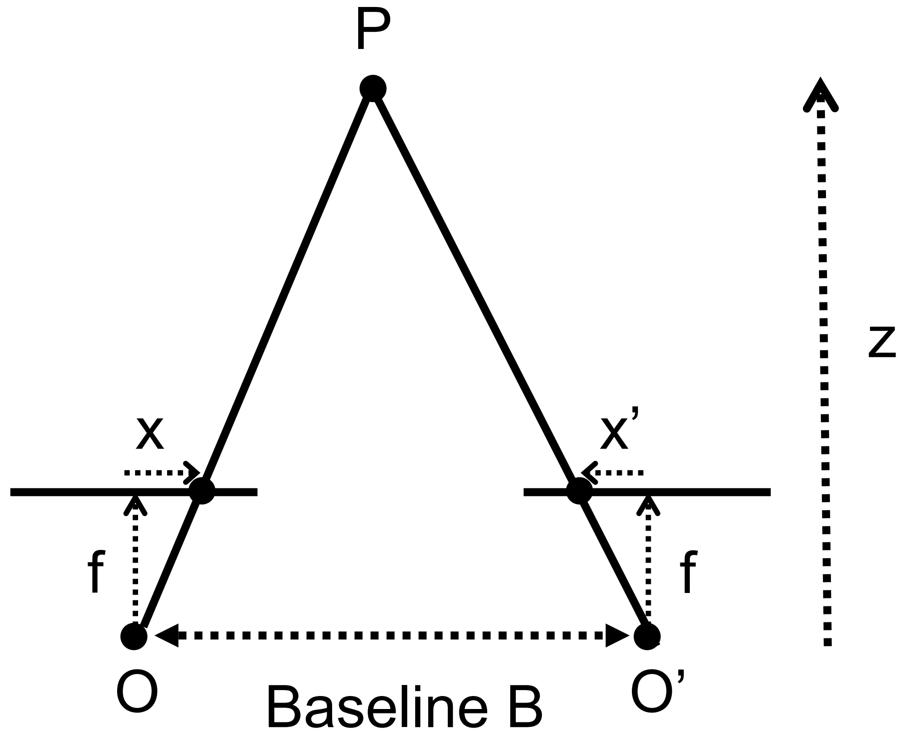
$$d^2(x_1, M_1 X) + d^2(x_2, M_2 X)$$



Computing depth



Computing depth



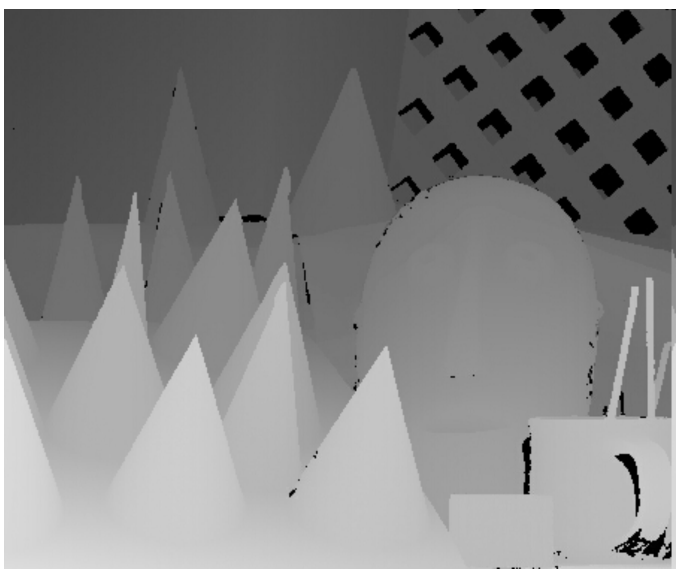
$$x - x' = \frac{B \cdot f}{z} = \text{disparity}$$

Note: Disparity is inversely proportional to depth

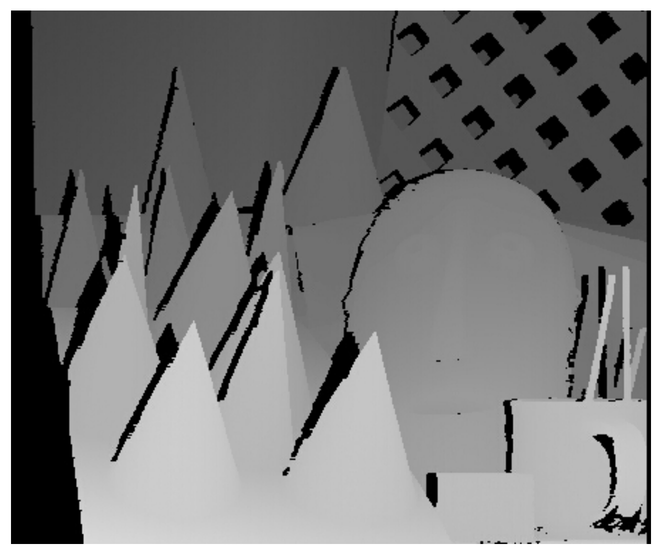
Disparity maps

$$x - x' = \frac{B \cdot f}{z}$$

Stereo pair



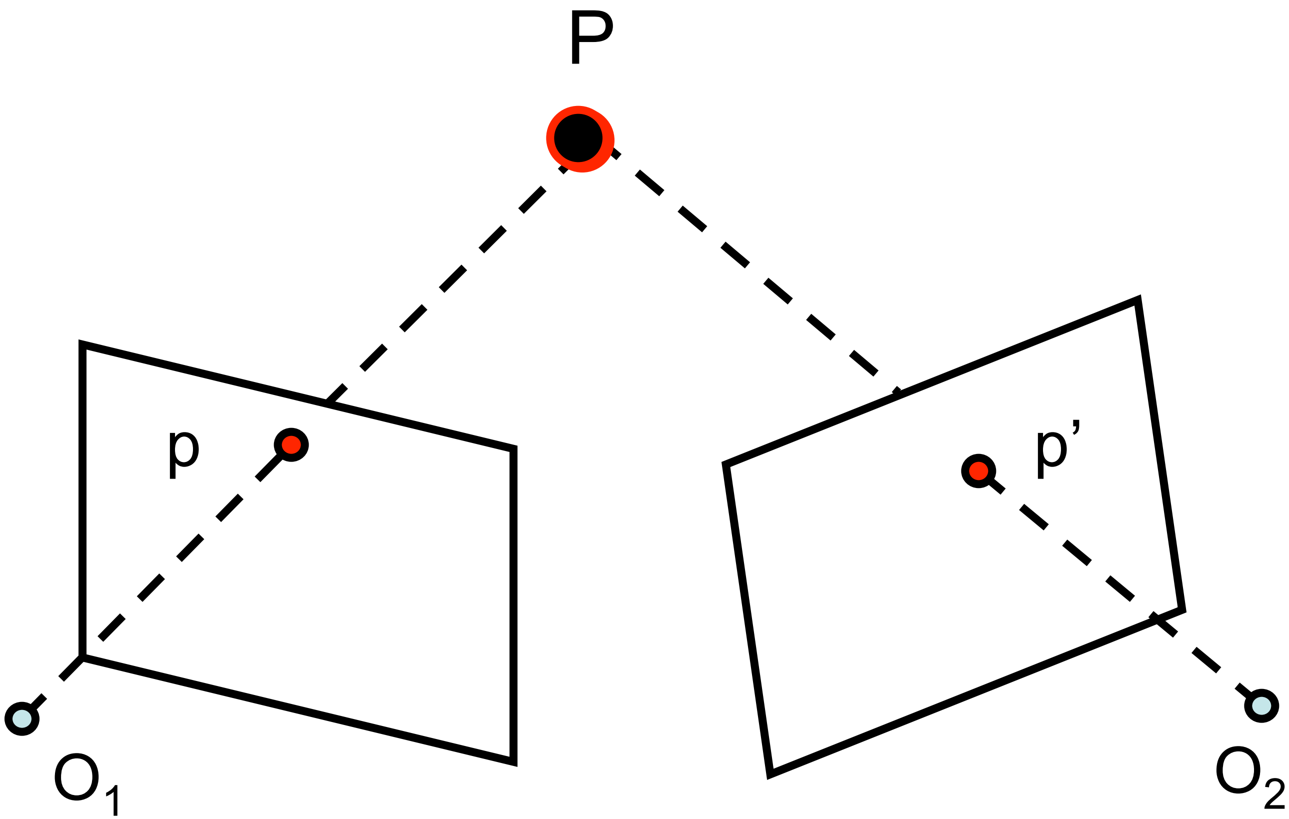
Disparity map / depth map



Disparity map with occlusions

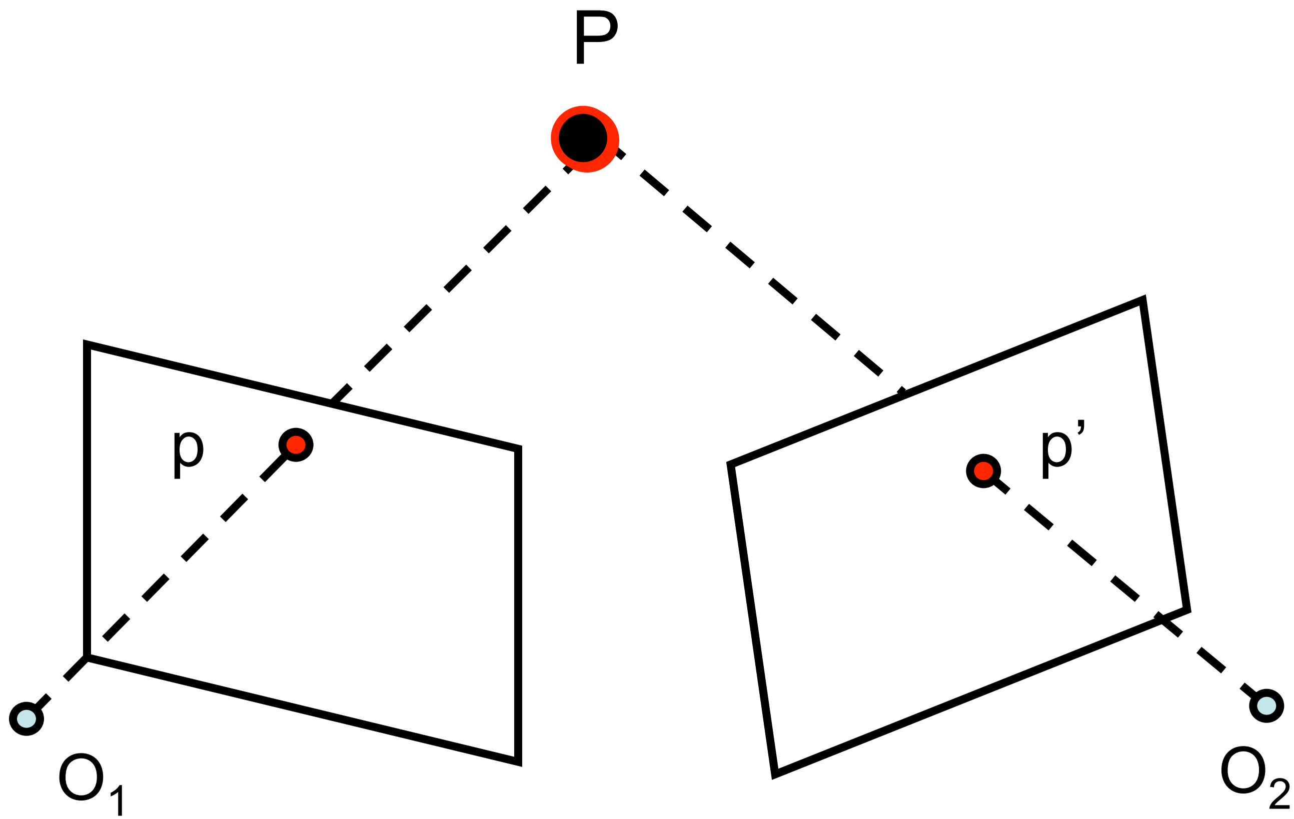
http://vision.middlebury.edu/stereo/

Stereo vision



Subgoals:
- Solve the correspondence problem
- Use corresponding observations to triangulate

Correspondence problem



Given a point in 3d, discover corresponding observations in left and right images [also called binocular fusion problem]

Correspondence problem

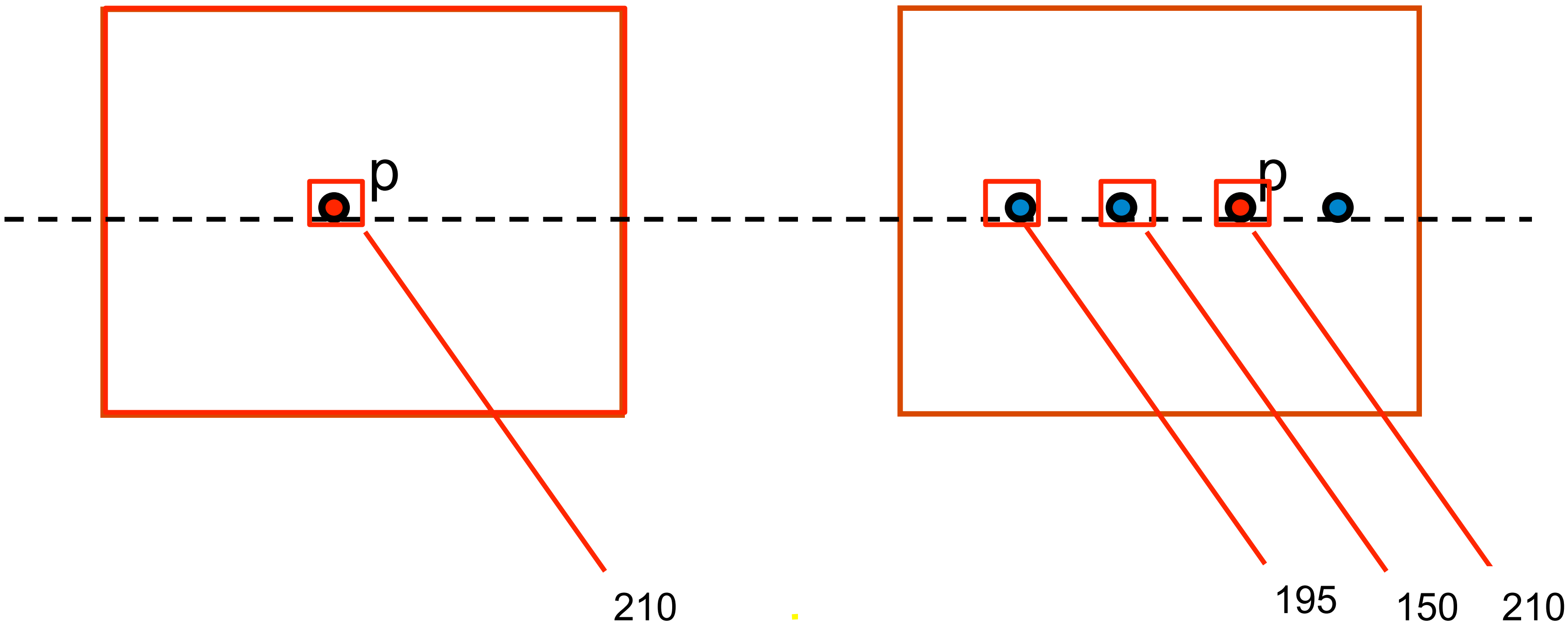
- A Cooperative Model (Marr and Poggio, 1976)

- Correlation Methods (1970--)

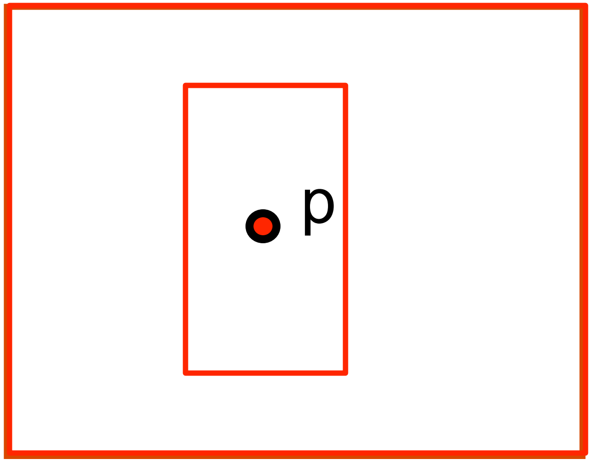
- Multi-Scale Edge Matching (Marr, Poggio and Grimson, 1979-81)

[FP] Chapters: 11

Correlation Methods (1970--)

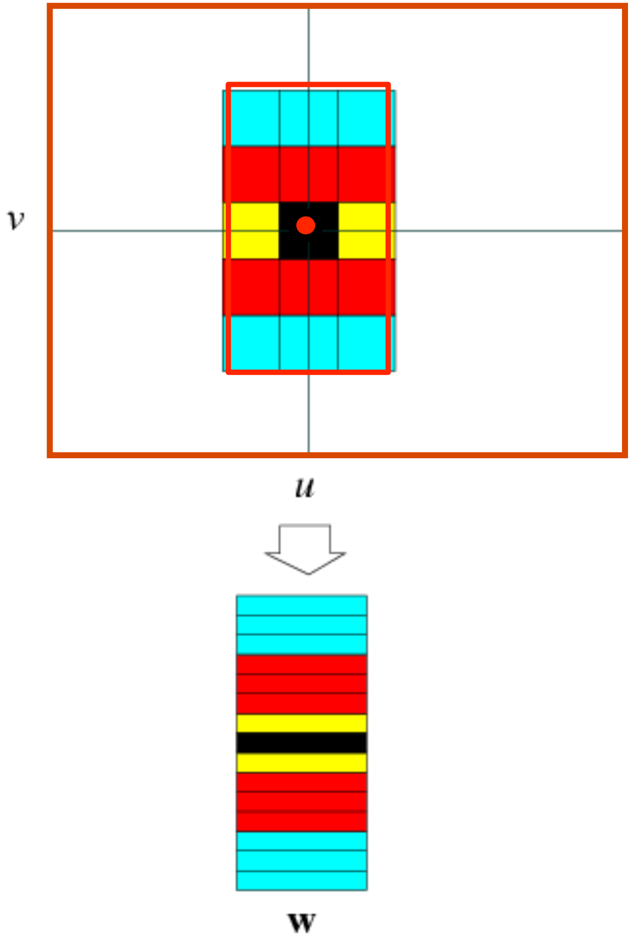


Correlation Methods (1970--)



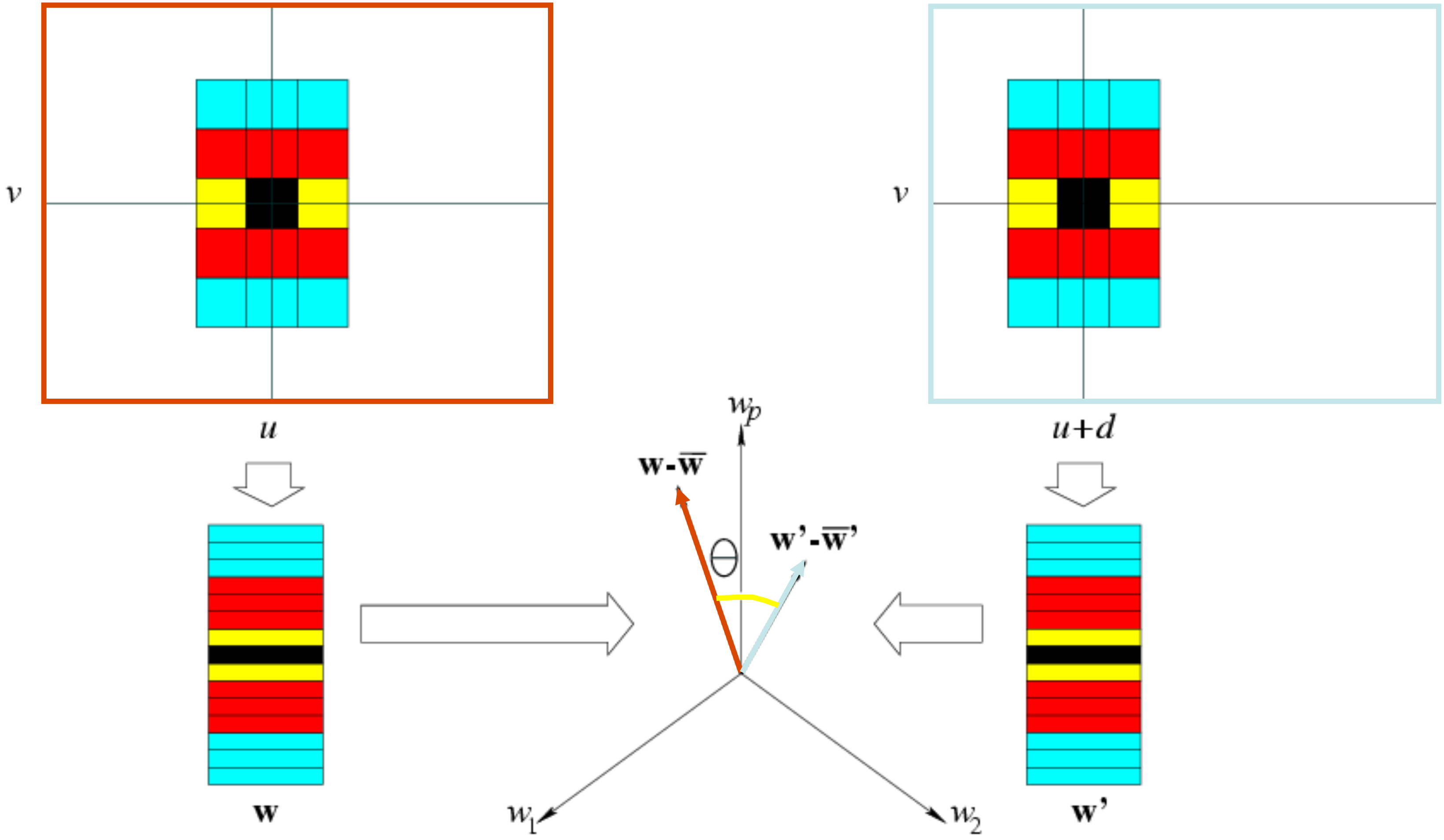
- Pick up a window around p(u,v)

Correlation Methods (1970--)



- Pick up a window around p(u,v)
- Build vector W
- Slide the window along v line in image 2 and compute w'
- Keep sliding until w·w' is maximized.

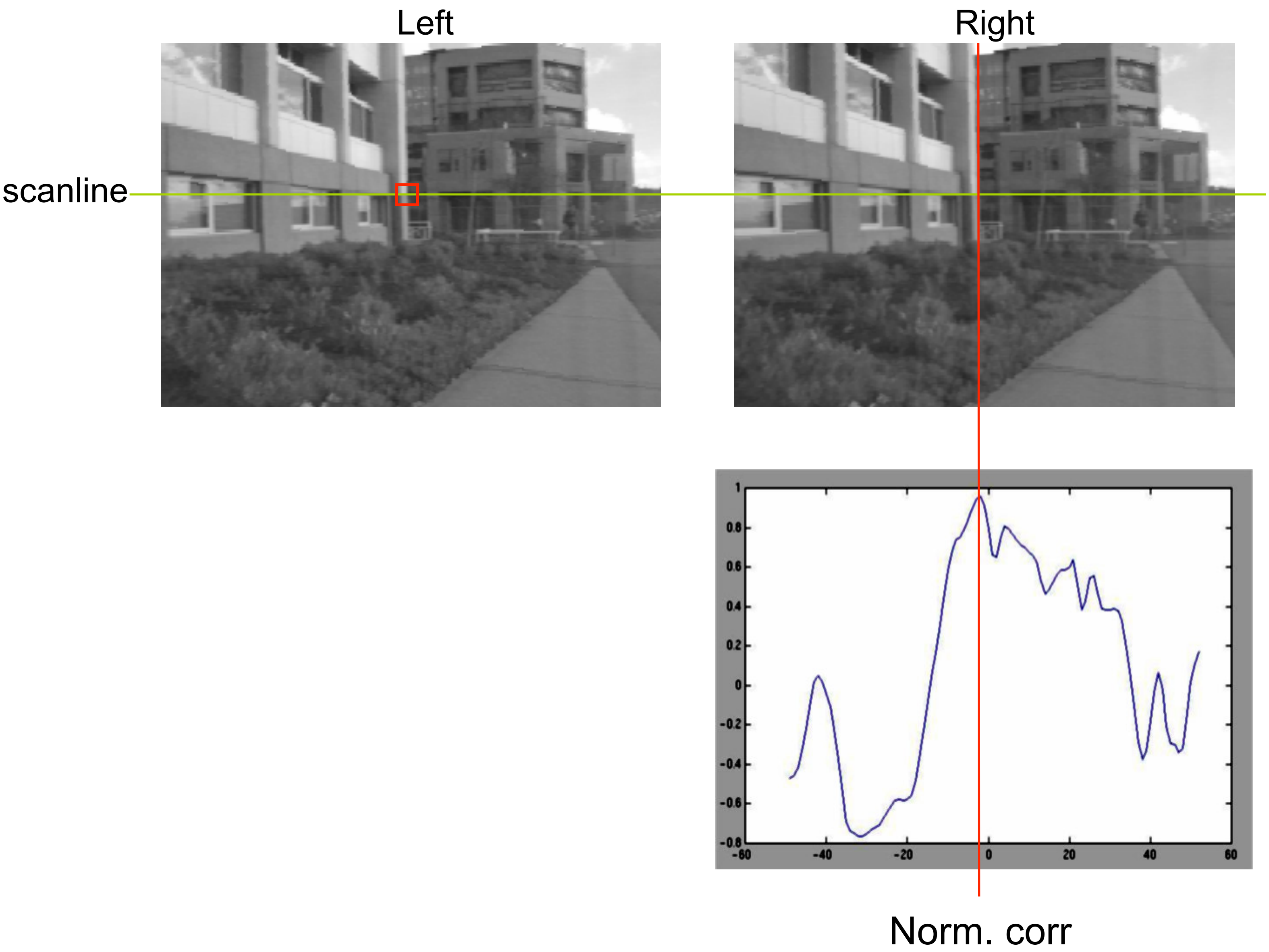
Correlation Methods (1970--)



Normalized Correlation; minimize:

$$\frac{(\mathrm{w}-\overline{\mathrm{w}})(\mathrm{w}'-\overline{\mathrm{w}}')}{\left\|(\mathrm{w}-\overline{\mathrm{w}})(\mathrm{w}'-\overline{\mathrm{w}}')\right\|}$$

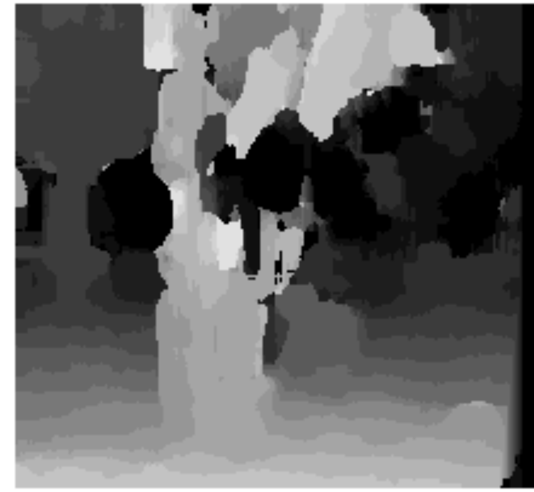
Correlation methods



Credit slide S. Lazebnik

Correlation methods

Window size = 3



Window size = 20

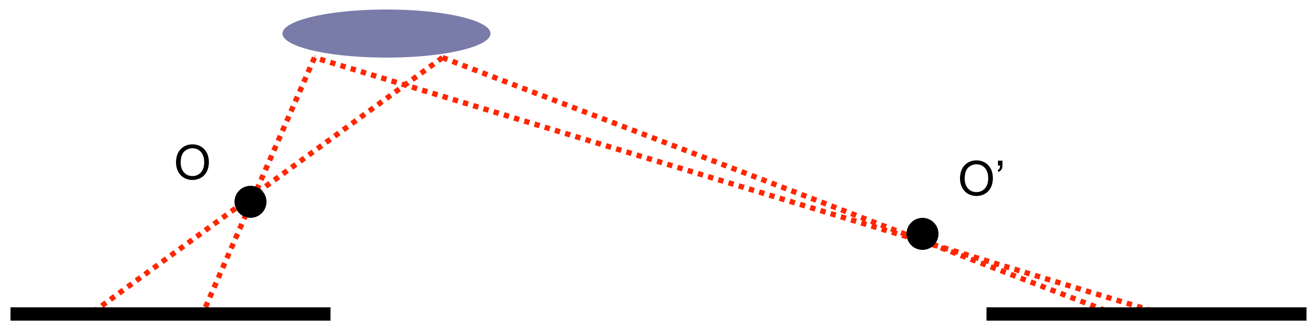
- Smaller window
  - More detail
  - More noise

- Larger window
  - Smoother disparity maps
  - Less prone to noise

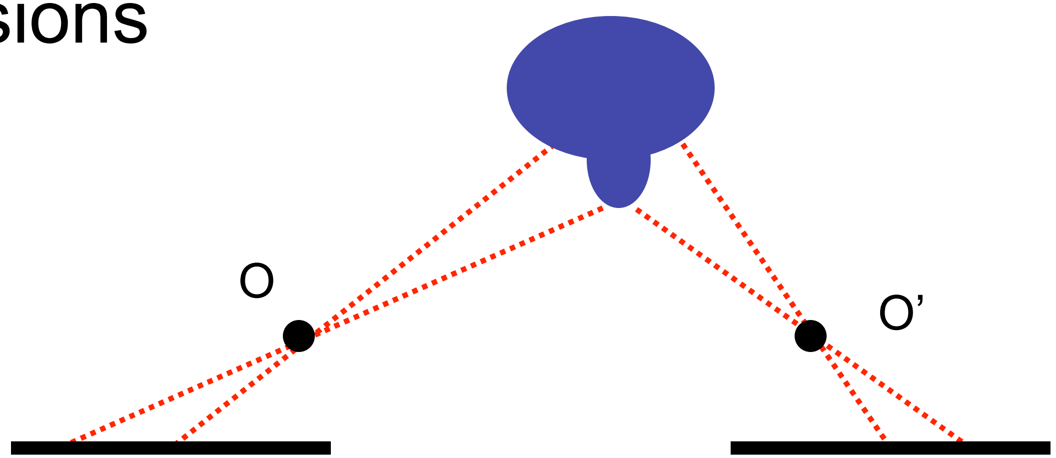
Credit slide S. Lazebnik

Issues

- Fore shortening effect

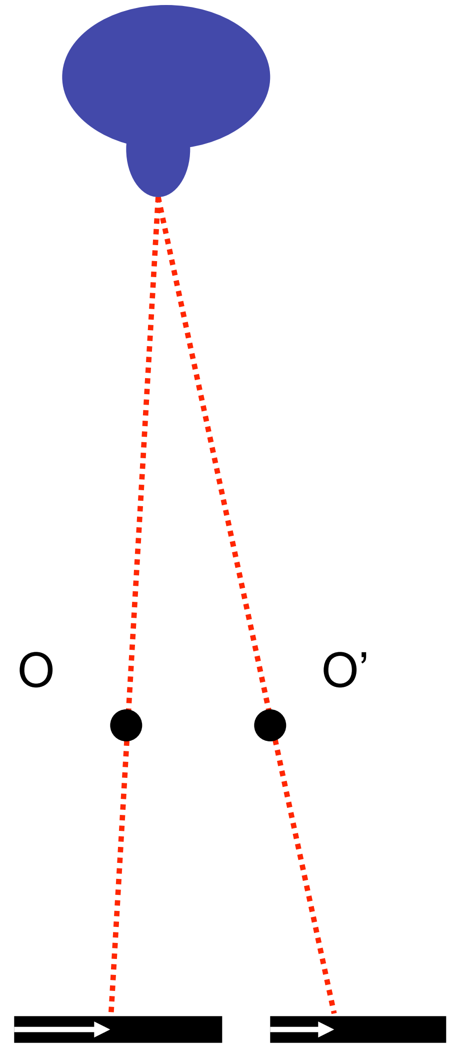


- Occlusions



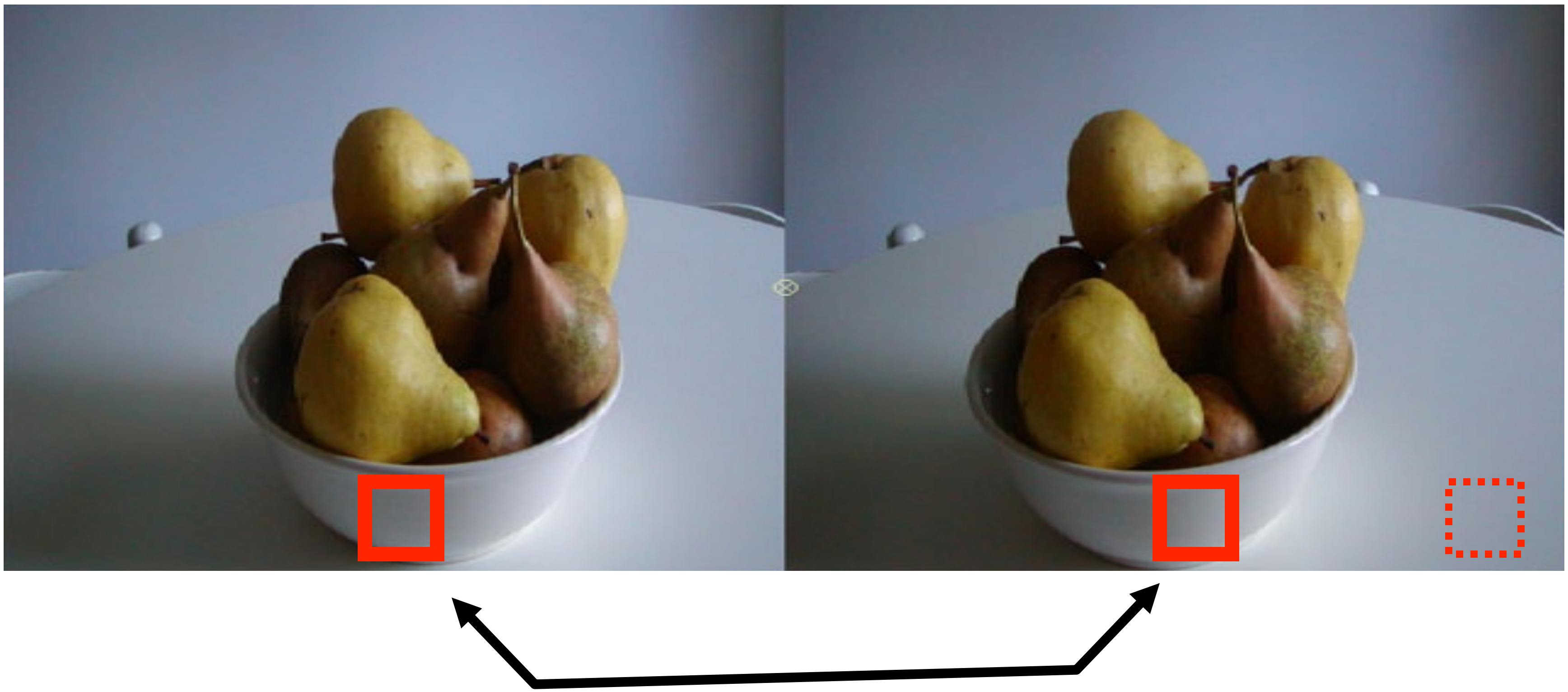
Issues

- It is desirable to have small B/z ratio!
- Small error in measurements implies large error in estimating depth



Issues

- Homogeneous regions



Hard to match pixels in these regions

Issues

- Repetitive patterns

Correspondence problem is difficult!

- Occlusions
- Fore shortening
- Baseline trade-off
- Homogeneous regions
- Repetitive patterns

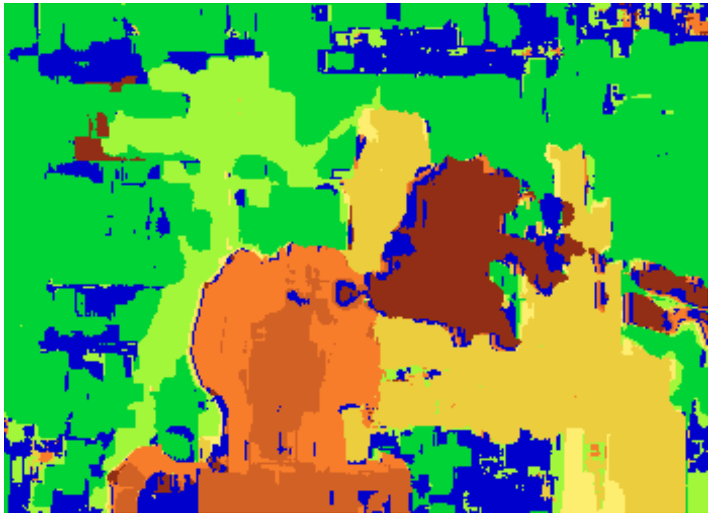
Apply non-local constraints to help enforce the correspondences

Results with window search

Data

Ground truth

Window-based matching



Credit slide S. Lazebnik

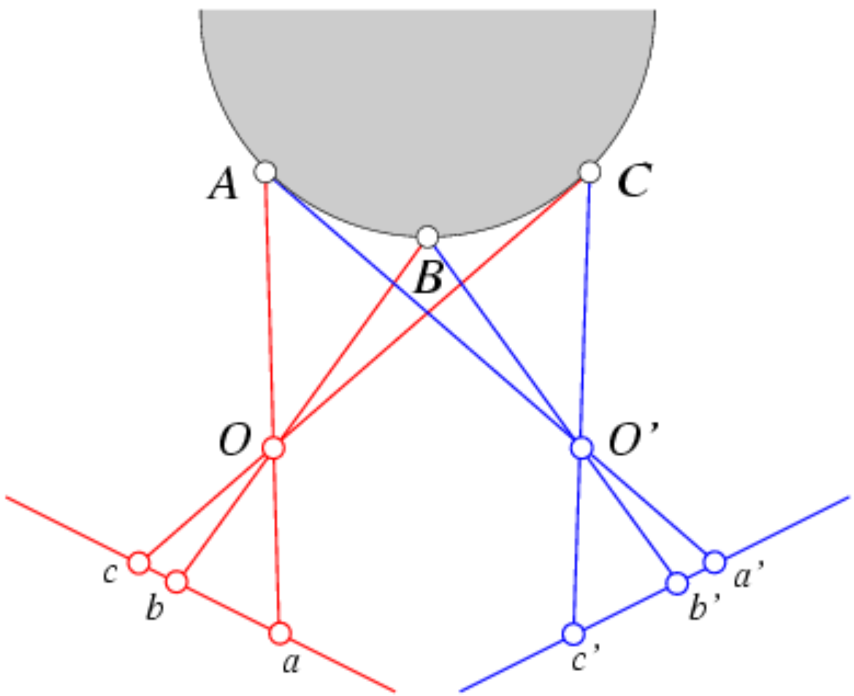
Improving correspondence: Non-local constraints

- Uniqueness
  - For any point in one image, there should be at most one matching point in the other image

Improving correspondence: Non-local constraints

- Uniqueness
  - For any point in one image, there should be at most one matching point in the other image

- Ordering
  - Corresponding points should be in the same order in both views

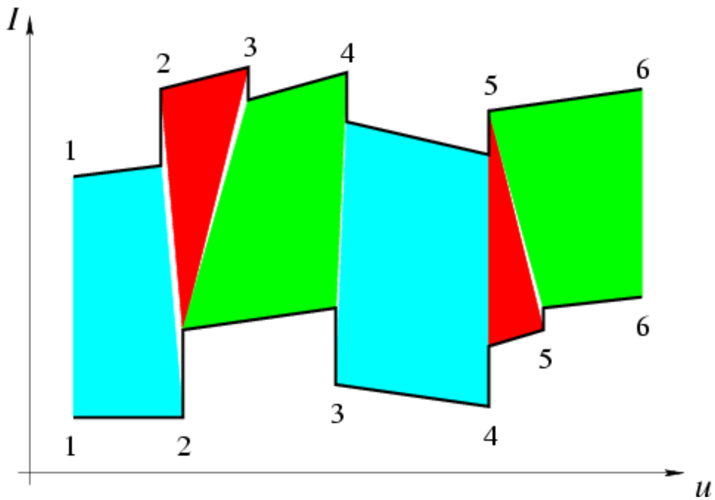


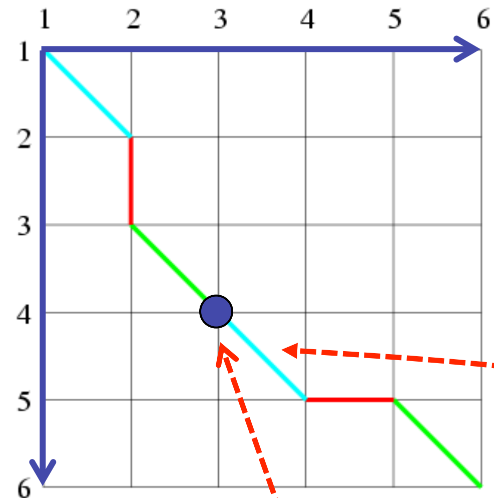
Not always in presence of occlusions!

courtesy slide to J. Ponce

Dynamic Programming (Baker and Binford, 1981)

[Uses ordering constraint]





•Nodes = matched feature points (e.g., edge points).

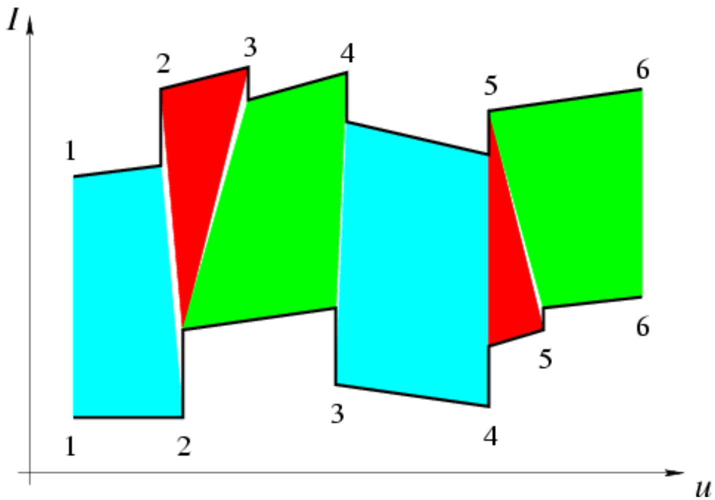
• Arcs = matched intervals along the epipolar lines.

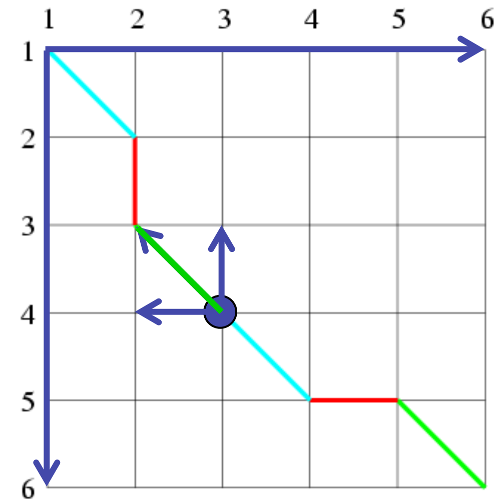
• Arc cost = discrepancy between intervals.

Find the minimum-cost path going monotonically down and right from the top-left corner of the graph to its bottom-right corner.

courtesy slide to J. Ponce

Dynamic Programming (Baker and Binford, 1981)

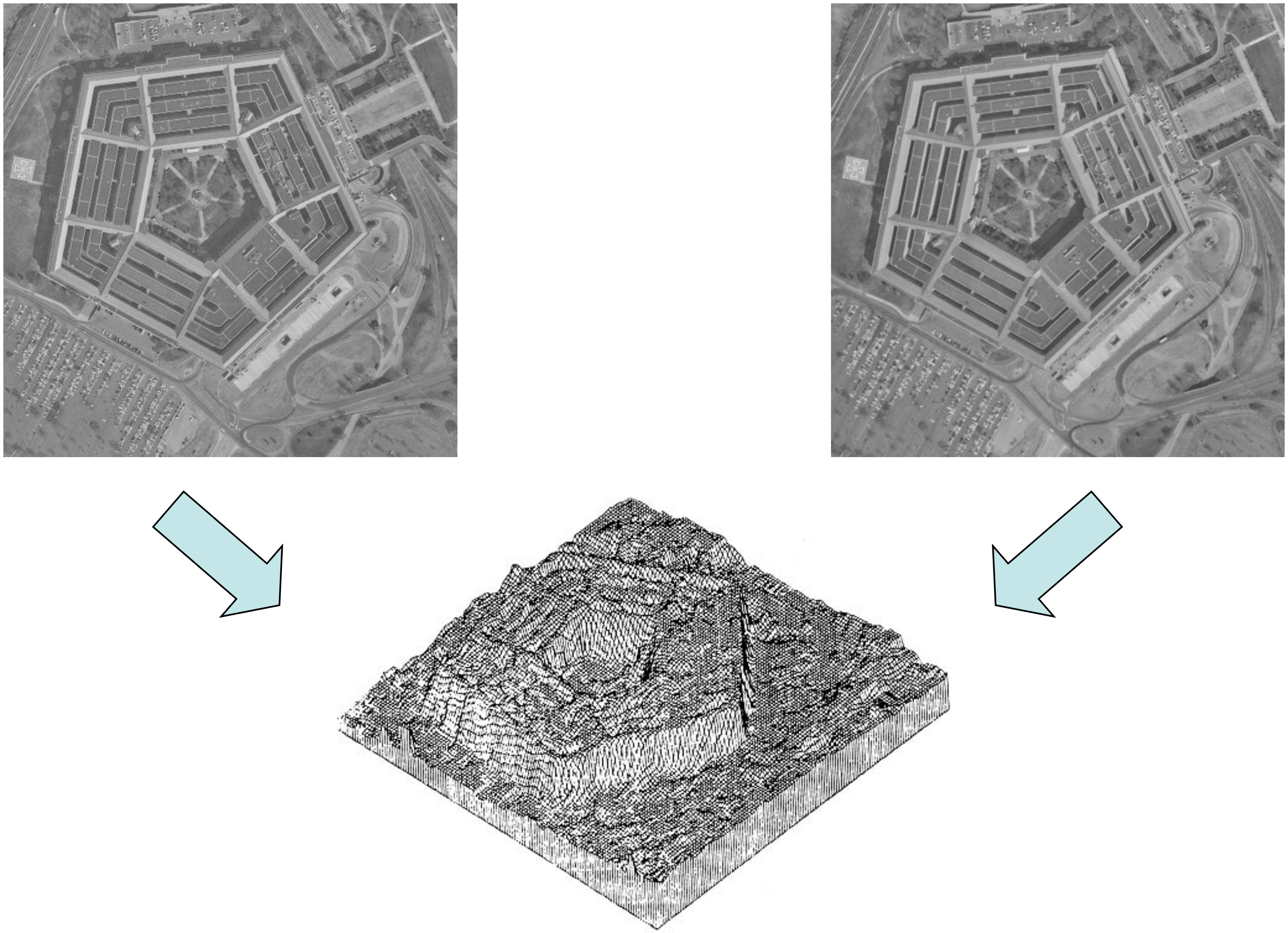




% Loop over all nodes (k, l) in ascending order.
for $k = 1$ to m do
  for $l = 1$ to n do
    % Initialize optimal cost $C(k, l)$ and backward pointer $B(k, l)$.
    $C(k, l) \leftarrow +\infty$; $B(k, l) \leftarrow$ nil;
    % Loop over all inferior neighbors (i, j) of (k, l).
    for $(i, j) \in$ Inferior $-$ Neighbors(k, l) do
      % Compute new path cost and update backward pointer if necessary.
      $d \leftarrow C(i, j) + \text{Arc} - \text{Cost}(i, j, k, l)$;
      if $d < C(k, l)$ then $C(k, l) \leftarrow d$; $B(k, l) \leftarrow (i, j)$ endif;
      endfor;
    endfor;
  endfor;
% Construct optimal path by following backward pointers from (m, n).
$P \leftarrow \{(m, n)\}$; $(i, j) \leftarrow (m, n)$;
while $B(i, j) \neq$ nil do $(i, j) \leftarrow B(i, j)$; $P \leftarrow \{(i, j)\} \cup P$ endwhile.

courtesy slide to J. Ponce

Dynamic Programming (Ohta and Kanade, 1985)

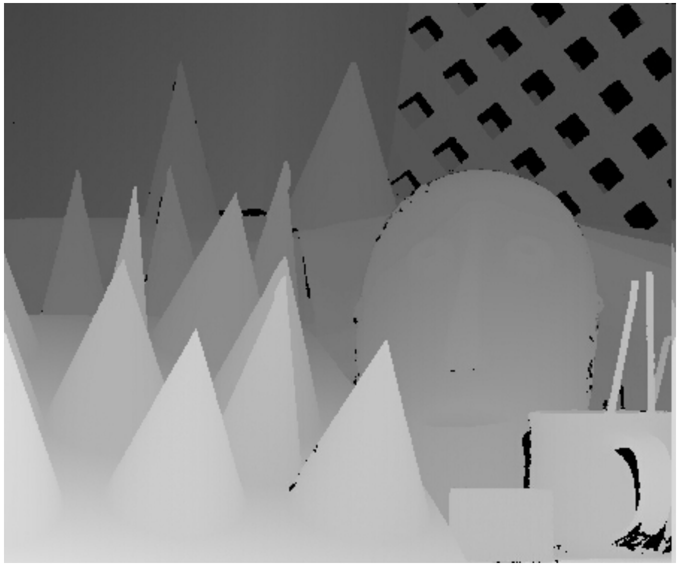


Reprinted from "Stereo by Intra- and Intet-Scanline Search," by Y. Ohta and T. Kanade, IEEE Trans. on Pattern Analysis and Machine Intelligence, 7(2):139-154 (1985). © 1985 IEEE.

Improving correspondence: Non-local constraints

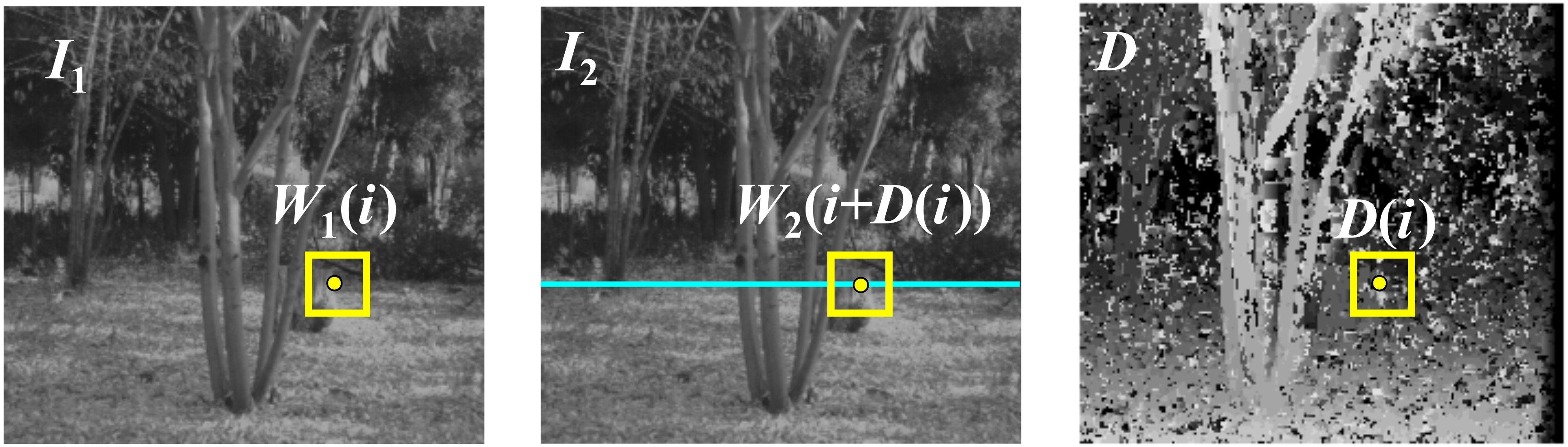
- Uniqueness
  - For any point in one image, there should be at most one matching point in the other image
- Ordering
  - Corresponding points should be in the same order in both views
- Smoothness
  - Disparity is typically a smooth function of x (expect in occluding boundaries)

Smoothness



Stereo matching as energy minimization

Y. Boykov, O. Veksler, and R. Zabih, Fast Approximate Energy Minimization via Graph Cuts, PAMI 01



$$E = \alpha \boxed{E_{\text{data}}(I_1, I_2, D)} + \boxed{\beta E_{\text{smooth}}(D)}$$

$$E_{\text{data}} = \sum_i \big(W_1(i) - W_2(i + D(i))\big)^2$$

$$E_{\text{smooth}} = \sum_{\text{neighbors } i,j} \rho\big(D(i) - D(j)\big)$$

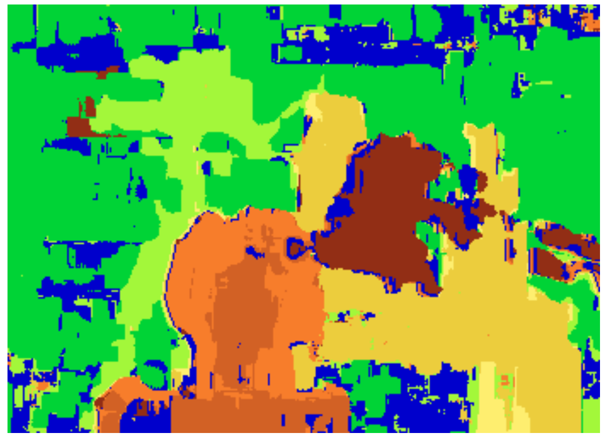
- Energy functions of this form can be minimized using *graph cuts*

Credit slide S. Lazebnik

Stereo matching as energy minimization

Y. Boykov, O. Veksler, and R. Zabih, Fast Approximate Energy Minimization via Graph Cuts, PAMI 01

Ground truth



Window-based matching

Graph cuts

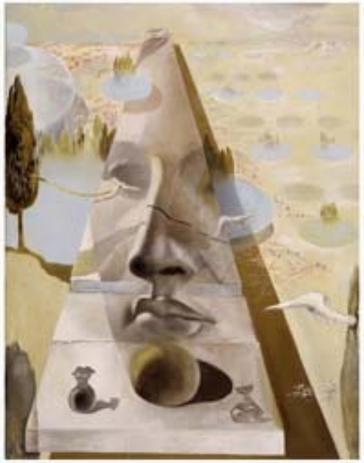
Two-frame stereo correspondence algorithms

Click here

http://www.middlebury.edu/stereo/

Stereo SDK stereo vision software development kit.

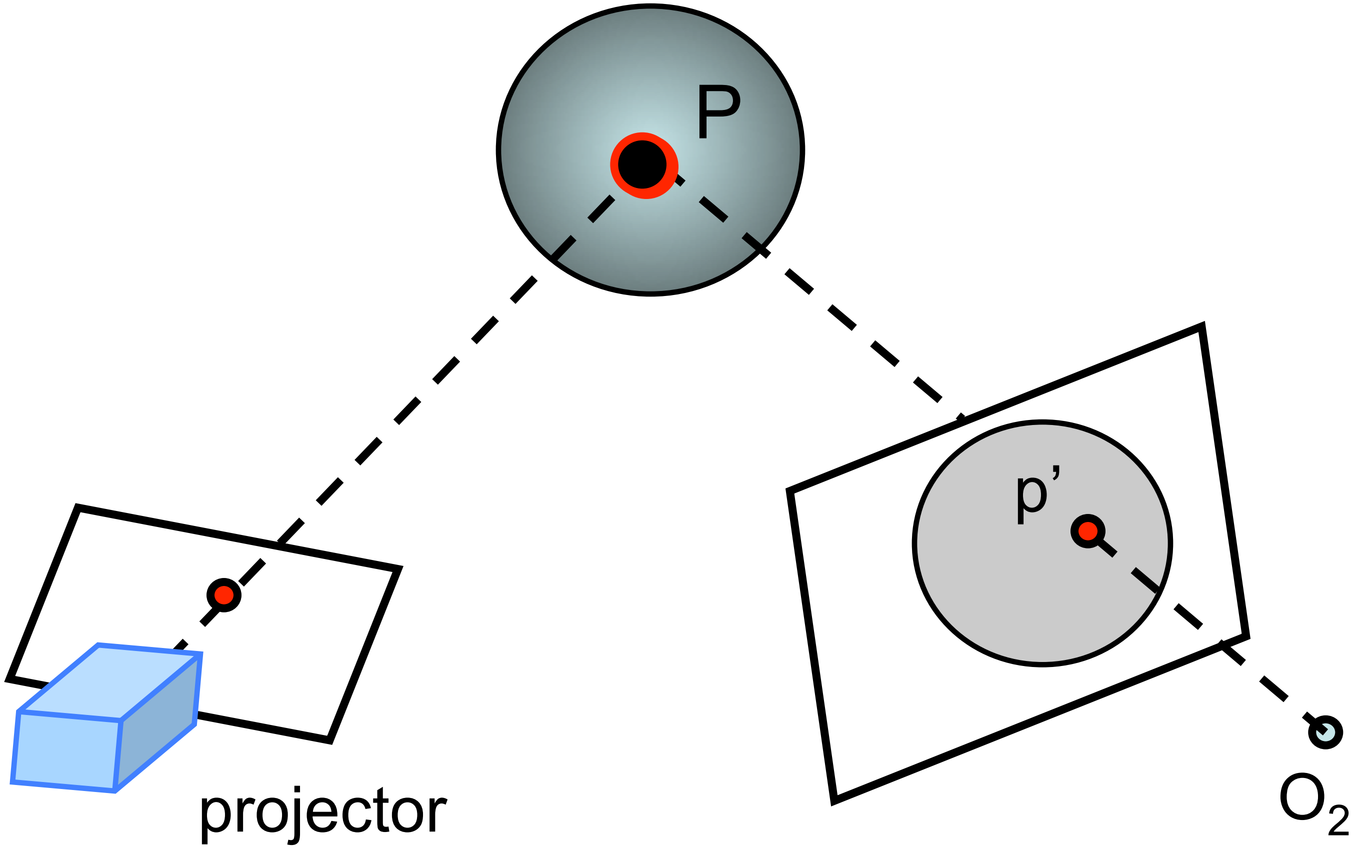
A. Criminisi, A. Blake and D. Robertson



Stereo systems

- Stereo vision
- Rectification
- Correspondence problem
- Active stereo vision systems

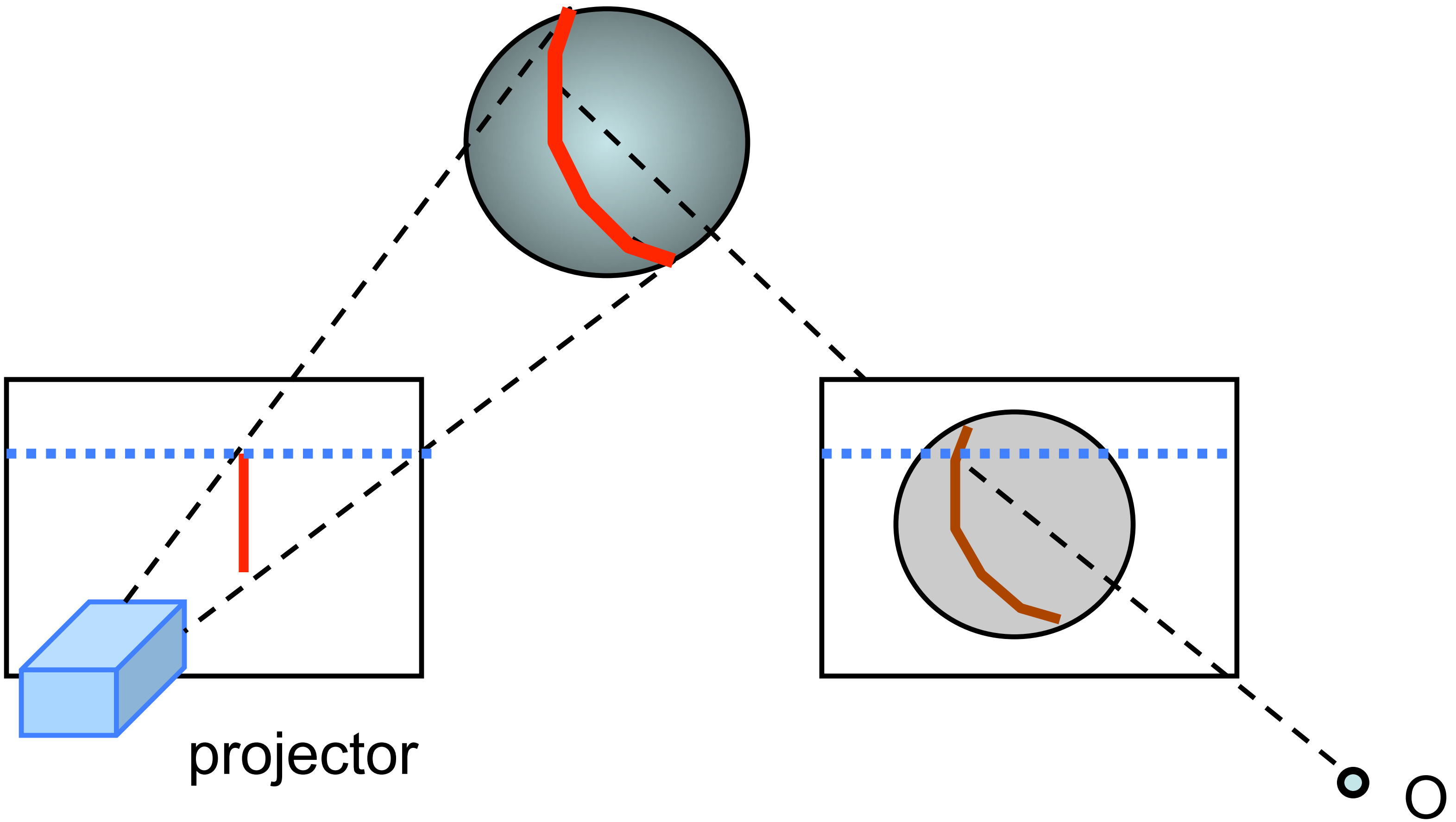
Active stereo (point)



Replace one of the two cameras by a projector

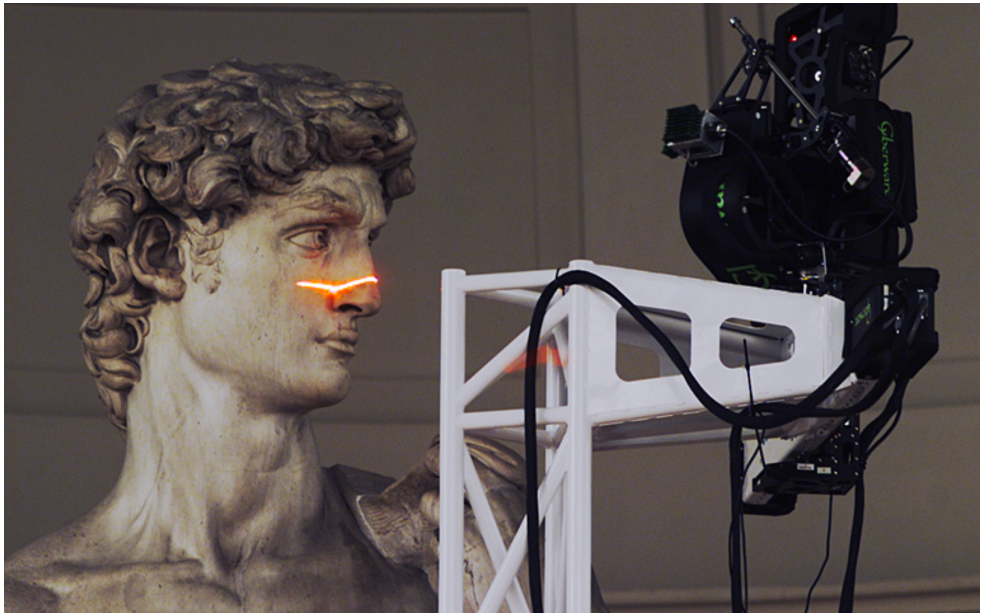
- Single camera
- Projector geometry calibrated
- What's the advantage of having the projector? Correspondence problem solved!

Active stereo (stripe)



-Projector and camera are parallel
- Correspondence problem solved!

Laser scanning



Digital Michelangelo Project
http://graphics.stanford.edu/projects/mich/

- Optical triangulation
  - Project a single stripe of laser light
  - Scan it across the surface of the object
  - This is a very precise version of structured light scanning

Source: S. Seitz

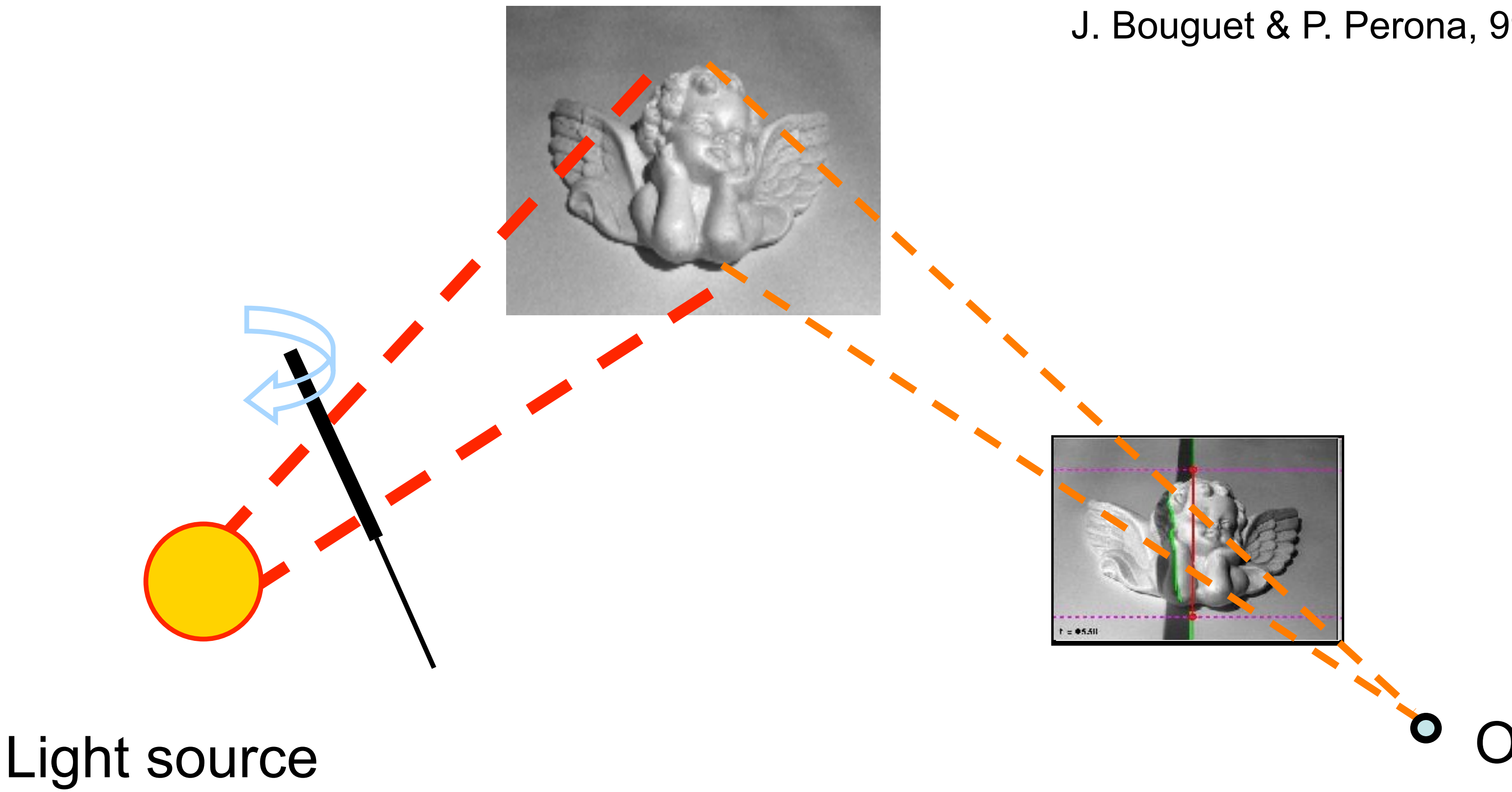
Laser scanning

The Digital Michelangelo Project, Levoy et al.

Source: S. Seitz

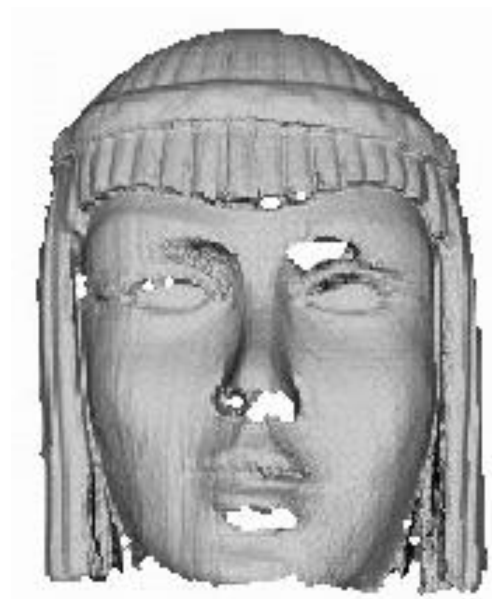
Active stereo (shadows)

J. Bouguet & P. Perona, 99



- 1 camera,1 light source
- very cheap setup
- calibrated the light source

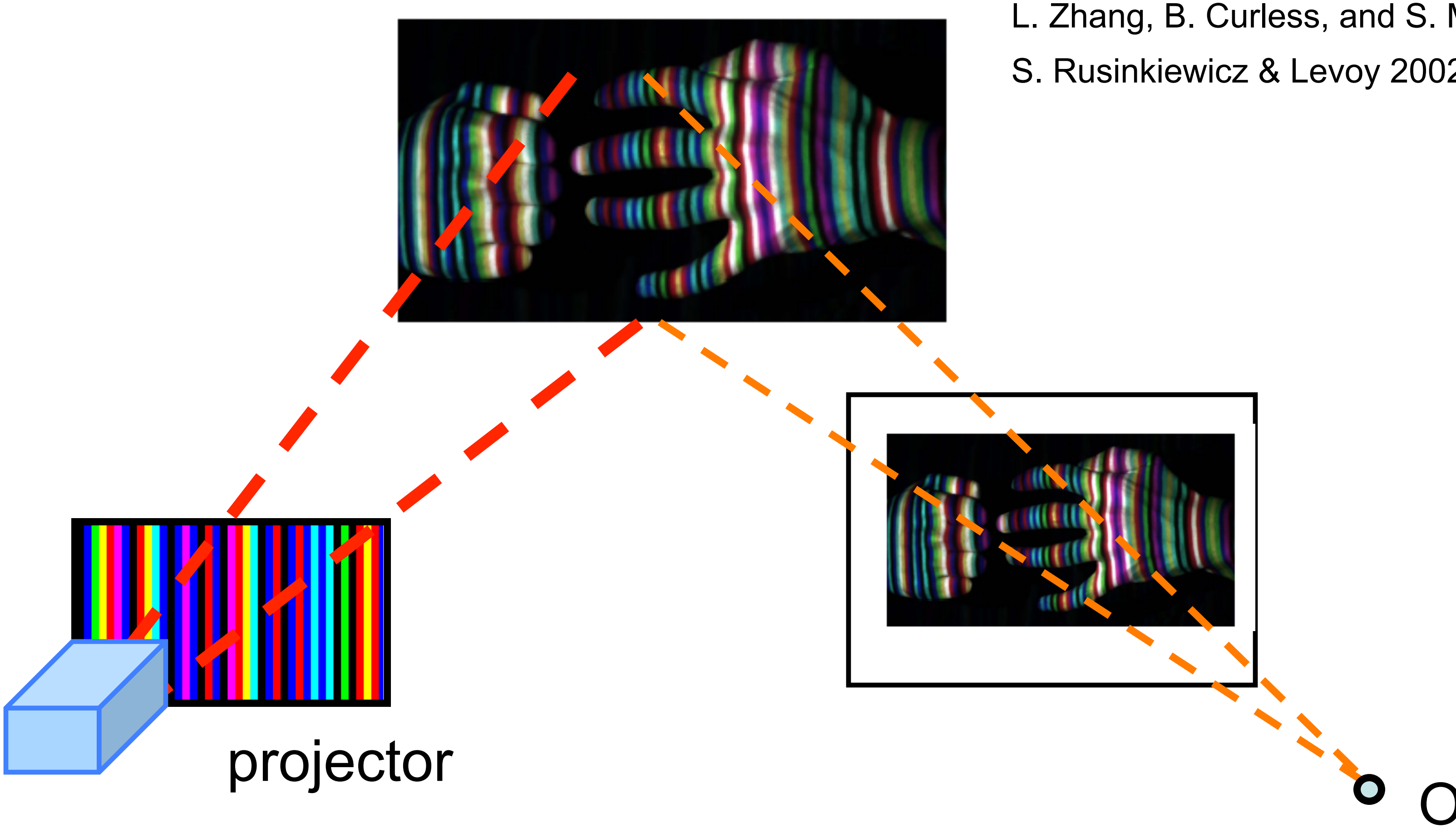
Active stereo (shadows)



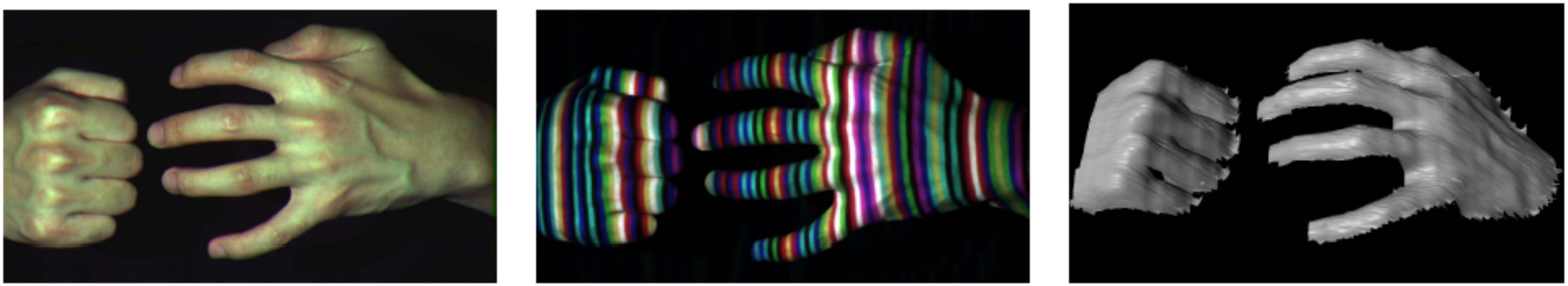
Active stereo (color-coded stripes)

L. Zhang, B. Curless, and S. M. Seitz 2002

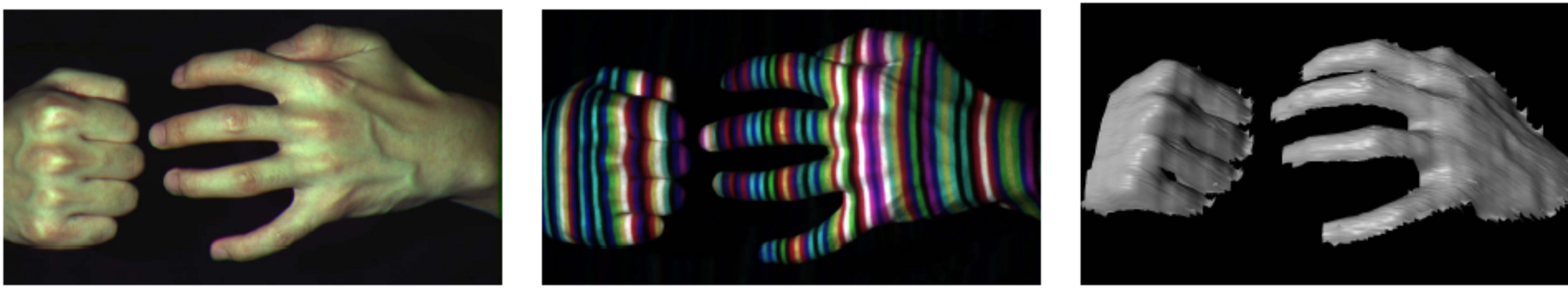
S. Rusinkiewicz & Levoy 2002



- Dense reconstruction
- Correspondence problem again
- Get around it by using color codes

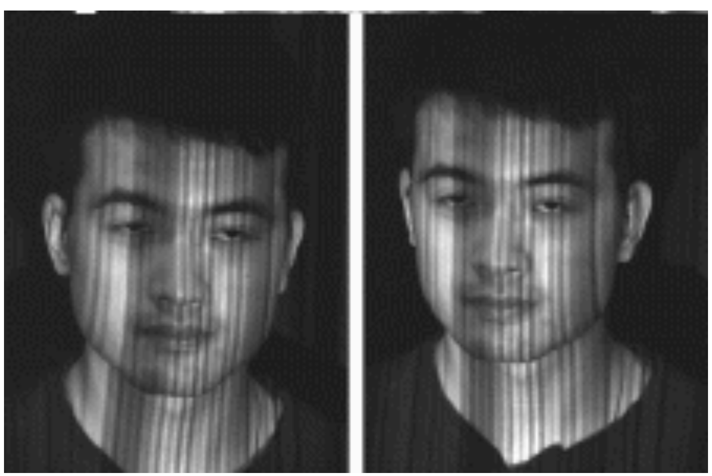


L. Zhang, B. Curless, and S. M. Seitz. Rapid Shape Acquisition Using Color Structured Light and Multi-pass Dynamic Programming. *3DPVT* 2002



L. Zhang, B. Curless, and S. M. Seitz. Rapid Shape Acquisition Using Color Structured Light and Multi-pass Dynamic Programming. *3DPVT* 2002

Rapid shape acquisition: Projector + stereo cameras

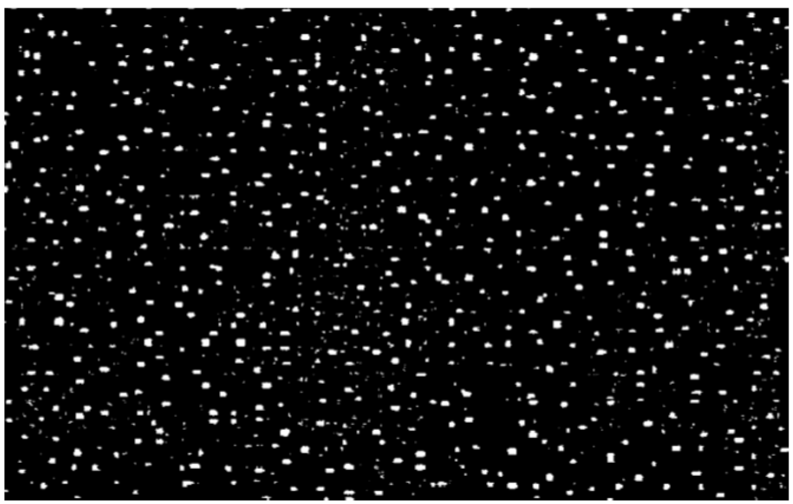


Microsoft Kinect

The Kinect combines structured light with two classic computer vision techniques: depth from focus, and depth from stereo.

Stage 1: The depth map is constructed by analyzing a speckle pattern of infrared laser light

The Kinect uses infrared laser light, with a speckle pattern



Shpunt et al, PrimeSense patent application US 2008/0106746

http://users.dickinson.edu/~jmac/selected-talks/kinect.pdf

Next lecture…

Affine Structure from Motion

Human Stereopsis

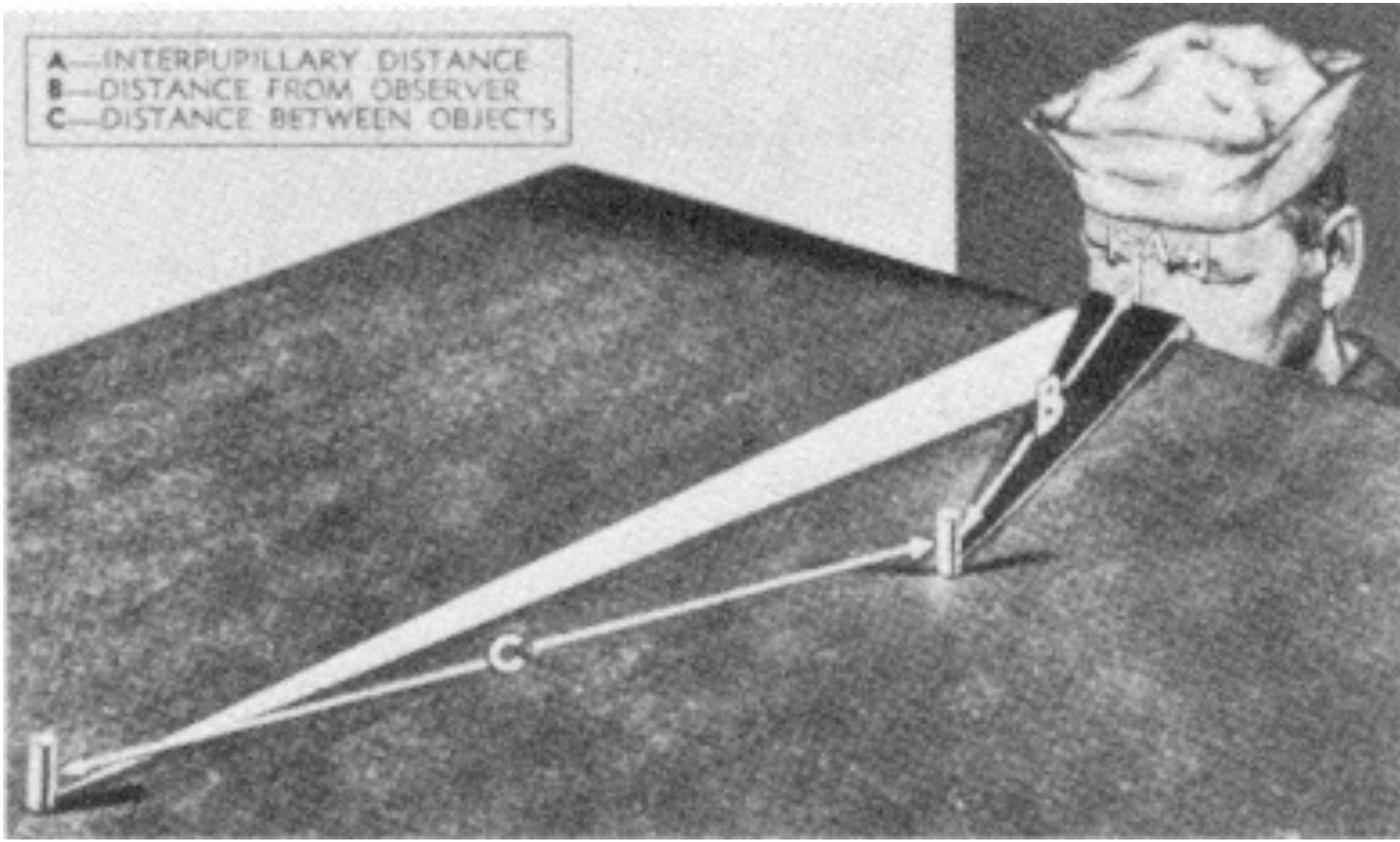
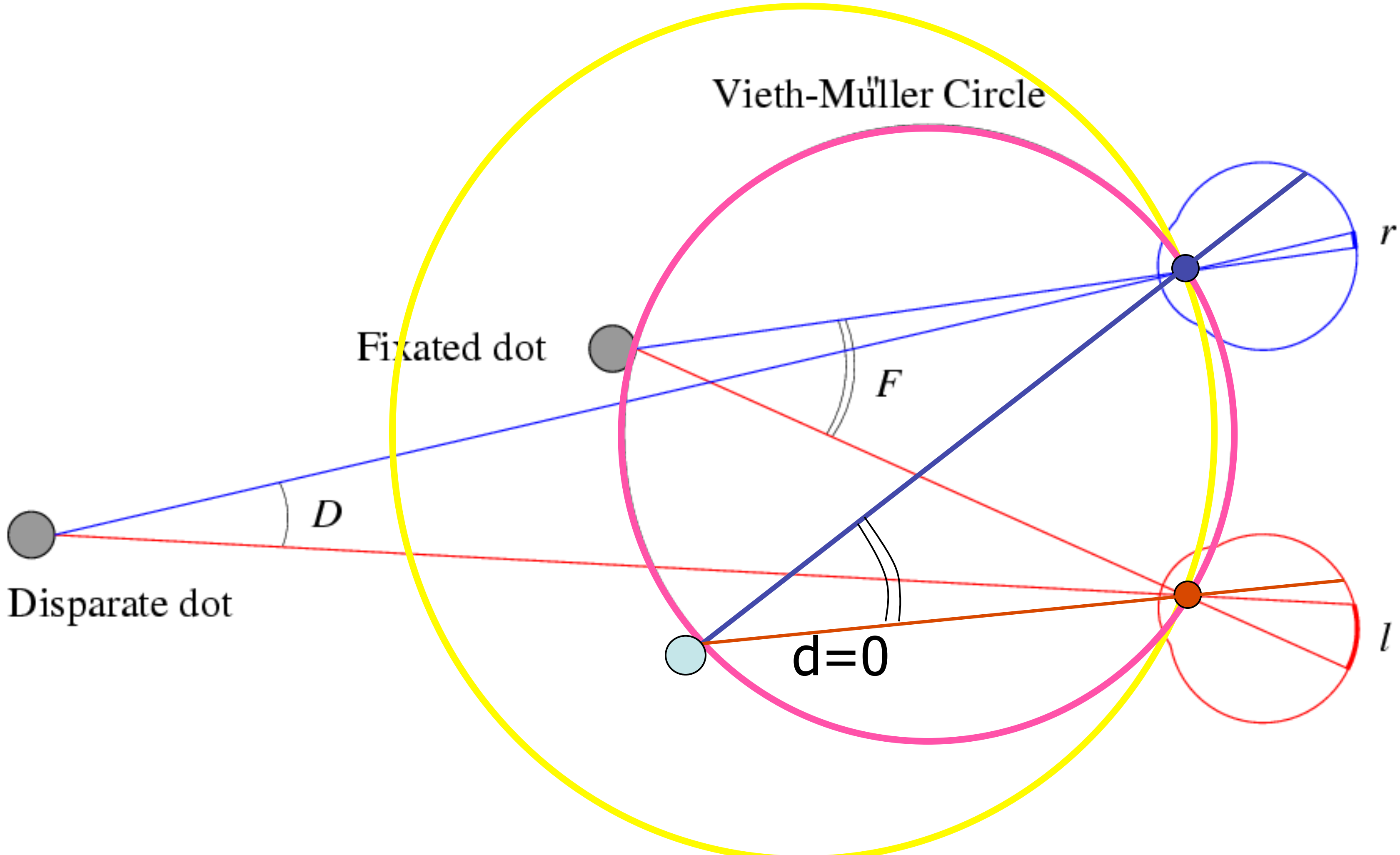


Figure from US Navy Manual of Basic Optics and Optical Instruments, prepared by Bureau of Naval Personnel. Reprinted by Dover Publications, Inc., 1969.

Credit slide J. Ponce

Human Stereopsis: Reconstruction



Disparity: d = r-l = D-F; d<0

In 3D, the horopter.

Credit slide J. Ponce

Human Stereopsis: Reconstruction

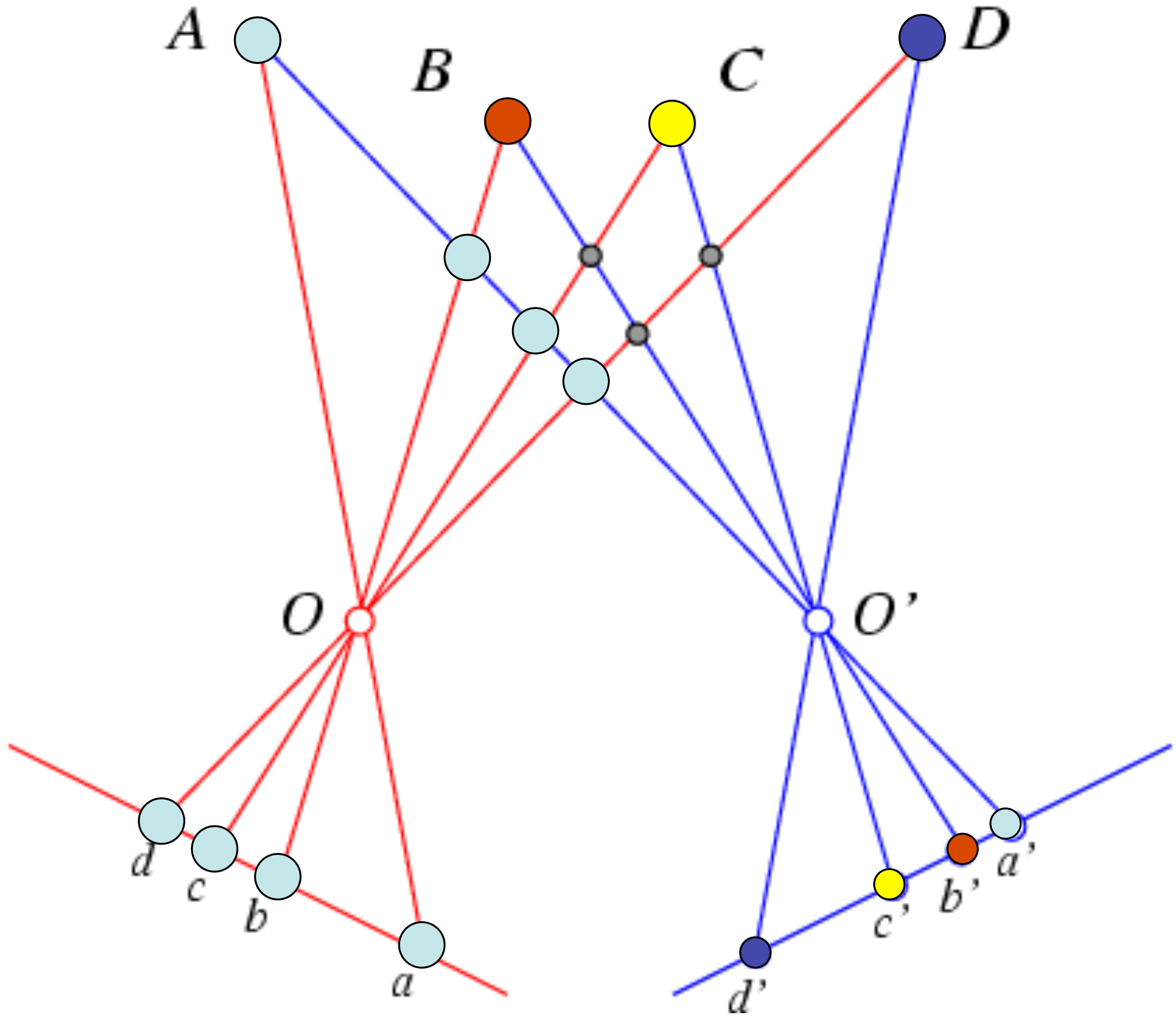
What if F is not known?

Helmoltz (1909):

- There is evidence showing the vergence angles cannot be measured precisely.

- Humans get fooled by bas-relief sculptures.

Human Stereopsis: Binocular Fusion



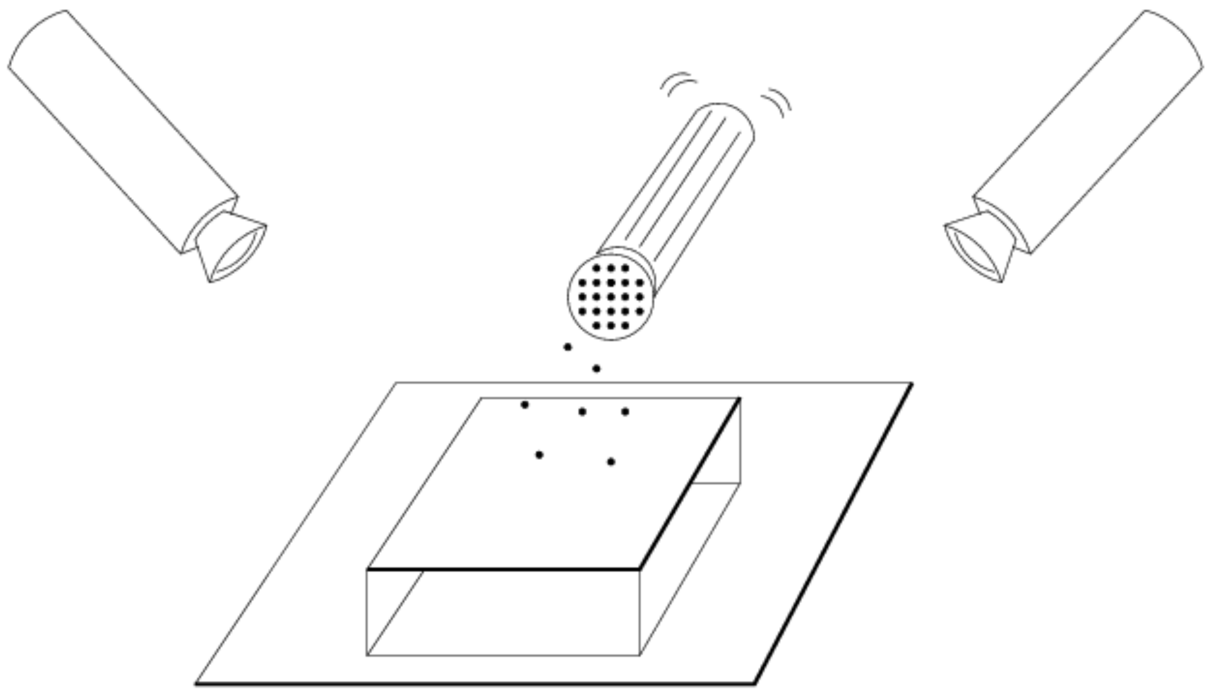
Credit slide J. Ponce

Human Stereopsis: Binocular Fusion

How are the correspondences established?

Julesz (1971): Is the mechanism for binocular fusion a monocular process or a binocular one??

- There is anecdotal evidence for the latter (camouflage).



- Random dot stereograms provide an objective answer

Credit slide J. Ponce

Issues